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Adaptive Nonlinear Vibration Damping Inspired by the Concept of Fractional Derivatives

József K. Tar[1], Imre J. Rudas[1], János F. Bitó[1], José A. Tenreiro Machado[2]

[1]Institute of Mathematical and Computational Sciences, John von Neumann Faculty of Informatics
Budapest Polytechnic
Népszínház utca 8, H-1081 Budapest, Hungary
tar.jozsef@nik.bmf.hu, rudas@bmf.hu, bito.janos@nik.bmf.hu

[2]Department of Electrotechnical Engineering, Institute of Engineering of Porto
Rua Dr. Antonio Bernardino de Almeida4200-072 Porto, Portugal
jtm@dee.isep.ipp.pt

Abstract *– In the paper a simple nonlinear, adaptive approach inspired by the CRONE method is presented for vibration control. It replaces the fractional derivatives with time-invariant Green functions. Being completed by a nonlinear feedback term it makes the positive definite weighted moving average of the square of the error converge to zero in the kinematic design of the desired damping the realization of which is guaranteed by the controller's adaptive nature. The burden of designing a sophisticated linear controller is evaded. The applicability of the approach is illustrated via simulations for a damped linear oscillator under external excitation at its resonance frequency. The adaptive loop simply successively maps the observed system behavior to the desired one without exerting any effort to identify the reasons of the differences. It is expected to be useful for solving even more complicated vibration damping problems with unmodeled and uncontrolled internal degrees of freedom.*

1 Introduction

Normally vibration is undesired, externally excited phenomenon that occurs in various physical systems therefore its efficient damping is of great practical significance. In a wider context this phenomenon can be modeled as linear or nonlinear interaction between various coupled subsystems having their own internal degrees of freedom. Normally the vibration of certain subsystems has to be reduced only while the other subsystems' vibration is not critical. For instance, in the case of a car the task is to minimize the vibration of the chassis while the wheels and other components of the suspension system may have even drastic vibration.

From the point of control technology the task has the „delicate" nature that the most of the internal degrees of freedom cannot be controlled directly, even the controller cannot be provided with their model or with information on their actual physical state. A novel branch of soft computing has recently been developed on the basis of the simultaneous using of the Modified Renormalization Transformation and simple ancillary methods [1] that is flexible enough to incorporate various algebraic blocks like Lorentz Transformations [2], special Symplectic Transformations [3] etc. It was shown that in the case of a wide class of physical systems with the aid of this method quite robust adaptive controllers can be developed for the control of very inaccurately and partially modeled physical systems which can have even unmodeled internal degrees of freedom [4]. As an input the method requires the desired trajectory of the generalized coordinates of the system which are to directly be controlled.

This approach should be very efficient for vibration control if the desired trajectory of the generalized coordinates of the susbsystems to be controlled could be prescribed. Unfortunately in the most cases juts this information is missing. For instance, in the case of a car, as it proceeds along an uneven, bumpy road crossing hills and valleys the absolute height of the chassis (that is its height with respect to the sea level) cannot be prescribed because the car's control system cannot be in the possession of such information. Instead of that some average distance between the chassis and the wheel can be prescribed because it is locally measurable quantity. The prescription of fixed rigid distance would result in pushing all the consequences of the bumps of the road to the chassis, that is in this way no any vibration caused by the uneven nature of the road could be suppressed. A feasible compromise is the application of some „forgetting integral" that does not allow abrupt changes with respect to the former values of this distance but allows the slow variation of this distance along a desired, prescribed mean value. Therefore the desired behavior of this distance can practically be prescribed to some extent by the terms used in the traditional linear controllers as frequency filters etc. The most plausible means would be the application of a simple PID type controller to keep a finite error at bay. However, the integrating term of this controller does not „forget" the past, and for an even small but constant error it generates infinite signal for feedback.

As the generalization of the concept of the derivative —remaining strictly within the frames of linear physics— the concept of fractional order integrals and derivatives found more and more phyisical applications to describe the „longer term memory" of various physical systems like in the case of visco-elastic phenomena [5, 6], seismic analysis [7], robotics [8], etc. In general the problem of designing fractional control systems remaining mainly within the frames of linear systems obtained considerable attention recently, e.g. [9, 10]. The French expression invented by Oustaloup „*CRONE: Commande Robuste d'Ordre Non Entier*" [11] almost became a „trademark" hallmarking a well elaborated design methodology that obtained application in vibration control, too [12]. Uderstanding and using this method requires deep engineering konwledge in the realm of linear systems, frequency spectrum analysis, the use of Laplace transforms and complex integrals, various typical diagrams as e.g. the Nichols plot, etc.

The aim of the present paper is to propose an alternative approach not strictly restricted to the way of thinking traditional in the case of linear systems. Tackling the problem from a more general nonlinear basis requires less amount of deep and specific engineering knowledge, the application of which can be evaded by the controller's adaptive nature or learning abilities. For this purpose the „long term memory" or slowly forgetting nature of the fractional order systems is considered in a more general view.

2 Fractional order derivatives and Green functions

In the case of a normal PID-type controller the desired trajectory reproduction can be prescribed in a purely kinematics based manner. For the second time-derivative of the actual coordinate errors the desired relation can be prescribed:

$$\left(\ddot{\mathbf{q}}^r - \ddot{\mathbf{q}}^N\right)^d = -P\left(\mathbf{q}^r - \mathbf{q}^N\right) - D\left(\dot{\mathbf{q}}^r - \dot{\mathbf{q}}^N\right) - I\int_{-\infty}^{t}\left(\mathbf{q}^r(t') - \mathbf{q}^N(t')\right)dt' \qquad (1)$$

which may be apt to result in too big desired joint coordinate acceleration $\ddot{\mathbf{q}}^d$. The main idea of this paper is based on the observation that the integrating term in (9) corresponds to a special form of the time-invarint causal Green functions that generally can be expressed as

$$y(t) := \int_{-\infty}^{t} G(t-\tau) f(\tau) d\tau \tag{2}$$

If $G(0)$ is considerable and $G(\xi)\to 0$ as $\xi\to\infty$ (2) describes a long term memory of slowly forgetting nature. If $G(\xi)=0$ for $\xi\le D>0$, than this function can also model the effect of delay.

It can clearly be observed that the Grünwald-Letnikov form of the fractional derivative of order a [13] also is similar to a finite element approximation of an integral with some Green-function if the $h\to 0$ limit is not exactly executed and some numerical approximation using finite elements for the variable of time time is used:

$$\frac{d^a f(t)}{dt^a} := \frac{\lim}{h\to 0}\left[(h)^{-a-1}\sum_{j=0}^{\infty}(-1)^j\left(\frac{\Gamma(a+1)}{\Gamma(j+1)\Gamma(a-j+1)}\right)f(t-jh)h\right] \tag{3}$$

Though the form invented by Caputo as

$$\frac{d^\beta}{dt^\beta}u(t) := \frac{1}{\Gamma(1-\beta)}\int_0^t\left[\frac{du(\tau)}{d\tau}\right](t-\tau)^{-\beta}d\tau, \quad \beta\in(0,1) \tag{4}$$

applies an integer order (more exactly 1st order) derivative re-integrated by some Green-function-like core function the finite element approximation of the derivative in it may result in similar expression. Eqs. (3) and (4) have the common property that they results are not rigorously restricted to the „zero distance” vicintity of the variable „t”: these operations have a more or less slowly forgetting „memory”.

However, besides this forgetting nature that seems to be the most important fact from „physical point of view” (3) and (4) contain complicated restrictions which stem from the requirement that the fractional order derivatives somehow must be related to the integer order ones in limit cases. In (3) the calculation of the values of the Γ functions means a numerical burden, while in (4) the numerically singular integrand, wihle a simple functon G in (2) may be exempt of such difficulties. These difficultes made the authors to consider a more general, less restricted problem-formulation as follows.

Considering the control problem in purely kinematic terms only, due to the laws of Classical Mechanics just the desired second order time-derivative of the error signal can be prescribed somehow, because it is this quantity that can directly be influenced by the control agent, which in its physical capacity can be torque or force. So let ν, $\mu>0$, and let the function $h(t)$ be the solution of the initial value problem

$$\ddot{h}(t) = -\nu h(t) + \mu\int_0^t h(\tau)G(\tau-t)d\tau + S(t) \text{ for } t\ge 0, h(0)=h_0, \dot{h}(0)=\dot{h}_0 \tag{5}$$

in which $G(\xi)$ has the following properties:

$$G(\xi)=0 \text{ } if \text{ } \xi\ge 0, G(\xi)\ge 0 \text{ } if \text{ } \xi\le 0, \int_{-\infty}^{0}G(\xi)d\xi = 1 \tag{6}$$

and let $G(\xi)$ be *continuously differentiable with the exception of certain discrete points of finite number*. It is expedient to introduce a “supplementary” term $S(t)$ that is expected to be necessary for maintaining the decreasing nature of the quadratic error integrated in the “moving window” in (7).

$$V(t) := \int_0^t h^2(\tau)G(\tau-t)d\tau \tag{7}$$

The function $G(\xi)$ in (5, 7) can be interpreted as a weight function of a weighted moving average within a „window" that picks up samples from $h(t)$ and $h^2(t)$ in certain vicinity of t. Since $h(t)$ represents some *error signal* then its convergence to zero is desired as $t\to\infty$. For $\mu=0$ and $S\equiv 0$ (5) evidently describes exponential error-relaxation. For small positive μ the 2nd term at the right hand side seems to decrease the speed of this relaxation. In comparison with the *common integrating term*, $G(\xi)$ represents some *short-term memory* because from (6) it can evidently be inferred that $G(\xi)\to 0$ as $\xi\to -\infty$, that is the long past's effects are forgotten. Furthermore, in $G(\tau-t)$ no any time instant is in „distinguished position". Its behavior is governed exclusively by the *difference of the various time-instants.*

To verify the desired „relaxing" nature of $h(t)$ consider the time-derivative of the „sample" of the square of the error. Because the upper limit of the integral explicitly depends on t, according to the Leibniz rule the derivative of $V(t)$ yields two terms:

$$\dot{V}(t) = h^2(t)G(0) + \int_0^t h^2(\tau)\frac{\partial G(\tau - t)}{\partial t}d\tau \tag{8}$$

Utilizing that

$$\frac{\partial G(\tau - t)}{\partial t} = -\frac{\partial G(\tau - t)}{\partial \tau} \tag{9}$$

and that $G(0)=0$ partial integration can be executed in (8):

$$-\int_0^t h^2(\tau)\frac{\partial G(\tau - t)}{\partial \tau}d\tau = -\int_0^t \frac{\partial}{\partial \tau}\left[h^2(\tau)G(\tau - t)\right]d\tau + \int_0^t 2h(\tau)\dot{h}(\tau)G(\tau - t)d\tau \tag{10}$$

In more details

$$\dot{V}(t) = -\left[\underbrace{h^2(t)G(0)}_{0} - h^2(0)G(-t)\right] + \int_0^t 2h(\tau)\dot{h}(\tau)G(\tau - t)d\tau \tag{11}$$

It is evident that near $t=0$ the first term causes increasing error but its significance decreases with the elapse of time since $G(-t)\to 0$ as $t\to\infty$. To utilize (5) we have to calculate the second derivative of V in its most convenient form in (11) by using again the Leibniz rule and the properties of G as given in (6) and in (9). Due to partial integration certain terms belonging to the upper limit cancel and the 2^{nd} derivative of the error h appear as

$$\ddot{V}(t) = h_0^2\dot{G}(-t) + 2\int_0^t \dot{h}^2(\tau)G(\tau - t)d\tau + 2\int_0^t \ddot{h}(\tau)h(\tau)G(\tau - t)d\tau\ . \tag{12}$$

Now (5) can be substituted into (12) resulting in

$$\begin{aligned}\ddot{V}(t) = &-2\nu V(t) + 2\mu\int_0^t\int_0^\tau h(\tau)G(\tau - t)h(\tau')G(\tau' - t)d\tau' d\tau + \\ &+ h_0^2\dot{G}(-t) + 2\int_0^t\left[\dot{h}^2(\tau) + S(\tau)h(\tau)\right]G(\tau - t)d\tau\end{aligned} \quad (13).$$

In this way a double integral is obtained in which between the variables of integration the following relations hold: $0\le\tau'\le\tau\le t$. Using the rectangular system of coordinates (τ,τ') this domain of integration corresponds to the *lower triangular half of the square shaped area* $[0,t]\times[0,t]$. Introducing the *operator of time ordering T* in the integrand in (13) with the definition

$$T\left[f(\tau)f(\tau')\right] := \begin{cases} f(\tau)f(\tau')\ if\ \ \tau \ge \tau' \\ f(\tau')f(\tau)\ if\ \ \tau < \tau' \end{cases} \tag{14}$$

the upper limit of integration according to τ' can be extended to the whole [0,t] interval. This exactly corresponds to extending the integration to the *upper triangular half of the square shaped area* [0,t]× [0,t] with *symmetric integrands.* That is this extension of the upper limit of integration with time-ordering exactly doubles the original integral. Since the integrands are common numbers satisfying a *commutative algebra* the operation of time-ordering can simply be omitted. Therefore by transforming the 2nd term in the right hand side of (13) it can be written that

$$\ddot{V}(t) = -2\nu V(t) + \frac{2\mu}{2}\int_0^t\int_0^t h(\tau)G(\tau - t)h(\tau')G(\tau' - t)d\tau' d\tau + \\ + h_0^2\dot{G}(-t) + 2\int_0^t\left[\dot{h}^2(\tau) + S(\tau)h(\tau)\right]G(\tau - t)d\tau \qquad (15).$$

It is reasonable to define the function $F(t)$ as

$$F(t) := \int_0^t h(\tau)G(\tau - t)d\tau \qquad (16).$$

By the use of which we obtain that

$$\ddot{V}(t) = -2\nu V(t) + \mu F^2(t) + h_0^2\dot{G}(-t) + 2\int_0^t\left[\dot{h}^2(\tau) + S(\tau)h(\tau)\right]G(\tau - t)d\tau \qquad (17)$$

To estimate the significance of the function $F(t)$ consider the following *non-negative expression*

$$0 \le \int_0^t [h(\tau) - F(t)]^2 G(\tau - t)d\tau = \\ = \int_0^t \left[h^2(\tau) - 2h(\tau)F(t) + F(t)^2\right]G(\tau - t)d\tau = \\ = V(t) + [g(t) - 2]F(t)^2 \qquad (18)$$

in which

$$0 \le g(t) := \int_0^t G(\tau - t)d\tau \xrightarrow[t\to\infty]{} 1 \qquad (19)$$

is a non-negative monotone increasing function approaching its upper limit 1. Therefore the inequality in (18) can be written as

$$0 \le V(t) + [g(t) - 2]F(t)^2 \le V(t) - F(t)^2 \Rightarrow \\ V(t) \ge F(t)^2 \qquad (20)$$

Taking into account (17) it would be a reasonable choice to so determine the supplementary term that it yields

$$S(\tau)h(\tau) := -\dot{h}(\tau)^2 - \rho\dot{h}(\tau)h(\tau) \qquad (21)$$

with a constant $\rho>0$ because its results in the appearance of the first time-derivative of $V(t)$ accoring to (11):

$$\ddot{V}(t) = -2\nu V(t) - \rho\dot{V}(t) + \mu F(t)^2 + h_0^2\left[\dot{G}(-t) + \rho G(-t)\right] \qquad (22)$$

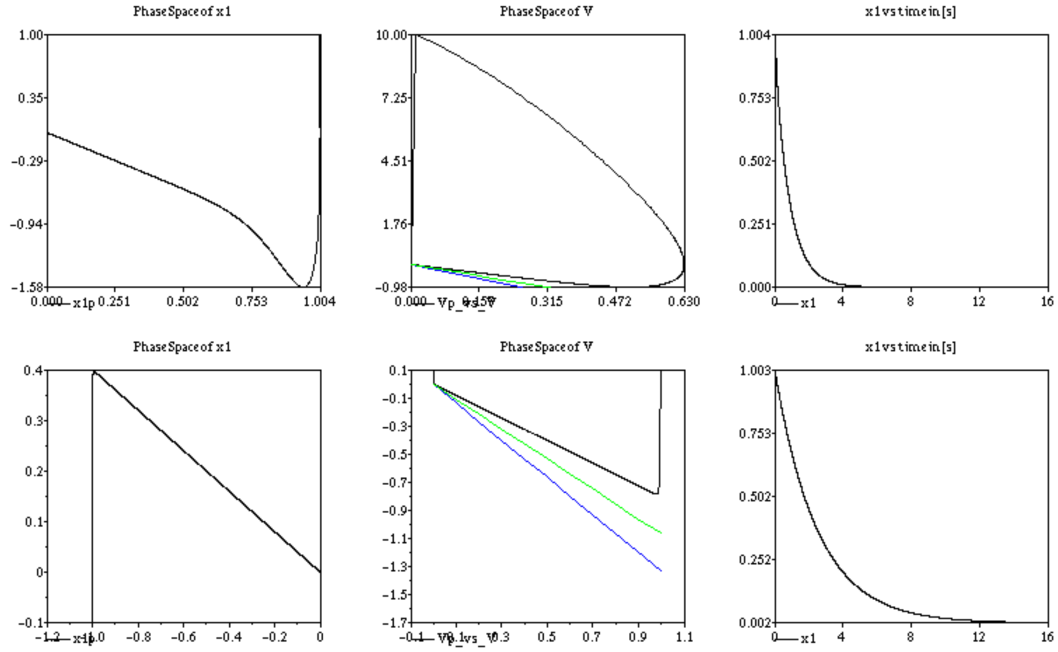

Figure 1: Typical behavior of the solution of (5) using (23) and (24) for h_0=1, $\dot{h}_0 = 1$, $D = 2$ *ms*, $\nu = 100\ s^{-2}$, $\mu = 40\ s^{-2}$, $\rho = 0.5\nu\ [s^{-1}]$, $\varepsilon = 10^{-6}$ *m*, and $exp(\beta\times 10^{-3}\ s)\approx 0.99$ (upper row), and for h_0=-1, $\dot{h}_0 = -1$, $D = 2$ *ms*, $\nu = 100\ s^{-2}$, $\mu = 40\ s^{-2}$, $\rho = 0.5\nu\ [s^{-1}]$, $\varepsilon = 10^{-6}$ *m*, and $exp(\beta\times 10^{-3}\ s)\approx 0.80$ (lower row).

Eq. (22) has a simple and lucid interpretation. Since G can be so constructed that both its value and and its derivative converges to zero as $t\to\infty$ the last term in it converges to zero and can be omitted following some transient phase. Regarding the terms remained, if the positive constant μ is small enough, in the phase space determined by v and dV/dt the location of the points in which the 2nd derivative of V can take the value 0 must be within a wedge shaped region in the V>0 half plane determined by the origin and the two straight loines of the equations $\dot{V} = -2\nu V/\rho, \dot{V} = (-2\nu + \mu)/\rho$. For a given V outside of the wedge to the dV/dt values bigger than that of the upper limit of the wedge $d^2V/dt^2<0$ belongs, while for the dV/dt values smaller than than the lower limit of the wedge $d^2V/dt^2>0$, and the solution cannot enter the $0>V$ half plane. This means that in the region outside of this wedge the wedge attracts the phase trajectories of V, and asymptotically leads them to the V=0, dV/dt=0 point. Therefore the moving average of the square of the error $h(t)$ approaches zero. This means that apart from an initial transient phase the error decreases in a longer time-interval. Since in the h=0 points (21) may yield infinite S via introducing a small positive value ε instead of its application that of the following approximation sems to be more expdient:

$$S(\tau) := -sign(h(\tau))\frac{\dot{h}(\tau)^2}{\varepsilon + |h(\tau)|} - \rho\dot{h}(\tau) \tag{23}$$

which for $|h| \rangle\rangle \varepsilon$ well approximates (21) and only for small h differs from it significantly. In choosing $G(\xi)$ we have a great extent of freedom. For instance, $G(\xi)$ can be the member of the set of the basic functions of fast decrease $\mathcal{D}$ on which the generalized functions (distributions) are defined as linear functionals [15]. These infinitely many times continuously differentiable functions have arbitrary but finite support guaranteeing exactly zero $G(-t)$ if t is greater than the upper limit of the support of this function.

For practical control applications further possibilites are offered by functions of simple parametric forms in which the parameters have lucid physical interpretation. Within the frames of the chosen form these parameters can be tuned in order to achieve "optimal" behavior. For instance, in the following choice with $D>0$, $\beta>0$

$$G(\xi):=\begin{cases}\beta e^{\beta D} e^{\beta\xi} \ \mathit{if} \ \xi \le -D \\ 0 \ \mathit{otherwise}\end{cases} \tag{24}$$

satisfies the general restrictions imposed in (6). Parameter D can be interpreted as the *delay time* of the moderation of the originally prescribed error relaxation. This function takes the value β at ξ=-D, it cannot be differentiated only in in ξ=-D. For illustrative purposes the behavior of the function $x_1(t)$ is described in Figure 1 for an appropriate parameter settings:

The figures well illustrate the expectation that over the wedge the 2nd derivative of V is negative while the error is great. The calculations reveal that the small error approximately exponentially approaches zero and that this exponent can be estimated as the slope of the wegde's lines. In the forthcoming part the method is used for active vibration damping. The application of the practically proposed (23) instead of the "theoretically desirable" (21) reveals itself in the "underestimation" of the coefficient of the term quadratic in dh/dt. Therefore in this example the phase curve of V cannot reach the wedge and d^2V/dt^2 becomes positive "over the wedge".

Furthermore, in (5) the presence of $-\nu h(t)$ near the "moderating integral" proportional to $\mu>0$ can also suggest a "physical interpretation", i.e. the application of a special case of the linear functionals, of some singular distribution in which the core function is similar to a Dirac delta. (From mathematical point of view this statement is not rigorously correct because all the theory of the distributions is based on the use of the functions belonging to $\mathcal{D}$ as the domain of definition, and strongly utilizes their zero value at the limits of their supports while operating with partial integration. Our error function $h(t)$ does not belong to $\mathcal{D}$ in a rigorous sense.) However, it can be expected that good results can be achieved if $\nu=0$ and $\mu<0$ are applied. This means a kind of elimination of the term belonging to the "singular" functional, and the direct use of a regular one for control purposes instead of moderating the action of the singular one. Appropriate counterparts of the results displayed in the 1st row of Figure 1 are given in Figure *2*. In this case the upper side of the wedge on the phase diagram of V becomes a horizontal line and essentially an error-relaxation similar to that of Figure 1 is obtained.

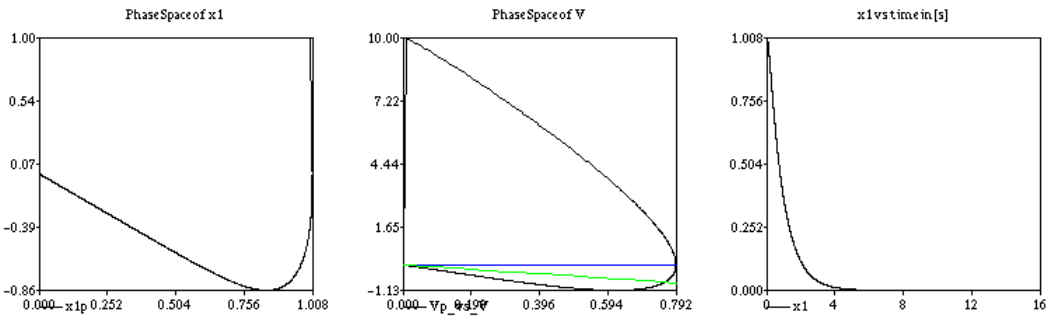

Figure 2: Typical behavior of the solution of (5) using (23) and (24) for h_0=1, $\dot{h}_0 = 1$, $D = 2$ *ms*, ν=0 s^{-2}, μ= -100 s^{-2}, ρ=100 s^{-1}, ε=10^{-6} *m*, and $exp(\beta \times 10^{-3}$ s)≈0.99

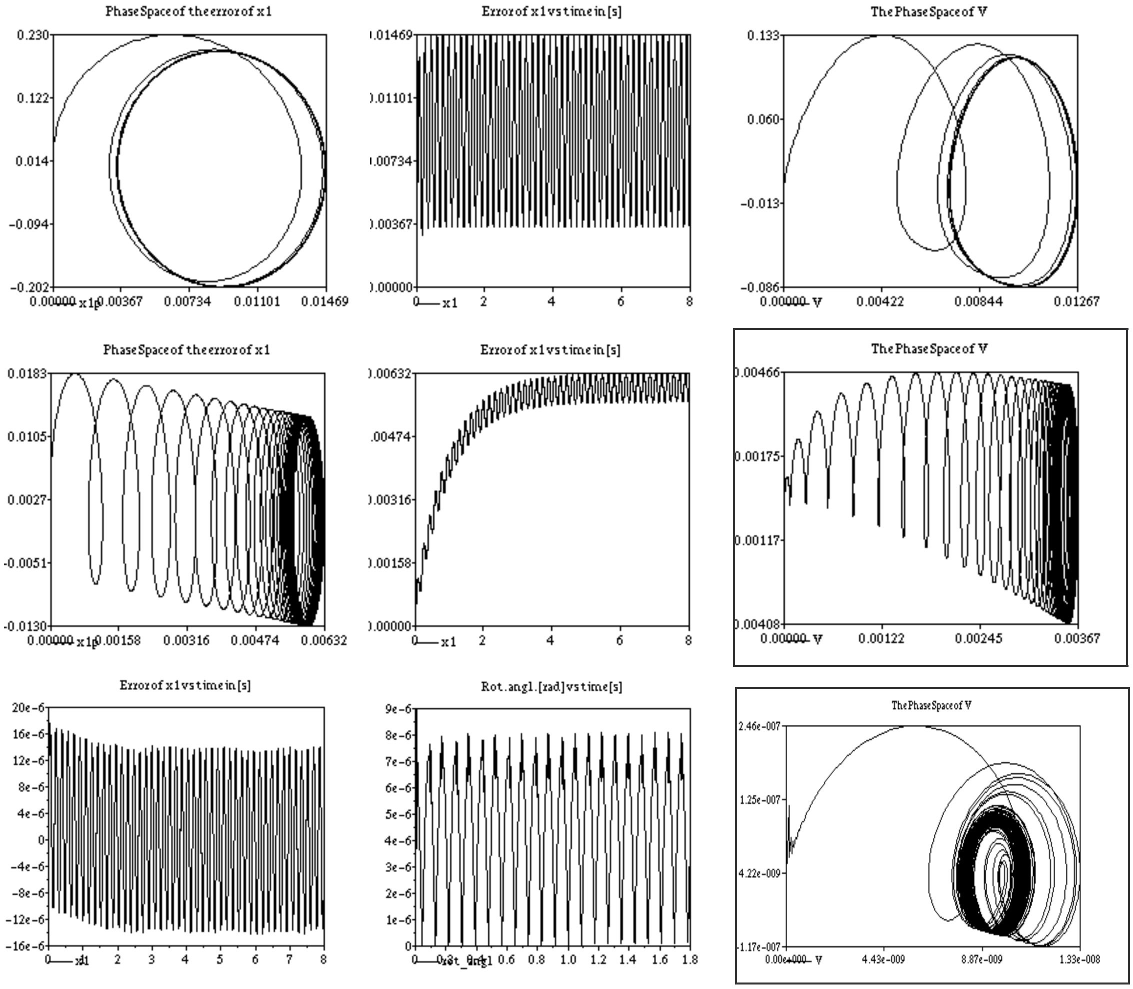

Figure 3: Vibration at the resonance frequency without active damping (upper row), with active damping with the parameters $D = 2$ ms, $\nu = 960\ s^{-2}$, $\mu = 384\ s^{-2}$, $\rho = 1.5\nu\ [s^{-1}]$, $\varepsilon = 10^{-6}$ m, and $\exp(\beta \times 10^{-3}\ s) \approx 0.99$ (middle row), and active damping with adaptivity (lower row). In this latter case instead of the phase trajectory of the error the abstract rotation angles applied by the adaptive controller are described in the figure.

3 Application to vibration control

For this purpose a simple paradigm, a damped linear oscillator consisting of a point-shaped body of mass m and a spring of spring constant (stiffness) k is considered. The spring's one end is fixed on the ceiling, and another spring is attached to it below, the end of which externally is moved along a prescribed height vs. time function (external excitation). This paradigm does contain some viscous damping, therefore it is expected that at its eigenfrequency the amplitude of its vibration is bounded. The results are given in Figure 3.

It can be seen that even the non-adaptive active vibration control designed on simple kinematic prescriptions shrinks the amplitude of the vibration decrease from the [3.67 *mm*, 14.69 *mm*] interval to the [~5 *mm*, ~6.5 *mm*] region. From purely mathematical point of view the adaptive control can be formulated as follows. There is given some imperfect model of the system on the basis of which some excitation is calculated to obtain a desired system response $\mathbf{i}^d$ as $\mathbf{e} = \varphi(\mathbf{i}^d)$. The system has its inverse dynamics described by the unknown function $\mathbf{i}^r = \psi(\varphi(\mathbf{i}^d)) = f(\mathbf{i}^d)$ and resulting in a realized response $\mathbf{i}^r$ instead of the desired one, $\mathbf{i}^d$. Normally one can obtain information via observation only on the function $f()$ considerably varying in time, and no any possibility exists to directly "manipulate" the nature of this function: only $\mathbf{i}^d$ as the input of $f()$ can be “deformed” to $\mathbf{i}^{d*}$ to achieve and maintain the $\mathbf{i}^d = f(\mathbf{i}^{d*})$ state. [Only the *model function* $\varphi()$

can directly be manipulated.] On the basis of the modification of the method of renormalization widely applied in Physics the following "scaling iteration" was suggested for finding the proper deformation:

$$\begin{gathered}\mathbf{i_0};\mathbf{S}_1\mathbf{f}(\mathbf{i_0})=\mathbf{i_0};\mathbf{i}_1=\mathbf{S}_1\mathbf{i_0};\ldots;\mathbf{S}_\mathrm{n}\mathbf{f}(\mathbf{i}_{n-1})=\mathbf{i_0};\\ \mathbf{i}_{n+1}=\mathbf{S}_{n+1}\mathbf{i}_n;\mathbf{S}_\mathrm{n}\xrightarrow[n\to\infty]{}\mathbf{I}\end{gathered} \tag{25}$$

in which the $\mathbf{S}_n$ matrices denote some linear transformations that map the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached therefore the controller „learns" the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (25) does not unambiguously determine the possible applicable quadratic matrices, we have additional freedom in choosing appropriate ones. At the present application a simple rotation was applied that turns the observed vector to the direction of the desired one in (25). Following that an appropriate shrink/dilatation is applied in only in the direction of the rotated vector to make the two vectors exactly equal to each other. This shrink/dilatation leaves the orthogonal subspace of the rotataed vector invariant. The appropriate rotational angles are described in Figure **3**. All the other algebraic and convergence aspects are detailed in the papers cited in the introduction. Returning back to the simulation examples due to the external disturbances the active controller designed on purely kinematic basis cannot realize this design. The adaptive law helps the active controller to approach the original "kinematic design" in a far more accurate manner. This shrinks the range of the vibration to the $\pm 1.6\times 10^{-5}$ *m*, which is a further drastic improvement.

4 Conclusions

In this paper a simple control design was proposed for vibration control purposes. The approach is inspired by the *CRONE* method and tackles the problem from the basis of linear control. Replacing the numerical approximation of the fractional derivatives with a more general concept of time-invariant Green functions, and via the application of a nonlinear feedback term it keeps the design of the desired damping within the simple realm of kinematic considerations. Following that the controller's adaptive nature guarantees the realization of this kinematic design. In this manner the burden of designing a controller based on more sophisticated linear considerations can be evaded and pushed to the controller. The applicability of the approach was illustrated via simulations based on the behavior of a simple paradigm, a passively damped linear oscillator near its resonance frequency. Since the operation of the adaptive controller simlpy consists of mapping the observed behavior to the desired one, and no any effort is invested into identifying the physical reasons that could explain these differences (e.g. inaccurate modeling and simultaneous external disturbances not modeled by the controller) it is also applicable when the observed system has internal degrees of freedom neither modeled nor controlled. On this basis it is expected that this approach can be useful for solving even more complicated vibration damping problems.

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