Flexibility in a Stackelberg leadership with differentiated goods

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We study the effects of product differentiation in a Stackelberg model with demand uncertainty for the first mover. We do an ex-ante and ex-post analysis of the profits of the leader and of the follower firms in terms of product differentiation and of the demand uncertainty. We show that even with small uncertainty about the demand, the follower firm can achieve greater profits than the leader, if their products are sufficiently differentiated. We also compute the probability of the second firm having higher profit than the leading firm, subsequently showing the advantages and disadvantages of being either the leader or the follower firm.

1. Introduction

The Stackelberg model [1] is one of the most widely used models in industrial organization to analyse the behaviour of the firms in a competitive environment. It models the strategic situation where firms sequentially choose their output levels in a market. The belief of first-mover advantage was widely held among entrepreneurs and venture capitalists, but is now questioned by numerous practitioners. There are examples of successful and unsuccessful pioneering firms as described, for instance, in Liu [2]: Dell was the first to introduce the direct-sale business model into the PC market, and it achieved great success; however, during the dot-com booming era, Pets.com, Webvan.com, Garden.com and eToys.com were all unsuccessful first movers in their respective market segments. The probability of success of pioneering in a market clearly depends on many factors, including technology, marketing strategy, market demand and product differentiation (see [1–18]). Liu [2] studied the effect of uncertainty in demand systems where only the leading firm is facing uncertainty in the demand parameter a that is considered to be uniformly distributed on an interval $[a_0, a_1]$. In this paper, we add the dimension of product differentiation to Liu's model and we show among other results that if their products are sufficiently differentiated, even with small uncertainty about the demand, the follower can achieve higher profits than the leader.

Our study focus on the influence of two parameters: the product differentiation $0 < \gamma \leq 1$ and demand uncertainty $\theta \geq 1$. When γ is close to one we have small

differentiation between the goods and when γ is small we have high differentiation between the goods. The demand uncertainty $\theta = a_1/a_0$ can be interpreted as the maximum value of demand using the minimum value of demand a_0 as the unit. In Theorem 1, we prove the existence of a unique perfect Bayesian equilibrium that requires a different reasoning from Liu's [2]. In Corollary 1, we compare the quantities produced by each firm and prove the existence of a threshold T for a, depending upon θ and γ, such that, bellow that threshold the leader is producing more than the follower and above that threshold the follower is producing more than the leader. In Theorem 2, we present the (ex-ante) expected profits of the two firms, depending upon the parameters θ and γ and we characterize two parameter regions A_1 and A_2 in the parameter space (γ, θ) with the following properties: in the region A_1 the ex-ante profits of firm F_1 are higher than ex-ante profits of firm F_2; in the region A_2 the ex-ante profits of firm F_2 are higher than ex-ante profits of firm F_1. In Theorem 3, we prove the existence of a threshold θ_0 for the demand uncertainty parameter θ, such that for θ greater than θ_0 the expected profit of the follower is always (i.e. for any γ) greater than the expected profit of the leader. In Theorem 4, we find the (ex-post) profits of the firms and we characterize three parameter regions P_1, P_2 and P_3 in the parameter space (γ, θ) corresponding to three distinct economic behaviours: a region P_1, where the leading firm always has a higher profit than the follower firm; a region P_2, where there exists R, depending upon γ and θ, with the property that (i) if $a < R$, the leading firm has a higher profit than the follower firm, and (ii) if $a > R$, the follower firm has a higher profit than the leading firm; autoedited1. a region P_3, where there exists L, depending upon γ and θ, with the property that (i) if $L < a < R$, the leading firm has a higher profit than the follower firm, and (ii) if $a < L$ or $a > R$, the follower firm has a higher profit than the leading firm. Hence, we show that the leading firm looses its advantage for high values of the demand intercept a, if the demand uncertainty parameter θ belongs to the union of regions P_2 and P_3. Furthermore, the leading firm also looses its advantage for low values of the demand intercept a, if the demand uncertainty parameter θ belongs to the region P_3. Furthermore, in Corollary 2, we compute the ex-ante probability $P(\pi_2^* > \pi_1^*)$ that the follower's ex-post profit is higher than the leader's ex-post profit. ex-ante expected profits and the ex-post profits of the leader firm and of the follower firm, and we also compute the probability $P(\pi_2^* > \pi_1^*)$ of the second firm having higher profit than the leading firm. We show that this probability increases both with the demand uncertainty and the degree of differentiation of the goods.

2. The model and the perfect Bayesian equilibrium

We start by describing the Stackelberg duopoly with product differentiation. We consider two firms, each producing a differentiated good. The demand, for simplicity, is linear

$$\begin{cases} p_1 = \alpha - q_1 - \gamma q_2 \\ p_2 = \alpha - \gamma q_1 - q_2 \end{cases}, \tag{1}$$

with $a > 0$ and $0 < \gamma \le 1$, where p_i is the price and q_i the amount of good produced by the firm F_i, for $i \in \{1, 2\}$. We note that the two products are substitutes and, since $\gamma \lessgtr$ 'cross effects' are dominated by 'own effects'. The value of γ expresses the degree of product differentiation. When γ is equal to one, the goods are homogeneous, and when γ tends to zero, we are close to independent goods (see [18]). We assume that the firms have the same constant marginal cost c. From now on, we consider prices net of marginal

costs. This is without any loss of generality because if the marginal cost is positive, we may replace a by $a-c$. We consider that the demand intercept is a random variable uniformly distributed in the interval $[a_0, a_1]$, with $a_1 > a_0 > 0$. We note that, in this case, the demand uncertainty parameter θ is equal to the ratio a_1/a_0 (can be interpreted as the maximum value of demand using the minimum value of demand a_0 as the unit). The distribution of a is common knowledge. Profit π_i of firm F_i is given by

$$\pi_i = p_i q_i = (\alpha - q_i - \gamma q_j)q_i. \tag{2}$$

As already stated in the Introduction, the timing of the game is as follows:

(i) Firm F_1 chooses a quantity level $q_1 \geq 0$ without knowing the value of the demand realization;

(ii) Firm F_2 first observes the demand realization and observes q_1, and then chooses a quantity level $q_2 \geq 0$.

In the next theorem, we show that this game has a unique perfect Bayesian equilibrium (q_1^*, q_2^*) and we give its explicit characterization. Let

$$K_\gamma = \frac{8 - 2\gamma - 3\gamma^2}{\gamma(2 - \gamma)},$$

$A = (4 - \gamma^2)^2 - 2\gamma^2(4 - 3\gamma)$, $B = 16(2 - \gamma^2)$ and $C = 2(8 - 3\gamma^3)$, and denote by Δ the expression

$$\Delta \equiv \Delta(\alpha_0, \gamma, \theta) = \alpha_0 \left(4 - 2\left(2 - \gamma^2\right)\theta + \sqrt{A\theta^2 - B\theta + C}\right).$$

We observe that $A\alpha_1^2 - B\alpha_0\alpha_1 + C\alpha_0^2 \geq 0$, for all $\theta = \alpha_1/\alpha_0 > 1$, and so Δ is well defined.

Theorem 1 *Consider a differentiated products Stackelberg duopoly facing the demand system* (1), *where the parameter a is uniformly distributed in the interval $[a_0, a_1]$. Then, there is a unique perfect Bayesian equilibrium (q_1^*, q_2^*) given as follows:*

(i) *If $\theta \leq K_\gamma$, then*

$$q_1^* = \frac{\alpha_0(2 - \gamma)}{4(2 - \gamma^2)}(\theta + 1) \text{ and } q_2^* = \frac{\alpha}{2} - \frac{\alpha_0\gamma(2 - \gamma)}{8(2 - \gamma^2)}(\theta + 1);$$

(ii) *If $\theta \geq K_\gamma$, then*

$$q_1^* = \frac{\Delta}{3\gamma^3} \text{ and } q_2^* = \begin{cases} 0, & \text{if } \alpha < \Delta/(3\gamma^2) \\ \frac{\alpha}{2} - \frac{\Delta}{6\gamma^2}, & \text{if } \alpha \geq \Delta/(3\gamma^2) \end{cases}.$$

Theorem 1 is proved in the Appendix. From this theorem, we conclude that when the interval of a is large and the products are not very differentiated (large γ), if the realized demand is small the follower may choose not to produce at all. The relevant parameters in terms of their economical effects are γ and θ. In the next corollary, we compare the quantities produced by each firm.
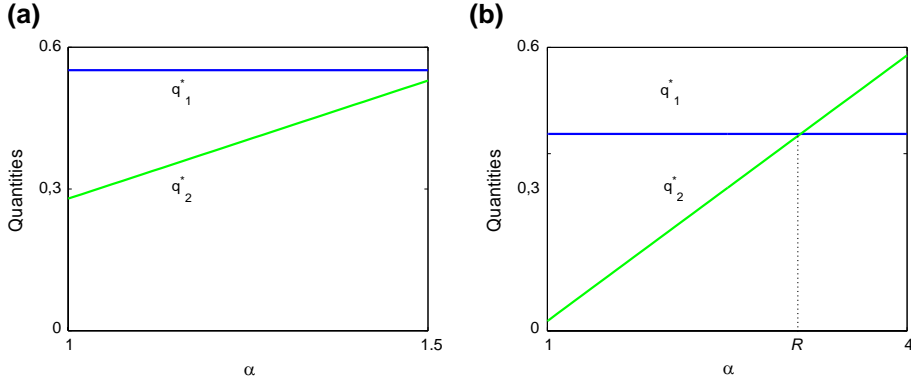
Figure 1. The quantities produced by each firm when the intervals of the uniform distribution of the parameter a are such that the ratio θ between its endpoints is as in each situation of Corollary 1, for $\gamma = 0.8, a_0 = 1$ and (a) $a_1 = 1.5$; (b) $a_1 = 4$.

Let $I_\gamma = (\gamma - 4)/(3\gamma^2 - 4)$.
Let

$$R(\alpha_0, \gamma, \theta) = \begin{cases} R_1 = \frac{\alpha_0(4-\gamma^2)}{4(2-\gamma^2)}(\theta + 1) & \text{if } \theta \le K_\gamma \\ R_2 = \frac{(2+\gamma)\Delta}{3\gamma^3} & \text{if } \theta \ge K_\gamma \end{cases}.$$

COROLLARY 1

(a) If $\theta < I_\gamma$, then, for all $\alpha \in [\alpha_0, \alpha_1]$, the leading firm produces more than firm F_2, i.e. $q_2^* < q_1^*$.
(b) If $\theta \ge I_\gamma$, then
   (i) if $\alpha \le R$, the leading firm produces more than firm F_2, i.e. $q_2^* \le q_1^*$;
   (ii) if $\alpha \ge R$, firm F_2 produces more than the leading firm, i.e. $q_2^* \ge q_1^*$.

Corollary 1 is proved in the Appendix. The three different situations described in this corollary are illustrated for some values of the parameters in Figure 1. We observe the existence of a threshold R for a such that for values of a smaller than that threshold the leader is producing more, and for values of a greater than that threshold the follower is producing more. This is due to the fact that the leader makes the decision based only on expectations. The follower is going to produce more if the demand turns out to be large (and possibly more than the leader), and less if the realized demand is small (and possibly less than the leader).

3. Ex-ante expected profits

In this section, we present an ex-ante analysis of the Stackelberg game previously described. In the next theorem, we explicitly show the profits that the firms can expect, before the knowledge of the demand realization a. For simplicity of notation, we denote by π_i^* the profit $\pi_i\left(q_1^*, q_2^*(q_1^*, a)\right)$, for $i \in \{1, 2\}$. Let $\gamma_0 : [3, +\infty) \to R_0^+$ be defined by

$$\gamma_0(\theta) = \begin{cases} 1, & \text{for } \theta = 3 \\[2mm] \frac{1+\theta-\sqrt{\theta^2-6\theta+25}}{\theta-3}, & \text{for } \theta > 3 \end{cases}$$

Let

$$Q = \{(\theta, \gamma) : \theta \geq 3 \wedge \gamma_0(\theta) \leq \gamma \leq 1\}$$

and

$$P = ([1, +\infty) \times (0, 1]) \setminus Q.$$

Let $\Sigma = \Delta/\alpha_0$.

THEOREM 2 *While the demand realization α is unknown for both firms, their expected profits $E(\pi_1^*)$ and $E(\pi_2^*)$ are given by*

$$E(\pi_1^*) = \begin{cases} \frac{\alpha_0^2(2-\gamma)^2(\theta+1)^2}{32(2-\gamma^2)}, & \text{if } (\theta, \gamma) \in P \\[3mm] \frac{\alpha_0\Delta(18\gamma^3(\theta^2-1)-12\Sigma(\theta-1)-9\gamma^4\theta^2+6\gamma^2\Sigma\theta-\Sigma^2)}{108\gamma^6(\theta-1)}, & \text{if } (\theta, \gamma) \in Q \end{cases}$$

and

$$E(\pi_2^*) = \begin{cases} \frac{\alpha_0^2((7\gamma^4+12\gamma^3-28\gamma^2-48\gamma+64)(\theta^2+1)-2\theta(\gamma^4-12\gamma^3-4\gamma^2+48\gamma-32))}{192(\gamma^2-2)^2}, & \text{if } (\theta, \gamma) \in P \\[3mm] \frac{\alpha_0^2(3\gamma^2\theta-\Sigma)^3}{324\gamma^6(\theta-1)}, & \text{if } (\theta, \gamma) \in Q \end{cases}$$

Theorem 2 is proved in the Appendix.

Let $\alpha_0 = 1$. Let $A_1 = \{(\gamma, \theta) : E(\pi_1^*) > E(\pi_2^*)\}$ and $A_2 = \{(\gamma, \theta) : E(\pi_2^*) > E(\pi_1^*)\}$. Then, line $E\left(\pi_1^*(\alpha)\right) = E\left(\pi_2^*(\alpha)\right)$ is given by $C(\theta, \gamma) = 0$, defined by

$$C(\theta, \gamma) = \begin{cases} \theta - \frac{4\gamma(2-\gamma^2)\sqrt{12\gamma-9\gamma^2-5\gamma^4+4(3\gamma^3-4\gamma^2+4)}}{13\gamma^4-12\gamma^3-16\gamma^2+16}, & \text{if } (\theta, \gamma) \in P \\[3mm] D\theta^5 + E\theta^4 + F\theta^3 + 2G\theta^2 + H\theta + 49\gamma^3 - 120, & \text{if } (\theta, \gamma) \in Q \end{cases}$$

where $D = 15\gamma^5 - 36\gamma^4 + 7\gamma^3 + 42\gamma^2 - 40$, $E = 15\gamma^5 - 6\gamma^4 + 49\gamma^3 - 138\gamma^2 + 200$, $F = 6\gamma^4 - 56\gamma^3 + 192\gamma^2 - 400$, $G = 3\gamma^4 - 49\gamma^3 + 120$ and $H = 49\gamma^3 - 96\gamma^2 + 120$ (Figure 2).

Theorem 3 *There is $\theta_0 \approx 7.30$ such that if the uncertainty parameter θ is greater than θ_0, then the expected profit of the follower firm is always greater than the expected profit of the leading firm.*

Theorem 3 is proved in the Appendix.

In Figure 3, we plot the firms' expected profits, $E(\pi_1^*)$ and $E(\pi_2^*)$, as functions of the demand uncertainty parameter θ and of the degree γ of product differentiation. Figure 4 illustrates cross-sections of Figure 3 at the degrees $\gamma = 0.9$ (Figure 4(a)) and $\gamma = 0.5$ (Figure 4(b)) of product differentiation. Figure 5 illustrates cross-sections of Figure 3 at the demand uncertainty parameter's values $\theta = 2$ (Figure 5(a)) and $\theta = 8$ (Figure 5(b)).
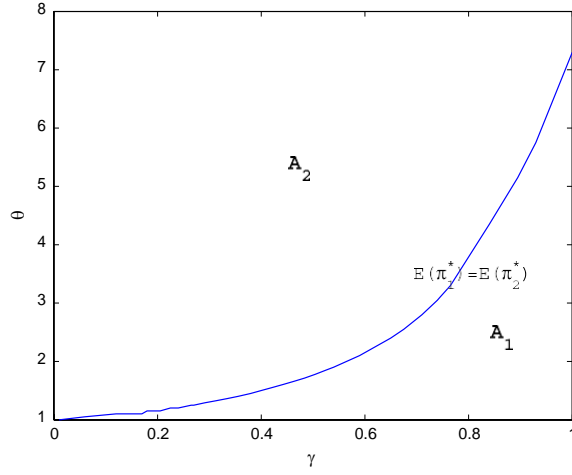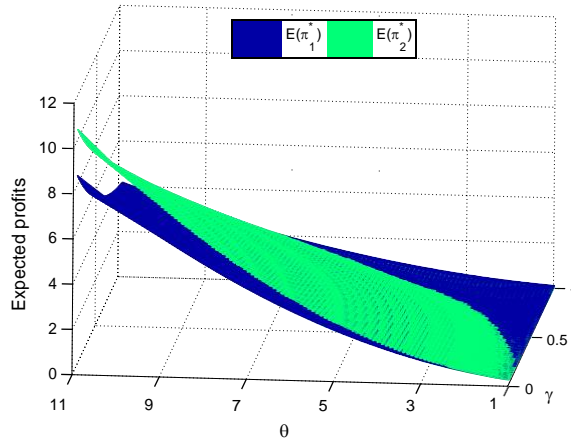
Figure 2. Plot of $C(\theta, \gamma)$.

Figure 3. Firms' expected profits varying with the demand uncertainty parameter θ and with the degree γ of the product differentiation, by taking $a_0 = 1$.

These figures illustrate that both parameters θ and γ are relevant to determine which firm has ex-ante higher expected profits.

4. Ex-post profits

In this section, we present the ex-post analysis of the same Stackelberg game. In Theorem 4, we explicitly present the ex-post profits π_1^* and π_2^* of firms 1 and 2, respectively, obtained after the observation of the demand realization. We describe three regions for the demand uncertainty parameter $\theta = a_1/a_0$ corresponding to three distinct profits relations between the leading and the follower firms (see Figure 6). The *low-medium uncertainty boundary value* is
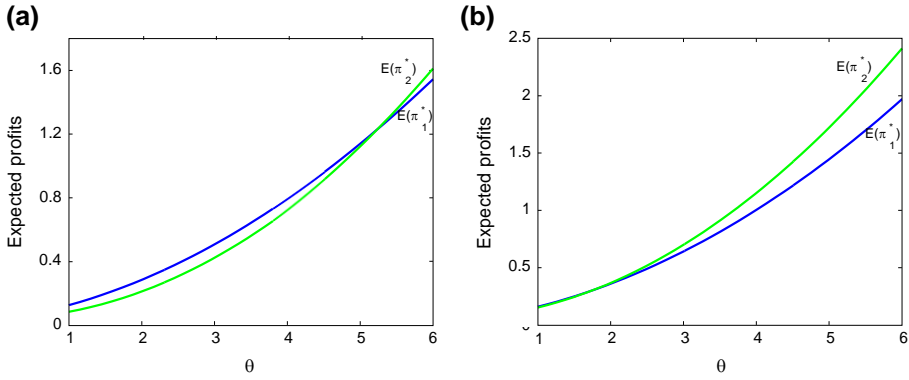
(a)

(b)

Figure 4. Cross-sections of Figure 3 at the degrees (a) $\gamma = 0.9$ and (b) $\gamma = 0.5$ of product differentiation.
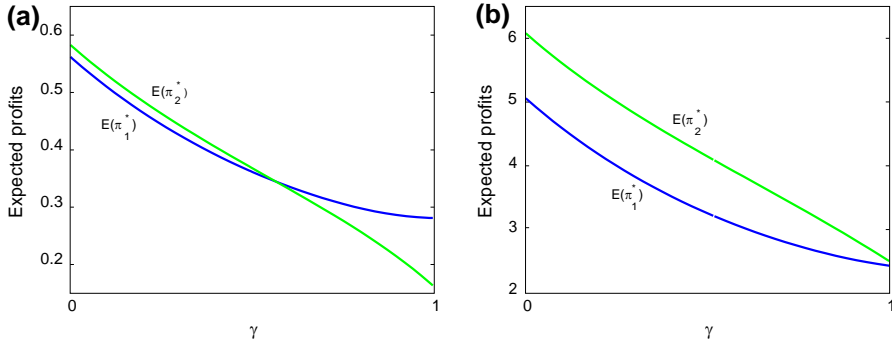

(a)

(b)

Figure 5. Cross-sections of Figure 3 at the uncertainty parameter's values (a) $\theta = 2$ and (b) $\theta = 8$.
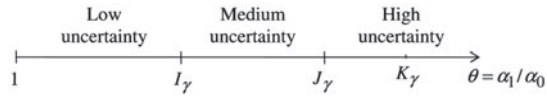

Figure 6. Three regions for the demand uncertainty parameter θ corresponding to three distinct profits relations between the leading and the follower firms.

$$I_\gamma = \frac{4 - \gamma^2}{4 - 3\gamma^2},$$

and the *medium–high uncertainty boundary value* is

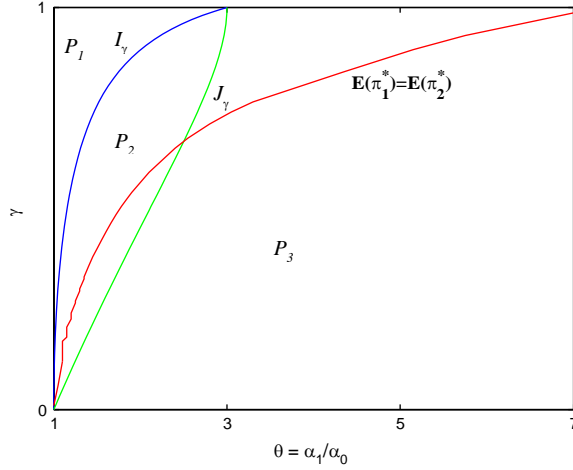$$J_\gamma = \frac{4 + 4\gamma - 5\gamma^2}{(2 - \gamma)^2}.$$

Figure 7. Plots of the functions I_γ and J_γ, and the regions P_1, P_2 and P_3.

Let $P_1 = \{(\theta, \gamma) : \theta < I_\gamma\}$, $P_2 = \{(\theta, \gamma) : I_\gamma \leq \theta \leq J_\gamma\}$ and $P_3 = \{(\theta, \gamma) : \theta > J_\gamma\}$ (see Figure 7).

The functions I_γ and J_γ characterize the demand uncertainty parameter θ for which the leading firm looses its advantage for some realizations of the demand random variable. In fact, in the next theorem, we will show that the leading firm looses its advantage for high values of the demand intercept, if the demand uncertainty parameter θ is greater than I_γ, and also for low values of the demand intercept, if the demand uncertainty parameter θ is greater than J_γ. Hence, for high values of the demand uncertainty parameter ($\theta > J_\gamma$) only in an intermediate zone of the realized demand does the first mover preserve its advantage. We observe that for homogeneous goods ($\gamma = 1$), the functions I_γ and J_γ coincide, i.e. $I_1 = J_1$ (Liu's case [2]), and for non-homogeneous goods ($0 \not\leq \gamma < 1$), we have that $I_\gamma < J_\gamma$. Observe that the curve given by $E(\pi_1^*) = E(\pi_2^*)$ crosses the curves P_1 and P_2 (see Figure 7).

We define L and R as follows:

$$L \equiv L(\alpha_0, \gamma, \theta) = \begin{cases} \frac{\alpha_0(2-\gamma)^2}{4(2-\gamma^2)}(\theta + 1), & \text{if } \theta \leq K_\gamma \\ \frac{(2-\gamma)\Delta}{3\gamma^3}, & \text{if } \theta \geq K_\gamma \end{cases}$$

and

$$R \equiv R(\alpha_0, \gamma, \theta) = \begin{cases} \frac{\alpha_0(4-\gamma^2)}{4(2-\gamma^2)}(\theta + 1), & \text{if } \theta \leq K_\gamma \\ \frac{(2+\gamma)\Delta}{3\gamma^3}, & \text{if } \theta \geq K_\gamma \end{cases}$$

THEOREM 4

(a) *Case $\theta \leq I_\gamma$. For all $\alpha \in [\alpha_0, \alpha_1]$, the leading firm has a higher profit than firm F_2, i.e. $\pi_2^* < \pi_1^*$.*
(b) *Case $I_\gamma < \theta \leq J_\gamma$. The value $R \in (\alpha_0, \alpha_1)$. Furthermore,*
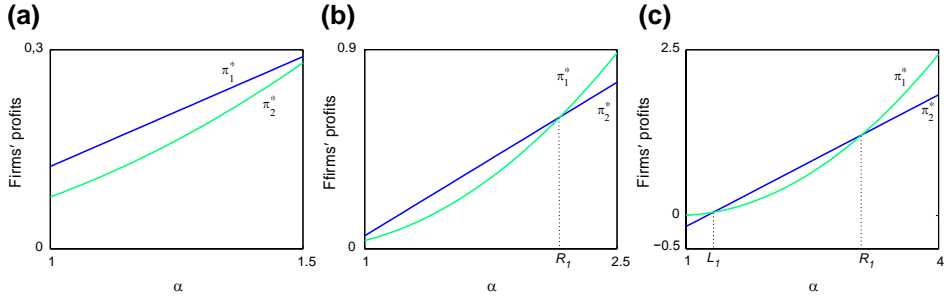
Figure 8. The profits of both firms when the intervals of the uniform distribution of the parameter a are such that the ratio θ between its endpoints is as in each situation of Theorem 4, for $\gamma = 0.8$, $a_0 = 1$ and (a) $a_1 = 1.5$; (b) $a_1 = 2.5$; (c) $a_1 = 4$.
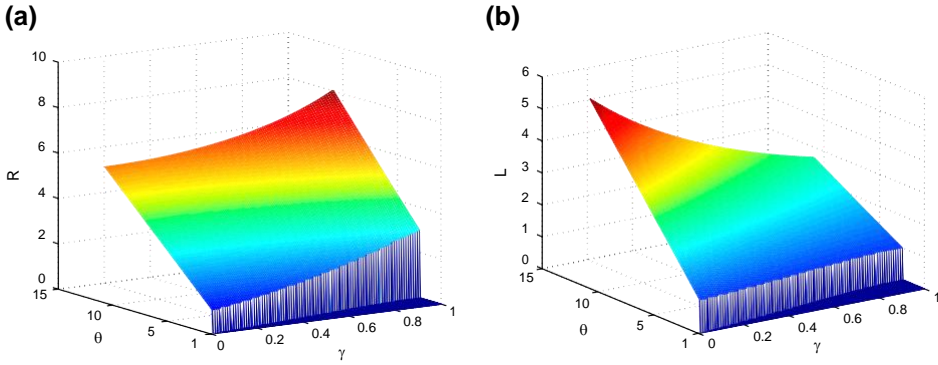

Figure 9. Plots of the functions (a) $R(a_0, \gamma, a_1)$; and (b) $L(a_0, \gamma, a_1)$, by considering $a_0 = 1$.

    (i)   *if $\alpha \leq R$, the leading firm has a higher profit than firm F_2, i.e. $\pi_2^* \leq \pi_1^*$;*

    (ii) *if $\alpha \geq R$, firm F_2 has a higher profit than the leading firm, i.e. $\pi_2^* \geq \pi_1^*$.*

(c)   *Case $\theta > J_\gamma$. The values $L, R \in (\alpha_0, \alpha_1)$. Furthermore,*

    (i)   *if $L \leq \alpha \leq R$, the leading firm has a higher profit than firm F_2, i.e.*
       $\pi_2^* \leq \pi_1^*$;

    (ii) *if either $\alpha \leq L$ or $\alpha \geq R$, firm F_2 has a higher profit than the leading firm,*
       i.e. $\pi_2^* \geq \pi_1^*$.

Theorem 4 is proved in the Appendix. This theorem allows us to say that even with small uncertainty about the demand, the follower can achieve greater profits than the leader, if their products are sufficiently differentiated (Figure 8). In Figure 9, we present the plots of R and L as functions of the demand uncertainty parameter θ and of the degree of differentiation γ. In Figure 10, we show the plots of R and L as functions of the demand uncertainty parameter θ, for some chosen values of the degree γ of differentiation. We take $a_0 = 1$ and, so, we observe that $a_1 = \theta$. We note that the values L and R are increasing with θ.
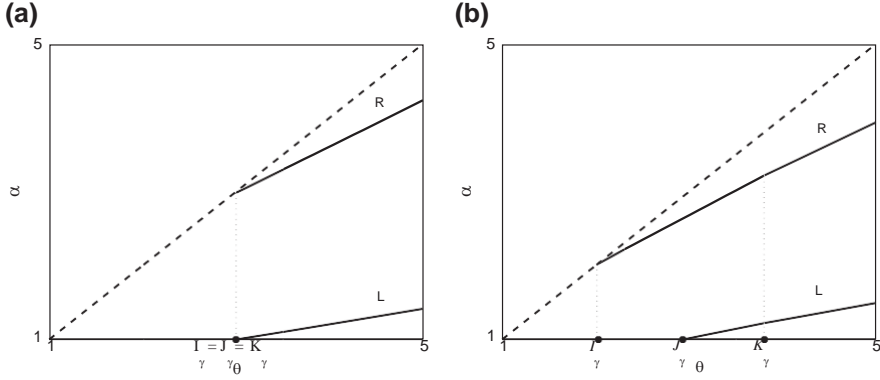
Figure 10. Plots of the functions $R(a_0, \gamma, a_1)$ and $L(a_0, \gamma, a_1)$, by considering $a_0 = 1$ and in the cases of (a) $\gamma = 1$; and (b) $\gamma = 0.9$.

5. Ex-ante probability of higher ex-post profits

Now, we are going to compute, in terms of the demand uncertainty parameter θ and of the product differentiation degree γ, the ex-ante probability $P\left(\pi_2^* > \pi_1^*\right)$ of the second firm having higher ex-post profit than the leading firm. Using the results presented in Theorem 4, we get the following corollary.

COROLLARY 2

(a) If $\theta < I_\gamma$, then $P\left(\pi_2^* > \pi_1^*\right) = 0$.

(b) If $I_\gamma \leq \theta \leq J_\gamma$, then

$$P\left(\pi_2^* > \pi_1^*\right) = \frac{(4 - 3\gamma^2)\theta - (4 - \gamma^2)}{4(2 - \gamma^2)(\theta - 1)}.$$

(c) If $\theta \geq J_\gamma$, then

$$P\left(\pi_2^* > \pi_1^*\right) = \begin{cases} \frac{(4-2\gamma-\gamma^2)\theta-(4+2\gamma-3\gamma^2)}{2(2-\gamma^2)(\theta-1)}, & \text{if } \theta \leq K_\gamma \\ 1 - \frac{2\Sigma}{3\gamma^2(\theta-1)}, & \text{if } \theta \geq K_\gamma \end{cases}.$$

The economic interpretations of this result include: (i) given the degree of the differentiation of the goods γ, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm having higher profit than the leading firm increases with the degree of the demand uncertainty; and (ii) given the demand uncertainty, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm having higher profit than the leading firm increases with the differentiation of the goods. Furthermore, when the goods are homogeneous, the probability $P\left(\pi_1^* > \pi_2^*\right)$ of the leading firm having higher profit than the follower firm is greater than the probability of the opposite situation. However, when the goods are sufficiently different, for sufficiently high level of uncertainty, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm having higher profit than the leading firm is greater than the probability of the opposite situation (see Figure 11).
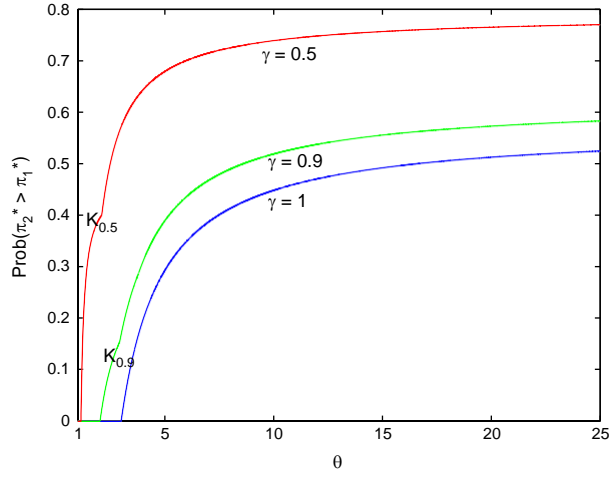
Figure 11. The probability of the second firm having higher profit than the leading firm, as a function of the demand uncertainty parameter θ, for different degrees γ of the differentiation of the goods, by taking $a_0 = 1$.
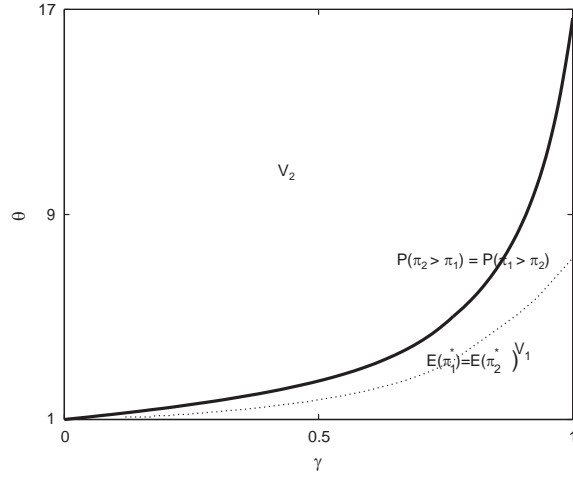

Figure 12. Solid line refers to the equality between the probabilities of the second firm having higher profit than the leading firm and the dashed line refers to the equality between the expected profits of both firms.

We observe that

$$
P\left(\pi_2^* > \pi_1^*\right) = \frac{1}{2} \Leftrightarrow
\begin{cases}
\theta = \frac{\gamma^2 - \gamma - 1}{\gamma - 1}, & \text{if } 0 \leq \gamma \leq 3 - \sqrt{5} \\[2mm]
\theta = \frac{32 - 5\gamma^2 - 4\gamma\sqrt{\gamma^2 + 32}}{32 - 32\gamma + 3\gamma^2}, & \text{if } 3 - \sqrt{5} \leq \gamma \leq 1
\end{cases}
$$

For a sufficiently high level of uncertainty ($\theta > 9+4\sqrt{33}/3$), the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm having higher profit than the leading firm is greater than the probability of the opposite situation, independently of the degree γ of differentiation. Let $V_1 = \{(\gamma, \theta) : P_1 > P_2\}$ and $V_2 = \{(\gamma, \theta) : P_1 < P_2\}$. In Figure 12, we show the region where the probability of the second firm having higher profit than the leading firm is greater than the probability of the opposite situation.

We also observe that

$$\lim_{\theta \to \infty} P\left(\pi_2^* > \pi_1^*\right) = \frac{8 - \gamma^2 - 2\sqrt{\gamma^4 + 6\gamma^3 - 16\gamma^2 + 16}}{3\gamma^2},$$

and

$$\frac{7 - 2\sqrt{7}}{3} \leq \frac{8 - \gamma^2 - 2\sqrt{\gamma^4 + 6\gamma^3 - 16\gamma^2 + 16}}{3\gamma^2} \xrightarrow[\gamma \to 0]{} 1.$$

6. Conclusions

In order to analyse the leadership and flexibility advantages, we considered three different situations, that depend upon the uncertainty parameter θ given by the ratio α_1/α_0 between the endpoints of the demand interval $[\alpha_0, \alpha_1]$ in which the parameter α is uniformly distributed. We found two functions I_γ and J_γ, that depend upon the degree γ of differentiation of the goods, such that (i) if $\theta < I_\gamma$, then the leading firm has a higher profit than the follower; (ii) if $I_\gamma \leq \theta \leq J_\gamma$, then when the realized demand is very high, the leading firm has a lower profits than the follower, otherwise the leading firm has a higher profit than the follower; and (iii) if $\theta > J_\gamma$, then when the realized demand is very low or very high, the leading firm has a lower profit than the follower, and when the realized demand is in an intermediate region, the leading firm has a higher profit than the follower. We observed that for homogeneous goods ($\gamma = 1$, Liu's case), the functions I_γ and J_γ coincide, i.e. $I_1 = J_1$, and for non-homogeneous goods ($0 \leq \gamma < 1$), we have that $I_\gamma < J_\gamma$.

We showed that, when the goods are homogeneous, the probability $P\left(\pi_1^* > \pi_2^*\right)$ of the leading firm having higher profit than the follower firm is greater than the probability of the opposite situation; however, if the goods are sufficiently different or have high level of uncertainty, the probability $P\left(\pi_2^* > \pi_1^*\right)$ of the follower firm having higher profit than the leading firm is greater than one half.

References

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Appendix

In this Appendix, we prove of the results presented throughout the paper.

Proof of Theorem 1 Using backwards-induction, we first compute firm F_2's reaction, $q_2^*(q_1, \alpha)$, to an arbitrary quantity q_1 fixed by firm F_1, and to the realized demand parameter α. The quantity $q_2^*(q_1, \alpha)$ is given by

$$\arg \max_{q_2 \geq 0} q_2(\alpha - \gamma q_1 - q_2),$$

which yields

$$q_2^*(q_1, \alpha) = \max \left\{ \frac{\alpha - \gamma q_1}{2}, 0 \right\}.$$

Therefore, firm F_1's problem in the first stage of the game amounts to determine

$$q_1^* = \arg \max_{q_1 \geq 0} E(q_1 \cdot p_1),$$

where $p_1 = \alpha - q_1 - \gamma q_2^*(q_1, \alpha)$ and $E(\bullet)$ is the expectation with respect to the demand intercept α. We are going to study separately the cases: (*I*) $\alpha_0 \geq \gamma q_1$ and (*II*) $\alpha_0 \leq \gamma q_1$. Note that the density function of α's distribution is $1/(\alpha_1 - \alpha_0)$.

Case I $\alpha_0 \geq \gamma q_1$ (see Figure 13). In this case, $\alpha \geq \alpha_0 \geq \gamma q_1$, and so $q_2^*(q_1, \alpha) = (\alpha - \gamma q_1)/2$. Therefore,
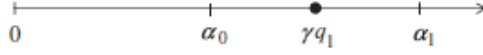
Figure 13. $\alpha_0 \geq \gamma q_1$ (*Case I*).

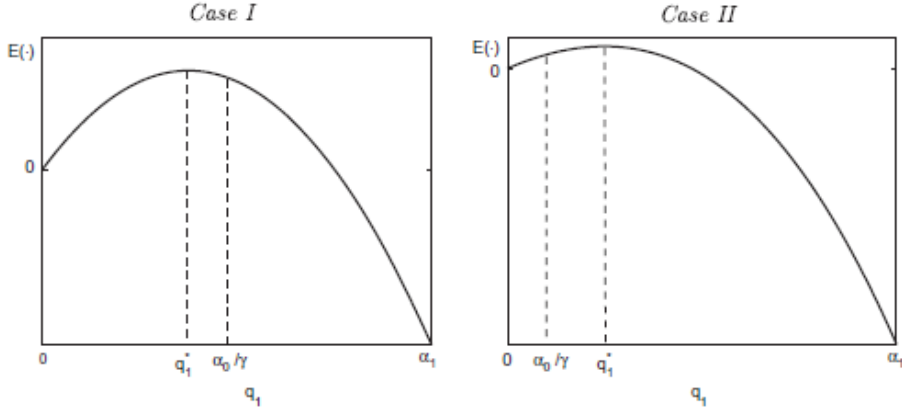

Figure 14. $\alpha_0 \leq \gamma q_1$ (*Case II*).

Figure 15. *Case I* ($\theta \leq K_\gamma$): $\arg\max_{q_1 \geq 0} E(q_1 \cdot p_1) \leq \alpha_0/\gamma$; *Case II* ($\theta \geq K_\gamma$): $\arg\max_{q_1 \geq 0} E(q_1 \cdot p_1) \geq \alpha_0/\gamma$.

We note that, in *Case I* ($\theta \leq K_\gamma$) we have that $\arg\max_{q_1 \geq 0} E(q_1 \cdot p_1) \leq \alpha_0/\gamma$, and in *Case II* ($\theta \geq K_\gamma$) we have that $\arg\max_{q_1 \geq 0} E(q_1 \cdot p_1) \geq \alpha_0/\gamma$ (see Figure 15).

Proof of Corollary 1   First, suppose that $\theta \leq K_\gamma$. By Theorem 1, we get that

$$q_1^* - q_2^* = \frac{4 - \gamma^2}{8(2 - \gamma^2)}(\alpha_1 + \alpha_0) - \frac{\alpha}{2}.$$

Therefore, $q_1^* - q_2^* \geq 0$ if, and only if,

$$\alpha \leq \frac{4 - \gamma^2}{4(2 - \gamma^2)}(\alpha_1 + \alpha_0).$$

Since $\frac{4 - \gamma^2}{4(2 - \gamma^2)}(\alpha_1 + \alpha_0) > \alpha_1$ if, and only if, $\theta < I_\gamma$, and $I_\gamma \leq k_\gamma$, we get the statement (a).
Since $\frac{4 - \gamma^2}{4(2 - \gamma^2)}(\alpha_1 + \alpha_0) \in [\alpha_0, \alpha_1]$ if, and only if, $\theta \geq I_\gamma$, we get the statement (b).
Now, suppose that $\theta \geq K_\gamma$. By Theorem 1, we get that

- if $\alpha < \Delta/(3\gamma^2)$, then

$$q_1^* - q_2^* = q_1^* = \frac{\Delta}{3\gamma^3},$$

which is positive;

- if $\alpha \geq \Delta/(3\gamma^2)$, then

$$q_1^* - q_2^* = \frac{(2+\gamma)\Delta}{6\gamma^3} - \frac{\alpha}{2}.$$

Therefore, $q_1^* - q_2^* \geq 0$ if, and only if,

$$\alpha \leq \frac{(2+\gamma)\Delta}{3\gamma^3},$$

which implies the statement (c). □

Proof of Theorem 2 Firm F_1's expected profit, $E(\pi_1^*(\alpha))$, is obtained by (3) and (6). Firm F_2's expected profit, $E(\pi_2^*(\alpha))$, is determined by

$$E(\pi_2^*(\alpha)) = \int_{\alpha_0}^{\alpha_1} q_2^* \left(\alpha - \gamma q_1^* - q_2^*\right) \frac{1}{\alpha_1 - \alpha_0} \, d\alpha,$$

with q_1^* and q_2^* given, respectively, by (4) and (5), in the case of $\theta \leq K_\gamma$, and given, respectively, by (7) and (8), in the case of $\theta \geq K_\gamma$. Finally, we note that the values (θ, γ) in the set P are the ones that satisfy $\theta \leq K_\gamma$, and the values (θ, γ) in the set Q are the ones that satisfy $\theta \geq K_\gamma$. □

Proof of Theorem 3 Here, we compute the exact value θ_0, when we take $\alpha_0 = 1$ and $\gamma = 1$, such that if the uncertainty parameter θ is greater than θ_0, then the expected profit of the follower firm is always greater than the expected profit of the leading firm.

Let

$$D = \frac{10348768573}{5971968} - \frac{8804787277\sqrt{17}}{23887872}, \quad E = \frac{10348768573}{5971968} + \frac{8804787277\sqrt{17}}{23887872},$$

$$F = \frac{118172131}{995328} - \frac{10655929\sqrt{17}}{373248} \quad \text{and} \quad G = \frac{118172131}{995328} + \frac{10655929\sqrt{17}}{373248}.$$

The value θ_0 is given by

$$\theta_0 = \frac{9}{4} + \sqrt{\frac{143}{48} - \left(\frac{3539}{3456} - \frac{3011\sqrt{17}}{13824}\right)^{1/3} - \left(\frac{3539}{3456} + \frac{3011\sqrt{17}}{13824}\right)^{1/3}}$$

$$+ \sqrt{\frac{143}{24} + \left(\frac{3539}{3456} - \frac{3011\sqrt{17}}{13824}\right)^{1/3} + \left(\frac{3539}{3456} + \frac{3011\sqrt{17}}{13824}\right)^{1/3} + \sqrt{H}},$$

where

$$H = D^{1/3} + E^{1/3} + F^{1/3} + G^{1/3} + \frac{19013}{576}.$$

Therefore, we observe that $\theta_0 \approx 7.30$. □

Proof of Theorem 4 By Theorem 1, in the case where $\theta \leq K_\gamma$, the profits π_1^* and π_2^* at equilibrium are given by

$$\pi_1^* = \frac{(2-\gamma)^2(4\alpha - \alpha_1 - \alpha_0)(\alpha_1 + \alpha_0)}{32(2-\gamma^2)}$$

and

$$\pi_2^* = \frac{\left(4\alpha(2-\gamma^2) - \gamma(2-\gamma)(\alpha_1 + \alpha_0)\right)^2}{64(2-\gamma^2)^2}.$$

So, we have that $\pi_1^* - \pi_2^* > 0$ if, and only if,

$$L_1 \equiv \frac{(2-\gamma)^2}{4(2-\gamma^2)}(\alpha_0 + \alpha_1) < \alpha < \frac{4-\gamma^2}{4(2-\gamma^2)}(\alpha_0 + \alpha_1) \equiv R_1.$$

Again by Theorem 1, in the case of $\theta \geq K_\gamma$, the profits π_1^* and π_2^* at equilibrium are given by

$$\pi_1^* = \begin{cases} \frac{(3\alpha\gamma^3 - \Delta)\Delta}{9\gamma^6}, & \text{if } \alpha < \Delta/(3\gamma^2) \\ \frac{(3\alpha\gamma^3(2-\gamma) - (2-\gamma^2)\Delta)\Delta}{18\gamma^6}, & \text{if } \alpha \geq \Delta/(3\gamma^2) \end{cases}$$

and

$$\pi_2^* = \begin{cases} 0, & \text{if } \alpha < \Delta/(3\gamma^2) \\ \frac{(3\alpha\gamma^2 - \Delta)^2}{36\gamma^4}, & \text{if } \alpha \geq \Delta/(3\gamma^2) \end{cases}$$

So, we have that $\pi_1^* - \pi_2^* > 0$ if, and only if,

$$L_2 \equiv \frac{(2-\gamma)\Delta}{3\gamma^3} < \alpha < \frac{(2+\gamma)\Delta}{3\gamma^3} \equiv R_2.$$

(a) For $\theta < I_\gamma$, we have that $L_1 < \alpha_0$ and $R_1 > \alpha_1$. Therefore, $\pi_2^* < \pi_1^*$ for all $\alpha \in [\alpha_0, \alpha_1]$.

(b) For $I_\gamma \leq \theta \leq J_\gamma$, we have that $L_1 < \alpha_0$ and $\alpha_0 \leq R_1 \leq \alpha_1$. Therefore, if $\alpha < R_1$ then $\pi_2^* < \pi_1^*$; and if $\alpha > R_1$ then $\pi_2^* > \pi_1^*$.

(c) For $J_\gamma < \theta \leq K_\gamma$, we have that $\alpha_0 \leq L_1$, $R_1 \leq \alpha_1$. Therefore, if $L_1 < \alpha < R_1$ then $\pi_2^* < \pi_1^*$; and if either $\alpha < L_1$ or $\alpha > R_1$ then $\pi_2^* > \pi_1^*$.
For $\theta > K_\gamma$, we have that $\alpha_0 \leq L_2$, $R_2 \leq \alpha_1$. Therefore, if $L_2 < \alpha < R_2$ then $\pi_2^* < \pi_1^*$; and if either $\alpha < L_2$ or $\alpha > R_2$ then $\pi_2^* > \pi_1^*$.

□