

# Robust energy scheduling for smart buildings considering uncertainty in PV generation

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**Abstract**—The fast growth of renewable energy sources in the residential building led to a complex problem related to the energy management system: the uncertainty associated with the forecast of photovoltaic power generation. To solve this challenge, this paper proposes a robust optimization model to obtain the optimal solution for the worst-case scenario of photovoltaic generation. A Mixed Binary Linear Programming problem is transformed into a trackable robust counterpart to provide immunity against the worst-case realization. Through the budget of uncertainty, the risk of the solution can be adjusted. The results demonstrate that the influence of Battery Energy Storage System and Electric Vehicles against uncertainties leads to higher economic gains up to 6% reduction.

**Index Terms**—Energy scheduling, PV Uncertainty, Robust Optimization, Smart Building

## I. INTRODUCTION

The increase of energy demand especially in residential buildings has led to a higher focus on energy efficiency methods [1]. As a result, the use of renewable energy sources in buildings is in rapid growth, allowing domestic consumers to benefit from more efficient and economic energy usage.

In this context, the home energy management system (EMS) arises to control small-scale energy resources of a building [2], such as Photovoltaic (PV) generation systems, Electric Vehicles (EVs), and Battery Energy Storage System (BESS). The EMS allows a significant bill decrease throughout efficient scheduling [3]. The use of BESS and EVs can be beneficial for energy management, especially during periods when there is no PV power generation [4].

However, renewable sources are characterized by a high level of uncertainty and variability and Robust Optimization (RO) is an effective technique to deal with uncertainty. For decision-making frameworks affected by uncertainty and where the decision-maker lacks complete information, RO models are proposed [5]. RO uses uncertainty sets to describe the uncertainty of parameters. This technique provides optimal solutions for the worst-case realization of the uncertain parameter [6].

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Several studies about the robust optimization problem under uncertainty have been stated in the recent literature. [7] proposes an adjustable robust optimization model to participate in day-ahead energy markets, considering uncertainty in energy prices, PV generation, and load. The results show that the robust formulation achieves a cost reduction of 5.7% in comparison with the deterministic solution. A robust approach is developed in [8] to deal with the uncertainty of PV power output regarding the load scheduling of a smart home. Further, the robust formulation is transformed into an equivalent quadratic programming problem. The simulation results confirm the validity and advantage of the proposed technique. [9] proposes a robust optimization model considering the randomness of electric and thermal loads and solar power generation and the coordination of several energy sources, such as electric grid, battery, and combined heat and power (CHP). The results demonstrate that the effectiveness of the CHP unit and battery in mitigating the influence of uncertainties in the operation of energy resources of the building.

A Mixed Binary Linear Programming (MBLP) problem was initially formulated to solve the energy resources scheduling problem. This paper proposes the transformation of the deterministic model in a robust optimization problem to deal with the uncertainty of PV power generation outputs. It is more common to find in the literature the use of a stochastic optimization model to solve these problems, so we intend to bring a new overview with RO. The main objective of the robust developed model is to minimize the energy costs of the residential building within the given uncertainty bounds. The conservatism of the solution can be adjusted by selecting an appropriate value of the budget of uncertainty.

This paper is organized as follows. In Section II, the problem statement is presented with a brief review about Robust Optimization and a description of the problem. Section III formulates the robust scheduling optimization problem. Section IV characterizes the scenarios of this case study and the results are discussed. Lastly, Section V draws the conclusions.

## II. PROBLEM STATEMENT

This section presents the problem statement, with a brief review of RO. Then, the optimization problem and the technique used to forecast the PV power generation are described.

### A. Robust Optimization

There are several uncertainty modeling techniques to deal with uncertainty in optimization problems, such as probabilistic approaches and robust optimization [6].

Stochastic Programming (SP) is a probabilistic approach and was the first method developed to deal with uncertainty [10]. However, this method presents some limitations, such as computational burden because the solution depends on the accuracy of the scenario generation technique. SP needs information about the probability density function of uncertainty [11].

Due to the SP constraints, RO gained popularity among the uncertainty modeling methods. The main objective of RO is to find the worst-case scenario that the energy management system might face, which is defined by the uncertainty set. RO focuses on minimizing the impact of the worst-case scenario, i.e., the obtained solutions are the best among the worst [6], [12].

The advantages of using RO are as follows [11]:

- Immunizes a solution against all possible realizations of the uncertain parameters within the uncertainty bounds;
- Formulate tractable models even for large problems without causing computational burden;
- Do not need to assume probability distributions;
- Only needs information about the upper and lower bounds of the uncertainty parameters.

### B. Problem description

This problem consists of optimizing the energy scheduling and energy costs of the EMS of a residential building composed of 6 apartments with a contracted power of 6.9 kVA. The building is equipped with a PV generation system, a BESS, and EVs.

An overview of the buildings' configuration is visualized in Fig. 1, in which the arrows indicate how the power flows from and to the building. The green arrows represent the energy supply and the red the energy demand.

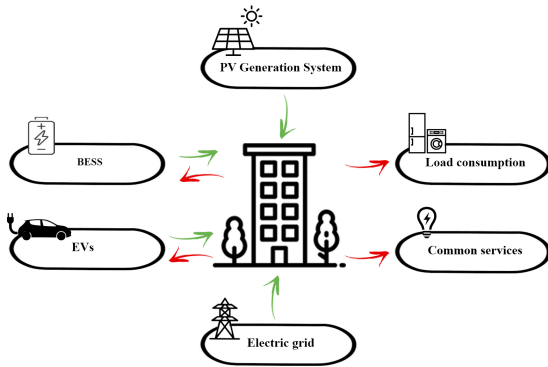


Fig. 1. Building's energy resources managed by the aggregator.

To define the model of the building energy resource management, some assumptions are made:

- An aggregator is responsible to manage all the building energy resources and for the energy exchanges between the building and the grid;
- The building is connected to the grid;
- Each apartment has its PV panel and EV;
- Each EV only charges once a day;
- The BESS and the EV's batteries can be charged or discharged;
- The power generated from PV panels is used to supply load demand from the apartments, charging EV batteries and the BESS.

For this problem, it is used a real dataset from 2019 of PV power generation, the energy consumption of each apartment, EV consumption, and energy consumption of common services.

### C. PV generation forecast

To optimize and manage the building energy management system, it is necessary to predict the PV power generation output. For this purpose, a multi-layer feed-forward Artificial Neural Network (ANN) is implemented. In this ANN topology, the information moves in only one direction (forward) through all the layers and, this way, there are no cycles or loops in the network.

Figure 2 shows the PV power generation forecast simulation results for the first day of September 2019. It is possible to observe that the curves of the actual and forecasted values are very similar, presenting low forecast errors.

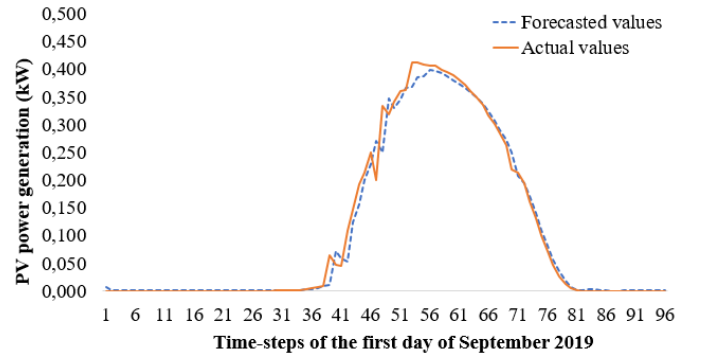


Fig. 2. Forecasting results for the first day of September 2019.

Further details of the forecast model are not going to be discussed because they are not the major focus of this paper.

## III. ROBUST OPTIMIZATION FORMULATION

The RO approach applied in this work is based on the theory presented in [11]. The deterministic model used is developed in [13] and was formulated to optimize the energy resource management of the residential building. Then, it is transformed into a robust counterpart whose main objective is to minimize the energy costs of the building considering PV uncertainty.

Figure 3 describes the essential stages of the RO formulation.

First, the uncertainty set is built with the values of the PV production forecast, and the upper and lower bounds are

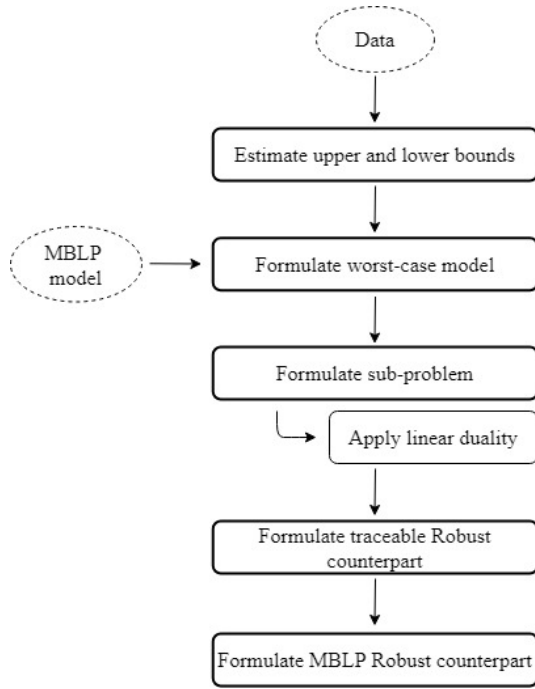


Fig. 3. Methodology used for the Robust Optimization formulation.

estimated. The deterministic model was already formulated, so the next step is to transform it into the worst-case model. The worst-case model contains a sub-problem that can be solved applying the linear duality theory. Finally, a tractable Robust counterpart is formulated to overcome the obstacles of uncertainties and it is included in the MBLP model.

#### A. Estimation of upper and lower bounds of PV forecasting

Before the formulation of the worst-case model realization, it is necessary to formulate the uncertainty bounds. Based on several literature, it was considered that the deviation range from the forecasted values ( $\Delta_{PV}$ ) is 20%. So, the upper and lower bounds will deviate 20% from the forecasted values [11].

To calculate the upper and lower bounds, (1) and (2) are used. To bound the uncertainty set, the prediction results are summed or subtracted by a "discount" of a certain percentage of the forecasted values.

$$\overline{P_{PV}} = \hat{P}_{PV} + \Delta_{PV} \cdot \hat{P}_{PV} \quad (1)$$

$$\underline{P_{PV}} = \hat{P}_{PV} - \Delta_{PV} \cdot \hat{P}_{PV} \quad (2)$$

where  $\Delta_{PV}$  is the deviation from the forecasted values of PV output.  $\overline{P_{PV}}$ ,  $\underline{P_{PV}}$  represents the lower and upper bounds of PV forecast, respectively.  $\hat{P}_{PV}$  is the forecasted value of PV power generation.

Fig. 4 shows the upper and lower bounds of PV power output forecasting, considering  $\Delta_{PV} = 20\%$ .

In this paper, the PV generation can reach its lower bound because that corresponds to the worst-case realization (maximum decrease of PV power generation).

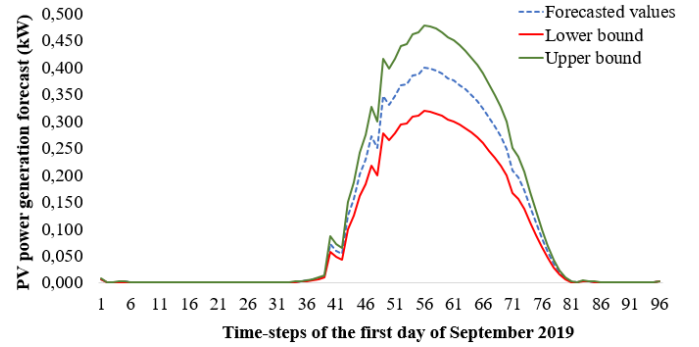


Fig. 4. Upper and lower bounds of PV power forecasting.

#### B. Uncertainty set

First, it is necessary to build an uncertainty set (denoted as  $U$ ), which is used to describe the uncertainties of PV power generation outputs, as shown in (3).

$$U = \left\{ \begin{array}{l} \hat{P}_{PV}(i, j) - \Delta_{PV}(i, j) \leq P_{PV}(i, j) \leq \hat{P}_{PV}(i, j) + \Delta_{PV}(i, j), \quad \forall i, j \\ \sum_i \frac{|P_{PV}(i, j) - \hat{P}_{PV}(i, j)|}{\Delta_{PV}(i, j)} \leq \Gamma_{PV}(j), \quad \forall j \end{array} \right\} \quad (3)$$

where  $P_{PV}(i, j)$  is the actual PV output which can vary within the limits  $[\hat{P}_{PV}(i, j) - \Delta_{PV}(i, j), \hat{P}_{PV}(i, j) + \Delta_{PV}(i, j)]$ .  $\Gamma_{PV}$  is the budget of uncertainty and it is used to adjust the level of robustness or conservatism of the solution.

#### C. Load balance equation

According to the deterministic model, the load balance constraint is subject to uncertainty. This constraint should be met when the worst scenario of uncertainties occurs, i.e., at the maximum decrease in the PV power generation.

According to this, the worst-case load balance is given by  $g(x)$  in (4).

$$g(x) = P_{G-A}(i) + \sum_{j=1}^J P_{EV-A}(i, j) + \sum_{j=1}^J P_{B-A}(i, j) + \sum_{j=1}^J P_{PV}(i, j) - P_{A-G}(i) - \sum_{j=1}^J P_A(i, j) - \sum_{j=1}^J P_{A-EV}(i, j) - \sum_{j=1}^J P_{A-B}(i, j) - P_{CS}(i) \quad (4)$$

where  $P_{G-A}$  is the power from the grid to the aggregator,  $P_{EV-A}$  is the power from the EV discharging process,  $P_{B-A}$  is the power from the BESS discharging process.  $P_{A-G}$  represents the power that the aggregator consumes from the grid,  $P_A$  is the power from each apartment and  $P_{CS}$  is the power consumed from the common services.  $P_{A-EV}$  and  $P_{A-B}$  correspond to the power consumed by the EV and the BESS, respectively, during the charging process.

#### D. Sub-problem and dual

After transforming the deterministic model into the worst-case model, the next step is to formulate the sub-problem and find the dual of the sub-problem. The sub-problem creates the worst scenario and the dual minimizes its impact.

The first objective is the maximization of the uncertainty factor ( $P_{PV}$ ) contained in the load balance equation. To formulate the sub-problem, it is required to transform (4) to the objective function  $g(x)$  and define the uncertainty bounds as constraints.

The formulation of the sub-problem is characterized in (5) and (6).

$$\text{maximize} = g(x) \quad (5)$$

s.t.

$$P_{PV} \in [P_{PV}, \overline{P_{PV}}] \quad (6)$$

Then, the sub-problem is converted into a dual problem and the linear duality theory is applied to solve it. The formulation of the dual of the sub-problem is characterized in (7) and (8), which is the dual objective function of the sub-problem.

$$\text{minimize} = \lambda^{\overline{P}} \cdot (P_{PV} - \underline{P_{PV}}) + \lambda^{\underline{P}} \cdot (\overline{P_{PV}} - P_{PV}) \quad (7)$$

s.t.

$$\lambda^{\overline{P}}, \lambda^{\underline{P}} \geq 0 \quad (8)$$

where  $\lambda^{\overline{P}}$ ,  $\lambda^{\underline{P}}$  are the dual variables for PV output.

#### E. Robust Counterpart

A robust counterpart is formulated to overcome the drawbacks introduced by uncertainty. The formulation of the robust counterpart consists of adding the sub-problem and dual as constraints of the MBLP model of [13].

### IV. SCENARIOS AND RESULTS

In this section, the methodology proposed in Section III is implemented and evaluated. The two scenarios under study for this work are described, and the respective results are presented.

#### A. Case study: Energy Smart Management considering PV Uncertainty

A case study for the energy resource management of a residential building was used to assess the proposed mathematical formulation, to minimize the energy costs of the building. To visualize the impact of PV uncertainty, 2 scenarios have been simulated in this paper.

In scenario 1, the developed robust scheduling problem is tested on the building energy system proposed in Fig. 1.

In this scenario, the following considerations are made:

- Each apartment can consume the PV power generation;
- The EV's battery can charge/discharge to its associated apartment.

In scenario 2, the only difference from scenario 1 is that a BESS is used.

#### B. Results

The RO scheduling results were obtained for the worst-case scenario. In this situation, the worst-case occurs when there is a maximum decrease of the PV power production, based on the condition that  $\Gamma_{PV}(j) = 1$ .

The analysis has been performed for a 24-hour scheduling horizon with a 15-minute interval, which corresponds to 96 periods. All the numerical simulations have been implemented in MATLAB.

Fig. 5 and Fig. 6 presents the scheduling results of the energy management system regarding scenario 1 and 2, respectively, considering the worst-case scenario.

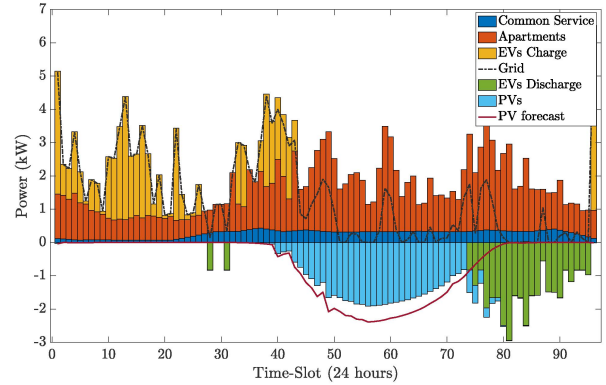


Fig. 5. Energy management system scheduling for scenario 1 considering the worst-case scenario.

It is possible to visualize in Fig. 5 that the EV's charge at low demand periods, during the night. Their discharging process occurs at higher demand periods, reducing the energy consumed by the grid. In some periods during the daytime, the PV power generation can meet most of the load demands, so the energy from the grid is lower.

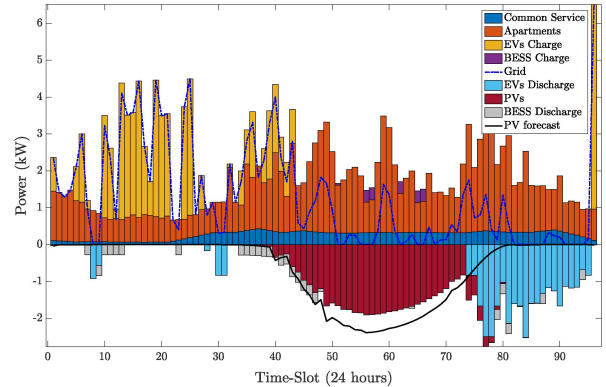


Fig. 6. Energy management system scheduling for scenario 2 considering the worst-case scenario.

Analyzing Fig.6, it is possible to observe that in some periods during the night, the EVs and the BESS are scheduled

to discharge for demand supply, even though the energy demand from apartments and common services is low. In some periods throughout the day, the energy demand is low and the PV power generation is high and, as a result, the BESS stores the surplus energy to be later discharged at periods with higher demand. The same happens to EVs.

### C. Analysis of budget of uncertainty

For this case study, it is considered 5 different values for the budget of uncertainty ( $\Gamma_{PV}(j)$ ), in a range from 0 to 1. The maximum value of  $\Gamma_{PV}(j)$  depends on the number of uncertain variables. In this case,  $\Gamma_{PV}(j) = 1$  because it is only considered one uncertainty source (PV power generation).

Table I presents the energy management system scheduling costs under five different budgets of uncertainty for the first day of September 2019, regarding scenarios 1 and 2.

TABLE I  
EMS SCHEDULING COSTS UNDER DIFFERENT  $\Gamma_{PV}(j)$ .

$\Gamma(j)$	Total cost for scenario 1 (€)	Total cost for scenario 2 (€)
0	19.12	17.80
0.25	19.41	18.17
0.5	19.73	18.57
0.75	20.09	18.98
1	20.47	19.40

For both scenarios, as the value of  $\Gamma_{PV}(j)$  increases, the total energy costs also increase. When  $\Gamma_{PV}(j) = 0$  no uncertainty is considered, so the real values of the PV generation are equal to the forecasted values.

When  $\Gamma_{PV}(j) = 1$ , the protection against uncertainty in solar power generation is completely ensured. The scheduling costs are higher due to the consideration of the worst scenario because when there is a maximum decrease of PV generation, the aggregator needs to buy energy from the grid to meet the demand. As a result, the EMS is scheduled to purchase more electricity from the grid to immunize it against PV uncertainty. In this case, each PV can reach its lower bound given in  $U$ , which might be over-conservative and lead to unnecessary costs.

Besides the fact that a higher value of  $\Gamma_{PV}(j)$  leads to higher conservatism solutions and, consequently, higher economic costs, it provides a better risk performance.

Analysing the results of both scenarios, it is noted that scenario 2 has lower energy costs than scenario 1. This way, it is possible to conclude that the use of a BESS leads to a decrease of almost 6% of the total costs in comparison with scenario 1, in which only EVs are used.

The best solution is to set  $\Gamma_{PV}(j)$  as a value within the range [0,1] to allow a compromise between the robustness and conservatism of the optimal solution. The decision-maker can choose a suitable value of  $\Gamma_{PV}(j)$  to adjust the robust scheduling.

## V. CONCLUSIONS

In this paper, a robust optimization method was proposed to manage the scheduling of the energy management system

of a residential building, to deal with the uncertainties of solar power forecast. The definition of RO has based on the realization of the worst-case scenario that in this case is the maximum decrease of the PV power generation.

The conservatism level of the solution against forecast errors can be adjusted by the budget of uncertainty. The simulation results show that the total energy costs for both scenarios increase as the budget of uncertainty also increases. Also, the use of BESS along with EVs offers a reduction of almost 6% of the energy cost for the building consumers, in comparison with scenario 1.

Therefore, a trade-off between the robustness and the energy costs can be managed by the decision-maker to achieve a good economic cost considering PV uncertainty.

## REFERENCES

- [1] Z. Luo, W. Gu, Z. Wu, Z. Wang, and Y. Tang, "A robust optimization method for energy management of CCHP microgrid," *J. Mod. Power Syst. Clean Energy*, vol. 6, no. 1, pp. 132–144, 2018, doi: 10.1007/s40565-017-0290-3.
- [2] Z. Foroozandeh, S. Ramos, J. Soares, Z. Vale, and M. Dias, "Single contract power optimization: A novel business model for smart buildings using intelligent energy management," *Int. Journal of Elect. Power Energy Sys.*, vol. 135, pp. 107534, 2022, doi: 10.1016/j.ijepes.2021.107534.
- [3] P. Zhao, H. Wu, C. Gu, and I. Hernando-Gil, "Optimal home energy management under hybrid photovoltaic-storage uncertainty: A distributionally robust chance-constrained approach," *IET Renew. Power Gener.*, vol. 13, no. 11, pp. 1911–1919, 2019, doi: 10.1049/iet-rpg.2018.6169.
- [4] Z. Foroozandeh, S. Ramos, J. Soares, and Z. Vale, "Energy management in Smart Building by a Multi-Objective Optimization Model and Pascoletti-Serafini Scalarization Approach," *Processes*, vol. 9, no. 2, 2021, doi: 10.3390/pr9020257.
- [5] M. Ghazvini, J. Soares, H. Morais, R. Castro, and Z. Vale, "Dynamic Pricing for Demand Response Considering Market Price Uncertainty," *Energies*, vol. 10, no. 9, 2017, doi: 10.3390/en10091245.
- [6] M. Aien, A. Hajebrahimi, and M. Fotuhi-Firuzabad, "A comprehensive review on uncertainty modeling techniques in power system studies," *Renew. Sustain. Energy Rev.*, vol. 57, pp. 1077–1089, 2016, doi: 10.1016/j.rser.2015.12.070.
- [7] C. A. Correa-Florez, A. Michiorri, and G. Kariniotakis, "Robust optimization for day-ahead market participation of smart-home aggregators," *Appl. Energy*, vol. 229, no. July, pp. 433–445, 2018, doi: 10.1016/j.apenergy.2018.07.120.
- [8] C. Wang, Y. Zhou, B. Jiao, Y. Wang, W. Liu, and D. Wang, "Robust optimization for load scheduling of a smart home with photovoltaic system," *Energy Convers. Manag.*, vol. 102, pp. 247–257, 2015, doi: 10.1016/j.enconman.2015.01.053.
- [9] P. Liu and Y. Fu, "Optimal operation of energy-efficiency building: A robust optimization approach," *IEEE Power Energy Soc. Gen. Meet.*, no. 1, pp. 1–5, 2013, doi: 10.1109/PESMG.2013.6673050.
- [10] H. Bakker, F. Dunke, and S. Nickel, "A structuring review on multi-stage optimization under uncertainty: Aligning concepts from theory and practice," *Omega (United Kingdom)*, vol. 96, p. 102080, 2020, doi: 10.1016/j.omega.2019.06.006.
- [11] A. Hussain, V. H. Bui, and H. M. Kim, "Robust optimization-based scheduling of multi-microgrids considering uncertainties," *Energies*, vol. 9, no. 4, 2016, doi: 10.3390/en9040278.
- [12] M. Beaudin and H. Zareipour, "Home energy management systems: A review of modelling and complexity," *Renew. Sustain. Energy Rev.*, vol. 45, pp. 318–335, 2015, doi: 10.1016/j.rser.2015.01.046.
- [13] Z. Foroozandeh, S. Ramos, J. Soares, F. Lezama, Z. Vale, A. Gomes, and R. L. Joenck, "A Mixed Binary Linear Programming Model for Optimal Energy Management of Smart Buildings," *Energies*, vol. 13, no. 7, p. 1719, doi: 10.3390/en13071719 .Apr. 2020.