

OPTIMAL CONTRACTS ALLOCATION USING MEAN VARIANCE OPTIMIZATION METHOD

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Abstract – The process of restructuration and liberalization of power systems are a constant all over the world. However, those processes, due to the specific characteristics of the “product” electricity, create uncertainty and new risks that did not exist when power systems were vertically integrated. Those changes origin the necessity of tools that allow the participants of the electricity markets to practice the hedge against the volatility of the System Marginal Price. In that sense, we present in this paper a decision-support application, based on a Mean Variance Optimization Method trying to give a response to the necessities of the electricity markets participants. The results show that the proposed method can be useful to producers and also to others participants of electricity markets like Brokers and Load Serving Entities (LSE).

Keywords: *Risk Management, Hedge, Electricity Markets, Contracts, Decision-Support System.*

I. INTRODUCTION

The power systems have suffered in the last decades profound changes in the way that they were operated. Those changes were caused by a restructuration process that includes a liberalization process. In countries, where these processes have taken place, the power systems, until then vertically integrated, were unbundled, leading to the complete separation of the several activities. In the new structure, only the transmission and the distribution activities maintain the natural monopoly status.

In part motivated by the airlines and telecommunications liberalization experiences, the liberalization and the correspondent restructuration processes of the electric sector did not have the same success due to the specific characteristics of the product “electricity”. To negotiate the energy was created a spot market, managed by the Market Operator (MO), where the Generators and the Load Serving Entities sell or buy, respectively, the energy on an hour or half-hour basis. The supply and demand bids are then aggregated creating the supply and demand curve, which sets the quantity of electric power to be traded and the equilibrium price designated by System Marginal Price (SMP). However, due to the demand characteristics and producer’s characteristics, that are described in [1], the SMP is very volatile, being difficult to the agents to predict that price. This characteristic leads the agents of those mar-

kets to search for hedging tools that help them to turn their results more predictable. Responding to that necessity, derivatives markets were created, in mature markets, which negotiate contracts with underlying active the electric energy. Those markets allow the electric energy market participants to practice the hedge against the volatility of the spot market and simultaneously allow them to eliminate the risk of credit and to turn the market more liquid.

Derivatives markets negotiate forward, futures and options contracts. The main difference between forward and futures contract is that forward contracts comprise the physical delivery of the electric energy amount negotiated and futures contracts are exclusively of financial type. The options contracts are similar to forward and futures. The options exercise could be of physical or financial type. When the option comprises the physical delivery, they are normally associated to forward contracts. The main difference that distinguishes options from forward or futures contracts is that they allow the buyer to exercise or not exercise the option but for that he has to pay previously a certain amount of money designated by premium.

To deal with those types of contracts, the participants of the electric markets need decision support tools that help them to decide which contracts to establish for a certain programming period, considering a certain objective function.

So, the present paper presents a decision support system that helps market agents (Generators, Load Serving Entities or Brokers) to decide what type of contracts to establish for a certain programming period. This decision support system also allows that entities to take advantage of the arbitrage opportunities.

II. PROBLEM FORMULATION

The problem related with the optimal allocation of contracts for the producers in a liberalized market is a very complex problem and has a very high importance. For a certain programming period, the producer has to decide which energy amount he should produce, the forms of the contracts to establish and the quantity of energy associated in order to satisfy the energy previously negotiated with a certain delivery price. For production excess, the producer has to decide which energy he should sell, and in what forms.

In the optimization method presented in this paper it was admitted that, all energy negotiated previously by the producer has the same delivery price and that the producer reserves a margin of production equal to a certain percentage θ of the maximum capacity plus the energy negotiated in the options that comprise the supply of energy that he will use to satisfy the energy previously negotiated. This margin of production is very important because it allows the producer to produce the additional energy if the options aren't exercised or if the incremental costs to produce that energy are higher than the SMP. The options positions that have to be considered in the margin of production are the long call and the short put.

The model developed in this paper is based on maximization of the Mean Variance of the profit (π) for a certain programming period i . The expected value of the return is calculated for a set of scenarios S and the variance is calculated for a set of scenarios T , where $T \subset S$. This assumption allows the producer to eliminate the risk associated to a wrong scenarios prediction.

A. Production to Satisfy Contracts Previously Negotiated

To satisfy the contracts previously established in earlier periods, the producer could produce that energy. In this case, it was considered that the delivery price of the energy previously negotiated was constant.

The revenue from the supply of the energy produced to satisfy the energy previously negotiated is given by

$$r_{ij}^P = e_i^P \times k_i \quad (1)$$

where,

- r_{ij}^P represents the revenue, in €, of the energy produced to satisfy the energy previously negotiated for the period i and scenario j
- e_i^P represents the energy produced, in MWh, to satisfy the energy that the producer had to supply and that was previously negotiated for the programming period i
- k_i represents the delivery price, in €/MWh, established for supply the energy amount e_i^P .

B. Spot Market

The producer could negotiate energy in the spot market to satisfy the energy previously negotiated and to sell the production excess energy capacity.

The purchase revenue for a certain amount of energy in the spot market used to satisfy the contracts previously established is given by

$$r_{ij}^{LS} = (k_i - SMP_{ij}) \times e_i^{LS} \quad (2)$$

where,

- r_{ij}^{LS} represents the revenue, in €, of the long position assumed in the spot market to satisfy the energy previously negotiated for the period i and scenario j
- e_i^{LS} represents the energy amount, in MWh, that the producer buy in the spot market to satisfy the contracts previously established for the period i
- k_i has the same meaning expressed in equation (1)
- SMP_{ij} represents the System Marginal Price (SMP), in €/MWh, for the period i and scenario j

The spot position revenue from the selling of the capacity of production that has not been used, is given by,

$$r_{ij}^{SS} = SMP_{ij} \times e_i^{SS} \quad (3)$$

where,

- r_{ij}^{SS} represents the revenue, in €, of the short position assumed in the spot market to sell the capacity of production don't used for the programming period i and scenario j
- e_i^{SS} represents the energy amount, in MWh, that the producer sell in the spot market for the programming period i
- SMP_{ij} has the same meaning expressed in equation (2)

C. Forward Contracts

In order to satisfy the energy previously negotiated, the producer could buy forward contracts with delivery date coincident with the programming period i , with a certain delivery price. The revenue of the long position assumed in forward contracts is given by

$$r_{ij}^{LF} = (k_i - \psi_i) \times e_i^{LF} \quad (4)$$

where,

- r_{ij}^{LF} represents the revenue, in €, of the long position assumed in the forward contract in order to satisfy the energy previously negotiated for the programming period i and scenario j
- e_i^{LF} represents the energy amount, in MWh, that

the producer buy in the forward contract for the programming period i

k_i has the same meaning expressed in equation (1)

ψ_i represents the delivery price, in €/MWh, of the forward contract established for the period i

The revenue of the contracts that the producer will establish in order to sell the excess energy capacity of production is given by

$$r_{ij}^{SF} = (k_i - \psi_i) \times e_i^{SF} \quad (5)$$

where,

r_{ij}^{SF} represents the revenue, in €, of the short position assumed in the forward contract in order to sell the excess energy capacity of production for the programming period i and scenario j

e_i^{SF} represents the energy amount, in MWh, that the producer buy in the forward contract for the programming period i

k_i has the same meaning expressed in equation (1)

ψ_i has the same meaning expressed in equation (4)

D. Options Contracts

When dealing with options it is necessary to distinguish two situations.

- Options positions assumed by the producer, and that he has to supply energy;
- Options positions assumed by the producer, and that permit him to sell energy.

The options that allow the supply of the energy for producer satisfy the energy previously negotiated. The positions in options contracts that permit that are: the long call and the short put. However, like was said previously, the options allow the buyer to not exercise those contracts. So, this characteristic turns the manipulation of the options very complex and with non-linear characteristics.

In this paper, we considered that the producer can take advantage of arbitrage opportunities. For each scenario, it is necessary to evaluate what is the best solution. So, for the long call and for the short put positions, it has to be evaluated, for each scenario j , if the option will be exercised, if the buyer should not exercise the option and buy the energy negotiated in that option in the spot market and if the producer should produce that energy.

The revenue of the long call options is given by

$$r_{ij}^{LC} = \max \begin{bmatrix} (k_i - \alpha_i^{LC} - p_i^{LC}) \times e_i^{LC} \\ (k_i - p_i^{LC}) \times e_i^{LC} - \partial C_i^{LC} \\ (k_i - SMP_{ij} - p_i^{LC}) \times e_i^{LC} \end{bmatrix} \quad (6)$$

where,

r_{ij}^{LC} represent the revenue, in €, for the scenario j of the assumed long call position in order to satisfy the energy previously negotiated the programming period i

k_i has the same meaning expressed in equation (1)

α_i^{LC} represents the exercise price, in €/MWh, of the long call option for the programming period i

p_i^{LC} represents the premium, in €/MWh, of the long call option for the programming period i

SMP_{ij} has the same meaning expressed in equation (2)

∂C_i^{LC} represents the impact, in €, at the costs of production, for the programming period i , if the producer decide to produce the energy negotiated in the long call option

e_i^{LC} represents the energy amount, in MWh, that the producer negotiate in the long call option with delivery date coincident with the programming period i .

For the short put option position it was considered that the put option buyer will exercise the option if the spot price is lower than the exercise price. If the spot price is higher than the exercise price, the producer has to decide if he should produce that energy or if he should buy it in the spot market.

The revenue of the short put options is given by,

$$r_{ij}^{SP} = \begin{cases} (k_i - \alpha_i^{SP} + p_i^{SP}) \times e_i^{SP} & \text{if } SMP_{ij} \leq \alpha_i^{SP} \\ \max \begin{bmatrix} (k_i + p_i^{SP}) \times e_i^{SP} - \partial C_i^{SP} \\ (k_i - SMP_{ij} + p_i^{SP}) \times e_i^{SP} \end{bmatrix} & \text{if } SMP_{ij} > \alpha_i^{SP} \end{cases} \quad (7)$$

where,

r_{ij}^{SP} represents the revenue, in €, for the scenario j , of the short put position assumed in order to satisfy the energy previously negotiated with delivery date coincident with the programming period i

k_i has the same meaning expressed in equation (1)

- α_i^{SP} represents the exercise price, in €/MWh, of the short put option for the programming period i
 p_i^{SP} represents the premium, in €/MWh, of the short put option for the programming period i
 SMP_{ij} has the same meaning expressed in equation (2)
 ∂C_i^{SP} Represents the impact, in €, at the costs of production, for the programming period i, if the producer decide to produce negotiated the energy negotiated
 e_i^{SP} Represents the energy amount, in MWh, that the producer negotiate in the short put option with delivery date coincident with the programming period i.

The producer could also negotiate the excess energy capacity in options contracts. The options positions that allow the producer to sell that energy are short call and long put. Like in previous positions, it is necessary to evaluate if the option should be exercised, if the producer should produce that energy and sell it in the spot market or if he should not do anything.

The revenue of the short call options is given by

$$r_{ij}^{SC} = \begin{cases} (\alpha_i^{SC} + p_i^{SC}) \times e_i^{SC} & \text{if } SMP_{ij} \geq \alpha_i^{SC} \\ \max \left[\begin{array}{l} p_i^{SC} \times e_i^{SC} + \partial C_i^{SC} \\ (SMP_{ij} + p_i^{SC}) \times e_i^{SC} \end{array} \right] & \text{if } SMP_{ij} < \alpha_i^{SC} \end{cases} \quad (8)$$

where,

- r_{ij}^{SC} Represents the revenue, in €, for the scenario j of the short call position assumed in order to sell the excess capacity of production for the programming period i
 α_i^{SC} Represents the exercise price, in €/MWh, of the short call option for the programming period i
 p_i^{SC} Represents the premium, in €/MWh, of the short call option for the programming period i
 SMP_{ij} has the same meaning expressed in equation (2)
 ∂C_i^{SC} Represents the impact, in €, at the costs of production, for the programming period i, if the producer decide to produce
 e_i^{SC} Represents the energy amount, in MWh, that the producer negotiate in the short call option with delivery date coincident with the programming period i.

The result of the short long put is given by,

$$r_{ij}^{LP} = \max \left[\begin{array}{l} (\alpha_i^{LP} - p_i^{LP}) \times e_i^{LP} \\ \partial C_i^{LC} - p_i^{LC} \times e_i^{LC} \\ (SMP_{ij} - p_i^{LP}) \times e_i^{LP} \end{array} \right] \quad (9)$$

where,

- r_{ij}^{LP} represents the revenue, in €, for the scenario j of the long put position assumed in order to sell the excess capacity of production for the programming period i
 α_i^{LP} represents the exercise price, in €/MWh, of the long put option for the programming period i
 p_i^{LP} represents the premium, in €/MWh, of the long put option for the programming period i
 SMP_{ij} has the same meaning expressed in equation (2)
 ∂C_i^{LP} represents the impact, in €, at the costs of production, for the programming period i, if the producer decide to produce
 e_i^{LP} represents the energy amount, in MWh, that the producer negotiate in the long put option with delivery date coincident with the programming period i

The options premium was calculated using the procedure known as *risk-neutral* valuation described in [2].

E. Optimization Problem

The optimization problem formulation that pretend to maximize a Mean Variance function is given by

$$\max_i U_i = E_i(\pi) - \frac{\delta}{2} \times Var_i(\pi) \quad (10)$$

$$s.t. \quad e_{ig}^{\min} \leq e_{ig} \leq e_{ig}^{\max} \quad (11)$$

$$e_i^P + e_i^{LS} + e_i^{LF} + e_i^{LC} + e_i^{SP} = E_i \quad (12)$$

$$e_i^{SS} + e_i^{SF} + e_i^{SC} + e_i^{LP} \leq e_{ig}^{\max} \times (1 - \theta) - (e_{ij}^{LC} + e_{ij}^{LP}) \quad (13)$$

$$e_{ig} = e_i^P + e_i^{SS} + e_i^{SF} + e_i^{SC} + e_i^{LP} \quad (14)$$

$$E_i(\pi) \geq 0 \quad (15)$$

where,

$$\pi = r_{ij}^P + r_{ij}^{SS} + r_{ij}^{LS} + r_{ij}^{SF} + r_{ij}^{LF} + r_{ij}^{SC} + r_{ij}^{LC} + r_{ij}^{SP} + r_{ij}^{LP} - C(e_{ig}) \quad (16)$$

and,

- π represents the profit, in €, for period i
- $E_i(\pi)$ represents the expected profit, in €, for the period I and is calculated for the scenarios $S_i = \{s_{i1}, \dots, s_{in}\}$ with probability $p_{is} = \{p_{is1}, \dots, p_{isn}\}$, for the programming period i
- $Var_i(\pi)$ represents the variance, in €, of the profit for the period i . These variance is calculated for a set of scenarios $T_i = \{t_{i1}, \dots, t_{im}\}$ which are not associated probabilities and $T \subset S$
- δ represents the aversion risk factor of the producer
- e_{ig} represents the energy generated, in MWh, for the period i
- e_{ig}^{min} represents the minimum energy, in MWh, that generators can produce
- e_{ig}^{max} represents the maximum energy, in MWh, that generators can produce
- θ represents the security reserve, in %, of the e_{ig}^{max} for the generator
- $C(e_{ig})$ represents the production costs, in €, of the energy e_{ig}

III. STUDY CASE

In this example, the goal is to pretend to calculate the optimal quantity of energy to produce and to buy in spot, forward or options contracts to satisfy the quantity of energy previously negotiated for a certain programming period i . We also want to calculate the optimal excess energy capacity to sell in spot, forward or options contracts for the same programming period i . It was been admitted that period i has the characteristics presented in Table I.

Table I- Characteristics of the programming period i

Duration (h)	0,5
Energy previously negotiated (MWh)	40
SMP scenario 1 (€/MWh; probability)	(26; 0,6)
SMP scenario 2 (€/MWh; probability)	(23; 0,4)

The characteristics of the options contracts negotiated, with delivery date coincident with the programming period i , are presented in Table II.

Table II- Characteristics of options contracts for period i

	Quantity/Contract (MWh)	Exercise Price (€/MWh)	Premium (€/MWh)
Long Call	2,40	24,00	2,00

Short Put	2,67	23,43	0,43
Short Call	2,11	25,20	0,80
Long Put	1,98	26,32	3,32

The characteristics of the forward contracts negotiated with delivery date coincident with the programming period i , are presented in Table III.

Table III- Characteristics of forward contracts for period i

Delivery Price (€/MWh)	26,00
Quantity/Contract (MWh)	1,12

The production cost function considered is equal to,

$$C(P_g) = 20 + 2 \times P_g + 0,1 \times P_g^2$$

with P_g in MW, C in €, $P_g^{min} = 100$ MW and $P_g^{max} = 5$ MW

It was admitted that the producer previously negotiated the supply of 40 MWh at the fixed price of 20 €/MWh. The risk aversion factor (δ) used was equal to one, the security reserve (θ) used was 5% and the scenario interval $T = [21, \dots, 36]$ (€/MWh) was considered.

A. Results

The results for the considered study case are presented in Tables IV and V.

Table IV- Optimal Energy to produce and to contract to satisfy the energy previously negotiated

Self Production (MWh)	3,845
Long Spot Position (MWh)	0,675
Long Forward Position (N.º of Contracts)	20
Long Call Position (N.º of Contracts)	1
Short Put Position (N.º of Contracts)	4

Table V- Optimal energy quantity of the excess capacity of production to negotiate

Short Spot Position (MWh)	0,527
Short Forward Position (N.º of Contracts)	13
Short Call Position (N.º of Contracts)	0
Long Put Position (N.º of Contracts)	0

The graphic of Figure 1 present the optimal allocation contracts to supply the energy previously negotiated and

the excess energy capacity of production that the producer should sell.

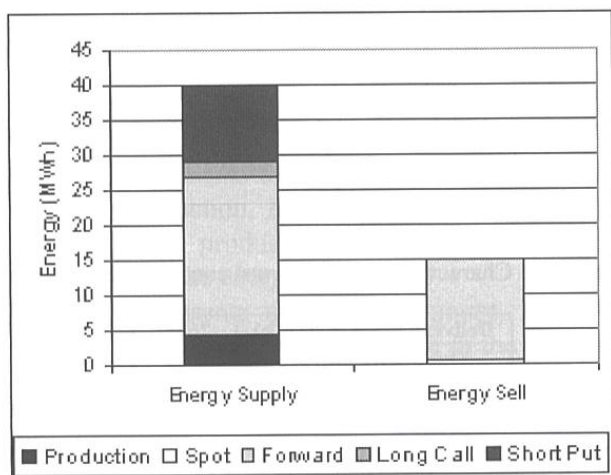


Figure 1- Optimal contracts allocation

Figure 2 presents the producer profit for the considered scenarios interval $T=[21, \dots, 36]$.

Analyzing the same picture we can see that the producer profit is stable for the T interval.

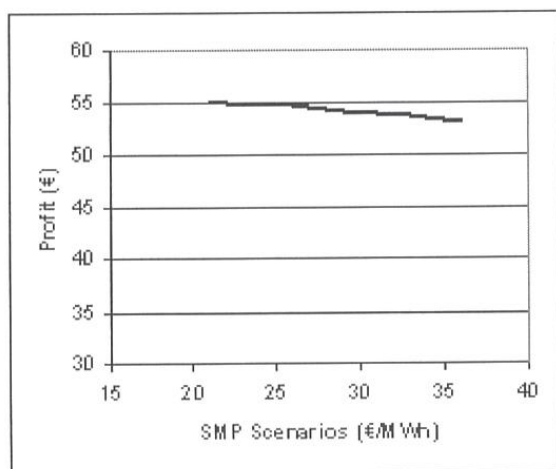


Figure 2- Producer profit for the interval $T=[21, \dots, 36]$

To achieve the optimal solution we use the following criteria. When, after approximately 1000 trials, the solutions obtained do not change, we increment the mutation rate with the hope to find better solutions than the obtained so far. Also, to obtain the better solution was realized several simulations and, from those simulations, was obtained the best solution, here designated by optimal.

IV. CONCLUSIONS

All over the world, the restructuring and liberalization processes of the electric sector, which culminate with the creation of electricity markets, have been very popular. However, those markets are not like the traditional markets due to the specific characteristics of the "product" negotiated – the electric energy. One of the characteristics of the electricity markets that take more

concerns to their participants is the volatility of the System Marginal Price (SMP). In order to turn those markets more liquid and to allow their participants to practice the hedge, in mature markets, were created derivatives markets, which introduce a set of tools (contracts) that allow the electric participants to protect themselves against the volatility of the spot market.

In this work we present a Mean Variance optimization method that allows the participants of electricity markets and in particular the producers, to practice the hedge against the volatility of the Market Clearing Price, using forward and options contracts.

Given the difficulties that the participants of the electricity markets must deal, with decision-support system here presented, can be useful to assist them in finding the optimal portfolio allocation and to manage, in a simple way, the risk associated to the volatility of the SMP.

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