22-24 May 1991

"Cankarjev dom" – Cultural and Congress Center

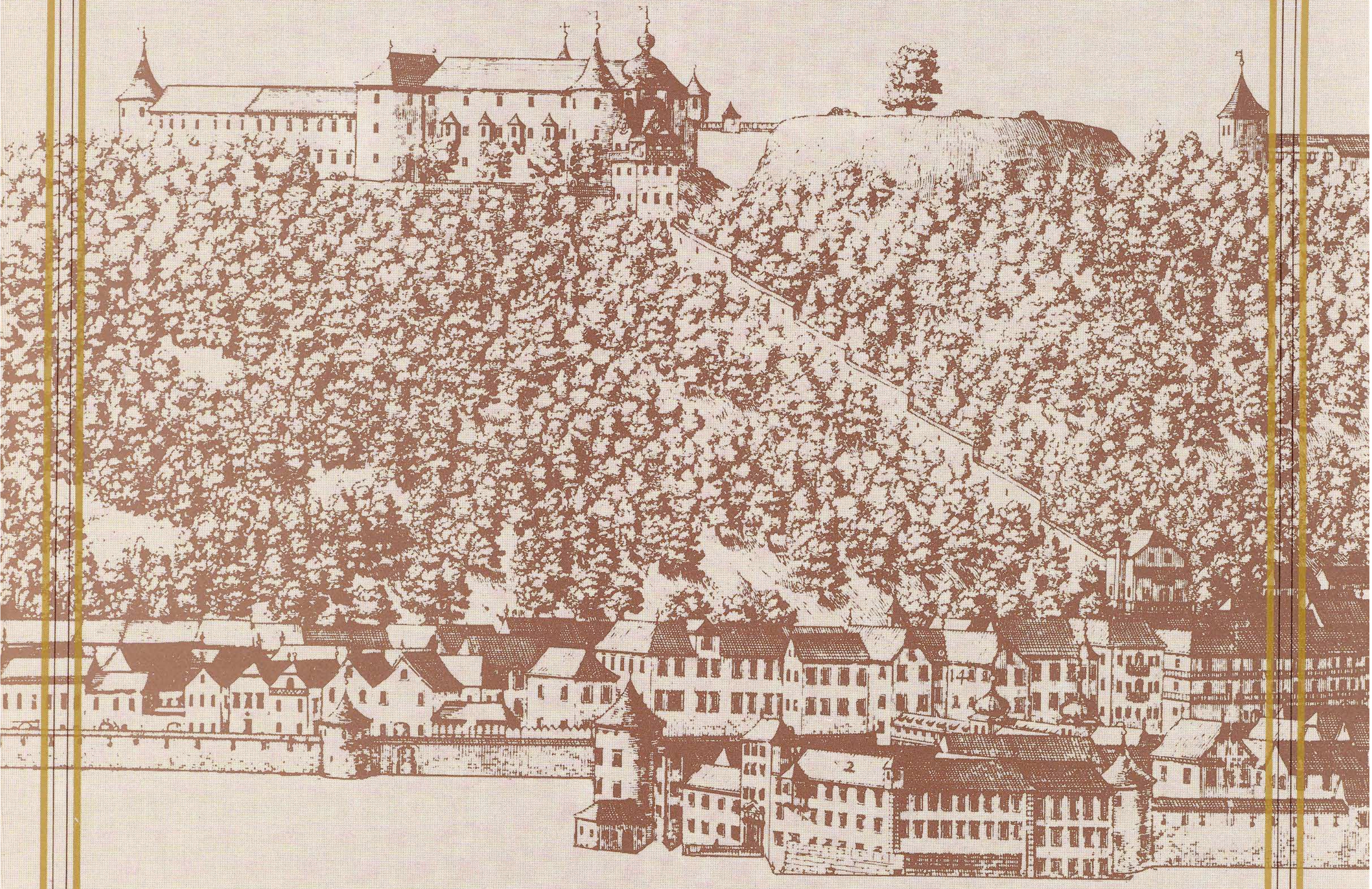
Ljubljana, Slovenia, Yugoslavia

melecon '91

MEDITERRANEAN ELECTROTECHNICAL CONFERENCE

ELECTRONICS, CONTROL AND ROBOTICS, TELECOMMUNICATIONS, COMPUTER TECHNIQUES, EDUCATION

Proceedings
Volume II



IEEE Catalog Number: 91CH2964-5
ISBN 0-87942-655-1 Softbound Edition
ISBN 0-87942-656-x Casebound Edition
ISBN 0-87942-657-8 Microfiche Edition
Library of Congress Number 90-86045

IEEE

Edited by

Baldomir Zajc
Franc Solina

TOWARDS THE STATISTICAL MODELLING OF ROBOTIC MANIPULATORS

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Abstract - A novel approach to the modelling of robot manipulators is presented. Usually, system descriptions are based on a set of differential equations which, due to their nature, lead to very precise results and strategies. However, they need laborious computations and, therefore, methods based on alternative mathematical concepts are required. The proposed statistical model overcomes these problems and gives clear guidelines towards the optimization of the robot performances.

1 INTRODUCTION

The area of application of robotics has widened to encompass a large range of industrial and scientific applications. However, many of them pose challenging issues to the available industrial manipulators. The superior performances of the human arm induced an extensive research towards better mechanical structures. Considerable research in this area has been done on the kinematics [1-2] and dynamics [3-5]. From those studies it is clear that a simple optimization criterion comprising both the kinematics and dynamics is still lacking. In this paper we address this problem, and our presentation is organized as follows. In section two the modelling problem and the corresponding mathematical concepts are discussed. In section three a statistical model which leads to a natural optimization criterion is formulated and applied to a 2R robot manipulator. Finally, in section four conclusions are drawn.

2 ON THE STATISTICAL MODELLING OF ROBOT MANIPULATORS

The classical modelling of robot manipulators is well known. For the kinematics the set of equations which relate the joint space and the operational space, can be found to be of the form:

$$q = \alpha(p) \quad (1a)$$
$$\dot{q} = \theta(p,\dot{p}) \quad (1b)$$
$$\ddot{q} = \Phi(p,\dot{p},\ddot{p}) \quad (1c)$$

where $\{q,\dot{q},\ddot{q}\}$ ($\{p,\dot{p},\ddot{p}\}$) are the n-vectors of positions, velocities and accelerations in the joint (operational) space for a n degrees of freedom (d.o.f.) manipulator. Associated with the kinematic model we have the statics model, that relates the operational space forces Γ with the joint actuator torques T:

$$T = J(q)^T\,\Gamma \quad (2)$$

where J(q) is the jacobian matrix corresponding to the differential relationship $\dot{p}=J(q)\dot{q}$. On the other hand, the dynamics is described by a nonlinear matrix differential equation:

$$T = I(q)\ddot{q}+C(q,\dot{q})+G(q) \quad (3)$$

having $T_I=I(q)\ddot{q}$, $T_C=C(q,\dot{q})$ and $T_G=G(q)$ for the n-vectors of inertial, Coriolis/centripetal and gravitational torques.

Based on these equations considerable research has been done on issues such as manipulator structure optimization [1-5] and path planning [6-7]. However, a more sound consideration of the whole theme reveals that these methods are far from achieving a comprehensive formulation. This observation motivates the re-evaluation of the approachs in use. In fact, expressions (1)-(3) show that the plethora of variables and parameters involved, gives rise to a cumbersome work both in the analysis and design stages. The huge number of possible combinations of values indicates that, in order to overcome implementation problems, alternative concepts are required. Statistics is a mathematical strategy well adapted to this type of problem. With this method, we lose the "certainty" of a deterministic model, however, we gain a simpler and more intuitive viewpoint. This approach has already been used by other researchers [8-9] in some restricted classes of problems. In the sequel we refer to the new approach, as the statistical model [10] to stress the contrast with the standard method.

Our modelling procedure comprises:

- The statistical description of a set of input variables, that is, variables that are free to change independently.
- The statistical description of a set of output variables, that is, variables that are functions of the previous ones.
- A set of parameters which are to be optimized in the design stage.

The above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is established as follows:

- $\{p,\dot{p},\ddot{p}\}$ act as input variables of the kinematic system. This option enables a definition of the required kinematic performances on the operational space which are more natural to the designer.
- $\{q,\dot{q},\ddot{q}\}$ act as output variables of the kinematic system, but play the role of input variables set in the dynamic model. Thereby, we arrive at a relationship between kinematics and dynamics in a form amenable to performance optimization criteria as defined in the sequel.
- The set of dynamic output variables consists of the required joint torques {T}.
- The parameter set consists of link lengths, masses and inertias.

In other words, we are stating that in the kinematics (dynamics), $\{p,\dot{p},\ddot{p}\}$ ($\{q,\dot{q},\ddot{q}\}$) are considered as independent random variables, its probability density functions (p.d.f.'s) being similar to the histograms of a long run sampling, while $\{q,\dot{q},\ddot{q}\}$ ({T}) are the corresponding random dependent variables. The statistical description of the involved variables, does not consider the (implicit) time variable. Therefore, variables that are related through the time derivative operator are considered independent of each other.

3 A STATISTICAL MODEL OF THE 2R ROBOT MANIPULATOR

Let us now adopt the 2R joint-actuated robot manipulator as the support for the development and implementation of the new modelling concepts. In subsection one we study the kinematics and, based on the results, in

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subsection two we analyse the total (i.e. kinematic and dynamic) system.

3.1 The Kinematics

The set of kinematic input variables consists of position, velocity and acceleration that our prototype manipulator is required to perform in the operational space. Therefore, it is necessary to characterize them in statistical terms, namely by defining appropriate p.d.f.'s for each variable. As there is no a priori knowledge about the typical behaviour we start with some reasonable assumptions. For the position variable $p=[x,y]^T$ we consider a bidimensional uniform p.d.f.

$$f_p(p)=\begin{cases} C & \text{if } (r_1-r_2)^2\le x^2+y^2\le(r_1+r_2)^2 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

with $C=1/\{\pi[(r_1+r_2)^2-(r_1-r_2)^2]\}$.

In the sequel we will see how to modify the input p.d.f. in order that the kinematic performances are optimized. It is also necessary to define the p.d.f.'s for $\dot{p}$ and $\ddot{p}$. By the same above arguments, we decided to use bidimensional Gaussian p.d.f.'s with zero mean

$$f_{\dot{p}}(\dot{p})=\mathrm{EXP}[-(\dot{x}^2+\dot{y}^2)/(2\sigma_{\dot{p}}^2)]/(2\pi\sigma_{\dot{p}}^2) \qquad (5)$$
$$f_{\ddot{p}}(\ddot{p})=\mathrm{EXP}[-(\ddot{x}^2+\ddot{y}^2)/(2\sigma_{\ddot{p}}^2)]/(2\pi\sigma_{\ddot{p}}^2) \qquad (6)$$

Moreover, using these p.d.f.'s we impose some useful properties, such as:

- The input random variables position, velocity and acceleration are independent of each other.
- The velocity and acceleration vectors are made of two independent components, that is, $\dot{x}$ ($\ddot{x}$) is independent of $\dot{y}$ ($\ddot{y}$).

The "excitation" of the (inverse) kinematic system produces output random variables $\{q,\dot{q},\ddot{q}\}$ with p.d.f.'s which are related to the previous ones by:

$$f_q(q) = |J_p| f_p(p) \qquad (7a)$$
$$f_{q\dot{q}}(q,\dot{q}) = |J_v| f_{p\dot{p}}(p,\dot{p}) \qquad (7b)$$
$$f_{q\dot{q}\ddot{q}}(q,\dot{q},\ddot{q}) = |J_A| f_{p\dot{p}\ddot{p}}(p,\dot{p},\ddot{p}) \qquad (7c)$$

where the jacobians J_p, J_v and J_A are:

$$J_p=\partial(p)/\partial(q)=r_1 r_2 S_2 \qquad (8a)$$
$$J_v=\partial(p,\dot{p})/\partial(q,\dot{q})=J_p(r_1 r_2 S_2) \qquad (8b)$$
$$J_A=\partial(p,\dot{p},\ddot{p})/\partial(q,\dot{q},\ddot{q})=J_v(r_1 r_2 S_2) \qquad (8c)$$

Each of the expressions (7) is made of two distinct factors:

- Weighting factors - J_p, J_v and J_A - which depend solely on the system kinematic properties
- The "excitation" p.d.f.'s - $f_p(p)$, $f_{p\dot{p}}(p,\dot{p})$ and $f_{p\dot{p}\ddot{p}}(p,\dot{p},\ddot{p})$ - which are a measure of the task requirements.

These factors can be interpreted in a system theoretic framework. The jacobians characterize the system intrinsic properties, while the excitation p.d.f.'s correspond to the system response to the input variables.

Bearing these facts in mind, several experiments were performed, having:

- The total link length constant, $L=0.6$.
- Seven robot configurations with ratios $\mu=r_1/r_2$ of 0.4, 0.6, 0.8, 1, 1.2, 1.4 and 1.6, respectively.
- Nine categories of requirements for $\dot{p}$ and $\ddot{p}$:
  1. $\sigma_{\dot{p}}=0.1$ $\sigma_{\ddot{p}}=0.1$, 4. $\sigma_{\dot{p}}=1$ $\sigma_{\ddot{p}}=0.1$, 7. $\sigma_{\dot{p}}=10$ $\sigma_{\ddot{p}}=0.1$
  2. $\sigma_{\dot{p}}=0.1$ $\sigma_{\ddot{p}}=1$, 5. $\sigma_{\dot{p}}=1$ $\sigma_{\ddot{p}}=1$, 8. $\sigma_{\dot{p}}=10$ $\sigma_{\ddot{p}}=1$
  3. $\sigma_{\dot{p}}=0.1$ $\sigma_{\ddot{p}}=10$, 6. $\sigma_{\dot{p}}=1$ $\sigma_{\ddot{p}}=10$, 9. $\sigma_{\dot{p}}=10$ $\sigma_{\ddot{p}}=10$
- Excitation of the kinematic system with a numerical random sample obeying the p.d.f.'s (4)-(6).
- Analysis of the resulting histograms of the output variables. Only marginal p.d.f.'s were considered, for the sake of simplicity.

After a large number of experiments using the numerical set of parameters depicted in Table 1, we concluded that the shape of the resulting p.d.f.'s varied significantly from variable to variable, but all of them showed symmetry around zero. Therefore, in order to characterize the resulting histograms by a scalar index, we decided to adopt for this index the difference between the 97.5% and 2.5% percentiles, that is, the 95%-inter-percentile range. The resulting histograms are condensed through this index and depicted in Fig. 1. We can observe in the majority of the charts a minimum about $\mu=1$ yet, this conclusion can be easily inferred from (7). Indeed, for symmetrical histograms about zero on the x-axis, having a peak on that point, a larger value of the jacobian corresponds to a smaller dispersion of the random variable. This, in turn, means average smaller amplitude requirements posed to that variable. Therefore, we have an optimization criterion which is based on the new statistical modelling concepts.

TABLE 1 Numerical values of the 2R manipulator

	Length	Radius	Mass	Inertia
Link 1	0.3 m	0.05 m	2.16 Kg	0.01755 Kgm²
Link 2	0.3 m	0.0389 m	1.68 Kg	0.01324 Kgm²

As the maximization of J_p, J_v and J_A requires the same steps, we have for:

$$L = r_1+r_2,\ \mu = r_1/r_2 \qquad (9)$$

that a maximum occurs when:

$$\mu = 1,\ q_2 = \pi/2 \qquad (10)$$

which coincide with the results obtained (using the classical approach) in previous studies [1,3]. Furthermore, our optimization criteria enables additional conclusions:

- Due to (2) the optimization of the kinematics is equivalent to the optimization of the statics.
- If further optimization is desired, then the next step will be the selection of an optimum "excitation" p.d.f.. This second step of optimization will define, in a statistical sense, an optimum kinematic class for the manipulator trajectories. Obviously, we can find a multitude of different p.d.f's obeying (10); yet, for the subsequent study a particular choice is of minor importance. Consequently, we decided to adopt the following family of position p.d.f.'s in the operational space (with $K\ge1$):

$$f_p(x,y)=\mathrm{cons}*|1-[(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2)]^2|^{(K-1)/2} \qquad (11)$$

which, in the joint space, corresponds to:

$$f_q(q_1,q_2)=\mathrm{constant}*|S_2|^K \qquad (12)$$

As extreme cases, we have that, for $K=1$ it becomes the uniform p.d.f. (4), while for $K\to\infty$ we get Dirac type optimum p.d.f. in the sense of (10). As far as $\dot{p}$ and $\ddot{p}$ are concerned we can see that the kinematic study does not point out any special class of p.d.f.'s. However, these variables are negatively affected by the position deviation from the optimum configuration $q_2=\pi/2$. Therefore, we decided to study the system behaviour both for performance requirements described by p.d.f.'s (12), (5) and (6) and for the alternative situation corresponding to p.d.f. (12) associated with the "enhanced" (q_2-dependent) p.d.f.'s for $\dot{p}$ and $\ddot{p}$:

$$f_{\dot{p}}(\dot{p},q_2)=\mathrm{EXP}\{-(\dot{x}^2+\dot{y}^2)/[2\sigma_{\dot{p}}^2(q_2)]\}/[2\pi\sigma_{\dot{p}}^2(q_2)] \qquad (13a)$$

$$\sigma_{\dot{p}}(q_2)=\begin{cases} 2\sigma_{\dot{p}}|q_2|/\pi & \text{if } 0<|q_2|\le\pi/2 \\ 2\sigma_{\dot{p}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\le\pi \end{cases} \qquad (13b)$$

$$f_{\ddot{p}}(\ddot{p},q_2)=\mathrm{EXP}[-(\ddot{x}^2+\ddot{y}^2)/[2\sigma_{\ddot{p}}^2(q_2)]\}/[2\pi\sigma_{\ddot{p}}^2(q_2)] \qquad (14a)$$

$$\sigma_{\ddot{p}}(q_2)=\begin{cases} 2\sigma_{\ddot{p}}|q_2|/\pi & \text{if } 0<|q_2|\le\pi/2 \\ 2\sigma_{\ddot{p}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\le\pi \end{cases} \qquad (14b)$$

To test numerically the above conjectures, the pre-

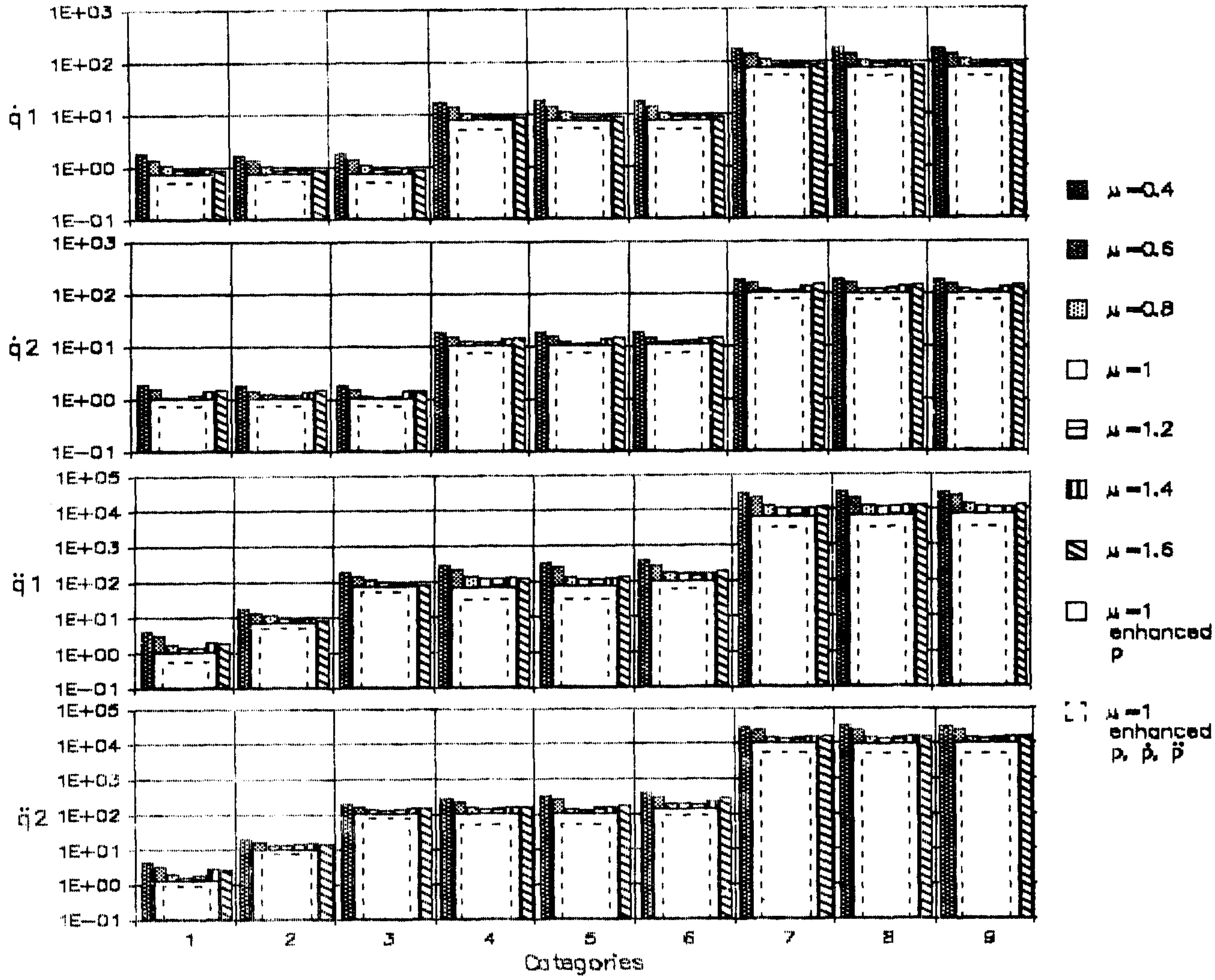

Figure 1: Comparison chart for the 2R joint-actuated robot kinematic performances. The narrow columns correspond to seven geometric configurations "excited" with p.d.f.'s (4) and (5)-(6). The wider columns correspond to the optimum geometric μ=1 "excited" with the enhanced p.d.f. (12) and (5)-(6) for the solid borders and enhanced p.d.f.'s (12) and (13)-(14) for the dotted borders.

vious results for μ=1 are compared with a new case using μ=1 and K=3 in (11)-(12). This has revealed a remarkable performance improvement as shown in Fig. 1, particularly for velocity-dependent requirements.

3.2 The Kinematics and Dynamics

The statistical description of the total system requires steps similar to those adopted in the kinematics, namely:

- Characterisation of the input variables ($\{p, \dot{p}, \ddot{p}\}$) through appropriate p.d.f.'s.
- "Stimulation" of the system behaviour through numerical experiments.
- Analysis of the histograms of the output variables ($\{T\}$).

The direct application of our optimizing method to the total system would require the mathematical and numerical treatment of 3n-dimensional p.d.f.'s. In order to avoid this intricate analysis, we decided to integrate, in our present investigation, the conclusions pointed out in the kinematic study and the results of several companion papers on the dynamics [11-14]. Hence we "excited" both the kinematics and dynamics with four different position p.d.f.'s (having K=3):

$$f_Q(q_1,q_2)=\text{constant}*|S_2|^K \quad (15)$$

$$f_Q(q_1,q_2)=\text{constant}*|S_1 S_2|^K \quad (16)$$

$$f_Q(q_1,q_2)=\text{constant}*|S_1 S_{12}|^K \quad (17)$$

$$f_Q(q_1,q_2)=\text{constant}*|C_2|^K \quad (18)$$

which are suggested by the optimization of the kinematics and dynamics namely:

- The kinematics.
- A compromise between kinematics and gravitational torques.
- The gravitational torques.
- Both the Coriolis/centripetal and inertial torques.

These position p.d.f.'s combined with the two alternative $\dot{p}$ and $\ddot{p}$ p.d.f.'s, namely (5)-(6) or (13)-(14) for the same nine categories of operational space variables reveals that (Fig. 2):

- For low $\dot{p}$ and $\ddot{p}$ the 95% index gives almost similar results for all p.d.f.'s, because the gravitational torques predominate.
- $\dot{p}$ has a much stronger influence than $\ddot{p}$.
- Kinematic effects prevail over the dynamic ones; therefore, the best results come from the "kinematic-dependent" p.d.f.'s (15)-(16).

In conclusion, the statistical analysis shows that mechanical joint-actuated manipulators are much more sensitive to $\dot{p}$ than to $\ddot{p}$. These facts indicate that we are dealing with "position and acceleration machines" rather than "velocity machines". Although obvious, this

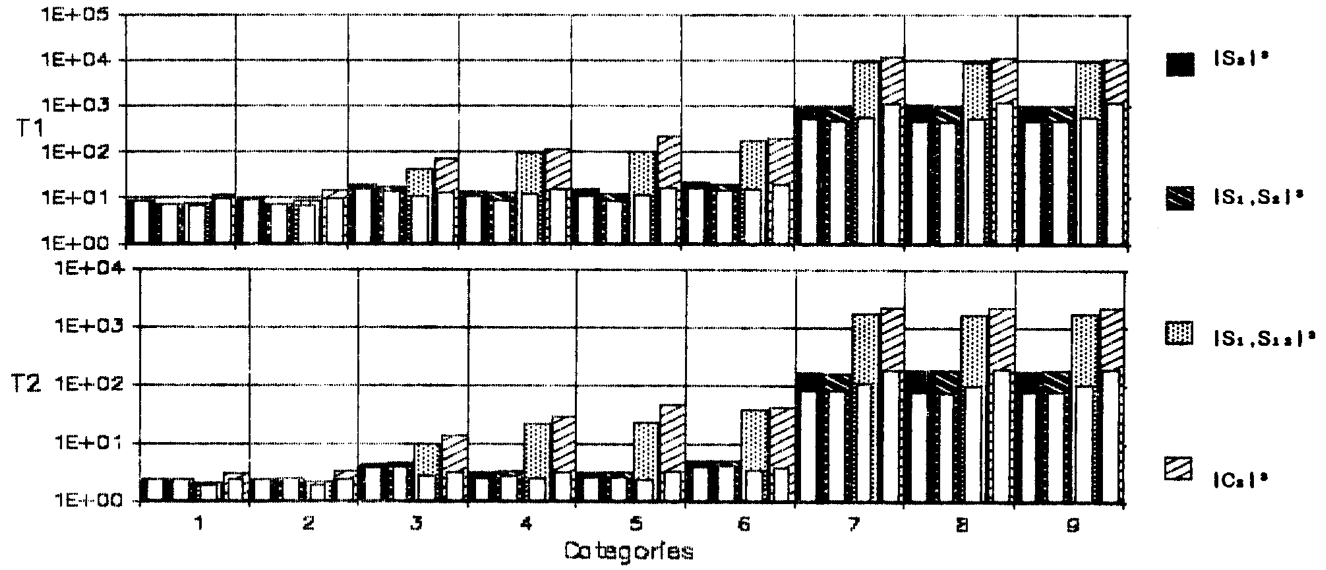

Figure 2: Kinematic/dynamic performances of the 2R manipulator ($\mu=1$) excited with the p.d.f.'s:
1st column: $f_e(q_1,q_2)=\text{constant}*|S_2|^a$ 3rd column: $f_e(q_1,q_2)=\text{constant}*|S_1S_{12}|^a$
2nd column: $f_e(q_1,q_2)=\text{constant}*|S_1S_2|^a$ 4th column: $f_e(q_1,q_2)=\text{constant}*|C_2|^a$
The back columns correspond to p.d.f.'s (5)-(6) and the front white columns correspond to the "enhanced" p.d.f.'s (13)-(14).

aspect has been somewhat overlooked. Moreover, it points out that standard robot actuators are not well adapted to robotic applications. Alternative solutions, such as muscle like actuators [15-17] will allow more efficient robot structures [18].

4 CONCLUSIONS

A new method for modelling robot manipulators is presented. Usually, system descriptions are based on a set of differential equations which, though leading to very precise results and strategies, are very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The new method provides a framework giving clear guidelines on the robot structure optimization. As a result, the manipulator design procedure leads to simple and intuitive conclusions. Furthermore, the inherent use of histograms allows not only fast calculation procedures but, above all, the use of experimental data; consequently, complex dynamic modelling exercises can be avoided. Furthermore, it should be highlighted the results pointing out some characteristics of the trajectory planning block and ideal-actuator properties. This observation is of utmost importance as it gives a net basis to new mechanical robot manipulator structures, with performances close to the muscle-actuated biological systems.

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