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TRAJECTORY CONTROL OF REDUNDANT MANIPULATORS

J.A.Tenreiro Machado* & Fernando B.M. Duarte**

* *Faculdade de Engenharia da Universidade do Porto, Dep. de Eng. Electrotécnica e de Computadores, Rua dos Bragas, 4099 Porto Codex, Portugal, Tel: +351-2-2009913, Fax: +351-2-319280, Email: jtm@fe.up.pt*

** *Esc. Sup. Tecnologia Viseu - Dep. Matemática, Campus Politécnico, 3510 Viseu, Portugal Tel: +351-32-4200500, Fax: +351-32-424651, Email: fduarte@graovasco.ipv.pt*

Abstract

This paper presents the fundamental aspects of the theory of the generalized inverses matrices and explores their application on the control of redundant manipulators. Redundant manipulators have some advantages when compared to classical arms because they allow a trajectory optimization, both on the free space and on the presence of obstacles, and the resolution of singularities. Nevertheless, for this type of manipulators the kinematic control adopts algorithms that use generalized inverses matrices. Consequently, the concepts associated with the control by generalized inverses are tested through several experiments that reveal the difficulties that often arise. In this perspective, it is studied the control of redundant and hyper-redundant manipulators namely through the analysis in points of singularity, showing that we may get non-optimal arm configurations.

Keywords

Kinematics, Redundant Manipulators, Pseudoinverses, Robots, Singularities.

1. INTRODUCTION

A kinematically redundant manipulator is a robotic arm possessing more axes, or degrees of freedom (*dof*), than those required to establish an arbitrary position and orientation of the end effector. Redundant manipulators offer several potential advantages over non-redundant arms. In a workspace with obstacles, the extra degrees of freedom can be used to move around or between obstacles and thereby to manipulate in situations that otherwise would be inaccessible. When a manipulator is redundant, it is anticipated that the inverse kinematics admits an infinite number of solutions. This implies that, for a given location of the manipulator's end effector, it is possible to induce a self-motion of the structure without changing the location of the gripper. Thus, the arm can be reconfigured to find better postures for an assigned set of task requirements. Of course, redundant manipulators have disadvantages too. Their structure is more complex and heavier, requiring sophisticated control algorithms that may lead to a higher computational load. A number of techniques for solving the kinematics of redundant manipulators have been suggested by researchers. Most proposed the control of the end effector motion by indirectly controlling the rates at which the individual joints are driven.

Having these ideas in mind, the paper is organized as follows. Section 2 presents an overview of the generalized matrices theory. The kinematics of redundant manipulators is developed in section 3. Section 4 presents several experiments with kinematics control schemes for redundant and hyper-redundant robots and, finally, section 5 draws the conclusions.

2. GENERALIZED INVERSES

2.1. Definitions

For $\mathbf{A} \in \mathfrak{R}^{m \times n}$ and $\mathbf{X} \in \mathfrak{R}^{n \times m}$, the following relations are used to define a generalized inverse (\mathbf{A}^-), a reflexive generalized inverse (\mathbf{A}_r^-) and a pseudoinverse ($\mathbf{A}^\#$) of \mathbf{A} :

$$\mathbf{AXA} = \mathbf{A} \quad (1)$$

$$\mathbf{XAX} = \mathbf{X} \quad (2)$$

$$(\mathbf{AX})^T = \mathbf{AX} \quad (3)$$

$$(\mathbf{XA})^T = \mathbf{XA} \quad (4)$$

Conditions (1) through (4) are called the *Penrose conditions*. A *generalized inverse* of matrix $\mathbf{A} \in \mathfrak{R}^{m \times n}$ is a matrix $\mathbf{X} = \mathbf{A}^- \in \mathfrak{R}^{n \times m}$ satisfying condition (1). On the other hand, a *reflexive generalized inverse* of matrix $\mathbf{A} \in \mathfrak{R}^{m \times n}$ is a matrix $\mathbf{X} = \mathbf{A}_r^- \in \mathfrak{R}^{n \times m}$ satisfying both conditions (1) and (2). Finally, a *pseudoinverse* of a matrix $\mathbf{A} \in \mathfrak{R}^{m \times n}$ (so-called *Moore-Penrose inverse*) is a matrix $\mathbf{X} = \mathbf{A}^\# \in \mathfrak{R}^{n \times m}$ satisfying conditions (1) through (4).

2.2. Generalized Inverses

Generalized inverses will be defined which satisfy only condition (1). The generalized inverse is not unique and, in general, if \mathbf{A}^- is a particular matrix satisfying (1), then all the generalized inverse of matrix \mathbf{A} are given by (5) where \mathbf{Y} varies over possible $n \times m$ matrices.

$$\mathbf{A}^- + \mathbf{Y} - \mathbf{A}^- \mathbf{A} \mathbf{Y} \mathbf{A} \mathbf{A}^- \quad (5)$$

Suppose that \mathbf{A} has rank r and that its rows and columns have been permuted (if necessary) to make the leading $r \times r$ submatrix non-singular. Therefore, to compute a generalized inverse of \mathbf{A} , apply row operations to the augmented matrix $\mathbf{A}_a = [\mathbf{A}, \mathbf{I}_m]$ (assuming $m \geq n$) to reduce it to the form $[\mathbf{B}, \mathbf{C}]$ where

$$\mathbf{B} = \begin{bmatrix} \mathbf{I}_r & \mathbf{B}_1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (6)$$

is $m \times n$. Then the first n rows of \mathbf{C} form a generalized inverse of \mathbf{A} . If $m < n$ the procedure can be applied to \mathbf{A}^T , leading to \mathbf{A}^- .

2.3. The Moore-Penrose Inverse

For a given $\mathbf{A} \in \mathfrak{R}^{m \times n}$ the pseudoinverse $\mathbf{A}^\# \in \mathfrak{R}^{n \times m}$ exists and is unique, whereas \mathbf{A}_r^- and \mathbf{A}^- are not necessarily unique. Let the sets of \mathbf{A}^- , \mathbf{A}_r^- and $\mathbf{A}^\#$ be S^- , S_r^- and $S^\#$, respectively. Then, the following inclusion relation holds:

$$S^\# \subset S_r^- \subset S^- \quad (7)$$

The following properties are analogous to those of the ordinary inverse:

- i) $(\mathbf{A}^\#)^\# = \mathbf{A}$
- ii) $(\mathbf{A}^T)^\# = (\mathbf{A}^\#)^T$
- iii) $\mathbf{A}^\# = (\mathbf{A}^T \mathbf{A})^\# \mathbf{A}^T = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^\#$.

For a matrix $\mathbf{A} \in \mathfrak{R}^{m \times n}$:

- i) If $m < n$ and $r(\mathbf{A}) = m$, then $\mathbf{A} \mathbf{A}^T$ is nonsingular and

$$\mathbf{A}^\# = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1} \quad (8)$$

- ii) If $m > n$ and $r(\mathbf{A}) = n$, then $\mathbf{A}^T \mathbf{A}$ is nonsingular and

$$\mathbf{A}^\# = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \quad (9)$$

- iii) If $m = n$ and $r(\mathbf{A}) = n$ then

$$\mathbf{A}^\# = (\mathbf{A})^{-1} \quad (10)$$

The matrices $\mathbf{A}^\# \mathbf{A}$, $\mathbf{A} \mathbf{A}^\#$, $\mathbf{I} - \mathbf{A}^\# \mathbf{A}$ and $\mathbf{I} - \mathbf{A} \mathbf{A}^\#$, where \mathbf{I} represents an identity matrix of appropriate dimension, are all idempotent and symmetric. If $\mathbf{A} \in \mathfrak{R}^{n \times n}$ is symmetric and idempotent then, for any matrix $\mathbf{B} \in \mathfrak{R}^{m \times n}$, the following condition holds

$$\mathbf{A} (\mathbf{B} \mathbf{A})^\# = (\mathbf{B} \mathbf{A})^\# \quad (11)$$

2.4. The Pseudoinverse of Full Rank Matrices

When the matrix $\mathbf{A} \in \mathfrak{R}^{m \times n}$ is full rank the pseudoinverse is computed using the regular inverse of a nonsingular matrix. From (8) and (9) the pseudoinverse is computed as follows:

- i) If $m < n$ and $r(\mathbf{A}) = m$ then

$$\mathbf{A}^\# = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1} \quad (12)$$

- ii) If $m > n$ and $r(\mathbf{A}) = n$ then

$$\mathbf{A}^\# = (\mathbf{A} \mathbf{A}^T)^{-1} \mathbf{A}^T \quad (13)$$

From (10) $\mathbf{A}^\# = \mathbf{A}^{-1}$, if $m = n$ and $r(\mathbf{A}) = m$. Note that (13) suggests that the pseudoinverse of a vector $\mathbf{a} \in \mathfrak{R}^n$ can be computed as

$$\mathbf{a}^\# = \frac{\mathbf{a}^T}{\|\mathbf{a}\|^2} \quad (14)$$

3. THE KINEMATICS OF REDUNDANT MANIPULATORS

We consider a manipulator with n degrees of freedom whose joint variables are denoted by q_i , $i = 1, 2, \dots, n$. We assume that a class of tasks we are interested in can be described by m variables x_j , $j = 1, 2, \dots, m$ ($m < n$) and that the relation between q_i and x_j are given by:

$$\mathbf{x} = f(\mathbf{q}) \quad (15)$$

where $\mathbf{q} = [q_1, q_2, \dots, q_n]^T$ is the joint vector, $\mathbf{x} = [x_1, x_2, \dots, x_m]^T$ is the manipulation vector. Differentiating (13) with respect to time yields

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \quad (16)$$

where $\dot{\mathbf{x}} \in \mathcal{R}^m$, $\dot{\mathbf{q}} \in \mathcal{R}^n$ and $\mathbf{J}(\mathbf{q}) = \partial f(\mathbf{q}) / \partial \mathbf{q} \in \mathcal{R}^{m \times n}$. Hence, from (14) it is possible to calculate a $\mathbf{q}(t)$ path in terms of a prescribed trajectory $\mathbf{x}(t)$. We assume that the following condition is satisfied:

$$\max \text{rank} \{ \mathbf{J}(\mathbf{q}) \} = m \quad (17)$$

Failing to satisfy this condition usually means that the selection of manipulation variables is redundant and the number of these variables m can be reduced. When condition (15) is satisfied, we say that the degree of redundancy of the manipulator is $n-m$. If, for same \mathbf{q}

$$\text{rank} \{ \mathbf{J}(\mathbf{q}) \} < m \quad (18)$$

then the manipulator is in a singular state. This state is not desirable because, in this region of the trajectory, the manipulating ability is very limited. To analyze this problem Yoshikawa (1988), proposed the expression $\mu = \sqrt{\det(\mathbf{J}\mathbf{J}^T)}$ as a measure of the manipulability at state \mathbf{q} .

3.1. Inverse Kinematics

Most of the approaches for solving redundancy that have been proposed (Klein & Huang, 1983; Lovass-Nagy, 1987; Nakamura, 1991) are based on the inversion of equation (16). A solution in terms of the joint velocities, is sought as

$$\dot{\mathbf{q}} = \mathbf{K}(\mathbf{q})\dot{\mathbf{x}} \quad (19)$$

where \mathbf{K} is a suitable ($n \times m$) control matrix based on the Jacobian matrix

$$\dot{\mathbf{q}} = \mathbf{J}^\#(\mathbf{q})\dot{\mathbf{x}} \quad (20)$$

where $\mathbf{J}^\#$ is one of the generalized inverses of the \mathbf{J} . If $\mathbf{J}^\#$ is the pseudoinverse, satisfying conditions (1) through (4), it can be easily shown that a more general solution to equation (16) is given by

$$\dot{\mathbf{q}} = \mathbf{J}^+(\mathbf{q})\dot{\mathbf{x}} + \left[\mathbf{I} - \mathbf{J}^+(\mathbf{q})\mathbf{J}(\mathbf{q}) \right] \dot{\mathbf{q}}_0 \quad (21)$$

where \mathbf{I} is $(n \times n)$ identity matrix and $\dot{\mathbf{q}}_0$ is a $(m \times 1)$ arbitrary joint velocity vector.

As can be seen, solution (21) is composed of two terms. The first term is relative to minimum norm joint velocities. The second term, the *homogeneous solution*, attempts to satisfy the additional constraints specified by $\dot{\mathbf{q}}_0$. The matrix $\mathbf{I} - \mathbf{J}^+(\mathbf{q})\mathbf{J}(\mathbf{q})$ is one of those matrices that allows the projection of $\dot{\mathbf{q}}_0$ in the null space of \mathbf{J} . A direct consequence is that it is possible to generate internal motions that reconfigure the manipulator structure without changing the gripper position and orientation (Siciliano, 1990). Another problem revealed by the solution of (20) is that repetitive trajectories in the operational space do not lead to periodic trajectories in the joint space. This is an obstacle for the solution of some problems because the resultant robot configurations have similarities with those of an unstable system.

4. APPLICATION TO REDUNDANT AND HYPER-REDUNDANT MANIPULATORS

In this section the mathematical formulation developed previously is employed for several manipulators. The direct kinematics and the Jacobian of the k -link manipulator are:

$$\mathbf{J} = \begin{bmatrix} l_1 C_1 + l_2 C_{12} + l_3 C_{123} + \dots + l_k C_{123\dots k} & -l_2 S_{12} - l_3 S_{123} \dots - l_k S_{123\dots k} & \dots & -l_k S_{123\dots k} \\ l_1 S_1 + l_2 S_{12} + l_3 S_{123} + \dots + l_k S_{123\dots k} & l_2 C_{12} + l_3 C_{123} + \dots + l_k C_{123\dots k} & \dots & +l_k C_{123\dots k} \end{bmatrix} \quad (22)$$

where $l_{i\dots k}$ are the links lengths, $S_{i\dots k} = \text{Sin}(q_1 + \dots + q_k)$ and $C_{i\dots k} = \text{Cos}(q_1 + \dots + q_k)$.

In the experiments, for all manipulators the total length of the links is $l_1 + l_2 + \dots + l_k = 3$.

The joint positions can be computed through the time integration of the velocities according with the block diagram of the inverse kinematics algorithm depicted in Figure 1.

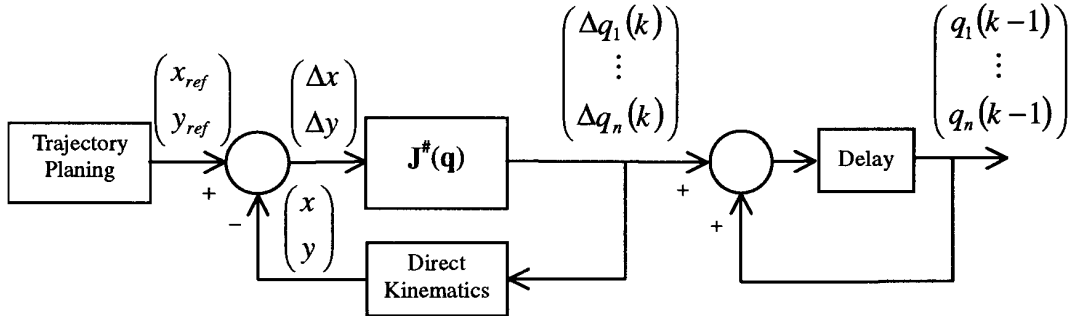


Figure 1: Block diagram of the inverse kinematics algorithm with generalized inverses

Based on this algorithm, in the next sub-sections we analyze its performances on the trajectory control of different types of manipulators. In this line of thought, we study the resulting joint trajectories for the 2R (non-redundant), 3R (redundant) and both the 4R and 5R (hyper-redundant) robots when subjected to a repetitive trajectory in the operational space according with the expression:

$$\mathbf{x}(t) = \begin{bmatrix} 0.5[1 - \cos(\pi t)] \\ 0.5[2 + \sin(\pi t)] \end{bmatrix}, \quad 0 \leq t \leq 30 \quad (23)$$

For all the experiments the initial end effector location is $\mathbf{x}(0) = [0 \quad 0.5]^T$.

4.1. Non-Redundant Manipulator

Consider the 2R robot with an initial posture $\mathbf{q}(0) = [2.8\pi \quad -2.46\pi]^T$. The results of Figure 2 show the joint positions for the inverse kinematics algorithm based on (20) using the ‘standard’ 2×2 Jacobian matrix.

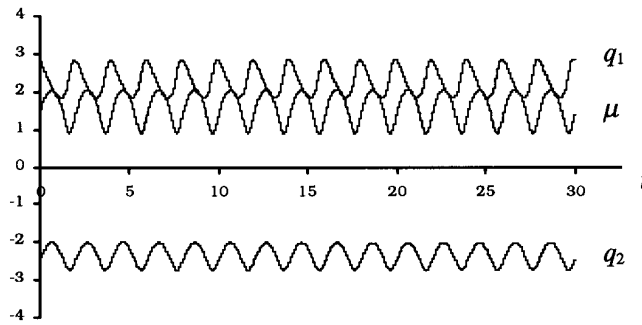


Figure 2: 2R-robot joint positions *versus* time using the ‘standard’ Jacobian matrix

As it is well known, in this case the joint trajectories and the manipulability index are repetitive along the xy circular motion or, by other words, there is a unique joint trajectory satisfying the prescribed cartesian path.

4.2. Redundant Manipulator

In this experiment we adopt the 3R arm with an initial posture $\mathbf{q}(0) = [\pi \quad -\pi/2 \quad -\pi/2]^T$. Figure 3 depicts the joint positions when adopting the generalized inverse matrix $\mathbf{J}^\# \equiv \mathbf{J}^-$ in the kinematic algorithm (20).

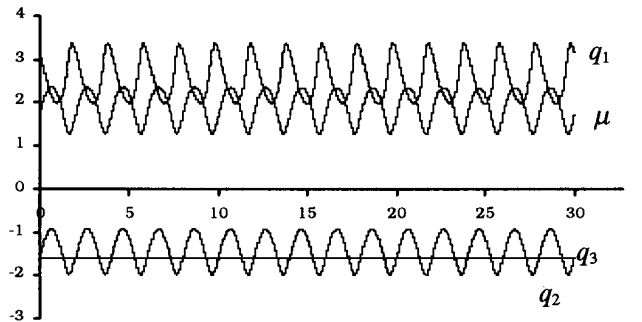


Figure 3: 3R-robot joint positions *versus* time using the generalized inverse $\mathbf{J}^\# \equiv \mathbf{J}^-$

In an alternative experiment, Figure 4 shows the joint positions for the inverse kinematic algorithm (20) using the pseudoinverse matrix $\mathbf{J}^\# \equiv \mathbf{J}^+$.

With these two experiments we have different results. When using $\mathbf{J}^\# \equiv \mathbf{J}^-$ the 3R-robot behaves like the non-redundant 2R-manipulator, that is, with joint 3 maintaining a constant (*i.e.* the initial) value. Therefore, we do not take advantage of the redundancy in order to optimize the performance and in the sequel this method is not further studied. In the second experiment, when adopting $\mathbf{J}^\# \equiv \mathbf{J}^+$, we use all the three joints but the performance is non-optimal. In fact, the joint trajectories exhibit sudden changes, which impose large joint velocity values, and, furthermore, they are not repetitive leading to a kind of chaotic/unstable performance.

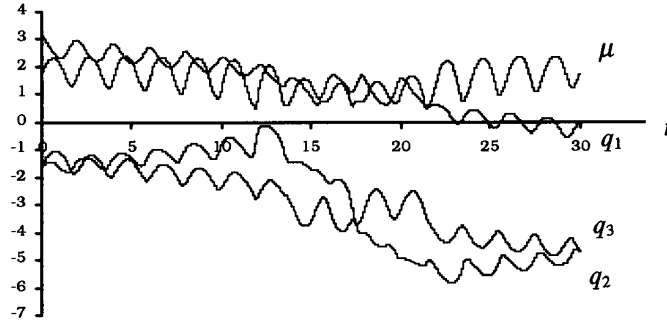


Figure 4: 3R-robot joint positions *versus* time using the pseudoinverse matrix $\mathbf{J}^\# \equiv \mathbf{J}^+$

4.3. Hyper-Redundant Manipulators

In this sub-section we consider the 4R and 5R hyper-redundant manipulators. In both cases the inverse kinematics algorithm based on (20) adopts the pseudoinverse matrix $\mathbf{J}^\# \equiv \mathbf{J}^+$.

The initial postures are $\mathbf{q}(0) = [3.05\pi \quad -0.87\pi \quad -1.29\pi \quad -1.24\pi]^T$ and $\mathbf{q}(0) = [-2.7\pi \quad -1.08\pi \quad -0.82\pi \quad -\pi/3 \quad -0.73\pi]^T$, respectively.

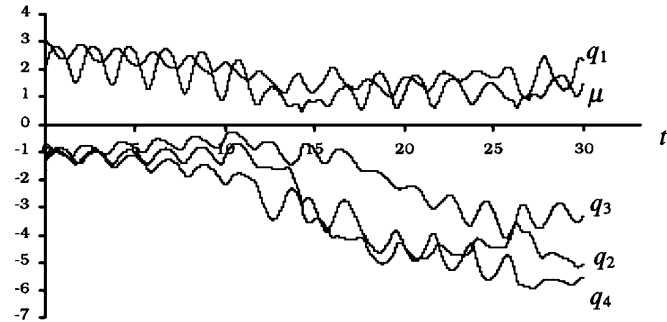


Figure 5: 4R-robot joint positions *versus* time using the pseudoinverse matrix $\mathbf{J}^\# \equiv \mathbf{J}^+$

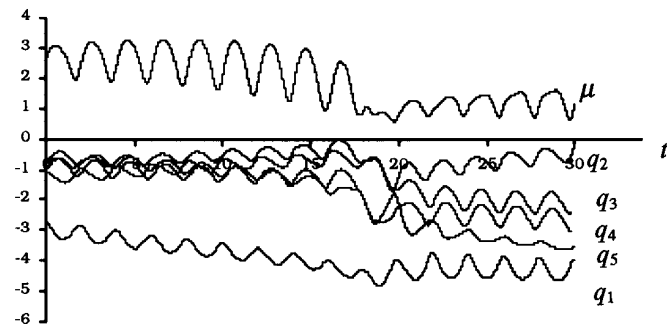


Figure 6: 5R-robot joint positions *versus* time using the pseudoinverse matrix $\mathbf{J}^\# \equiv \mathbf{J}^+$

In both cases we observe a non-optimal performance similar to the 3R-robot experiment. Nevertheless, the manipulability index and the peak of joint velocities reveal better performances the higher the number of *dof*. Therefore, the adoption of hyper-redundant structures deserves to be further investigated.

5. CONCLUSIONS

This paper presented the main aspects of the theory of the generalized inverses and its application to the control of redundant manipulators. In this line of thought, several methods were analyzed for the calculation of generalized inverse and pseudoinverses in order to solve the inverse kinematics. Stemming from the mathematical formulation, these techniques were applied to the trajectory control of redundant and hyper-redundant manipulators revealing that such algorithms may lead to non-optimal performances both from the manipulability and repeatability viewpoints. Therefore, alternative algorithms with superior performances must be further investigated.

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