Wilfried Elmenreich, J. Tenreiro Machado and Imre J. Rudas (Eds)

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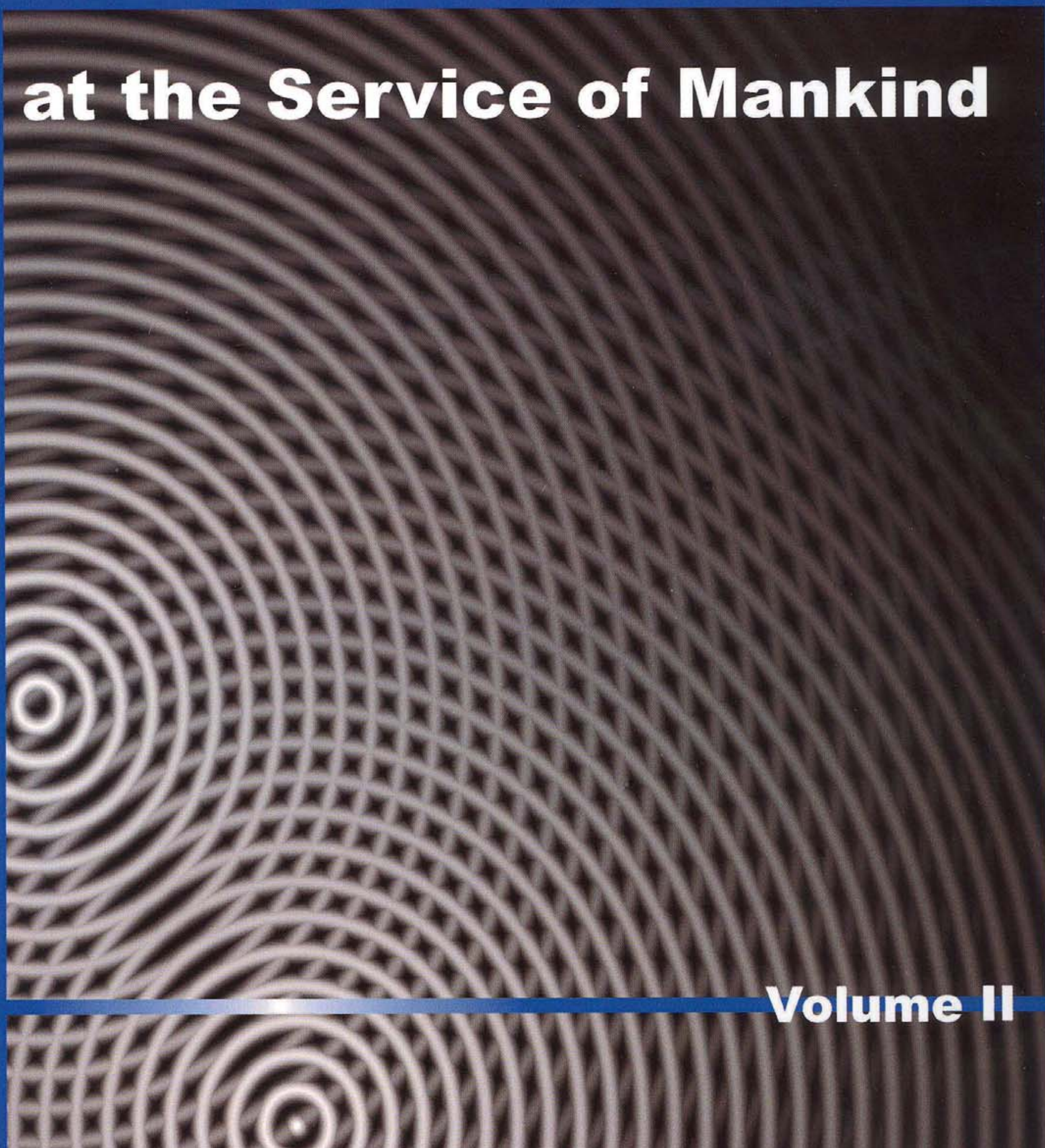

Volume II

Simple Kinematic Design for Evading the Forced Oscillation of a Car-Wheel Suspension System

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Abstract — *An adaptive control damping the forced vibration of a car while passing along a bumpy road is investigated. It is based on a simple kinematic description of the desired behavior of the damped system. A modified PID controller containing an approximation of Caputo's fractional derivative suppresses the high-frequency components related to the bumps and dips, while the low frequency part of passing hills/valleys are strictly traced. Neither a complete dynamic model of the car nor 'a priori' information on the surface of the road is needed. The adaptive control realizes this kinematic design in spite of the existence of dynamically coupled, excitable internal degrees of freedom. The method is investigated via Scicos-based simulation in the case of a paradigm. It was found that both adaptivity and fractional order derivatives are essential parts of the control that can keep the vibration of the load at bay without directly controlling its motion.*

1 Introduction

Externally excited vibration normally is an undesired phenomenon that occurs in various physical systems, therefore its efficient damping is of great practical significance. In the case of actively controlled solutions these tasks have the "delicate" nature that the controller cannot be provided with the "exact" model of the system and the forced oscillation to be reduced, and/or with complete information on its actual physical state. A novel adaptive branch of soft computing was developed to solve such problems [1, 2]. It was shown that in the case of a wide class of physical systems this method can result in quite robust adaptive control for very inaccurately and partially modeled physical systems that can have even unmodeled internal degrees of freedom [3]. As an input the method requires the desired trajectory of the generalized coordinates of the subsystem that has to directly be controlled.

In the case of Classical Mechanical Systems this approach can be useful if the desired trajectory of the generalized coordinates can be prescribed with respect to an inertial system of reference. However, in the most cases the system considered is a part of a moving object that does not serve as a basis of an inertial frame, as, e.g., a car proceeding along a bumpy road passing hills and valleys. In this case some slow motion along certain average distance between the chassis and the wheel can be prescribed for the controller because it is locally measurable quantity. The desired behavior of this distance can practically be prescribed to some extent by the terms used in the traditional linear controllers as frequency filters etc. The most plausible means would be the application of a simple PID type controller to keep the error at bay. The small integrating term of this controller can be used for eliminating small, constant trajectory tracking errors. For the compensation of "abrupt" changes in the tracking error the proportional and the derivative terms are responsible. Due to these terms the vibration of the wheel also is transmitted to the chassis.

As generalizations of the concept of the integer order derivatives, fractional order derivatives were introduced. The problem of designing fractional order control systems within the frames of linear control obtained considerable attention recently, e.g. [4]. The French expression invented by Oustaloup *"CRONE: Commande Robuste d'Ordre Non Entier"* [5] became a "trademark" hallmarking a well-elaborated design methodology that obtained application in vibration control [6]. Understanding and application of this method requires deep engineering knowledge in the realm of linear systems, frequency spectrum analysis, the use of Laplace transforms and complex integrals, various typical diagrams as, e.g., the Nichols plot.

However, tackling the problem from a more general nonlinear and adaptive basis may require less amount of profound and specific engineering knowledge. The aim of the present paper is the detailed investigation of an alternative approach that was roughly outlined and approximately investigated in [7]. For this purpose, the frequency-filtering nature of the fractional order system is considered in a more general view.

Regarding the basic idea of the novel adaptive control, we refer to our paper entitled "Improved Numerical Simulation for a Novel Adaptive Control Using Fractional Order Derivatives" [Chapter 2: "The Control Problem in General"]. In similar manner, Chapter 4 entitled "The Fractional Order Derivatives" contains all the "obligatory survey" parts on the fractional order derivatives that are necessary for convenient readability. It also describes in details the particular numerical approximation of Caputo's fractional order derivative that is also used in the present paper. Consequently, in this paper we restrict ourselves to the particular phenomenology of the car plus payload system, and to the presentation of the simulation results that were obtained by the scientific co-simulator program of INRIA's SCILAB called Scicos that we applied here to replace the preliminary Scilab programs on which the conclusions of [7] were based. In our mentioned excerpt in this book, Scicos is also introduced briefly.

2 The dynamic model of the car-payload system

The model of the system considered is described in Figure 1. The mass of the wheel was supposed to be negligible with respect to that of the chassis of mass $M_c = 100$ kg the model value of which was supposed to be 150 *kg*. (This value cannot exactly be known *a priori* since one or more than one traveller of even 100 *kg* weight each can sit in the car.) The passive suspension system consisted of a spring of stiffness $k = 2 \times 10^4$ *N/m* and viscosity of $v = 1 \times 10^4$ *N/(m/s)*. The payload of the car was supposed to be comprised in a box holding it via a spring of stiffness $k_2 = 3 \times 10^4$ *N/m* and variable viscosity v_2 *N/(m/s)* between the "ceiling" and the "floor" of the box. Its mass $M_p = 160$ *kg* was also unknown by the controller. For fixing it in the case of very drastic vibration the floor and the ceiling are covered by very stiff elastic bumpers modeled by sharp, conservative, very stiff potential functions. The force of the active suspension F^a was supposed to be generated according to the control law. The coordinates x_c and x_w in *m* units describe the height of the chassis and the wheel, respectively, with respect to an inertial frame, i.e. with respect to the sea level, so they are not available as direct data for the controller. The nominal height of the chassis while rigidly following the wheel was prescribed to be $x_{cnom} = x_w + L_0$ with $L_0 = 0.5$ *m*. In the case of loose trajectory tracking little humps and dips need not be traced by the chassis, but while climbing a higher hill or deeper valley it must be traced with limited error.

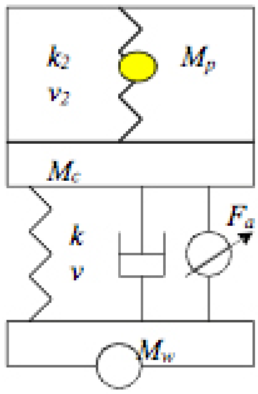

Figure 1: The rough model of the suspension system

However, the error of his trajectory tracking is available via local measurements within the car as

$$e = x_c - x_{cnorm} = (x_c - x_w) - L_0 \tag{1}$$

The 1$^{\text{st}}$ order time-derivative of the error can be numerically estimated by finite element methods. Because x_w and x_c are measured with respect to an inertial frame their second traditional time-derivatives also are measurable even by the use of micro-sensors developed on a chip. Therefore the desired acceleration of the chassis can be written as:

$$\ddot{x}_c{}^d = \frac{1}{\Gamma(1-\beta)} \int_0^t \ddot{x}_w(\tau)(t-\tau)^{-\beta} d\tau - Pe - \frac{D}{\Gamma(1-\beta)} \int_0^t \dot{e}(\tau)(t-\tau)^{-\beta} d\tau - I \int_0^t \dot{e}(\tau) d\tau \tag{2}$$

in which, according to Caputo, the first integral corresponds to the $(1+\beta)^{\text{th}}$ derivative, while the 2$^{\text{nd}}$ one represents the β^{th} order of derivation. The long and slowly decreasing tail of the function $(t-\tau)^\beta$ is expected to behave as a frequency filter to suppress the feedback for high frequency components and strengthen it for low frequencies. For the numerical approximation of the integrals with singular integrands the following formula can be used:

$$\frac{d^{\beta}}{dt^{\beta}}u(t) \cong \frac{u'(t)\delta^{-\beta+1}}{\Gamma(2-\beta)} + \sum_{0<s \text{ while } s\delta<T} \frac{\delta^{-\beta+1}\left[(s+1)^{-\beta+1} - s^{-\beta+1}\right]}{\Gamma(2-\beta)} u'(t-s\delta) \quad (3)$$

In this form the full interval of the integration of length *T s* is divided into small ones of length *sδ s*, during which the reintegrated derivative is supposed to be approximately constant. This numerical approximation of the fractional order derivatives therefore has three free parameters: β means the order of derivation, *T* means the "length of the memory" of this operation, and *δ* describes the time-resolution in this case. Eq. (2) represents a tracing requirement expressed by the use of *purely kinematic terms*. The main expectation behind it is the supposition that for small proportional coefficient *P* some loose tracking can be achieved the accuracy of which is increased by the "filtered" integrals at low frequency (that is for hill climbing), while for the higher frequency components occurring when small dips are passed it remains loose. By the use of the approximate dynamic model of the system the ap propriate active force can be estimated.

Due to the approximate nature of the dynamic model exertion of this force will not result in the desired acceleration of the chassis. For the realization of (2) adaptive control is needed. In this case we simply utilized the standard scheme developed for a 3 degree of freedom system (as in our other paper in this book already referred to) by writing zeros into the non-existing degrees of freedom in the case of the *desired acceleration of the chassis* as a data *controlled by the controller*:

$$[\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3, \mathbf{m}_4, \mathbf{m}_5] = \begin{bmatrix} \ddot{x}_c^d & -\ddot{x}_c^d & e_1^{(3)} & e_1^{(4)} & e_1^{(5)} \\ 0 & 0 & e_2^{(3)} & e_2^{(4)} & e_2^{(5)} \\ 0 & 0 & e_3^{(3)} & e_3^{(4)} & e_3^{(5)} \\ d & -d & e_4^{(3)} & e_4^{(4)} & e_4^{(5)} \\ D & \frac{\left(\ddot{x}_c^d\right)+d^2}{D} & e_5^{(3)} & e_5^{(4)} & e_5^{(5)} \end{bmatrix} \quad (4)$$

The *observed* acceleration was written into a similar scheme. Of course (4) does not have room for the generalized coordinate of the payload that is "unmodeled" by the controller. In the sequel various simulation results are presented.

3 Simulation results

In the forthcoming simulations $\beta = 0.05$ and $T = 30 \times \delta$ were chosen. The maximal time step of the numerical integration was limited to be 0.0001 *s* even if the estimated absolute and relative errors of the integrations remained under the prescribed limits 0.0001 and 0.00001, respectively.

Figure 2 was calculated for a very viscous payload-fixing system of $\nu_2 = 5\times10^4$ [*Ns/m*]. It reveals that $\delta = 6$ *ms* (that in the same time is the cycle time of the external adaptive loop of the controller) is too clumsy and results in bad tracking of the kinematically prescribed trajectory tracking. Step-by-step decrease in its values eventually lead to $\delta = 2$ *ms* that seems to result in accurate, acceptable tracking. As it can be seen in Figure 3, the strong

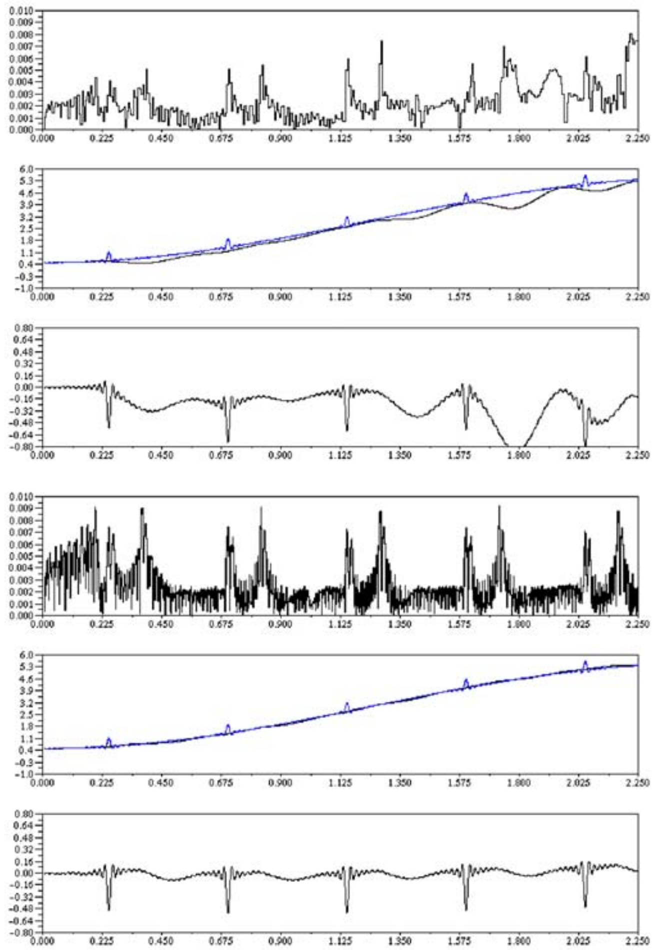

Figure 2: Adaptive active fractional order vibration control for $\nu_2 = 5 \times 10^4$ [*Ns/m*]: 1st box: the norm of the **S-I** matrices; 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [*m*] (upper line); 3rd box: tracking error $(x_c - L_0 - x_w)$ [*m*] vs. time [*s*]: In the upper set $\delta = 6$[*ms*], in the lower set $\delta = 2$ [*ms*]

viscous forces practically rigidly fixed the payload making the chassis rather similar to a rigid body. However, fast adaptivity considerably reduced the excitation of this coupled, unmodeled degree of freedom. It is also interesting to see the effect of adaptivity besides the non-integer derivatives based control. In Figure 5 the non-adaptive counterpart of the results presented in Figure 4 are given. Systematic decrease in the viscosity of fixing the payload leads to considerable variation in the adaptive signal while the trajectory tracking properties of the adaptive control remain good as it described in Figure 4 belonging to $\nu_2 = 2 \times 10^3$ [*Ns/m*]. As it can well be seen, decreased viscosity leads to a considerable increase in the displacement of the payload, and as a consequence, the elastic spring force trying to fix it. Increased stretch or compression of this spring means higher potential energy in it, i.e., the energy exchange between the modeled and the unmodeled subsystems considerably increased, too.

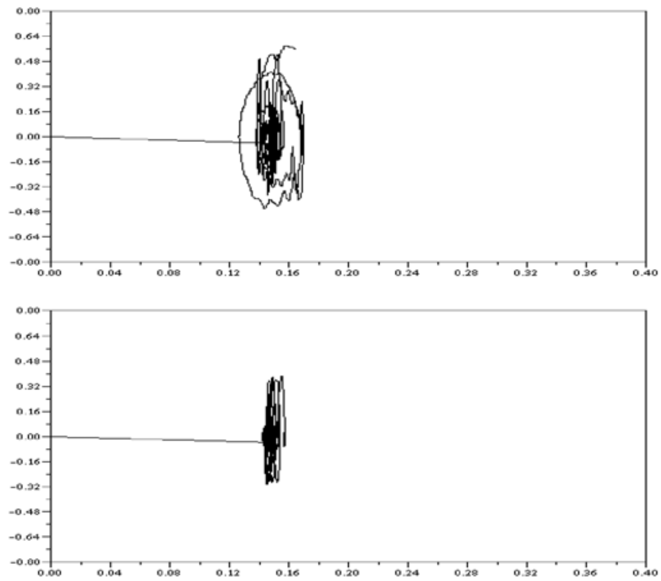

Figure 3: The phase space of the payload-floor distance in the case of $\delta = 6$ [*ms*] (upper box), and $\delta = 2$ [*ms*] (lower box) [*m, m/s*].

Within a short time-interval the difference in trajectory tracking does not seem to be significant. However, for longer time a characteristic drift can be observed: the tracking error slowly increases. The phase space of the payload is also considerably modified. Though the amplitude of the oscillation of the payload has not been increased, the shape of the phase trajectory well reveals the differences. Typical upper and lower regions can be revealed within which the velocity of the payload's motion is decelerated and commences to alter its direction. Though this alteration is not completely abrupt, it is quite significant.

Similar tendencies can be revealed if the viscosity of the payload fixing further is decreased to $\nu_2 = 4 \times 10^2$ [*Ns/m*]. In Figure 6 the adaptive and the non-adaptive control's outputs are described. It can well be observed that the adaptive signal varies in a wider interval than in the case of greater viscosity values. This reveals the more drastic effect of dynamic coupling the effect of which needs compensation by the controller. Furthermore, the essential role of adaptivity in preventing the drift in tracking error again is well revealed.

The increased absolute values of the tracking errors of the appropriate figures represent that the active suspension system is able to expand or shrink in order to avoid transmission of the oscillation of the road surface to the car. Figure 7 reveals the appropriate active forces that are needed to complete the contribution of the passive suspension in order to achieve the necessary kinematic design (this signal corresponds to the component of high frequency and great amplitude). The other line corresponds to the model mass of the chassis multiplied by the desired acceleration of the chassis minus the gravitational acceleration. In the present approximation in which the mass of the wheel is neglected this corresponds to an overall suspension force. If this value is negative, this means the car has to be pulled to the road which cannot be achieved with a common wheel system.

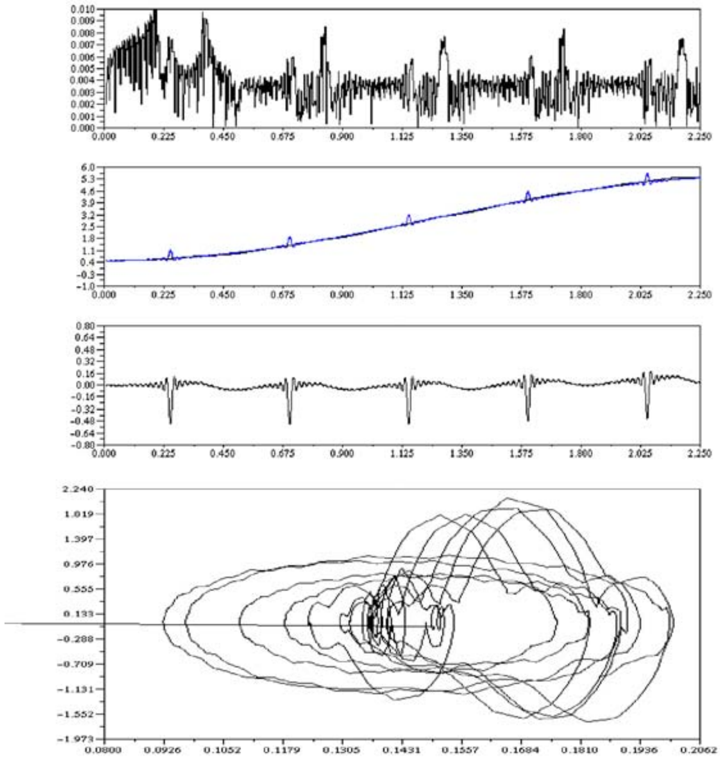

Figure 4: Adaptive active fractional order vibration control for $\nu_2 = 2 \times 10^3$ [*Ns/m*]: 1st box: the norm of the **S-I** matrices; 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [*m*] (upper line); 3rd box: tracking error $(x_c - L_0 - x_w)$ [*m*] vs. time [*s*]; The phase space of the payload-floor distance [*ms*].

However, in practice sometimes it may occur that the car flies, that is leaves the road. This certainly corresponds to ”extreme” conditions.

4 Conclusions

Regarding the model of the road considered, in the simulation examples the car was supposed to move with a velocity of 10 *m/s* (36 *km/h*) while climbing a hill covered by a bumpy road as given in the figures presenting the results. The bumps/dips were modeled by a Fourier series containing $\omega = 1, 2, \dots, 314\ s^1$ circular frequencies with equal weights. For the highest frequency component this corresponds to $T_{min} = 2 \times 10^{-2}\ s$ period (frequency of 50 Hz), which, at 10 *m/s* velocity means a combination of pairs of 10 *cm* wide dips and bumps that roughly corresponds to a road built up of granite blocks of this size. The lower frequency components represent an uneven surface of dips/bumps of higher characteristic size. These frequencies roughly correspond to the calamitous situation prevailing at the old Thököly Street of Budapest that waits for reconstruction for a long time. However, the superposition of these frequency components, periodically repeated from zero to the end of a finite time-interval, results in rather extreme conditions in which our approach was investigated. It was found that the active suspension system is

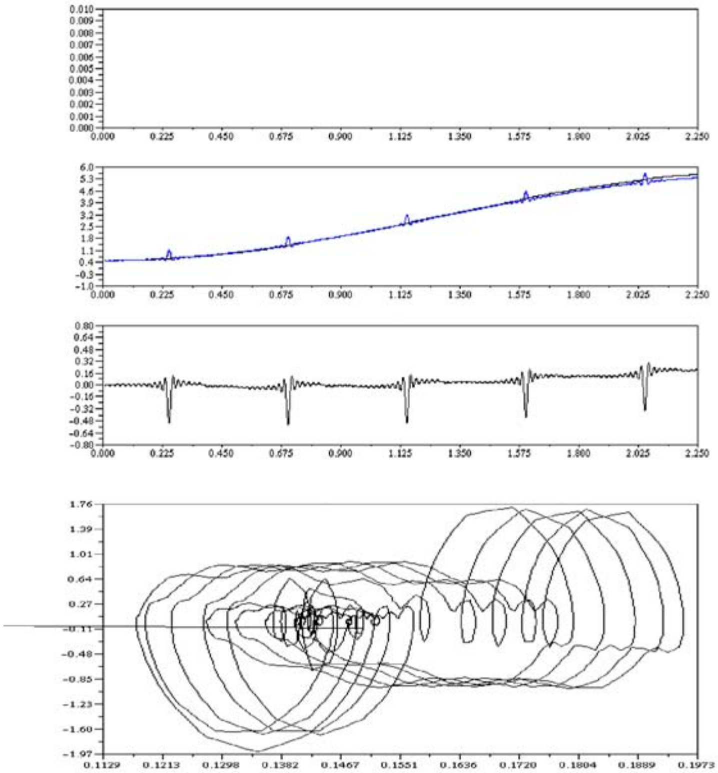

Figure 5: Non-adaptive active fractional order vibration control for $\nu_2 = 2 \times 10^3$ [*Ns/m*]: 1st box: the norm of the **S-I** matrices (in this case this signal is necessarily identical to zero); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [*m*] (upper line); 3rd box: tracking error $(x_c - L_0 - x_w)$ [*m*] vs. time [*s*]; The phase space of the payload-floor distance [*ms*].

able to expand or shrink in order to avoid transmission of the oscillation of the road surface to the chassis of the car. The fractional order design well filters the high-frequency components while the adaptive loop well compensates the effects of the rather rough modeling uncertainties including the existence of dynamically coupled subsystems not at all modeled by the controller.

In this approach no particular suppositions are needed for the nature of the vibration that assumptions used to be typical in the traditional control literature, e.g. that vibration can be treated with low order Taylor series expansion around some equilibrium position, or that the suspension system and the external excitation have characteristic eigenfrequencies or peaks in their Fourier spectra for the absorption of which the eigenfrequency of the damping medium has to be properly tuned.

While the basic idea seems to be good and useful, for further research it seems to be necessary to consider the problem of leaving the road, i.e., flying of the car. This normally occurs due to two definite reasons: the combination of the extremely bad quality of the road (that is the existence of very sharp dips) combined with high speed and relatively stiff prescribed PID tracking rules that can result in an acceleration that is greater that can

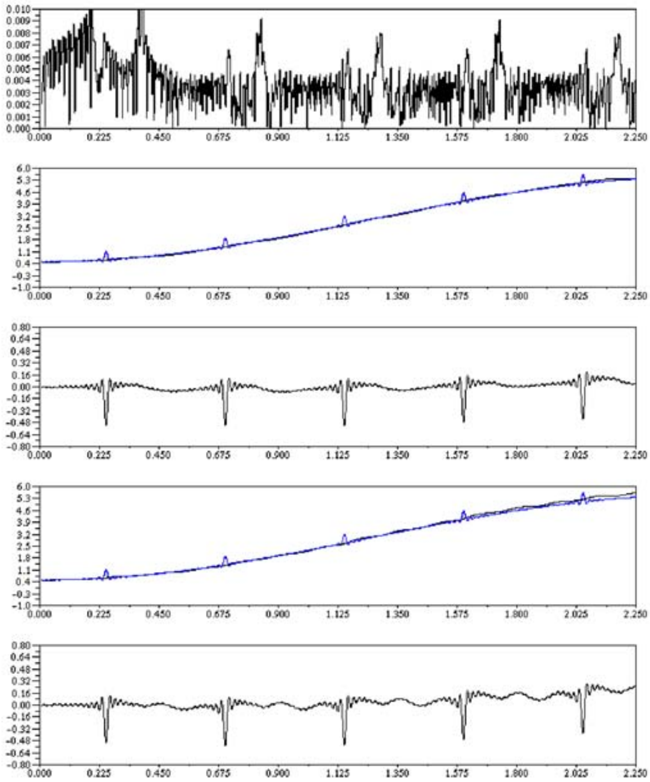

Figure 6: Adaptive (upper set) and non-adaptive (lower set) active fractional order vibration control for $\nu_2 = 4 \times 10^2$ [*Ns/m*]: 1st box: the norm of the **S**-**I** matrices (missing from the non-adaptive results); 2nd box: the position of the wheel (lower line) and that of the chassis minus L_0 [*m*] (upper line); 3rd box: tracking error $(x_c - L_0 - x_w)$ [*m*] vs. time [*s*].

be produced by the gravitation. For this purpose the modification of the road model, that of the original PID parameters to be softened by the fractional order derivation, the order of the derivations applied may simultaneously be needed.

Acknowledgments

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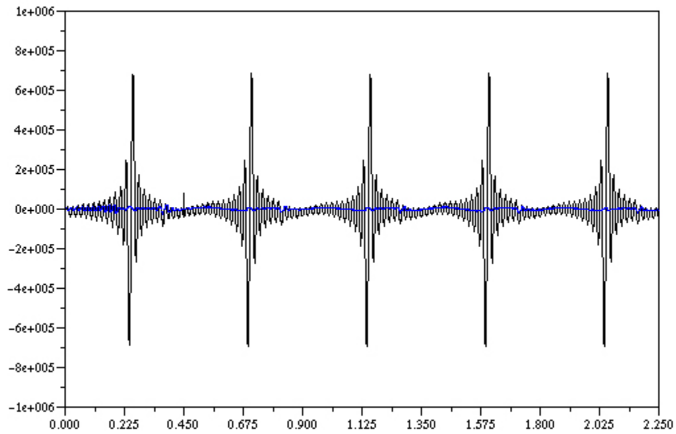

Figure 7: Adaptive (upper set) and non-adaptive (lower set) active fractional order vibration control for $\nu_2 = 4 \times 10^2$ [*Ns/m*]: 1st box: the norm of the **S-I** matrices (missing from the non-adaptive results); 2nd box: the position of the wheel (lower line) and that of the chassis minus L0 [m] (upper line); 3rd box: tracking error $(x_c - L_0 - x_w)$ [*m*] vs. time [*s*].

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