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On the Statistical/Harmonic Modelling of Mechanical Manipulators

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1. Introduction

Mechanical manipulators are developed according to engineering and scientific principles which are based on fundamental concepts such as those arising from mathematics and physics. Based on these formulations, the first step on the study of a physical phenomena is the development of an adequate model. Usually the fundamental concepts are the differential and matrix calculus and the classical newtonian physics, while the model consists on a set of differential equations. Nevertheless, several phenomena, such as quantum physics and thermodynamics, may be studied through different mathematical tools namely using statistical methods. These facts suggest that, for a given problem, we may adopt different mathematical models, each with its own merits and drawbacks. The second step on the study of the physical phenomena is the analysis of the properties revealed by the model. For a model consisting on a set of linear differential equations we can adopt simple strategies (*e.g.* the Fourier analysis) but, for a non-linear model these tools are not adequate and the analysis becomes complex and difficult to generalise. In fact, experience demonstrates that for cases, such as the kinematic and dynamic models arising in robotics, efficient tools capable of rendering clear results are still lacking.

This paper developes a new modelling formalism based on the embedding of statistical and Fourier transform concepts. These concepts are then illustrated on several experiments. The examples reveal not only the capabilities of the new method but also the limitations of standard robot structures and path planning algorithms. Consequently, in order to develop the new formalism the paper is organised as follows. Section two starts by presenting the fundamental modelling concepts. Based on the new concepts, section three illustrates its application on the kinematic and trajectory planning analysis of mechanical manipulators. Finally, section four outlines the main conclusions.

2. Embedding Statistics and Fourier Transform for the Modelling of Robots

For a robot having n degrees of freedom (dof) the classical direct kinematic model is described by a set of equations $\mathbf{p} = \psi(\mathbf{q})$ relating the operational and the joint spaces, where $\mathbf{p}$ and $\mathbf{q}$ are the $n \times 1$ vectors of position in the operational and joint spaces, respectively. Based on these equations considerable research has been done on issues such as the optimisation of the manipulator structure [1,2] and the development of efficient path planning algorithms [3,4]. However, the kinematic equations usually are non-linear and reveal a plethora of variables and parameters that give rise to a cumbersome work both in the analysis and design stages. Therefore, in order to overcome these problems alternative concepts are required. Statistics is a mathematical tool well adapted to handle a large volume of data that has already been used in some restricted classes of robotic problems [5,6]. Nevertheless, for the kinematic modelling, statistics is not capable of dealing with time-dependent relations. Therefore, to overcome the limitations of statistics [7], the new method will also take advantage of the Fourier transform by embedding both tools in a broader formalism. In this line of thought, the first stage of the new modelling formalism starts by comprising:

- A set of input variables (*ivs*), that is, variables that are free to change independently.
- A set of output variables (*ovs*), that is, variables that depend on the *ivs*.
- A set of parameters to be optimised in the design stage.

As usually, in the direct kinematics the *ivs* and *ovs* are established by the relation $\psi: \{\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}\} \rightarrow \{\mathbf{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}}\}$, while for the inverse kinematics we get the reverse relation. In both cases, the set of parameters depend on the manipulator structure and the time/space evolution of the trajectories.

The second stage of the formalism consists on the embedding of the statistical analysis into the Fourier transform through the algorithm:

i) A statistical sample for the variables is obtained by driving the manipulator through a large number of trajectories (generated according with adequate statistics) having appropriate time/space evolutions. All the variables (*i.e.* the *ivs* and the *ovs*) are calculated, sampled in the time domain, and the resulting numerical values are stored in arrays.

ii) For each of the previous arrays, the Fourier transform is computed and the corresponding frequency spectrum is stored in a second class of arrays.

iii) After concluding the statistical sample of trajectories, for all the variables and for each frequency within the spectrum range under study, several statistical indices of the amplitudes and/or phases of the arrays obtained in *ii)* are calculated. The statistics of the frequency spectrum is stored in a third class of arrays.

iv) For all the variables and for each frequency, the values of the statistical indices calculated in *iii)* are collected on a 'composite' frequency spectrum and stored in a forth class of arrays.

v) The procedure *i)* to *iv)* is repeated for different numerical values of the link lengths. The numerical results for the forth class of arrays obtained in *iv)* are compared and analytical expressions, that fit the numerical data, are extrapolated.

vi) The algorithm *i)* to *v)* is repeated for different time/space trajectories.

vii) The algorithm *i)* to *vi)* is repeated for different robot structures.

viii) The partial conclusions drawn by the analytical expressions obtained in *v)*, *vi)* and *vii)* are integrated and final conclusions are drawn.

3. A New Model for the Robot Kinematics

The application of the formalism defined previously requires the development of numerical calculations for the statistics. Therefore, before proceeding, we need to establish the different experiments according with the guidelines:

i) Modelling case: *IK* = {Inverse Kinematics}
DK = {Direct Kinematics}

ii) Type of trajectory

ii.1) Time acceleration profile:
O = {On/Off acceleration},
T = {Triangular acceleration},
P = {Parabolic acceleration},
S = {Sinusoidal acceleration}

ii.2) Total time definition:
MAL = {Maximum Acceleration Limitation},
MVL = {Maximum Velocity Limitation},
RAL = {Random Acceleration Limitation}

ii.3) Space evolution:
SL = {Straight Line},
DP = {Direct Parabolic},
IP = {Inverse Parabolic}

iii) Type of robot mechanical structure:
RR = {joint 1 Rotational, joint 2 Rotational},
RP = {joint 1 Rotational, joint 2 Prismatic}

The type of "time acceleration profile" leads to different formulae for the "total time definition" according with:

Acceleration $O \propto MAL$: $t_{max} = 2\sqrt{dist/A_{max}}$;

$O \propto MVL$: $t_{max} = 2\,dist/V_{max}$;

$O \propto RAL$: $t_{max} = 2\sqrt{dist/random(A_{max})}$

Acceleration $T \propto MAL$: $t_{max} = \sqrt{8dist/A_{max}}$;

$T \propto MVL$: $t_{max} = 2\,dist/V_{max}$;

$T \propto RAL$: $t_{max} = \sqrt{8\,dist/random(A_{max})}$

Acceleration $P \propto MAL$: $t_{max} = \sqrt{8dist/A_{max}}$;

$P \propto MVL$: $t_{max} = 2\,dist/V_{max}$;

$P \propto RAL$: $t_{max} = \sqrt{8\,dist/random(A_{max})}$

Acceleration $S \propto MAL$: $t_{max} = \sqrt{2\pi dist/A_{max}}$;

$S \propto MVL$: $t_{max} = dist/V_{max}$;

$S \propto RAL$: $t_{max} = \sqrt{2\pi dist/random(A_{max})}$

where $0 \le random(A_{max}) \le A_{max}$, *dist* is the total distance along the trajectory, t_{max} is the total time of duration of the movement and A_{max} and V_{max} are the maximum allowed values for the acceleration and the velocity, respectively.

For example, Figs. 1 and 2 show the Statistics of the Harmonic Content (SHC) of several variables for a sample of 500 trajectories and the experiment {*IK-RR*, *O*, *MAL*, *SL*} with $l_1 = 1$, $l_2 = 0.1$, $A_{max} = 10$.

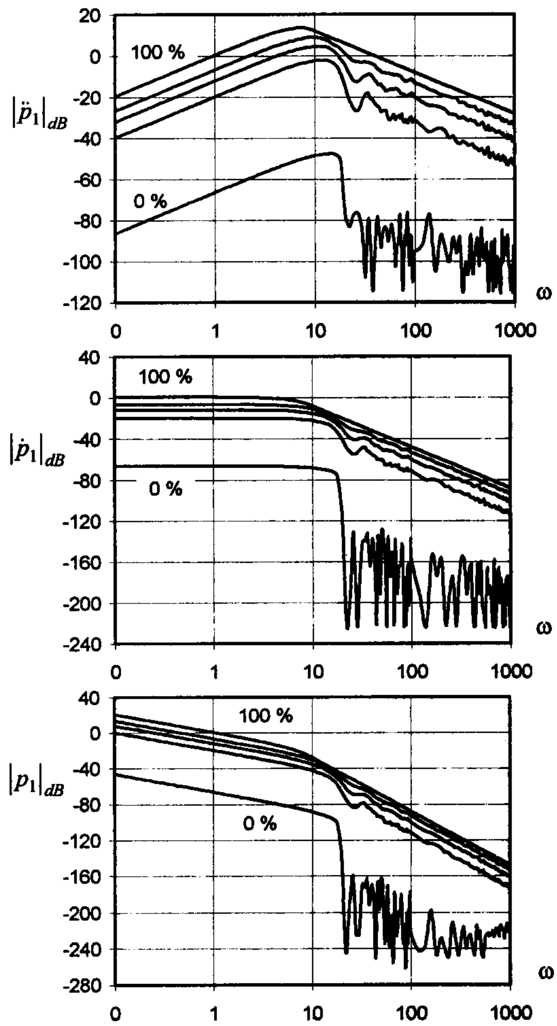

Fig. 1 Percentiles of {0,25,50,75,100} of the SHC-amplitude *versus* the frequency ω for several *ivs*.

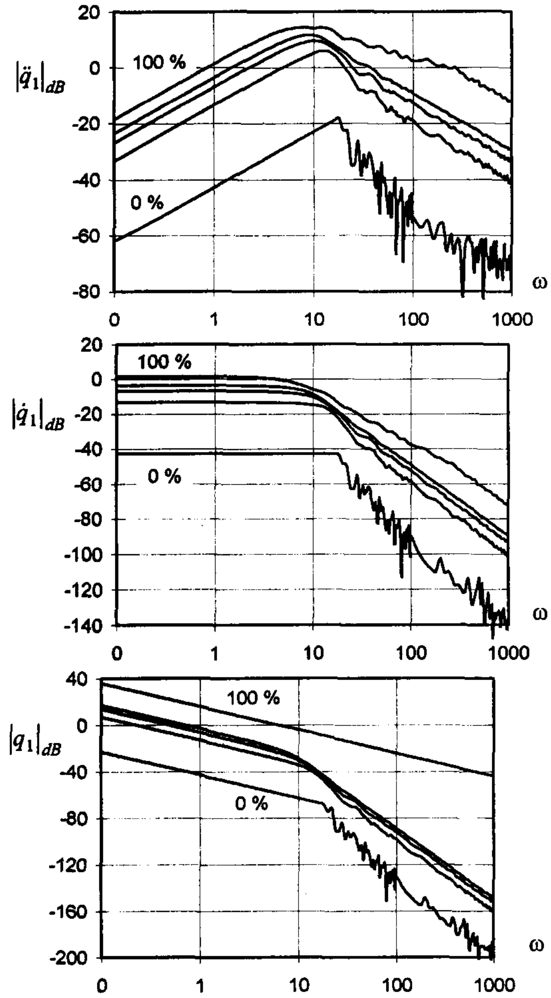

Fig. 2 Percentiles of {0,25,50,75,100} of the SHC-amplitude *versus* the frequency ω for several *ovs*.

Applying a frequency-domain identification algorithm [8] to estimate the SHCs from the different experiments, we get analytical expressions that point out the properties:

- Numerical convergence - After repeating a large number of numerical experiments the charts with the SHCs do not change significantly.
- Derivative/integral sensitivity - Although being composite curves, the SHC still obey the 'standard' *jω* operator for variables that have a time-derivative relation.
- Analytical coherence - The numerical data that results from the experiments 'fits' the analytical expressions that lead to clear conclusions. For example, the *DK*, that calculates the trajectories in the joint space, leads to expressions for the poles that do not depend on the length of the workspace, while the *IK*, that calculates the trajectories in the operational space, leads to poles that are sensitive to the workspace length.
- Generality - While for the classical models we can not find clear relations between different robot structures, with the SHC general characteristics are highlighted. For example, joint 1 of the RR robot and joint 1 of the RP structure reveal almost similar SHCs which reflects that, in both cases, we have a rotational joint. On the other hand, for joint 2, we have different mechanical articulations in the two cases and, therefore, the experiments lead to distinct results.
- Compatibility - The conclusions based on the analysis of the SHC are coherent with the results of previous studies using different mathematical tools [1,2,8]. For example, if $l_1 + l_2 =$ constant we verify that the maximum gain and bandwidth of the SHC occurs for $l_1 = l_2$. Also, the 'lower-elbow' and 'upper-elbow' solutions for the *IK* where analysed revealing similar properties.

4. Conclusions

A new method for the analysis and design of manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. The algorithm, by embedding the statistical analysis into the Fourier transform, gives clear guidelines towards the optimisation both of the path planning algorithm and the robot structure and a deeper understanding of the actuator limitations and requirements.

5. References

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