NONLINEAR DYNAMICS, CHAOS, CONTROL, AND THEIR APPLICATIONS TO ENGINEERING SCIENCES

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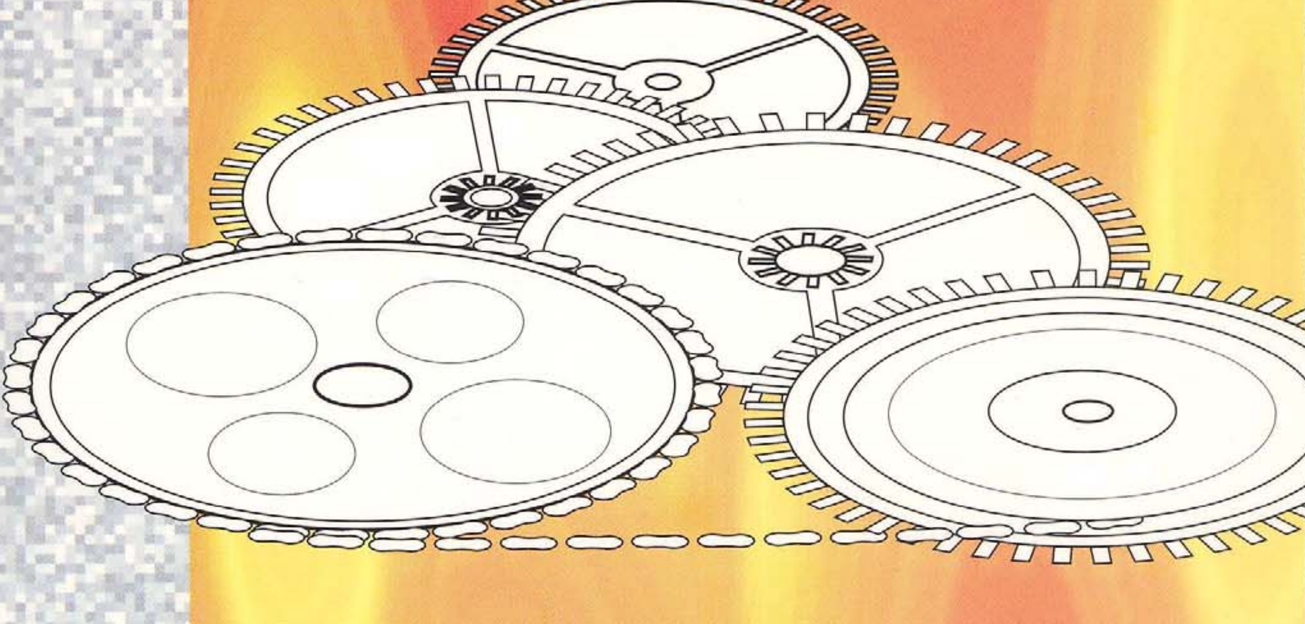

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Dynamic Simulation of Nonrigid Link Manipulators

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ABSTRACT

This paper deals with the dynamic simulation of a flexible link robot manipulator. The mechanical structure is composed of flexible links, where each one is composed of several rigid segments connected through rotational joints. These segments are considered actives or passives. In the passive segments, deflection in one or two planes as well as torsion may be present. These deformations are simulated by the passive joints. The present conception leads to a inverse dynamic system algorithm with a Newton-Euler recursive numerical formulation, similar to rigid systems. With the adequate manipulation of the input vectors, it is possible to obtain the mass matrix and the vectors that consider the gravitational, Coriolis and centripetal effects. In this way, the algorithm allows the computation of the direct dynamics through the adoption of different numerical integration schemes. For this propose, the Runge-Kutta methods of order two, four and ten are analysed and compared. The system efficiency and its precision are evaluated and, finally, some simulation results are presented.

1 - INTRODUCTION

A robot manipulator may be considered as an electro-mechanical mechanism which has the function of positioning and orientating another mechanism which is on its tip. The objective of the tip mechanism is to perform the orientation and the adequate fixing tools defined by the foregone task. The manipulator kinematics defines its workspace. The whole structure may be considered in two parts: The first one is the waist/arm, which has at least three degrees of freedom (dof), used for positioning the main concentration point of the orientation frames. The second one is the grasp, normally constituted of another three rotational dof used for the orientation of the tools. Neglecting the joints deformation, the waist member and the members responsible for the orientation may be considered rigid because their frames are located in a common point.

In the mathematical model of the structure, it is considered that the link deformations are restricted to the angular variation of the passive joints. Thus, all link segments can be considered rigid. In this way, it is possible to develop a mathematical model like the Newton-Euler (N-E) recursive method, that is used to solve the inverse dynamics of a rigid system [1]. Having these ideas in mind, the dynamic system of the structure can be established with any number of links and segments per link. On the other hand, the proposed model may be applied with a high precision through the adequate selection of segments per link. Obviously, the higher the number of segments per link, the higher the system complexity. Therefore, this quantity needs to be "engaged" in order to give an adequate precision to the system simulation without requiring high computational resources.

In this line of thought, the paper is organized as follows. Section two begins by presenting the fundamental aspects of the manipulator structure. Section three develops the algorithms for the simulation of the inverse and direct dynamics and section four shows some simulation results. Finally, section five outlines the main conclusions.

2 - MECHANICAL STRUCTURE

The proposed nonrigid links manipulator structure consists in an open loop mechanism, composed of n links connected through rotational joints, that are considered active dof at the points where the motor torques are applied, and $nseg$ rotational joints that correspond to $(nseg-1)$ segments per link. In each segment it can be considered one, two or three passives dof. The link geometry is shown in figure 1, where it can be seen that the coordinate systems of local active frame $(X_{i,0}, Y_{i,0}, Z_{i,0})$ and of the passive joints $(X_{i,j}, Y_{i,j}, Z_{i,j})$, where $j = 1, \ldots, nseg-1$. The origin of the frame $(X_{i,nseg}, Y_{i,nseg}, Z_{i,nseg})$ is coincident with the origin of the active joint frame of link $i+1$. The Z axis of inertial frame is in the same direction of the gravity action. The first link joint is an active one and it is on the base of the manipulator. It is considered that the Z axis of that link frame is coincident with the Z axis of the inertial frame and it is rigid. In the second link, the first segment has an active joint in the $Z_{2,0}$ axis of its local frame. The oncoming segments frames are $(X_{2,j}, Y_{2,j}, Z_{2,j})$, where $j = 1,\ldots,nseg-1$. The same convention is applied to the next links.

In Figure 1, it can be seen that the active local frames have their dof in the Z axis and the displacement of the links occurs on the XY planes of their local frames. In the flexible link frames, rotation in the X axis denote segment twist, while rotations in the Y and Z axis denote flexibility in the planes XZ and XY of the considered segment frame.

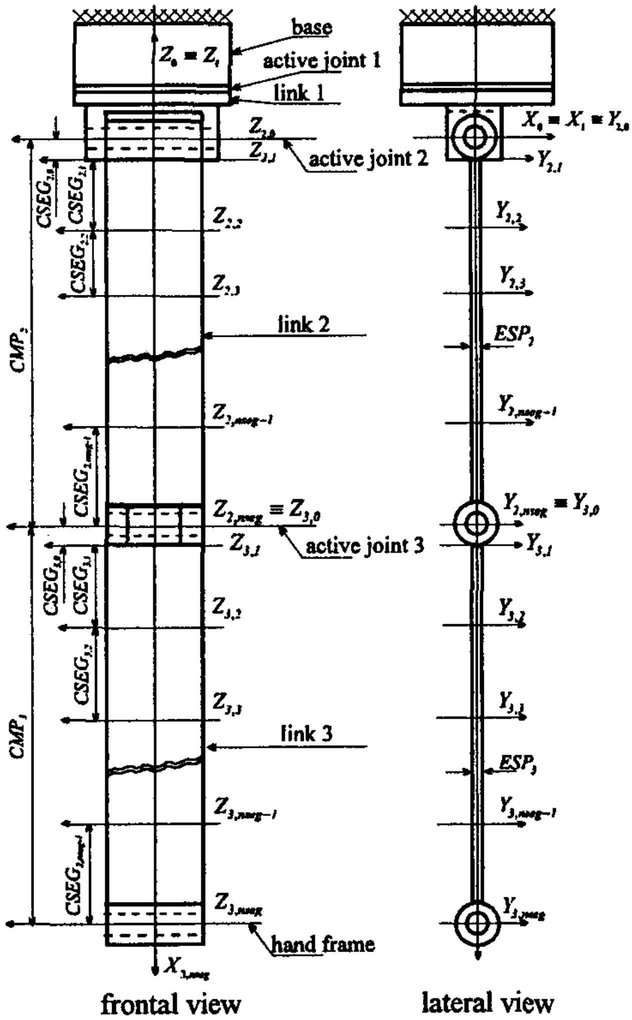


Figure 1: Manipulator with three active joints and *nseg*−1 segments in each link for the simulation of flexibility

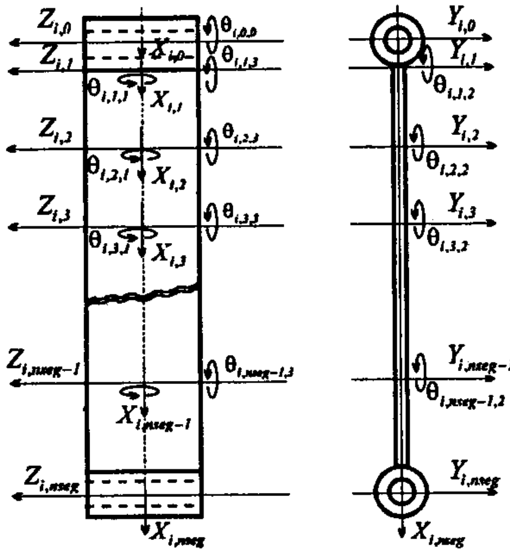
The deformation analysis is done considering the approximation of each flexible link through an adequate number of rigid segments connected in the joints of fictitious frames [1,2]. They concentrate the effects due to bending and/or twist deformation of each segment. It is not considered the longitudinal deformation effect of the segments. In this way, it is possible to analyze the passive segments with three analyses types. In the first type (T1) it is assumed that there is deformation only in the Z axis (bending), *tip=1*. In the second type (T2), deformation in Y and Z axis is considered (bending in two planes), *tip=2*. In the last type (T3), it is considered deformation in all three axis of each segment (twist and bending in two planes twist), *tip=3*. So, the total number of dof of whole structure is defined as:

$$ngl = n\left[tip\left(nseg-1\right)+1\right]-tip\left(nseg-1\right) \qquad (1)$$

The calculation of the total number of dof is done because the dynamic evaluation works with two vector fields. In the subroutines that calculate the forces and torques (gyroscopic, centripetal and gravitation) and for the vectors used to build the system mass matrix, the definition of independent variables vectors need to be in three dimension. These dimensions correspond to the link, the segment of the link and to the frame axis in consideration. Out of that subroutines, it is more convenient to work with the vectors elements in one dimension, that is, adopting a sequential orientation. In this perspective, were developed subroutines that have the objective of transforming the reference vectors.

In figure 2 it can be seen the independent variables for each type of analysis of the structure, where $\theta_{i,0,0}$ is the active angle of link i (measured over $Z_{i,0}$ axis between $X_{i-1,nseg-1}$ and $X_{i,0}$) and $\theta_{i,j,k}$ is the passive angle of the j passive segment, of link i (measured between k axis of $j-1$ and j segments). It is considered that $i = 1,\ldots,n$ and $j = 0,\ldots,nseg-1$. In the case of $k = 1,2,3$, the evaluation done in the system is of type T3, i.e., there are rotation in all axis frame. For $k = 2,3$ it is done analysis of type T2, so, it is considered rotation in Y and Z axis. For $k = 3$ the rotation occurs in Z axis and it corresponds to analysis type T1. It can be seen that the total number of segments per flexible link has to be at least two, because each link is composed of one rigid segment followed by the corresponding flexible segments.

This methodology was developed considering the necessity of high, medium and low precision in the analysis like types T3, T2 and T1, respectively. So, it is possible to do the analysis with better security conditions with the possibility of simplifications in the structural analysis. The related simplifications are dependent of the manipulator characteristics and of tasks to be done in the workspace.



frontal view lateral view

Figure 2: Convention adopted for the independent variables.

In the manipulator structure kinematics analysis it is adopted the methodology used in [3,4], where it is considered that the position vector and the orientation matrices are obtained through a recursive mathematical model from the base to the tip. Optionally, the established conventions rules of Denavit *et al* [5] can be used to define the homogeneous matrices transformation. The local and inertial frames with respective vectors are shown in figure 3, where the $\mathbf{vloc}_{i,j}$ are local vectors and the $\mathbf{vin}_{i,j}$ are inertial vectors.

3 - DYNAMIC SYSTEM

To establish the dynamic system of the manipulator structure the equations, in symbolic form, may be written as:

$$\mathbf{M}\ \ddot{\mathbf{q}} + \mathbf{T}_{IN} = \mathbf{T}_{EX} \qquad (2)$$

where

$$\mathbf{T}_{IN} = \mathbf{T}_{GC} + \mathbf{T}_{FL} + \mathbf{T}_{FR} \qquad (3)$$

In the dynamic equations of the proposed system (2), $\mathbf{M}$ is the inertia matrix of the manipulator, which is dependent of position vector $\mathbf{q}$, non singular, symmetric, positive definite and with dimension $ngl \times ngl$. The vector $\ddot{\mathbf{q}}$ (dim. $ngl \times 1$) represents the acceleration in the active and passive joints. The vector $\mathbf{T}_{IN}$ is composed of the vectors $\mathbf{T}_{GC}$, $\mathbf{T}_{FL}$ and $\mathbf{T}_{FR}$. The torque $\mathbf{T}_{GC}$ (dim. $ngl \times 1$,) is a vector dependent of position vector $\mathbf{q}$ and velocity vector $\dot{\mathbf{q}}$ that considers the effects of centrifugal, Coriolis and gravity forces. $\mathbf{T}_{FL}$ (dim. $ngl \times 1$) is a vector dependent of $\mathbf{q}$ that corresponds to the rigidity of the passive segments of the structure and $\mathbf{T}_{FR}$ (dim. $ngl \times 1$.) is a vector dependent of velocity vector $\dot{\mathbf{q}}$ that represents the effects of friction in active and passive joints. The vector $\mathbf{T}_{EX}$ (dim. $ngl \times 1$) is the torque to be applied in the joint motors.

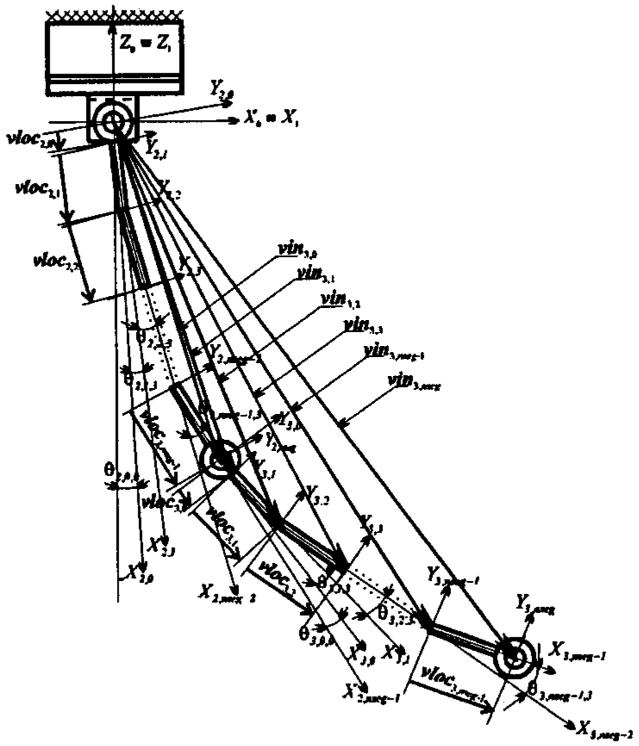


Figure 3: Local and inertial vectors of the direct kinematics

The vector $\ddot{\mathbf{q}}$ is represented as:

$$\ddot{\mathbf{q}} = \left\{ \ddot{\theta}_{1,0,0} \quad \ddot{\mathbf{q}}_{1,1}{}^{T} \quad \ddot{\mathbf{q}}_{1,2}{}^{T} \quad \cdots \quad \ddot{\mathbf{q}}_{1,nseg-1}{}^{T} \quad \cdots \quad \cdots \quad \ddot{\theta}_{n,0,0} \quad \ddot{\mathbf{q}}_{n,1}{}^{T} \quad \ddot{\mathbf{q}}_{n,2}{}^{T} \cdots \quad \ddot{\mathbf{q}}_{n,nseg-1}{}^{T} \right\}^{T} \qquad (4)$$

where the sub vectors $\ddot{\mathbf{q}}_{i,j}$ depend of the type of analysis. For T1, T2 or T3 it comes $\ddot{\mathbf{q}}_{i,j} = \{\theta_{i,j,3}\}$, $\ddot{\mathbf{q}}_{i,j} = \{\theta_{i,j,2} \quad \theta_{i,j,3}\}^{T}$ and $\ddot{\mathbf{q}}_{i,j} = \{\theta_{i,j,1} \quad \theta_{i,j,2} \quad \theta_{i,j,3}\}^{T}$, respectively.

To calculate the inertia matrix $\mathbf{M}(\mathbf{q})$ it is adopted the methods presented by Walker *et al* [2]. In this way, the elements of $\mathbf{M}(\mathbf{q})$ are calculated using the algorithm of inverse dynamic of Newton-Euler (IDNE), where the effects of gravity, velocities, elastic forces and external forces are not considered. In the vector $\ddot{\mathbf{q}}$ all elements are zero except the element $\ddot{\theta}_{i,j,k} = 1$. Therefore, the $ngl \times ngl$ elements of that matrix are obtained calculating the vector of each column, like in the first method, the elements of principal diagonal downwards, from the last element, like in the second method or of principal diagonal upward, from the last element, like the third method. The vector $\mathbf{T}_{IN}(\mathbf{q},\dot{\mathbf{q}})$, is obtained setting all the elements of $\ddot{\mathbf{q}}$ equal to zero. Thus, in this vector are considered the effects of gravity, velocities, elastic forces and external forces applied in the hand frame. The vector $\mathbf{T}_{IN}(\mathbf{q},\dot{\mathbf{q}})$ is the sum of $\mathbf{T}_{GC}$, $\mathbf{T}_{FL}$ and $\mathbf{T}_{FR}(\dot{\mathbf{q}})$. $\mathbf{T}_{GC}$ is obtained with the recursive algorithm IDNE, where the elements of the acceleration vector $\ddot{\mathbf{q}}$ are set to zero. It can be noted that, in the sub routine elaborated to calculate $\mathbf{T}_{GC}(\mathbf{q},\dot{\mathbf{q}})$, it is possible to evaluate the effects of gravity, independently, doing $\dot{\mathbf{q}}(t) = \mathbf{0}$. The vector $\mathbf{T}_{FL}$ is obtained considering the gravity action equal zero in $\mathbf{T}_{GC}(\mathbf{q},\dot{\mathbf{q}})$. The vector $\mathbf{T}_{FR}(\dot{\mathbf{q}})$ represents the influence of viscous and Coulomb friction [12]. All IDNE sub routines that calculate the elements of $\mathbf{M}(\mathbf{q})$ and $\mathbf{T}_{GC}(\mathbf{q},\dot{\mathbf{q}})$ were optimized, i.e., eliminating 'redundant' operations such as products with zero or one and sums with zero. The torque $\mathbf{T}_{EX}$ is dependent of system state, where the vector **u** is obtained from the control algorithm.

It is shown in figure 4 a system simulation flowchart. In order to solve the dynamic system it is used a LU decomposition of $\mathbf{M}(\mathbf{q})$ so that it is not required the inversion of the mass matrix to obtain $\ddot{\mathbf{q}}$ [11].

For the direct dynamics, with the integration of $\ddot{\mathbf{q}}$ obtained from the resolution of (2), one can obtain the velocity and displacement vectors for the next time step. Among the available methods, were investigated three numerical algorithms in order to compare the precision and the computational effort and to conclude about the best scheme for the simulation.

One simple integration procedure [10,12] is the second order Runge-Kutta method (RK2). A more efficient and popular method [13] is the fourth order formulation of Runge-Kutta (RK4), which requires the evaluation of four parameter functions vectors. Another efficient integration method is the tenth order Runge-Kutta (RK10) developed by Hairer [14], where it is necessary the evaluation of seventeen parameter functions vectors.

The general form to use the Runge-Kutta methods with a fixed time step, for the second order differential equations, can be written as follows:

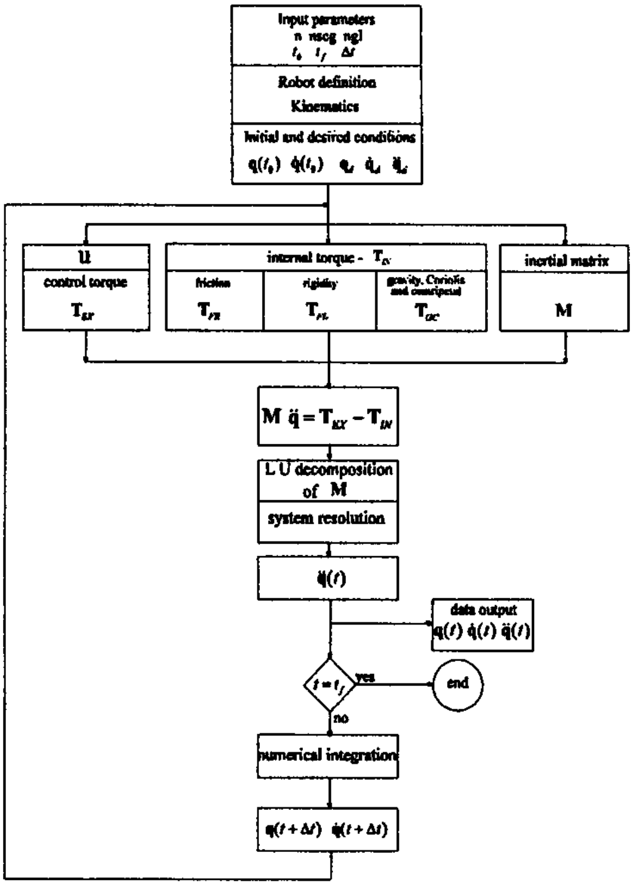


Figure 4: Flowchart of the system simulation.

a) It is known the acceleration in the instant t. Therefore, it is possible to establish the first stage of the calculation of the parameter vectors in the form:

$$\mathbf{K}_1 = \ddot{\mathbf{q}}(t)\,\Delta t \tag{5}$$

$$\mathbf{L}_1 = \dot{\mathbf{q}}(t)\,\Delta t \tag{6}$$

The next step is to calculate the auxiliary displacement and velocity vectors $\mathbf{q}_{aux}$ and $\dot{\mathbf{q}}_{aux}$ as:

$$\dot{\mathbf{q}}_{aux}(t) = \dot{\mathbf{q}}(t) + \sum_{j=2}^{i} a_{ij-1}\mathbf{K}_{j-1} \tag{7}$$

$$\mathbf{q}_{aux}(t) = \mathbf{q}(t) + \sum_{j=2}^{i} a_{ij-1}\mathbf{L}_{j-1} \tag{8}$$

b) It is necessary to evaluate the system once more for the conditions $\mathbf{q}_{aux}$ and $\dot{\mathbf{q}}_{aux}$ in each integration step. This means that it is required to evaluate $\mathbf{M}(\mathbf{q}_{aux})$ and $\mathbf{T}_{IN}(\mathbf{q}_{aux},\dot{\mathbf{q}}_{aux})$ and to obtain $\ddot{\mathbf{q}}_{aux}(t)$ in those conditions. Therefore, the parameter vectors $\mathbf{K}_{i+1}$ and $\mathbf{L}_{i+1}$ are calculated as:

$$\mathbf{K}_{i+1} = \ddot{\mathbf{q}}_{aux}(t)\,\Delta t \tag{9}$$

$$\mathbf{L}_{i+1} = \dot{\mathbf{q}}_{aux}(t)\,\Delta t \qquad (10)$$

Calculating all parameter vectors of the last step, it is possible to obtain the approximate values of the vectors $\dot{\mathbf{q}}(t+\Delta t)$ and $\mathbf{q}(t+\Delta t)$ in the form:

$$\dot{\mathbf{q}}(t+\Delta t) = \dot{\mathbf{q}}(t) + \sum_{i=1}^{s} b_i \mathbf{K}_i \qquad (11)$$

$$\mathbf{q}(t+\Delta t) = \mathbf{q}(t) + \sum_{i=1}^{s} b_i \mathbf{L}_i \qquad (12)$$

where i and the coefficients a_{ij} and b_i change with the order of the employed Runge-Kutta method.
To establish a coherent next time step Δt, in the foregoing methods, it is necessary to try an initial value and to check the response convergence. The time increment Δt in the numerical integration scheme should be sufficiently large to obtain a fast simulation, but, sufficiently small for the simulation to converge. Considering those factors, it is also convenient to take into account the accumulation of round off error per iteration.

4 - SIMULATION RESULTS

Studying the integration methods referred previously, the simulation results show that, as expected, the methods RK4 and RK10 are more efficient than the RK2. Nevertheless, due to the simplifications imposed to the system, the RK10 adoption. was not justified and, in fact, the RK4 revealed a sufficient performance over all the cases. A large number of simulations were developed for the manipulator with three links and a varied number of segments per link.
The simulation example shown in figure 5 represents the temporal response of the angular displacement (in degrees) and the inertial cartesian coordinates of the gripper (in meters) during a time period of five seconds. They show the behavior of a simplified system, where it is considered that the dof of the first link is a fixed one, the active joints vector control is zero and there is neither friction, nor external actions applied on the manipulator tip frame, i.e., $\mathbf{T}_{EX} = \{\mathbf{0}\}$, $\mathbf{T}_{FR} = \{\mathbf{0}\}$, $\mathbf{Ftot}_{n+1,0} = \{\mathbf{0}\}$ and $\mathbf{Mtot}_{n+1,0} = \{\mathbf{0}\}$.
In the figure can be seen the responses of the active joints and the passive joints in each segment. Each link is composed of one active joint and four passive joints, that is, the manipulator is analyzed with T1 type, with four vibration modes per link and one vibration mode per segment (rotation in local Z segment axis).

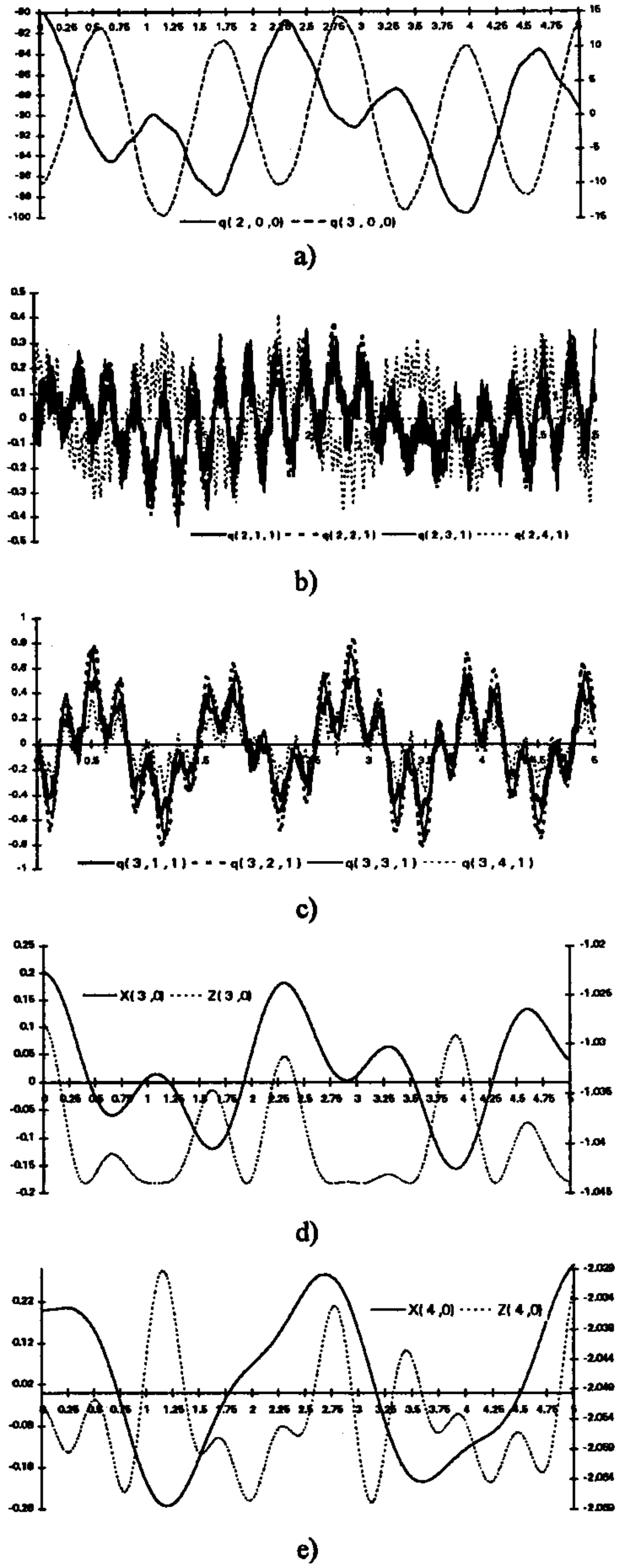


Figure 5: Angular displacements for the:
a) Active joints.
b) Passive joints of the second link.
c) Passive joints of third link.
d) Cartesian coordinates of the third active joint.
e) Cartesian coordinates of the hand.

The links are built with an aluminum alloy and each segment has length of 0,25 m and a strength section of $0.04 \times .004\ m^2$, as shown in figure 1.

The initial conditions are $\dot{q}(0) = \{0\}$ and. $q(0) = \{-80^\circ \ 0 \ 0 \ 0 \ 0 \ -10^\circ \ 0 \ 0 \ 0 \ 0\}^T$.

5 - CONCLUSIONS

In modeling a general mechanical linkage, it was assumed that the link deformation is concentrated in the frame rotation of the passive joints. Thus, each segment can be considered rigid. In this way, it was possible to develop a model such as the recursive Newton-Euler algorithm for rigid systems, to solve the inverse dynamics. With the adaptation of this method, it is possible to establish the linkage dynamic system with any number of links and segments.. Consequently, the algorithm can be applied in different precision levels, choosing the adequate number of segments per link.

The resultant dynamic model is highly non linear and complex. It is necessary to take into account the inertia variation with the robot configuration, to ensue the tasks needed in the workspace. On the other hand, the elastic deformations can be analyzed in one, two or three rotation axis or, by other words, it is possible to analyze the structure in three different ways.

In this line of thought, this study adopted the Newton-Euler recursive algorithm for the computation of the inverse dynamics, in order to model all the dynamic phenomena present in flexible manipulators. On the other hand, several Runge-Kutta algorithms were tested for the evaluation of the direct dynamics. The comparison of the precision and the computational load, for different orders of the integration scheme, revealed that the standard RK4 leads to good results without requiring high computational resources. Based on this package trajectory planning and control algorithms for nonrigid manipulators are presently under evaluation.

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