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APPLICATION OF FRACTIONAL ALGORITHMS IN CONTROL OF A QUAD ROTOR FLIGHT

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Abstract: This paper studies the application of fractional algorithms in the control of a quad-rotor rotorcraft machine. The main contribution of this paper focuses in the development a flight simulator to provide the evaluation model of the quad-rotor. Several basic maneuvers are investigated, namely the elevation and the position control.

Keywords: Rotorcraft flight, mathematical model, position control, fractional control, nonlinear.

1. Introduction

A full-scale four-rotor helicopter was built by *De Bothezat* in 1921. This idea of using four rotors is not new. Rotary wing aerial vehicles have distinct advantages over conventional fixed wing aircrafts on surveillance and inspection tasks, since they can take-off land in limited spaces and easily fly above the target (Barnes and McCormick *et al*., 1995), Gordon Leishman *et al*., 1995. A quad-rotor is a four rotor helicopter, and are example is shown in Figure 1. Helicopters are dynamically unstable and, therefore; suitable control methods is needed to make them stable (Etkin, B. and Reid L. R. *et al*., 1996). Although an unstable dynamics is not desirable, it is good in the viewpoint of agility (Singh, and Schy *et al*. 1978), (Romero, Benosman, Lozano *et al*. 2006). The instability comes from the changing of the helicopter parameters and from the disturbances such as the wind (Salazar, Palomino, Lozano et al.2005, Drouin, Ramos, Camino et al. 2006). A quad–rotor helicopter is controlled by varying the rotors speed, thereby changing the lift forces. It is an under-actuated dynamic vehicle with four input forces and six outputs coordinates (Asep, Shen, Achaíbou and Camino et al. 1993, Ghosh, and Tomlin, et al. 2000). One of the advantages of using a multi-rotor helicopter is the increased payload capacity. It has more lift and before, heavier weights can be carried (Hoffmann, Rajnarayan,

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Waslander, Dostal, Jang, and Tomlin *et al*. 2004). The quad–rotors are highly manoeuvrable, which enables vertical take-off/landing, as well as flying into hard to reach areas, but the disadvantages are the increased helicopter weight and increased energy consumption due to the extra motors (Salazar, Palomino, Lozano *et al*. 2005), (Castillo, Dzul, Lozano, *et al*. 2003, 2004, 2005a, 2005b). Since the machine it is controlled with rotor-speed changes, it is more suitable to adapt electric motors. Large helicopter engines, that which a have slow response, may not be satisfactory without, incorporating a proper gear-box system (Castillo, Lozano, Garcia, Albertos *et al*. 2005). The main contribution of this study concerns the use of non-linear control techniques to stabilize and to perform output tracking control of the helicopter.

1.1. *Helicopter Model*

Unlike regular helicopters, that have variable pitch angles, a quad rotor has fixed pitch angle rotors and the rotor speeds are controlled in order to produce the desired lift forces. Basic motions of a quad rotor can be described using Figure 1. In the first method, the vertical motion of the helicopter can be achieved by changing all of the rotor speeds at the same time. Motion along the *x-axis* is related to tilt around the *y-axis*.

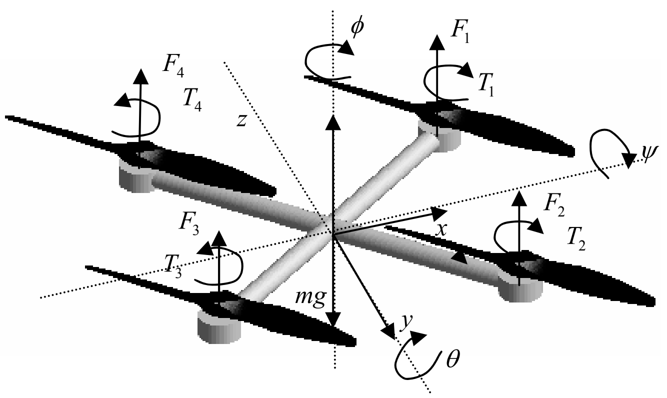

Fig. 1. The quad-rotor helicopter.

This tilt can be obtained by decreasing the speeds of rotors 1 and 2 and by increasing speeds of rotors 3 and 4. This tilt also produces acceleration along the *x-axis*. Similarly y-motion is the result of the tilt around the *x-axis*.

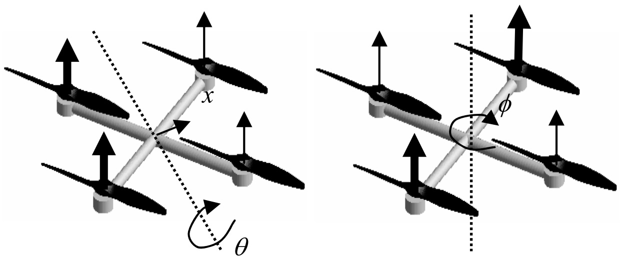

Fig. 2. The first tilting method.

It was tested another tilting method which consists in decreasing only the speed of rotor 1 and increasing his opposite rotor speed, the rotor 3 (for example) , however this method led to less power to actuate the tilting motions, making the platform more difficult to control.

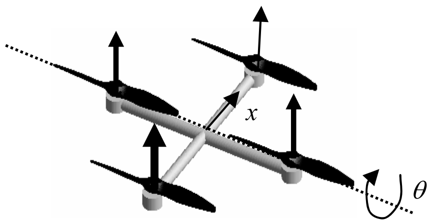

Fig. 3. The second tilting method.

The yaw motions are obtained using the moments that are created as the rotors spin. Conventional helicopters have the tail rotor in order to balance the moments created by the main rotor. With the four-rotor machine, spinning directions of the rotor are set to balance and to cancel these moments. This is also used to produce the desired yaw motions.

To turn in a clock-wise direction, the speed of rotor's 2 and 4 should be increased to overcome the moments created by rotors 1 and 3. A good controller should be able to reach a desired yaw angle while keeping the tilt angles and the height constant. A body fixed frame is assumed to be at the center of gravity of the quad-rotor, where the *z-axis* is pointing upwards. This body axis is related to the inertial frame by a position vector (x, y, z) and three Euler angles, (θ, ψ, ϕ), representing pitch, roll and yaw, respectively.

A ZYX - Euler angle representation given in Equation 1, has been chosen for the representation of the rotations.

$$R = \begin{pmatrix} c_\varphi c_\theta & c_\varphi s_\theta s_\psi - s_\varphi c_\psi & c_\varphi s_\theta c_\psi + s_\varphi s_\psi \\ s_\varphi c_\theta & s_\varphi s_\theta s_\psi - c_\varphi c_\psi & s_\varphi s_\theta c_\psi + s_\varphi s_\psi \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi \end{pmatrix} \tag{1}$$

Where c_θ and s_θ represent $\cos(\theta)$ and $\sin(\theta)$ respectively.

Each rotor produces moments as well as vertical forces. These moments were observed experimentally to be linearly dependent on the forces at low speeds. There are four input forces and six output states (*x, y, z,* θ, ψ, ϕ) and, therefore the quad-rotor is an under-actuated system. The rotation direction of two of the rotors are clockwise while the other two are counter clockwise, in order to balance the moments and to produce yaw motions as needed.
The equations of motion can be written using the force and moment balance, yielding:

$$\ddot{x} = \frac{\left(\sum_{i=1}^{4} F_i\right)\left(c_\phi s_\theta c_\psi + s_\phi s_\psi\right) - K_1\dot{x}}{m} \tag{2}$$

$$\ddot{y} = \frac{\left(\sum_{i=1}^{4} F_i\right)\left(s_\phi s_\theta c\psi + c_\phi s_\psi\right) - K_2\dot{y}}{m} \tag{3}$$

$$\ddot{z} = \frac{\left(\sum_{i=1}^{4} F_i\right)\left(c_\phi c_\psi\right) - mg - K_3\dot{z}}{m} \tag{4}$$

$$\ddot{\theta} = \frac{l\left(-F_1 - F_2 + F_3 + F_4 - K_4\dot{\theta}\right)}{J_1} \tag{5}$$

$$\ddot{\psi} = l\frac{\left(-F_1 + F_2 + F_3 - F_4 - K_5\dot{\psi}\right)}{J_2} \tag{6}$$

$$\ddot{\phi} = \frac{l\left(-M_1 + M_2 + M_3 - M_4 - K_6\dot{\phi}\right)}{J_3} \tag{7}$$

The factors Ki (i = 1, 2, …, 6) given above are the drag coefficients. In the following we assume the drag is zero, since drag is negligible at low speeds. By convenience, we will define the inputs to be:

$$u_1 = \frac{F_1 + F_2 + F_3 + F_4}{m} \tag{8}$$

$$u_2 = \frac{-F_1 - F_2 + F_3 + F_4}{J_1} \tag{9}$$

$$u_3 = \frac{-F_1 + F_2 + F_3 - F_4}{J_2} \tag{10}$$

$$u_4 = C\frac{(F_1 - F_2 + F_3 - F_4)}{J_3} \tag{11}$$

where J1, J2 and J3 are the moment of inertia with respect to the axes and C is the force-to-moment scaling factor. The variables u_1 represents a total thrust on the body in the *z-axis*, u_2 and u_3 are the pitch and roll inputs and u_4 is a yawing moment. Therefore, the equations of motion become:

$$\ddot{x} = u_1\left(c_\phi s_\theta c_\psi + s_\phi s_\psi\right) \tag{12}$$

$$\ddot{y} = u_1\left(s_\phi s_\theta c_\psi + c_\phi s_\psi\right) \tag{13}$$

$$\ddot{z} = u_1\left(c_\theta c_\psi\right) - g \tag{14}$$

$$\ddot{\theta} = u_2 l \tag{15}$$

$$\ddot{\psi} = u_3 l \tag{16}$$

$$\ddot{\phi} = u_4 \tag{17}$$

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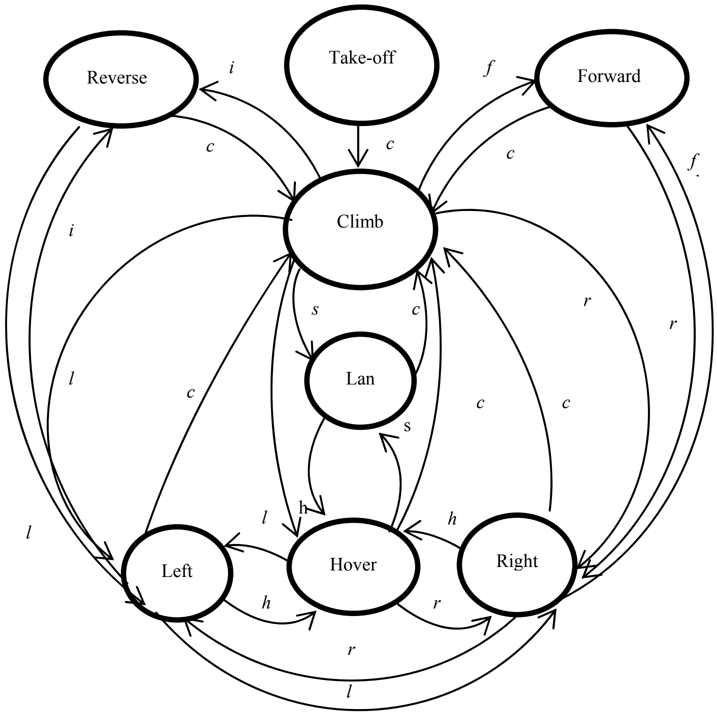

Fig. 4. The model of the quad-rotor helicopter.

Table. 1. The state of the quad-rotor helicopter.

State	***Input force***	***Forces***
Climb	*c*	$F_1=F_2=F_3=F_4>F_{hover}$
Reverse	*i*	$F_1=F_3>F_2=F_4$
Left	*l*	$F_1=F_2>F_3=F_4$
Forward	*f*	$F_1=F_3<F_2=F_4$
Land	*s*	$F_1=F_2=F_3=F_4<F_{hover}$
Hover	*h*	$F_1=F_2=F_3=F_4=F_{hover}$
Right	*r*	$F_1=F_2<F_3=F_4$

Figure 4 shows state diagram of the quad-rotor model.
Table 1 shows the attitude of the forces produced by the four rotors, for each state of figure 4.

3. *Fractional Control*

In this section we present the Fractional Order algorithms inserted at the position loops.
The mathematical definition of a derivative of fractional order α has been the subject of several different approaches (Podlubny et al. 1999, Miller and Ross et al. 1993, Oustaloup et al. 1991and Machado et al. 2001). For example, we can mention the Laplace and the Grünwald-Letnikov definitions:

$$D^{\alpha}[x(t)] = L^{-1}\{s^{\alpha} X(s)\} \tag{18}$$

$$D^{\alpha}[x(t)] = \lim_{h \to 0}\left[\frac{1}{h^{\alpha}}\sum_{k=1}^{\infty}\frac{(-1)^{k}\,\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}x(t-kh)\right] \tag{19}$$

where Γ is the gamma function and h is the time increment.

In our case, for implementing *FO* algorithms of the type:

$$C(s) = K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{T_i s} + K_{\mathrm{D}}\, s^{\alpha}, -1 < \alpha < 1 \tag{20}$$

we adopt a 4^{th}-order discrete-time Pade approximation (a_i, b_i, c_i, $d_i \in \Re$, $k = 4$):

$$C_{\mathrm{Ph}}(z) \approx K_{\mathrm{Pi}}\frac{a_0 z^{k} + a_1 z^{k-1} + .. + a_k}{b_0 z^{k} + b_1 z^{k-1} + ... + b_k} \tag{21}$$

where K_{Pi} are the position gains, respectively.

Table. 2. The parameters of the controllers.

i	K_p	K_d	α
1 - Pitch Control	15	25	0.95
2 - Roll Control	15	25	0.95
3 - Yaw Control	30	40	0.95
4 - X Control	10	57	0.95
5 - Y Control	10	57	0.95
6 - Z Control	25	102	0.95

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Table 2 shows the PD^{α} tuning parameters implemented on the attitude and position controllers (N. M. Fonseca Ferreira et al. 2007 and 2008).

4. *The Flight Simulator*

We developed a flight simulator (Fig.5.) to provide a test bed for evaluating models. The simulator is written in Matlab and all the model parameters are stored in files.

When using the simulator, the first thing that is obvious is how difficult it is to get the simulated quad-rotor helicopter to stop rising or falling. To get it to hover at one height you have to adjust the throttle until both velocity and acceleration in the z direction are zero.

In a first phase we consider the vertical motion of the helicopter starting in {x, y, z} ≡ {0, 0, 10} [m] to {x, y, z} ≡ {0, 0, 20} [m]. In a second phase we consider the horizontal motion of the helicopter starting in {x, y, z} ≡ {0, 0, 10} [m] up to {x, y, z} ≡ {10, 0, 10} [m]. In a third phase we consider a circular trajectory centered at {x, y, z} ≡ 0, 0, 10} [m] with a 5 meter radius. The time responses show us that the quad-rotor it is very complex due to the several couplings effects caused by the several propellers drag moments. Nevertheless it reveals an high maneuverability, which enables a quick vertical take-off and landing.

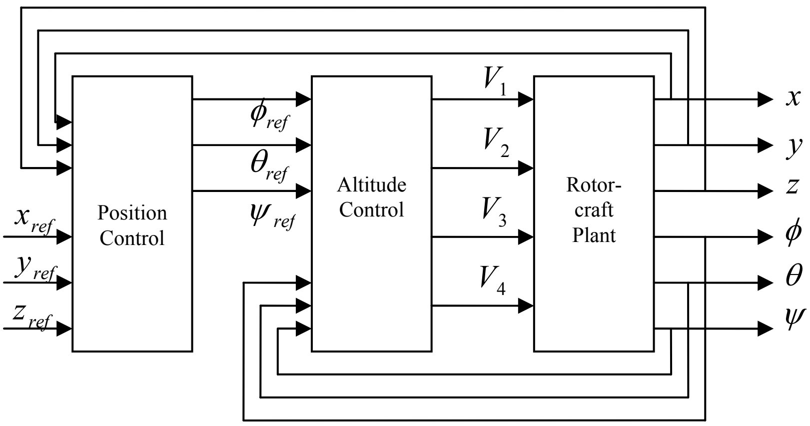

Fig. 5. The control diagram of the quad-rotor *helicopter*.

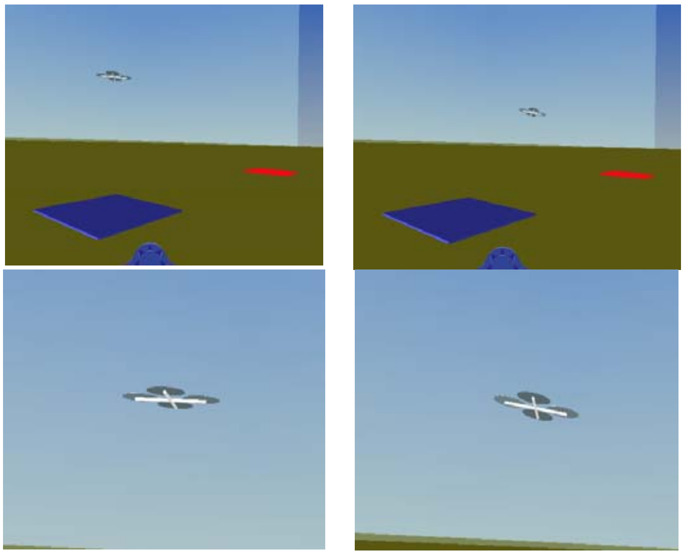

Fig. 6.The animation of the quad-rotor *helicopter.*

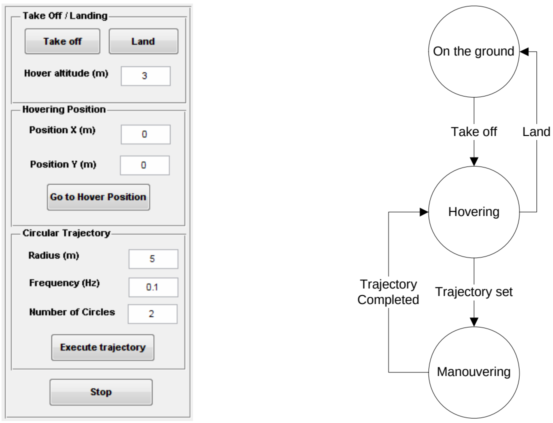

Fig. 7. The flight simulator of the quad-rotor *helicopter.*

Figure 7 show the high level commands and interface of the flight simulator, were we can set the parameters of pre-programmed trajectories, such as a circular trajectory centered at hovering position. Figures 10 and 12 show that the applied forces for the lifting are not the same for each motor due to the controller corrections required of the controllers to keep the quad-rotor near the desired references in order to compensate the coupling effects

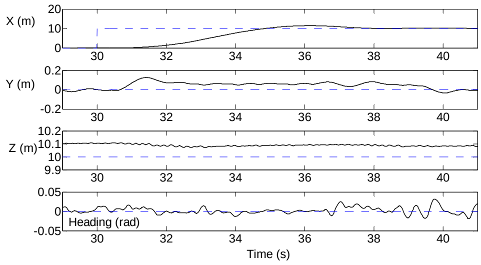

Fig. 8. Time response of the quad-rotor's position, considering horizontal motion in the X axis of the helicopter.

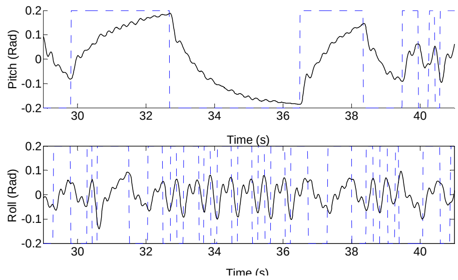

Fig. 9. Time response for attitude of the quad-rotor, considering horizontal motion in the *x* axis of the helicopter.

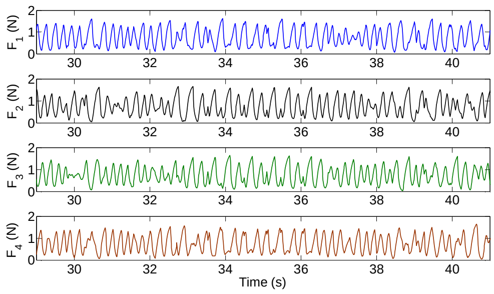

Fig. 10. Time response of the forces applied in the quad-rotor for the horizontal motion in the *x* axis of the helicopter.

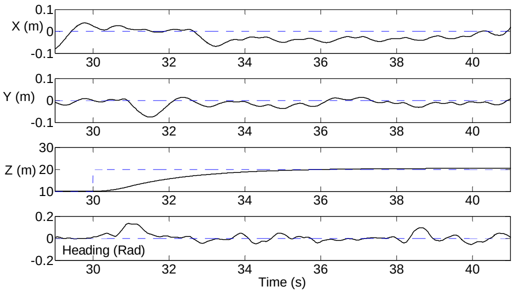

Fig. 11. Time response of the quad-rotor's position, considering vertical motion in the z axis of the helicopter.

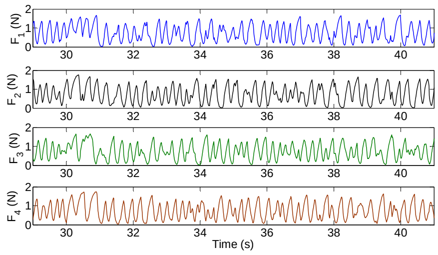

Fig. 12. Time response of the forces applied in the quad-rotor for the vertical motion in the z axis of the helicopter.

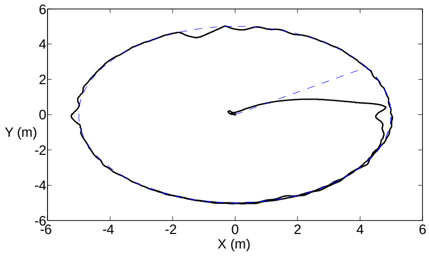

Fig. 13. Time response for a circular trajectory centered at $\{x, y, z\} \equiv \{0, 0, 10\}$ [m] with a 5 meter radius in the x-y plane.

5. *Conclusions*

Our analysis has shown that the quad-rotor helicopter is a complex system. In this analysis we have developed a model of the quad-rotor and we tested some basic maneuvers. We have explored the resulting forces and moments applied to the vehicle and through these, investigated their impact on elevation and position control.

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