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PERFORMANCE OF FRACTIONAL-ORDER CONTROL OF COOPERATIVE ROBOTS

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Abstract. *In this paper it is studied the implementation of fractional-order algorithms in the position/force control of two cooperating robotic manipulators. The system performance is analyzed in the time and frequency domains. The effect of backlash and flexibility at the robot joints is also investigated.*

Keywords: fractional calculus, fractional derivative, robots, cooperation, position, force, task.

1 INTRODUCTION

Two robots carrying a common object are a logical alternative for the case in which a single robot is not able to handle the load. Nevertheless, with two cooperative robots the resulting interaction forces have to be accommodated and, in addition to position feedback, force control is also required [1, 2]. There are two basic methods for force control, namely the hybrid position/force and the impedance schemes. The first method, proposed by Raibert and Craig [3], separates the task into two orthogonal subspaces corresponding to the force and the position controlled variables. Once established the subspace decomposition two independent controllers are designed. The second method was first proposed by Siciliano and Villani [2]. In this scheme, by a proper choice of the arm mechanical impedance the interaction forces can be controlled to obtain an adequate response. This paper studies the position/force control of two cooperative manipulators, using fractional-order (FO) algorithms [4-6].

In this line of thought the paper is organized as follows. Section two presents the controller architecture for the position/force control of two robotic. Section three develops several experiments for the performance evaluation of the controllers, for robots having several types of dynamic phenomena at the joints. Finally, section four outlines the main conclusions.

2 POSITION FORCE CONTROL OF TWO ARMS

When two robots grasp an object (Figure 1), that is moved it from one location to another, a coordinated motion is required. In order to get good performances it is necessary to specify no only the desired motion of each robot but also the corresponding handling force. In the system under study the contact of the robot gripper with the load is modeled through a linear system with a mass M, a damping B and a stiffness K. It is also considered that the load has length l_0 and orientation θ_0. On the other hand, we consider two manipulators each with two rotational joints the robots have link lengths l_1 and l_2 and the shoulders are separated by the distance l_b. The dynamics of a robot with n links interacting with the environment is modeled as:

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) - \mathbf{J}^{\mathbf{T}}(\mathbf{q})\mathbf{F} \quad (1)$$

where $\boldsymbol{\tau}$ is the $n \times 1$ vector of actuator torques, $\mathbf{q}$ is the $n \times 1$ vector of joint coordinates, $\mathbf{H}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the $n \times 1$ vector of centrifugal/Coriolis terms and $\mathbf{G}(\mathbf{q})$ is the $n \times 1$ vector of gravitational effects. The matrix $\mathbf{J}^{\mathbf{T}}(\mathbf{q})$ is the transpose of the Jacobian matrix and $\mathbf{F}$ is the force that the load exerts in the robot gripper.

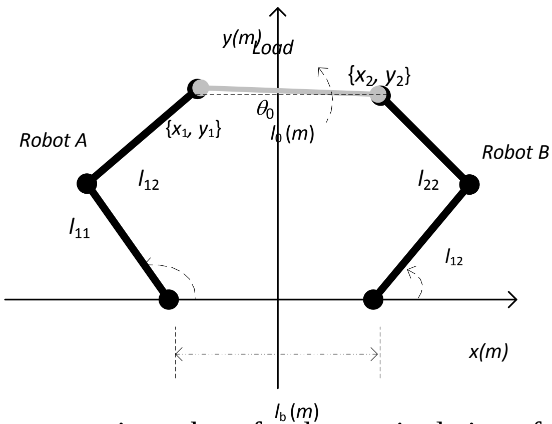

Figure 1. Two cooperating robots for the manipulation of an object.

The numerical values adopted for the robots and the object are $m_1 = 0.5$ *kg*, $m_2 = 6.25$ *kg*, $r_1 = 1.0$ *m*, $r_2 = 0.8$ *m*, $J_{1m} = J_{2m} = 1.0\ kgm^2$, $J_{1g} = J_{2g} = 4.0\ kgm^2$ $l_b = l_0 = 1.0$ *m* and $\theta_0 = 0$ *deg*, $B_1 = B_2 = 1\ Ns.m^{-1}$ and $K_1 = K_2 = 10^4\ Nm^{-1}$

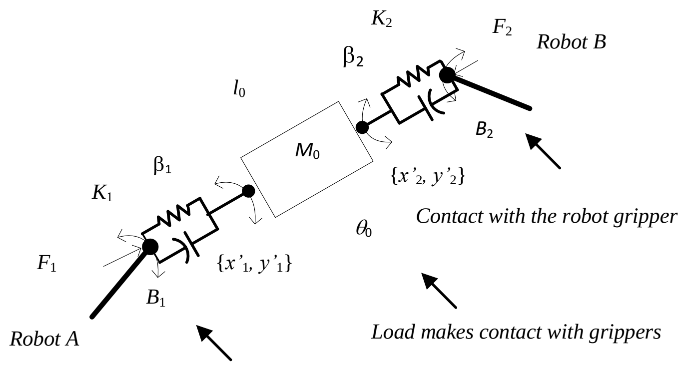

Figure 2. The contact between the robot gripper and the object.

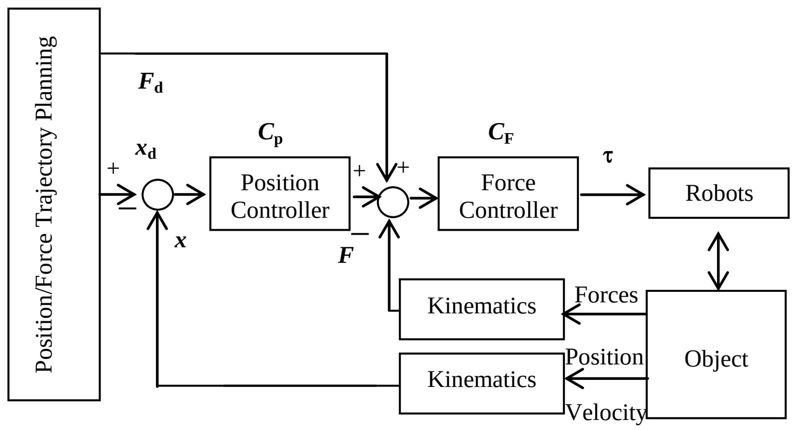

Figure. 3. The position/force controller architecture.

The controller architecture (Figure 3) is inspired on the impedance and compliance schemes. Therefore, we establish a cascade of force and position algorithms as internal an external feedback loops, respectively, where x_d and F_d are the payload desired position coordinates and contact forces.

We consider fractional controllers (*FO)* both in the position and force control loops we adopt the Grünwald-Letnikov definition:

$$D^{\alpha}[x(t)] = \lim_{h \to 0} \left[\frac{1}{h^{\alpha}} \sum_{k=1}^{\infty} \frac{(-1)^k \Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)} x(t-kh) \right] \quad (2)$$

where Γ is the gamma function and h is the time increment. In our case, for implementing the algorithms of the type $C(s) = K s^{\alpha}$, $-1 < \alpha < 1$, we adopt discrete – time approximations using 4th–order Pade fractions.

3. SYSTEMS PERFORMANCES

This section analyzes the system performance both for robots ideal transmissions and robots with dynamic phenomena at the joints, such as backlash and flexibility. Moreover, we compare the response of integer and fractional algorithms, namely $C_{\mathrm{P}}(s) = K_{\mathrm{P0}} s^{\alpha_{\mathrm{P}}}$: $C_{\mathrm{P}}(s) = K_p\ (1 + T_d\ s)$ and $C_{\mathrm{F}}(s) = K_{\mathrm{F0}} s^{\alpha_{\mathrm{F}}}$: $C_{\mathrm{F}}(s) = K_F\ [1 + (T_i\ s)^{-1}]$, in the position and force loops, respectively [8, 9]. Both algorithms were tuned by trial and error having in mind getting a similar performance in the two cases. The resulting parameters yield for fractional algorithm: $\{K_{\mathrm{P}},\alpha_{\mathrm{P}}\} \equiv \{10^4, 1/2\}$, $\{K_{\mathrm{F}}, \alpha_{\mathrm{F}}\} \equiv \{2, -1/5\}$ and for the integer algorithm: $\{K_p,K_d\} \equiv \{10^4,10^2\}$, $\{K_p,K_i\} \equiv \{10,10^4\}$ in the position and force loops, respectively.
It is adopted the operating point at the center of the object $\mathrm{A} \equiv \{x,y\} \equiv \{0,1\}$ and a M object surface with parameters $\{\theta,M,B_j,K_j\} \equiv \{0,10,1.0,10^3\}$.
In order to study the system dynamics we apply, separately, small amplitude rectangular pulses, at the position and force references. Therefore, we perturb the references with $\{\delta x_{\mathrm{d}}, \delta y_{\mathrm{d}}, \delta Fx_{\mathrm{d}}, \delta Fy_{\mathrm{d}}\} = \{10^{-3}, 0, 0, 0\}$, $\{\delta x_{\mathrm{d}}, \delta y_{\mathrm{d}}, \delta F_{\mathrm{d}}, \delta Fy_{\mathrm{d}}\} = \{0, 10^{-3}, 0, 0\}$,
$\{\delta x_{\mathrm{d}}, \delta y_{\mathrm{d}}, \delta Fx_{\mathrm{d}}, \delta Fy_{\mathrm{d}}\} = \{0, 0, 1, 0\}$, $\{\delta x_{\mathrm{d}}, \delta y_{\mathrm{d}}, \delta Fx_{\mathrm{d}}, \delta Fy_{\mathrm{d}}\} = \{0, 0, 0, 1\}$.

In order to evaluate the performance of the proposed algorithms we compare the response for robots with dynamical phenomena at the joints.
In all experiments the controller sampling frequency is $f_{\mathrm{c}} = 10$ kHz for the operating point A of the object and a contact force of each gripper of $\{Fx_j,Fy_j\} \equiv \{0.5,5\}$ Nm for the jth ($j = 1, 2$) robot.
In a first phase figures 4 and 5 we consider robots with ideal transmissions at the joints depict the time response of the robots 1 and 2, under the action of the fractional and the integer algorithms.
In a second phase (Figure 6) we analyze the response of robots with dynamic backlash at the joints [10, 14]. For the ith joint gear, with clearance h_i, the backlash reveals impact phenomena between the inertias, which obey the principle of conservation of momentum and the Newton law:

$$\dot{q}_i' = \frac{\dot{q}_i\left(J_{ii} - \varepsilon J_{im}\right) + \dot{q}_{im} J_{im}\left(1+\varepsilon\right)}{J_{ii} + J_{im}} \tag{3}$$

$$\dot{q}_{im}' = \frac{\dot{q}_i J_i\left(1+\varepsilon\right) + \dot{q}_{im}\left(J_{im} - \varepsilon J_{ii}\right)}{J_{ii} + J_{im}} \tag{4}$$

where $0 \le \varepsilon \le 1$ is a constant that defines the type of impact namely $\varepsilon = 0$ for the inelastic impact, and $\varepsilon = 1$ for the elastic impact, and $\dot{q}'_i$ and $\dot{q}'_{im}$ are the velocities of the ith joint and motor after the collision, respectively. The parameters J_{ii} and J_{im} stand for the link and motor inertias of joint i, respectively. The numerical values adopted are $h_i = 1.8\ 10^{-4}$ rad and $\varepsilon_i = 0.8$ (i = 1, 2).

In a third phase (Figure 7) we study the performance of robots with compliant joints. For this case the dynamic model corresponds to model (1) augmented by the equations:

$$\boldsymbol{\tau} = \mathbf{J_m}\ddot{\mathbf{q}}_\mathbf{m} + \mathbf{B_m}\dot{\mathbf{q}}_\mathbf{m} + \mathbf{K_m}(\mathbf{q_m} - \mathbf{q}) \quad (5)$$

$$\mathbf{K_m}(\mathbf{q_m} - \mathbf{q}) = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \quad (6)$$

where $\mathbf{J_m}$, $\mathbf{B_m}$ and $\mathbf{K_m}$ are the $n \times n$ diagonal matrices of the motor and transmission inertias, damping and stiffness, respectively. In the simulations we adopt $K_{mi} = 2\ 10^6$ Nm rad^{-1} and $B_{mi} = 10^4$ Nms rad^{-1} (i = 1,2). The time responses (Tables I, II, III and VI), namely the percent overshoot $PO\%$, the steady-state error e_{ss}, the peak time T_p and the settling time T_s, reveal that, the fractional is superior to the integer controller in the cases with dynamical phenomena at the robot joints.

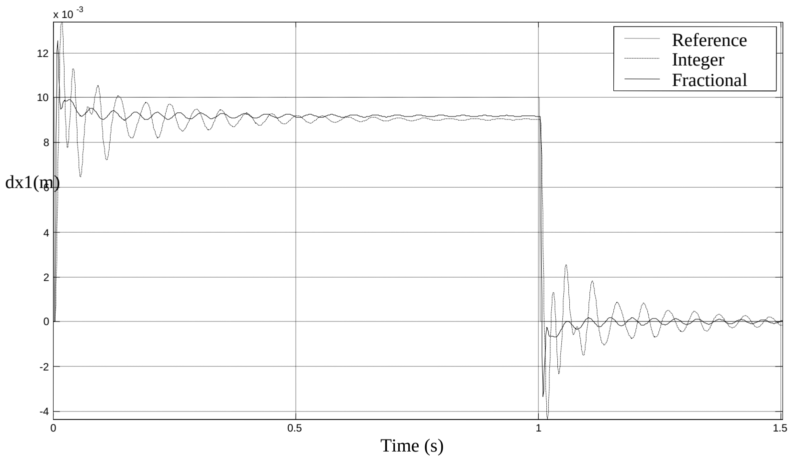

Figure 4 – Time response for the robot 1 under the action of the fractional and the integer algorithms, for a pulse perturbation at the position reference $\delta x_d = 10^{-3}$m.

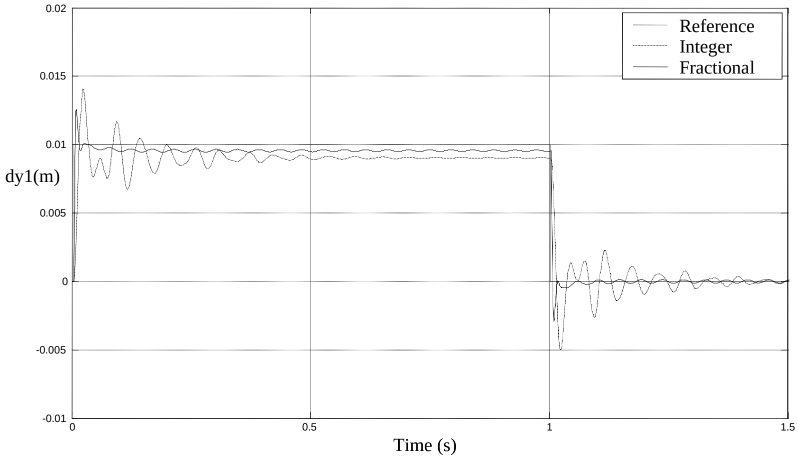

Figure 5 – Time response for the robot 1 under the action of the of fractional and the integer algorithms, for a pulse perturbation at the robot 1 force reference $\delta F_{\mathrm{yd}} = 1\mathrm{N}$

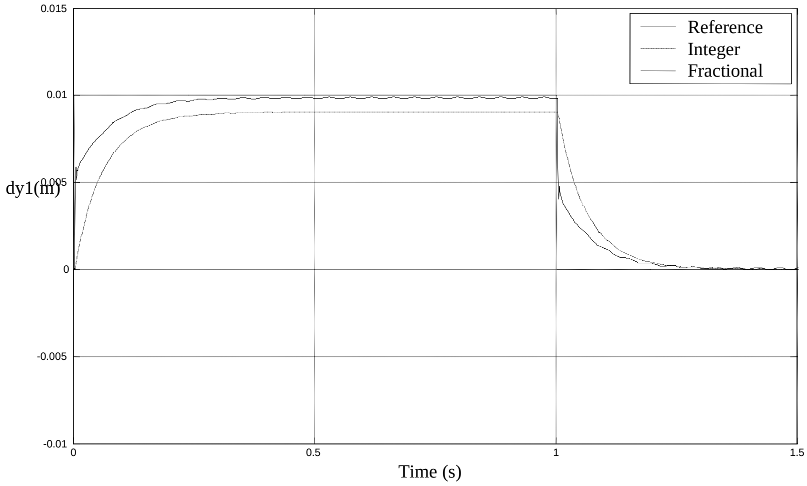

Figure 6 – Time response for a robot with joints having backlash under the action of fractional and the integer algorithms, for a pulse perturbation at the robot 1 position reference $\delta_{\mathrm{yd}} = 10^{-3}\mathrm{m}$.

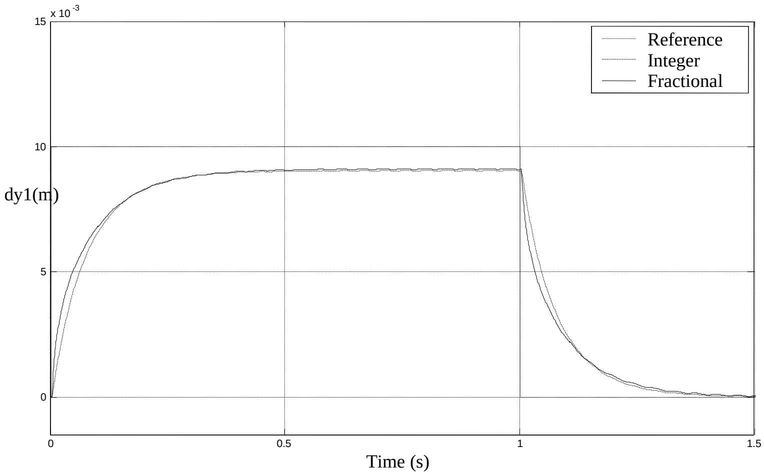

Figure 7 – Time response for a robot with joints having flexibility under the action of of fractional and the integer algorithms, for a pulse perturbation at the robot 1 position reference $\delta x_d = 10^{-3}$m.

Table I – Time response parameters for a rectangular pulse δx_d the robot 1 position reference**.**

Joint		*PO%*	e_{ss}	T_p	T_s
Ideal	*Integer*	33.89	$9.8\ 10^{-4}$	$17\ 10^{-3}$	$70\ 10^{-2}$
	Fractional	25.39	$8.3\ 10^{-4}$	$9\ 10^{-3}$	$50\ 10^{-2}$
backlash	*Integer*	4.5	$5.3\ 10^{-3}$	$17\ 10^{-3}$	$31\ 10^{-3}$
	Fractional	1.05	$2.2\ 10^{-4}$	$13\ 10^{-3}$	$30\ 10^{-3}$
Flexible	*Integer*	4.87	$10\ 10^{-2}$	$35\ 10^{-3}$	$71\ 10^{-3}$
	Fractional	2.51	$2.2\ 10^{-3}$	$33\ 10^{-3}$	$60\ 10^{-2}$

Table II – Time response parameters for rectangular pulse δyd the robot 1 position reference.

Joint		*PO%*	e_{ss}	T_p	T_s
Ideal	*Integer*	40.6	$9.7\ 10^{-4}$	$23\ 10^{-2}$	$70\ 10^{-2}$
	Fractional	25.87	$4.7\ 10^{-4}$	$9\ 10^{-1}$	$45\ 10^{-2}$
backlash	*Integer*	9.5	$9.6\ 10^{-3}$	$66\ 10^{-2}$	$91\ 10^{-2}$
	Fractional	9.7	$9.7\ 10^{-3}$	$80\ 10^{-3}$	$90\ 10^{-3}$
Flexible	*Integer*	9.6	$9.7\ 10^{-3}$	$98\ 10^{-2}$	$98\ 10^{-2}$
	Fractional	8.8	$2.2\ 10^{-3}$	$93\ 10^{-3}$	$93\ 10^{-2}$

Table III – Time response parameters for rectangular pulse δFx_{d} at the robot 1 force reference.

Joint		*PO%*	e_{ss}	T_p	T_s
Ideal	*Integer*	–78.54	$102\ 10^{-2}$	$11\ 10^{-3}$	$23\ 10^{-3}$
	Fractional	–90.32	$94\ 10^{-2}$	$99\ 10^{-3}$	$199\ 10^{-3}$
backlash	*Integer*	–89.85	$93\ 10^{-2}$	$10\ 10^{-2}$	$20\ 10^{-2}$
	Fractional	–92.32	$93\ 10^{-2}$	$27\ 10^{-2}$	$55\ 10^{-2}$
Flexible	*Integer*	–89.51	$94\ 10^{-2}$	$5.6\ 10^{-2}$	$11\ 10^{-2}$
	Fractional	–91.76	$93\ 10^{-2}$	$23\ 10^{-2}$	$47\ 10^{-2}$

Table IV – Time response parameters for rectangular pulse δFy_{d} at the robot 1 force reference.

Joint		*PO%*	e_{ss}	T_p	T_s
Ideal	*Integer*	–78.96	$10\ 10^{-1}$	$14\ 10^{-3}$	$29\ 10^{-3}$
	Fractional	–90.69	$9.4\ 10^{-2}$	$99\ 10^{-3}$	$199\ 10^{-3}$
Backlash	*Integer*	–90.63	$93\ 10^{-2}$	$23\ 10^{-2}$	$46\ 10^{-2}$
	Fractional	–92.01	$93\ 10^{-2}$	$72\ 10^{-2}$	$145\ 10^{-2}$
Flexible	*Integer*	–90.67	$93\ 10^{-2}$	$14\ 10^{-2}$	$29\ 10^{-2}$
	Fractional	–91.97	$93\ 10^{-2}$	$41\ 10^{-2}$	$82\ 10^{-2}$

6 SUMMARY AND CONCLUSIONS

This paper compared the position/force control of two robots working in cooperation using fractional and integer order control algorithms. The dynamic performance of two arms holding an object was analyzed in the time domain. Moreover, the manipulating system was also tested for several types of nonlinear phenomena at the joints. The results demonstrate that the fractional-order algorithm is superior, revealing a good performance and an high robustness.

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