

Hermínio Duarte-Ramos
Amadeu Leão Rodrigues
Coordenadores

ENGENHARIA ELECTROTÉCNICA LUSO-ESPANHOLA

Volume 4

**Instalações Eléctricas
Instrumentação e Controlo
Automação e Robótica
Posters**

EDIZOON
Lisboa
1999

Multifrequency Computational Optimization of Robot Path Planning Algorithms

J. Tenreiro Machado⁽¹⁾, Carla Cordeiro⁽²⁾, L. Horváth, József Tar⁽³⁾

⁽¹⁾ Polytechnic Inst. of Porto
Dept. of Electrical
Engineering
Porto, Portugal

⁽²⁾ Modern University
Dept. of Automation and
Control Engineering
Porto, Portugal

⁽³⁾ Bánki Donát Polytechnic
Dept. of Information
Technology
Budapest, Hungary

Abstract

The computational burden of robot trajectory planning algorithms may be significant, namely when the inverse kinematics is required. Nevertheless, the variables involved have distinct bandwidths. Therefore, they have different effects upon the final result and we can adopt a multifrequency strategy for the calculations, namely using different frequencies for the robot joint positions, velocities and accelerations. In this line of thought, this paper compares a multirate trajectory planning scheme in the cartesian space in contrast with the classical monofrequency calculation strategy.

Key Words: Path planning, multirate sampling, computational load, optimization

1. Introduction

In the last decades robotics has been a major area of research and development in industrial automation [1-5]. The kinematics, dynamics and control of mechanical manipulators are presently standard issues when thinking on practical implementations. Nevertheless, these algorithms have aspects and details still unexplored that deserve further attention.

Bearing these facts in mind, this paper studies the multirate computation of the robot kinematics, namely its influence on the trajectory evolution and the associated processing time. This strategy is motivated by the different bandwidths of the signals involved (*i.e.*, positions, velocities and accelerations) that lead to distinct effects upon the resulting trajectory. However, the non-linear nature of the kinematic equations makes difficult to predict the numerical errors due to a finite sampling rate. Consequently, the definition of a optimal multifrequency scheme, establishing a compromise with a given computational load, must be analyzed. In fact, some authors

have proposed different calculation frequencies [6] for the signals without considering, neither distinct multirate schemes, nor the effects upon the final results. Therefore, a deeper investigation must take into account all the sides of the problem, before establish of a given computer implementation.

In this line of thought the paper is organized as follows. Section 2 starts by defining the statistical measures for the analysis of robot trajectory errors arising in a finite sampling evaluation. Section 3 presents the other side of the problem, that is, the computational burden associated with the kinematic algorithm. Section 4 develops optimization methods for a compromise between the trajectory errors and the corresponding computational load. Finally, section 5 outlines the main conclusions.

2. Trajectory Errors

The goal of this study consists in determining a compromise between the trajectory errors for the positions, velocities and accelerations and the corresponding computational burden, when adopting different sampling frequencies.

The work adopts a two link planar robot arm (RR) and straight-line trajectories, assuming rectangular on-off acceleration profiles. The trajectories are defined in the Cartesian space, which implies the calculation of the inverse kinematics to obtain the corresponding joint values.

Due to the infinite number of possible robot trajectories, it is used a statistical perspective to determine the root-mean-square (RMS) errors for different calculation frequencies. The statistical results of a random sample of distinct trajectories are presented in percentiles, in detriment of others statistical measures, due to their robustness, namely for experiments passing near singular points. In this perspective, it is adopted the notation ${}^{\text{RMS}}E_p(q_i)$, ${}^{\text{RMS}}E_p(\dot{q}_i)$ and ${}^{\text{RMS}}E_p(\ddot{q}_i)$ for the P-percentiles of the RMS-errors for the i th joint position, velocity and acceleration, respectively. Furthermore, it is also investigated the influence of different robot link lengths in order to determine their influence on the results.

Fig. 1 shows a typical example of the results, namely the 50%-percentiles of the RMS trajectory errors for the joint 2 velocity and acceleration *versus* the calculation frequencies. For other percentiles of the RMS errors we have charts of the same type.

As expected, the results demonstrate that the RMS error for the positions, velocities and accelerations depend only on $\{f_p\}$, $\{f_v, f_p\}$ and $\{f_a, f_v, f_p\}$, respectively. In the present case, we can obtain numerically heuristic formulae for the errors such as:

$${}^{\text{RMS}}E_{50\%}(q_i) = \frac{K_{pi}}{f_p} ; {}^{\text{RMS}}E_{50\%}(\dot{q}_i) = \frac{K'_{pi}}{f_p} + \frac{K'_{vi}}{f_v} ; {}^{\text{RMS}}E_{50\%}(\ddot{q}_i) = \frac{K''_{pi}}{f_p} + \frac{K''_{vi}}{f_v} + \frac{K''_{ai}}{\sqrt{f_a}} \quad (1)$$

where K_{pi}, \dots, K''_{ai} are system dependent constants for the i th robot joint. The expressions for other percentiles are similar except for the percentile of 0% due to numerical errors resulting from very small values.

The parameters K_{pi}, \dots, K''_{ai} depend on the arm link lengths l_1 and l_2 . For example, for the 50% percentile we have the approximate expressions:

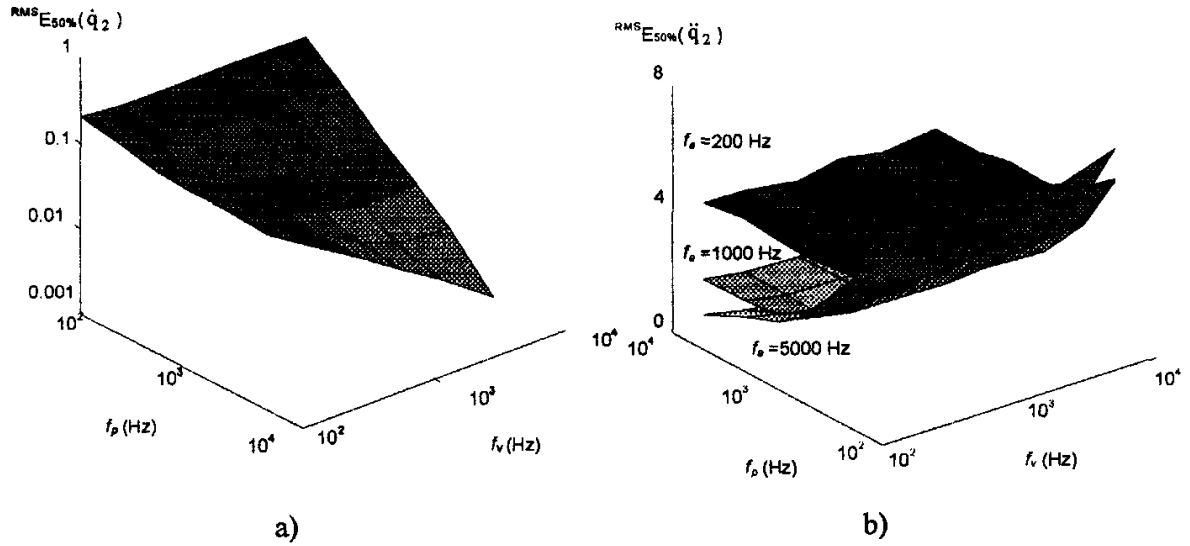


Fig 1: Percentile of 50% of the RMS error for joint 2 ($l_1=1.8$ m, $l_2=0.2$ m)
a) Velocity
b) Acceleration

$$K_{p1} = 1.10 - 0.63 \frac{l_1 - l_2}{l_1 + l_2} \quad K_{p2} = 1.722 - 0.247(l_1 + l_2) \quad (2.a)$$

$$K'_{p1} = \frac{7.28}{l_1 + l_2} - 4 \frac{|l_1 - l_2| + (l_1 - l_2)}{(l_1 + l_2)^2} \quad K'_{v1} = \frac{13}{l_1 + l_2} - 8 \frac{l_1 - l_2}{(l_1 + l_2)^2} \quad (2.b)$$

$$K'_{p2} = \frac{18}{l_1 + l_2} + 18 \frac{l_1 - l_2}{(l_1 + l_2)^2} \quad K'_{v2} = \frac{11}{l_1 + l_2} + 311 \frac{(l_1 - l_2)^4}{(l_1 + l_2)^5} \quad (2.c)$$

3. Computational Burden *Versus* Trajectory Errors

In a multifrequency scheme a compromise between the trajectory errors and the computational burden [7,8] determine an 'optimal' distribution. For example, in a first experience, Fig. 2 compares the time evolution of the joint velocities and accelerations for a monofrequency calculation (with $f_a = f_v = f_p = 10^3$ Hz) *versus* a multifrequency scheme (with $f_a = 10^3$ Hz, $f_v = 200$ Hz, $f_p = 10^2$ Hz).

Due to the low sampling frequencies the trajectory errors are significant and may not be acceptable. This means that there is the need for speeding-up the calculation rate.

In a second experiment, Fig. 3 compares the position and velocity trajectories for the monofrequency processing *versus* a new multifrequency distribution with $f_a = 500$ Hz, $f_v = 250$ Hz, $f_p = 200$ Hz. Obviously, in this case we have smaller trajectory errors but an higher computational load.

For the three cases the total computational time T for a 80486DX2@100MHZ is:

- Monofrequency, $f_a = f_v = f_p = 10^3$ Hz : $T = 16.3 \times 10^{-3}$ sec
- Multifrequency, $f_a=500$ Hz, $f_v=200$ Hz, $f_p=100$ Hz : $T = 4.2 \times 10^{-3}$ sec
- Multifrequency, $f_a=500$ Hz, $f_v=250$ Hz, $f_p=200$ Hz : $T = 5.0 \times 10^{-3}$ sec

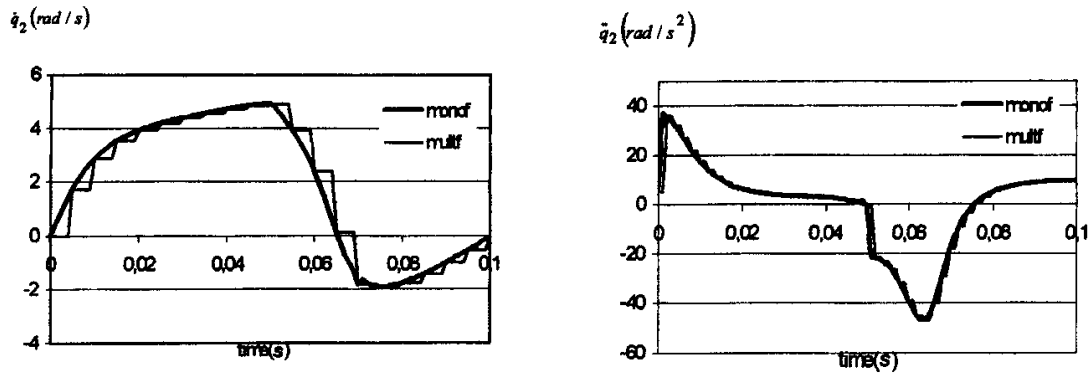


Fig. 2: Time evolution of the joint 2 velocity and acceleration ($l_1 = l_2 = 1.0$ m)
 Monofrequency: $f_a = f_v = f_p = 10^3$ Hz
 Multifrequency: $f_a = 500$ Hz, $f_v = 200$ Hz, $f_p = 10^2$ Hz

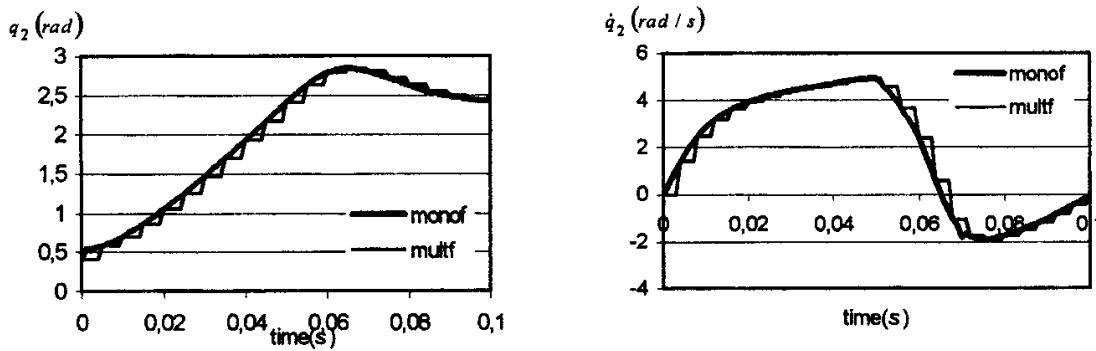


Fig. 3: Time evolution of the joint 2 position and velocity ($l_1 = l_2 = 1.0$ m)
 Monofrequency: $f_a = f_v = f_p = 10^3$ Hz
 Multifrequency: $f_a = 500$ Hz, $f_v = 250$ Hz, $f_p = 200$ Hz

The importance of multirate schemes is related with the computational time saving. Therefore, we need to identify the mathematical operations involved in the inverse kinematics and to cluster them in different groups according to its processing time. According with [7] the groups are {multiplication/division, sum/subtraction, sine/cosine, arctang, square root}. Each one of these groups has a different processing time resulting, for the multifrequency scheme, an average total calculation time T_{av} for each trajectory point (*i.e.*, for a point in the position, velocity and acceleration trajectories) given by an expression:

$$T_{av} = \frac{1}{\max(f_a, f_v, f_p)} \left\{ f_a \sum_i K_a^i T_i + f_v \sum_j K_v^j T_j + f_p \sum_k K_p^k T_k \right\} \quad (3)$$

where T_i are processing times for different floating point operations and K_a^i, K_v^i, K_p^i are the corresponding number of operations involved. Consequently, the sampling period T_s adopted for the highest calculation frequency in the multifrequency scheme must be:

$$T_s \geq T_{av} \quad (4)$$

Using a 80486DX2 computer, with a frequency clock of 100 MHz, we get:

$$\begin{aligned} T_a &= \sum_i K_a^i T_i \approx T_v = \sum_j K_v^j T_j = 4.8 \times 10^{-6} \text{ sec} \\ T_p &= \sum_k K_p^k T_k = 3.77 \times 10^{-5} \text{ sec} \end{aligned} \quad (5)$$

Due to the reduced computational load of the RR-robot kinematics, for simulation purposes it is considered that for calculations we have merely 10% of the computer processing capability presented in (5).

4. Optimization

Our main concern is to determine a set of values for f_a, f_v and f_p that guarantee, on one side, small errors for the positions, velocities and accelerations and, on the other side, a low calculation time. Those goals are obviously contradictory. Therefore we have a problem with multiple goals and there is no unique optimal solution.

For a P-percentile our multi-objective problem can be formulated as follows ($i = 1, 2$):

$$\text{Min}_i \left\{ \text{RMS } E_p(q_i) \right\}; \text{Min}_i \left\{ \text{RMS } E_p(\dot{q}_i) \right\}; \text{Min}_i \left\{ \text{RMS } E_p(\ddot{q}_i) \right\}; \text{Min} \{T_s\} \quad (6)$$

$$\text{Subjected to: } f_a, f_v, f_p \geq 0 \quad (7.a)$$

$$T_s \geq T_{av} \quad (7.b)$$

The methodology used to solve this problem was based on the minimization of an aggregated cost index, J_k , ($k \in \mathbb{N}$) defined as

$$J_k = \sum_i w_{pi} \left[\text{RMS } E_p(q_i) \right]^k + \sum_i w_{vi} \left[\text{RMS } E_p(\dot{q}_i) \right]^k + \sum_i w_{ai} \left[\text{RMS } E_p(\ddot{q}_i) \right]^k + w_t T_s \quad (8)$$

where w_{pi} , w_{vi} , w_{ai} and w_t are weighting factors, that is, priorities associated to each objective. Therefore, the new problem can be formulated as follows:

$$\begin{aligned} &\text{Min}\{J_k\} \\ &\text{Subject to: } f_a, f_v, f_p \geq 0 \\ &T_s \geq T_{av} \end{aligned} \quad (9)$$

To solve this problem it was adopted a numerical method based on successive approximations. In this line of thought, the admissible frequency range is divided in a discrete grid, with increasing accuracy and closer upper/lower limits for successive

iterations, until converging to an optimal solution in the sense of (9). During any iteration of the algorithm, if the candidate solution is located at the grid border then it is not guaranteed the algorithm convergence and, therefore, we must enlarge the frequency limits. The flowchart of the algorithm is represented in Fig. 4.

Several situations were investigated in order to determine the optimal calculation frequencies $\{f_a, f_v, f_p\}$ for different weighting factors in (8). The results indicate that in all cases the multifrequency strategy should obey the relationship $f_a \geq f_v \geq f_p$. This result is easily explained in a signal processing perspective, since the accelerations may suffer sudden variations while the positions are the smoothest [9,10].

The minimization of T_s and the minimization of the trajectory errors have opposite effects. Therefore, we analyze (Figs. 5 and 6) the evolution of the cost index J_k (for $k=1$ and $k=2$) and the corresponding optimal solution in terms of $\{f_a, f_v, f_p\}$ for weighting factors $w_{pi}=w_{vi}=w_{ai}=1$ and $w_{pi}=1, w_{vi}=0.5, w_{ai}=0.25$ and $w_t > 0$.

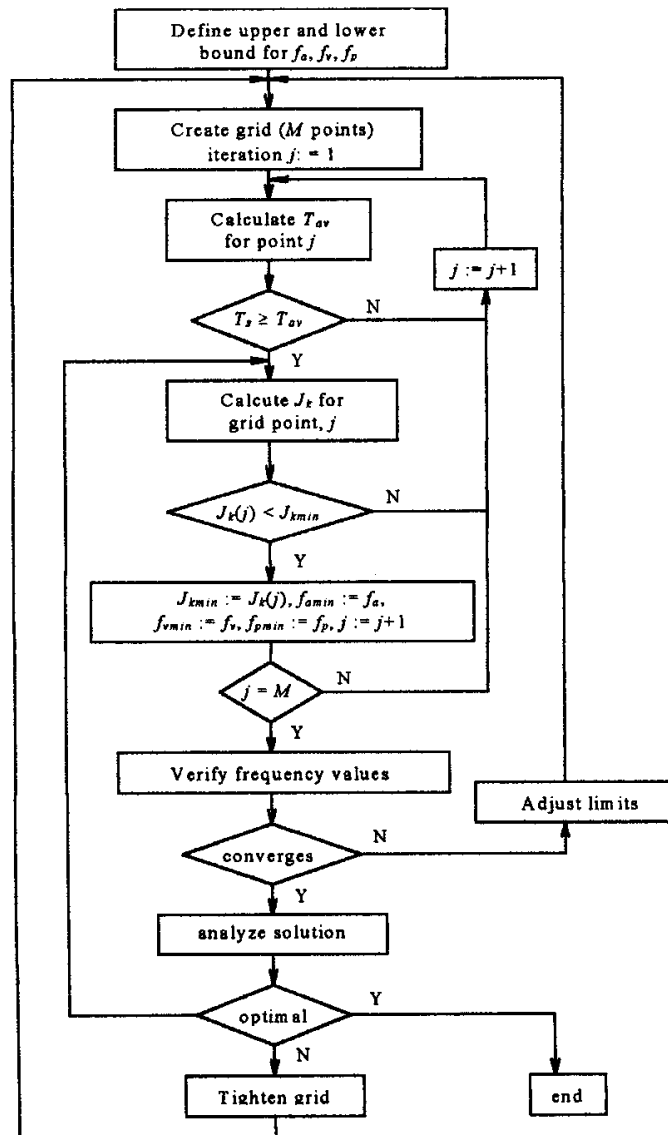


Fig. 4: Flowchart of the optimization algorithm.

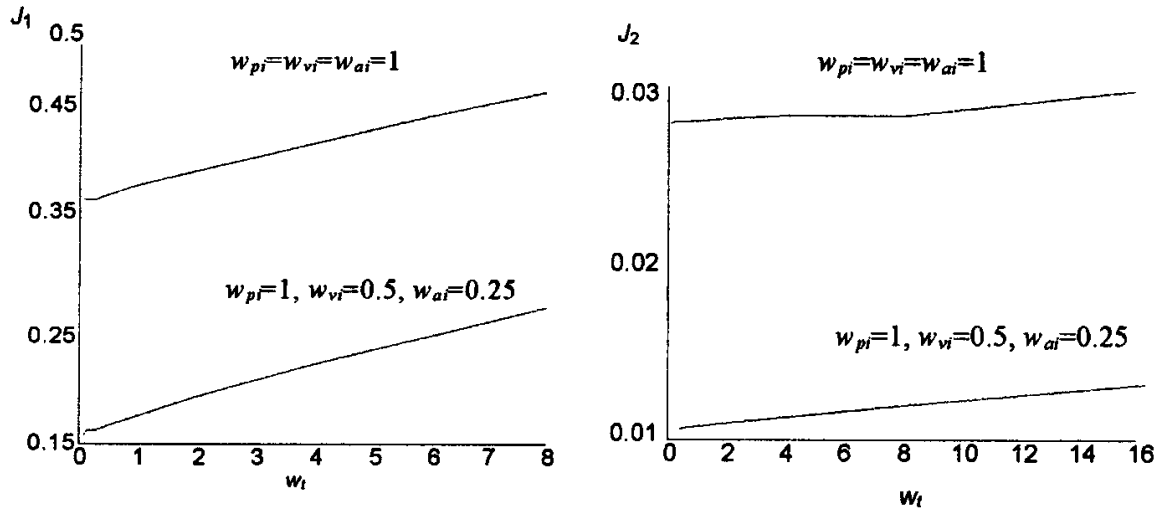


Fig 5: Evolution of the cost index J_1 and J_2 for $w_{pi}=w_{vi}=w_{ai}=1$ and $w_{pi}=1, w_{vi}=0.5, w_{ai}=0.25$ versus w_t .

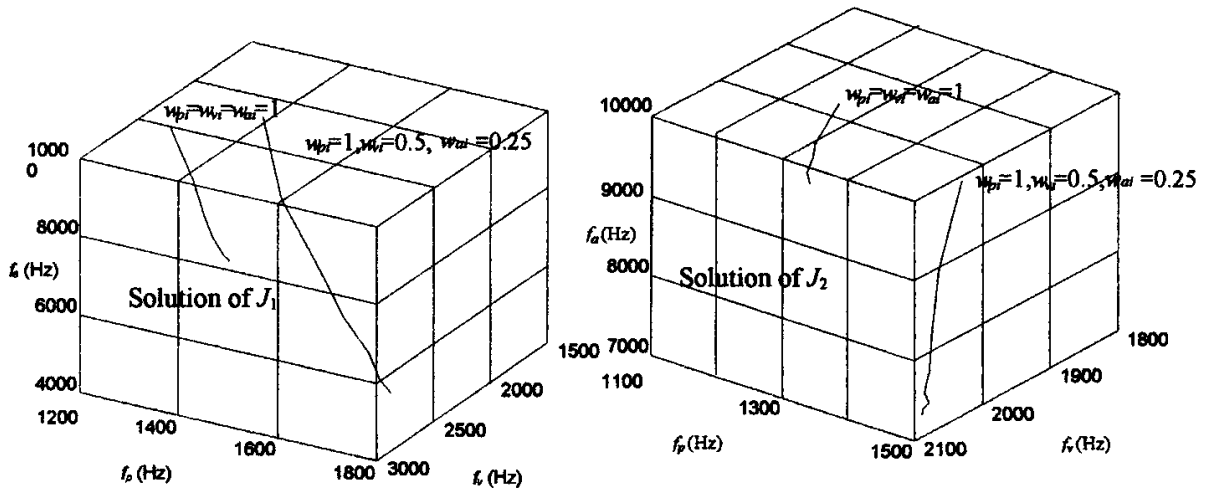


Fig 6: Locus of the optimal solution of J_1 and J_2 for $w_{pi}=w_{vi}=w_{ai}=1$ and $w_{pi}=1, w_{vi}=0.5, w_{ai}=0.25$ with $w_t > 0$ versus $\{f_a, f_v, f_p\}$.

In all cases we get $f_a \geq f_v \geq f_p$ which is in accordance with the intuition of assigning higher sampling frequencies the higher the bandwidth. However, we do not have fixed ratios f_v / f_a and f_p / f_a as proposed by some authors and, in fact, the results are:

- For $w_{pi}=w_{vi}=w_{ai}=1$ and $k = 1$
 $f_a / f_v = 2.8, f_a / f_p = 5.1$ (for $w_t = 1/16$) ; $f_a / f_v = 4.2, f_a / f_p = 7.7$ (for $w_t = 8$)
- For $w_{pi}=1, w_{vi}=0.5, w_{ai}=0.25$ and $k = 1$
 $f_a / f_v = 1.8, f_a / f_p = 2.7$ (for $w_t = 1/16$) ; $f_a / f_v = 4.8, f_a / f_p = 7.3$ (for $w_t = 8$)
- For $w_{pi}=w_{vi}=w_{ai}=1$ and $k = 2$
 $f_a / f_v = 4.6, f_a / f_p = 7.4$ (for $w_t = 1/16$) ; $f_a / f_v = 5.0, f_a / f_p = 8.0$ (for $w_t = 16$)
- For $w_{pi}=1, w_{vi}=0.5, w_{ai}=0.25$ and $k = 2$
 $f_a / f_v = 3.5, f_a / f_p = 4.8$ (for $w_t = 1/16$) ; $f_a / f_v = 4.7, f_a / f_p = 6.6$ (for $w_t = 16$)

We conclude that both J_1 and J_2 have a linear dependence with w_i , with similar slopes for the two sets of weighting factors $\{w_{pi}, w_{vi}, w_{ai}\}$.

In what concerns the locus of optimal frequencies $\{f_a, f_v, f_p\}$, in the sense of J_1 and J_2 , we have also nearly straight lines. Nevertheless, for $w_{pi}=w_{vi}=w_{ai}=1$ the locus reveals a smaller range of variation than the one corresponding to $w_{pi}=1, w_{vi}=0.5, w_{ai}=0.25$. This means that in this case we have a higher sensitivity. Moreover, comparing the results for the optimization with $k=1$ and $k=2$, we observe that the second case is more conservative which is important when thinking on a real implementation, where the frequency ratios must be approximated through integer values.

5. Conclusions

The effect of the sampling frequency upon the performance of a computer control system is very important. For a robot controller having several algorithms and models it is natural to expect distinct speeds for each case and, therefore, it is reasonable to allocate them different sampling and computing rates. In this perspective, for a given algorithm, the multirate scheme assigns the computing power to each variable in accordance to its needs leading to a more rational management of the system resources. In this paper this strategy was investigated in the calculation of the inverse kinematics for the RR robot trajectories. A compromise between accuracy and computational burden lead to the development of a simple optimisation algorithm. The results are consistent with intuition of assigning higher calculation frequencies the higher the signal bandwidth while giving a formal basis for the problem definition.

References

- [1] Richard P. Paul, *Robot Manipulators: Mathematics, Programming and Control*, The MIT Press, 1981.
- [2] K. S Fu.; R. C Gonzalez; C. S. G. Lee, *Robotics: Control, Sensing, Vision and Intelligence*, McGraw-Hill, 1987.
- [3] Saïd M. Megahead, *Principles of Robot Modelling and Simulation*, J. Wiley & Sons, 1993.
- [4] J. Robert Schilling, *Fundamentals of Robotics: Analysis and Control*, Prentice-Hall, 1990.
- [5] Mark Spong, W.; M. Vidyasagar, *Robot Dynamics and Control*, J. Wiley & Sons, 1989.
- [6] M. Kircanski, M. Vukobratovic, N. Kircanski and T. Timcenko, *A New Program Package for the Generation of Efficient Manipulator Kinematic and Dynamic Equations in Symbolic Form*, *Robotica*, 1988, vol. 6, pp. 311-318
- [7] J.A. Tenreiro Machado and Alexandra M. Galhano, *Benchmarking Computer Systems for Robot Control*. IEEE Trans. on Education, Aug. 1995, vol. 38, no. 3, pp. 205-210.
- [8] J.A. Tenreiro Machado, Carla S. M. Cordeiro and Imre J. Rudas, *On the Multifrequency Calculation of Robot Trajectories*, IEEE Int. Conf. On Intelligent Engineering Systems, September 1998, Vienna, Austria.
- [9] J.A. Tenreiro Machado and J.L. Martins Carvalho, *Engineering Design of a Multirate Nonlinear Controller for Robot Manipulators*, *Journal of Robotic Systems*, vol. 6, n. 1, John Wiley & Sons, 1989.
- [10] J.A. Tenreiro Machado and Abilio Azenha, *Variable Structure Position/Force Hybrid Control of Manipulators*, 4th IFAC Workshop on Algorithms and Architectures for Real-Time Control, April 1987, Vilamoura, Portugal, pp. 347-352.