IFAC Tallinn '90

PREPRINTS

11th IFAC WORLD CONGRESS

«Automatic Control in the Service of Mankind»

TALLINN, ESTONIA, USSR
August 13—17, 1990

Edited by

V. UTKIN
Ü. JAAKSOO

VOLUME 9

Subject Areas
9.1 AUTOMATIC CONTROL IN MANUFACTURING
9.2 ROBOT CONTROL

ON THE ANALYSIS AND DESIGN OF ROBOT MANIPULATORS: A STATISTICAL APPROACH

Alexandra M.F. Galhano, J.A. Tenreiro Machado, J.L. Martins de Carvalho

Faculty of Engineering of the University of Porto,
Dept. of Electrical and Computer Engineering, 4099 Porto Codex, Portugal

Abstract. A new approach to the analysis and design of robot manipulators is presented. The novel feature resides on a non-standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but in general lead to laborious computations. This motivates the need of alternative models based on other mathematical concepts. The proposed statistical method is a step in this direction which gives clear guidelines towards the robot kinematic and dynamic optimization. Furthermore, the use of histograms allows not only fast design procedures but above all, the use of experimental data; consequently, complex dynamic modelling exercises can be avoided.

Keywords. Robots; modelling; models; robot design; statistics.

INTRODUCTION

In the last years the area of application of robot manipulators has widened to a large range of industrial and scientific purposes. However, many of the recent applications pose challenging issues to the available industrial manipulators. The relatively poor performance of today's industrial manipulators when compared with the human arm, motivated an extensive research towards the development of better mechanical structures and actuators. The present paper deals with the problem of the manipulator performance optimization. Considerable research in this area has already been done leading to the implementation of direct drive mechanical arms and to the study of lightweight flexible manipulators. A different approach consisted in the development of mathematical and computer tools to study the kinematic and dynamic properties of robotic systems. Examples of such research on kinematics are the works of Tsai and Soni (1981) and Yoshikawa (1985). The dynamic characterization has also been investigated by Asada (1983), Homsup and Anderson (1986), Khatib (1986), Peltomaa and Koivo (1987), Salisbury and Abramowitz (1985), Yang and Tzeng (1986), Yoshikawa (1986) and Youcef-Toumi and Asada (1987).

From those studies it is clear that a simple optimization criterion comprising both the kinematics and dynamics is still lacking. In this paper we address this problem, and our presentation is organized as follows: in section 2 the robot manipulator model and the associated optimization criterion are discussed, in section 3 a new model which leads to a natural optimization criterion is formulated and applied to a 2R robot manipulator, and finally, in section 4, future developments of this work are discussed and some conclusions are drawn.

ON THE ROBOT MANIPULATOR MODEL

The modelling of rigid robot manipulator systems is well known. For the kinematic model, a set of equations relating the joint space {q} and the operational space {p}, can be found to be of the form (Brady, 1982; Paul, 1981):

$$q=\alpha(p) \tag{1a}$$
$$\dot{q}=\beta(p,\dot{p}) \tag{1b}$$
$$\ddot{q}=\gamma(p,\dot{p},\ddot{p}) \tag{1c}$$

Associated with the kinematic model we have the statics model, that relates the operational space forces $\mathcal{F}$ with the joint actuator torques T:

$$T=\mathcal{J}(q)^T\mathcal{F} \tag{2}$$

where $\mathcal{J}(q)$ is the jacobian matrix corresponding to the differential relationship $\dot{p}=\mathcal{J}(q)\dot{q}$. The dynamic description neglecting non-rigid link behaviour, actuator dynamics saturation, backlash and other dynamic phenomena, is described by a set of nonlinear differential matrix equations

$$T=I(q)\ddot{q}+C(q,\dot{q})+G(q) \tag{3}$$

where $T_I=I(q)\ddot{q}$, $T_C=C(q,\dot{q})$ and $T_G=G(q)$ are the n-dimensional vectors of the inertial, Coriolis/centripetal and gravitational torques.

If an "optimal performance" manipulator is to be designed then we should bear in mind the following points:

- Which "optimal performance" criterion is to be obeyed? In fact, there is a plethora of proposed criteria such as maximum workspace, kinematic and dynamic manipulability measures, minimization of power requirements, minimization of torque requirements, minimization of the nonlinear components of the torque requirements etc. Moreover, some criteria such as the dynamic manipulability measure, still allow several different numerical indices.

- Kinematic criteria may be inconsistent, or difficult to relate with dynamic criteria. If an optimization algorithm is to be chosen, it is imperative to find some form of relation between them.
- If possible, such criteria should provide insight into related areas such as trajectory planning or controller architectures.
- An optimization criterion, should provide a clear and easy to use method of manipulator design; moreover, it should be well adapted to a CAD environment. Methods failing to provide any structural guidelines, e.g. not surpassing the level of mere hints, should be avoided.
- Although there is a large number of determining parameters, like masses, inertias, link lengths etc., the optimization algorithm should be computationally efficient, if it is to be used in a CAD environment.

To obey the above considerations a non-standard formulation of the optimization criterion should be sought after. As this criterion must be closely related to the robot manipulator model, we conclude that different modelling concepts are required.

In this perspective, a closer look at expressions (1)-(3) shows that the large number of parameters involved, gives rise to a cumbersome work either in a design or in an analysis stage. On the other hand, an alternative model, based on a statistical description of the robot manipulator system is more in the line of thought of the above requirements. If, with this strategy, we lose the "certainty" of the deterministic model, we gain a simpler and more intuitive viewpoint. This approach, in a simplified way, has already been used by Mooring and Pack (1987) for the kinematics and by Scheinman and Roth (1984) for the dynamics. In the sequel we refer to the new approach, as the statistical model in contrast with the standard method. Our model comprises:

- The statistical description of a set of input variables, that is variables that are free to change independently.
- The statistical description of a set of output variables, that is, variables that are functions of the the previous ones.
- A set of parameters which are to be optimized in the design stage.

One should note that the above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is established as follows:

- $\{p, \dot{p}, \ddot{p}\}$ act as input variables of the kinematic system. This option enables a definition of the required kinematic performances on the operational space which are more natural to the designer.
- $\{q, \dot{q}, \ddot{q}\}$, act as output variables in the kinematic system, but play the role of input variables set in the dynamic model. In this way we arrive at a relationship between kinematics and dynamics in a form amenable to performance optimization criteria as defined in the sequel.
- The set of dynamic output variables consists of the required joint torques $\{T\}$
- The parameter set consists of link lengths, masses and inertias.

For the statistical description of the involved variables, one could think of a stochastic model. Nevertheless, such a model would be more complex than the deterministic one. Therefore, to achieve, as desired, a simple to use model, the implicit time variable is not considered, that is variables which are related through the time derivative operator (p, $\dot{p}$, $\ddot{p}$ and q, $\dot{q}$, $\ddot{q}$), are considered independent of each other.

Let us now look at the performance optimization procedure. As mentioned previously it is difficult to find the best option. For this reason, instead of starting by defining an optimization criterion, we look ahead of the model to gain a feel for such a criterion. A more precise statement of these issues is presented in the next section with the help of a 2R robot manipulator (Fig. 1).

A STATISTICAL MODEL FOR THE 2R ROBOT MANIPULATOR

In this section we use the 2R robot manipulator as the support for the development and implementation of the concepts presented in section 2. This strategy is justified because this manipulator configuration already includes all the kinematic and dynamic phenomena that appear in manipulators with more degrees of freedom.

Let us start by introducing our approach in the kinematic case; in the second subsection we shall analyse the dynamic case.

Kinematics

The set of kinematic input variables consists of position, velocity and acceleration that our prototype manipulator is required to perform in the operational space. Therefore, it is necessary to characterize them in statistical terms, namely by defining appropriate probability density functions (p.d.f.'s) for each one. As there is no a priori knowledge about the typical behaviour we start with some reasonable assumptions namely, for the position variable $p=[x,y]^T$ we consider a bidimensional uniform p.d.f.

$$f_p(p)=\begin{cases} C & \text{if } (r_1-r_2)^2 \leq x^2+y^2 \leq (r_1+r_2)^2 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

with $C=1/\{\pi[(r_1+r_2)^2-(r_1-r_2)^2]\}$.

In the sequel we will see how to modify the input p.d.f. in order that the kinematic performances are optimized. It is also necessary to define the p.d.f.'s for velocity and acceleration. By the same above arguments, we decided to use bidimensional Gaussian p.d.f.'s with zero mean

$$f_{\dot{p}}(\dot{p})=1/(2\pi\sigma_{\dot{p}}^2)\ \exp[-(\dot{x}^2+\dot{y}^2)/(2\sigma_{\dot{p}}^2)] \qquad (5)$$

$$f_{\ddot{p}}(\ddot{p})=1/(2\pi\sigma_{\ddot{p}}^2)\ \exp[-(\ddot{x}^2+\ddot{y}^2)/(2\sigma_{\ddot{p}}^2)] \qquad (6)$$

Moreover, using these p.d.f.'s we impose some interesting properties, such as:

- The random variables position, velocity and acceleration in the operational space are independent of each other.
- The velocity and acceleration vectors are made of two independent components, that is $\dot{x}$ is independent of $\dot{y}$ and $\ddot{x}$ is independent of $\ddot{y}$.

The "excitation" of the (inverse) kinematic system produces output random variables q, $\dot{q}$ and $\ddot{q}$ (i.e. the position, velocity and acceleration in the joint space), with p.d.f.'s which are related to the previous ones by:

$$f_q(q)=J_P\ f_P(p) \qquad (7a)$$
$$f_{q\dot{q}}(q,\dot{q})=J_V\ f_{p\dot{p}}(p,\dot{p}) \qquad (7b)$$
$$f_{q\dot{q}\ddot{q}}(q,\dot{q},\ddot{q})=J_A\ f_{p\dot{p}\ddot{p}}(p,\dot{p},\ddot{p}) \qquad (7c)$$

where the jacobians J_P, J_V, J_A are

$$J_P=\frac{\partial(p)}{\partial(q)}=r_1r_2S_2 \qquad (8a)$$
$$J_V=\frac{\partial(p,\dot{p})}{\partial(q,\dot{q})}=J_P(r_1r_2S_2) \qquad (8b)$$
$$J_A=\frac{\partial(p,\dot{p},\ddot{p})}{\partial(q,\dot{q},\ddot{q})}=J_V(r_1r_2S_2) \qquad (8c)$$

Each of the expressions (7) is made of two distinct factors:

- Weighting factors - J_P, J_V and J_A - which depend solely on the system kinematic properties
- The "excitation" p.d.f.'s - $f_P(p)$, $f_{p\dot{p}}(p,\dot{p})$ and $f_{p\dot{p}\ddot{p}}(p,\dot{p},\ddot{p})$ - which are a measure of the task performances.

These factors can be interpreted in a dynamical system theoretic framework. The jacobians characterize the system intrinsic properties, while the excitation p.d.f.'s correspond to the system response to the input variables. Note the similarities of these interpretations with the well known free and forced response of linear systems.

Bearing these facts in mind, several experiments were performed, having:

a) The total link length constant, L=1.8.
b) Several ratios $\mu=r_1/r_2$, namely 0.4, 0.6, 0.8, 1, 1.2, 1.4 and 1.6
c) Operational space categories corresponding to four distinct requirements:
  1. $\sigma_{\dot{p}}$=0.1 m/s and $\sigma_{\ddot{p}}$=0.1 m/s²
  2. $\sigma_{\dot{p}}$=0.1 m/s and $\sigma_{\ddot{p}}$=1 m/s²
  3. $\sigma_{\dot{p}}$=1 m/s and $\sigma_{\ddot{p}}$=0.1 m/s²
  4. $\sigma_{\dot{p}}$=1 m/s and $\sigma_{\ddot{p}}$=1 m/s².
d) Excitation of the kinematic system with a numerical random sample of 4000 operational space variables obeying the p.d.f.'s (4)-(6).
e) Analysis of the resulting histograms of the output variables amplitude. In order to simplify matters, only marginal p.d.f.'s were considered.

After a large number of experiments using the numerical set of parameters depicted in Table 1, we concluded that the shape of the resulting p.d.f.'s varied significantly from variable to variable, but all of them showed symmetry around zero. For this reason, and in order to characterize the resulting histograms by a scalar index, we decided to adopt for this index the upper limit of the integral which corresponds to 95% of probability (this definition coincides with two standard deviations for Gaussian type p.d.f.'s). The resulting histograms are condensed through this index and depicted in Fig. 2. We can observe in the majority of the charts a minimum about μ=1; nevertheless, this conclusion can be easily inferred from (7). In fact, for symmetrical histograms about zero on the x-axis, a larger value of the jacobian corresponds to a

TABLE 1 Numerical values of the 2R Robot

r_i=0.9 m, R_i=0.15 m, m_i=0.5 Kg, m_e=0 Kg
$I_i=m_i(r_i^2/12+R_i^2/4)$, I_i=0.0366 Kgm²; i=1,2

smaller dispersion of the random variable, which in turn means average smaller amplitude requirements posed to that variable. Therefore, we found an algorithm that leads naturally to an optimization criterion, in the sense that it is not forced externally, but instead it arises naturally from the model.

As the maximization of J_P, J_V and J_A requires the same steps, we have for:

$$r_1+r_2=L,\ r_1/r_2=\mu \qquad (9)$$

that a maximum occurs when

$$\mu=1,\ q_2=\pi/2 \qquad (10)$$

which coincide with the results obtained (using different methodologies) by Tsai and Soni (1981) and Yoshikawa (1985). Furthermore, our optimization criteria enables additional conclusions and procedures:

- Because J_P, J_V and J_A are consecutive powers of $r_1r_2S_2$, we see that for a given deviation from the optimal values (10) we have an increasingly degradation of the cost function with the powers of $r_1r_2S_2$. In other words this means that for a given deviation, we have, by increasing order of sensitivity, position, velocity and acceleration.
- Due to (2) a kinematic optimization is equivalent to a static optimization.
- If further optimization is desired, then the next step will be the selection of an optimum "excitation" p.d.f.. This second step of optimization will define, in a statistical sense, an optimum kinematic class for the manipulator trajectories. Obviously, we can find a multitude of different p.d.f's obeying (10); nevertheless, for the subsequent study a particular choice is of minor importance. Consequently we decided to adopt in our case study the following family of p.d.f.'s in the operational space (with K≥1):

$$f_P(x,y)=\text{constant}*\{1-[(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2)]^2\}^{(K-1)/2} \qquad (11)$$

which, in the joint space, corresponds to:

$$f_q(q_1,q_2)=\text{constant}\ *\ S_2^K \qquad (12)$$

As extreme cases, we have that for K=1 it becomes the uniform p.d.f. (4), while for K→∞ we get Dirac type p.d.f. (δ(.)) optimum in the sense of (10):

$$f_P(x,y)=\delta[x^2+y^2-(r_1^2+r_2^2)] \qquad (13a)$$
$$f_q(q_1,q_2)=1/2\ [\delta(q_2+\pi/2)+\delta(q_2-\pi/2)] \qquad (13b)$$

As far as velocity and acceleration are concerned we can see that the kinematic study does not point out any special class of p.d.f.'s.

To test numerically the above conjectures, the previous results for μ=1 are compared with a new case using μ=1 and K=3 in (11)-(12). This has revealed a remarkable performance improvement as shown in Fig. 2, particularly for high velocity requirements (categories 3 and 4).

Dynamics

In order to gain a deeper insight for the subsequent study we decided to consider, in a first stage, as dynamic output variables the components of the joint torques, that is the gravitational, Coriolis/centripetal and inertial torques. Based on this preliminary analysis then, in a second stage, we consider the total joint torques. In the first stage we have:

$$f_G(q)=J_G\ f_Q(q) \qquad (14a)$$
$$f_C(q,\dot{q})=J_C\ f_{Q\dot{Q}}(q,\dot{q}) \qquad (14b)$$
$$f_I(q,\dot{q},\ddot{q})=J_I\ f_{Q\dot{Q}\ddot{Q}}(q,\dot{q},\ddot{q}) \qquad (14c)$$

where

$$J_G=\frac{\partial(q)}{\partial(T_G)}=[(m_1/2+m_2+m_0)(m_2/2+m_0)g^2r_1r_2S_1S_{12}]^{-1} \qquad (15a)$$

$$J_C=\frac{\partial(q,\dot{q})}{\partial(T_G,T_C)}=J_G\{[2(m_2/2+m_0)r_1r_2S_2]^2\dot{q}_1\dot{q}_{12}\}^{-1} \qquad (15b)$$

$$J_I=\frac{\partial(q,\dot{q},\ddot{q})}{\partial(T_G,T_C,T_I)}=J_C\{[(m_1/4+m_2+m_0)r_1^2+I_1+(m_2/2+m_0)r_1r_2C_2][(m_2/4+m_0)r_2^2+I_2]- -[(m_2/4+m_0)r_2^2+I_2+(m_2/2+m_0)r_1r_2C_2](m_2/2+m_0)r_1r_2C_2\}^{-1} \qquad (15c)$$

and $f_G(q)$, $f_C(q,\dot{q})$, $f_I(q,\dot{q},\ddot{q})$ represent the p.d.f.'s of the gravitational, Coriolis/centripetal and inertial torques, respectively. Unlike the kinematic situation, where the optimization was similar for all the jacobians, now their effects differ according to each dynamic term. Analysing the jacobians (15) we conclude that:

• The maximizing of J_G stipulates that q_1 and q_{12} should have p.d.f.'s with maxima at 0 or π The observation of histograms resulting from "excitation" p.d.f.'s obeying these conditions showed an interesting result. As expected the (symmetrical) histograms resembled Dirac plots; however, those peaks were located at non-zero values. In fact, the plots showed two sharp symmetrical peaks located at the maxima (positive and negative) values attained by the gravitational torques. This means that the optimization procedure used in the kinematics, should be carefully adapted to the present circumstances. For this particular case, the optimization procedure must avoid those Diracs, that is to say we must minimize J_G (Fig. 3).

• The maximizing of J_C implies that q_2 must have a p.d.f. with a maximum on 0 or π. Numerical experiments showed that in this case the resulting histograms of the Coriolis/centripetal terms tended, as desired, towards a Dirac on zero.

• The analytical expression of J_I is more complex. Nevertheless, its analysis revealed a maximizing condition similar to the previous one (i.e. q_2 should have a p.d.f. with maxima at 0 or π), which was indeed confirmed by numerical results.

The integration of these (partial) results gives several guidelines for the study of the joint torques (i.e. $T=T_G+T_C+T_I$). In this perspective, the optimizing directives either for J_G or for J_C and J_I were considered in order to "excite" the dynamics with suitable p.d.f.'s. In a case we considered (with K=3)

$$f_Q(q_1,q_2)=\text{constant} * (S_1S_{12})^K \qquad (16)$$

which minimizes J_G and, in an alternative case, (with K=3)

$$f_Q(q_1,q_2)=\text{constant} * C_2^K \qquad (17)$$

which comes in the line of maximizing both J_C and J_I. In order plot histograms of the torques, and due to non-existence of optimization guidelines on $\dot{q}$ and $\ddot{q}$, we decided to consider the following Gaussian "excitation" p.d.f.'s in the joint space:

$$f_{\dot{Q}}(\dot{q})=1/(2\pi\sigma_{\dot{q}}^2)\exp[-(\dot{q}_1^2+\dot{q}_2^2)/(2\sigma_{\dot{q}}^2)] \qquad (18)$$
$$f_{\ddot{Q}}(\ddot{q})=1/(2\pi\sigma_{\ddot{q}}^2)\exp[-(\ddot{q}_1^2+\ddot{q}_2^2)/(2\sigma_{\ddot{q}}^2)] \qquad (19)$$

and the four different categories:

1. $\sigma_{\dot{q}}=0.1$ rad/s and $\sigma_{\ddot{q}}=0.1$ rad/s^2
2. $\sigma_{\dot{q}}=0.1$ rad/s and $\sigma_{\ddot{q}}=10$ rad/s^2
3. $\sigma_{\dot{q}}=10$ rad/s and $\sigma_{\ddot{q}}=0.1$ rad/s^2
4. $\sigma_{\dot{q}}=10$ rad/s and $\sigma_{\ddot{q}}=10$ rad/s^2.

Figure 4 depicts the results of T_G, T_C, T_I and T when the 95% index is applied to the corresponding histograms. These charts revealed several important properties:

• The relative magnitudes of T_G, T_C and T_I are, as expected, dependent on $\sigma_{\dot{q}}$ and $\sigma_{\ddot{q}}$:
-low (high) requirements on $\sigma_{\dot{q}}$ correspond to low (high) values on T_C
-low (high) requirements on $\sigma_{\ddot{q}}$ correspond to low (high) values on T_I.
Nevertheless, this correspondence between $\dot{q}$ ($\ddot{q}$) and T_C (T_I) can not be generalized to the operational space. In fact, high values of $\dot{p}$ are costly both for T_C and T_I due to the strong kinematic influence of $\dot{p}$ both on $\dot{q}$ and $\ddot{q}$. We believe that this conclusion clarifies the discussion about the importance of the velocity dependent terms (Paul, 1981; Hollerbach, 1984).

• J_G defines a "rest region" while J_C and J_I define an "active region". In each of these regions we have the minimization - in a statistical sense - of the corresponding torques. Consequently, the total torque T is minimized for low velocities/accelerations through p.d.f.'s like (16) and in the opposite case by p.d.f.'s obeying the same guidelines as (17).

• A dynamic trajectory planning algorithm developed according to these principles must perform a "switching" between the different p.d.f.'s. This "switching" must be driven accordingly the required $\dot{p}$ and $\ddot{p}$

• The same conclusions were attained for a non-zero payload mass m_0.

• A manipulator, which is a machine that mimics the human arm, is by consequence very sensitive to velocity requirements in the joint space. This indicates that we are dealing with a "position/acceleration machine" and not a "velocity machine". Although obvious, this aspect is somewhat overlooked in the literature. Moreover, it points out that the usual electrical, hydraulic or pneumatic robotic actuators, which are developments of standard "velocity machines" are not well adapted to robotic applications. Alternative solutions such as muscle like "position/acceleration" actuators (Akazawa and Fujii, 1986; Hirose and others, 1989; Tatara, 1987) will allow more efficient robot structures.

CONCLUSIONS

A new approach to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, on the other hand, can be very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The proposed statistical scheme is a step in that direction which has been shown to present interesting properties and to enable new procedures. It provides a framework giving clear guidelines towards the robot structure optimization. As a result, the manipulator design procedure, both kinematic and dynamic, leads to simple and intuitive conclusions. Previously proposed schemes gave similar results for the simple kinematic case; unfortunately, they are difficult to apply in the (more complex) dynamic stage. In the proposed algorithm both situations appear as natural and clear extensions of a common and systematic methodology. Furthermore, the inherent use of histograms allows not only fast calculation procedures but, above all, the use of experimental data; consequently, complex dynamic modelling exercises can be avoided. Also, it should be highlighted the results pointing out some characteristics of the trajectory planning block, defining optimal rest and active regions, and ideal-actuator properties, as "position/acceleration" devices instead of "velocity" machines. This observation is of utmost importance as it gives a clear basis to new mechanical robot manipulator structures, with performances close to the muscle-actuated biological systems.

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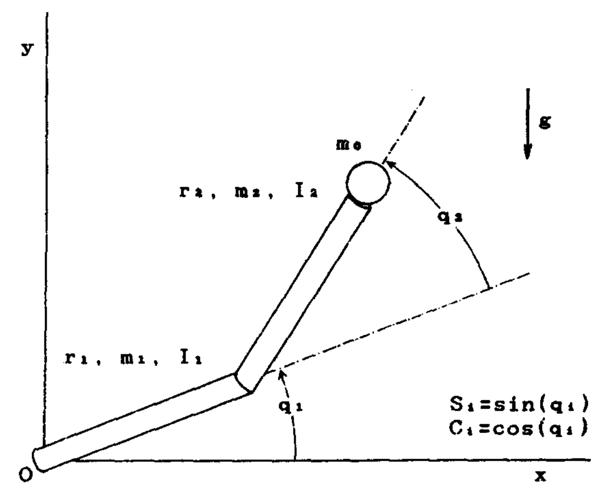

Fig. 1. The 2R robot manipulator

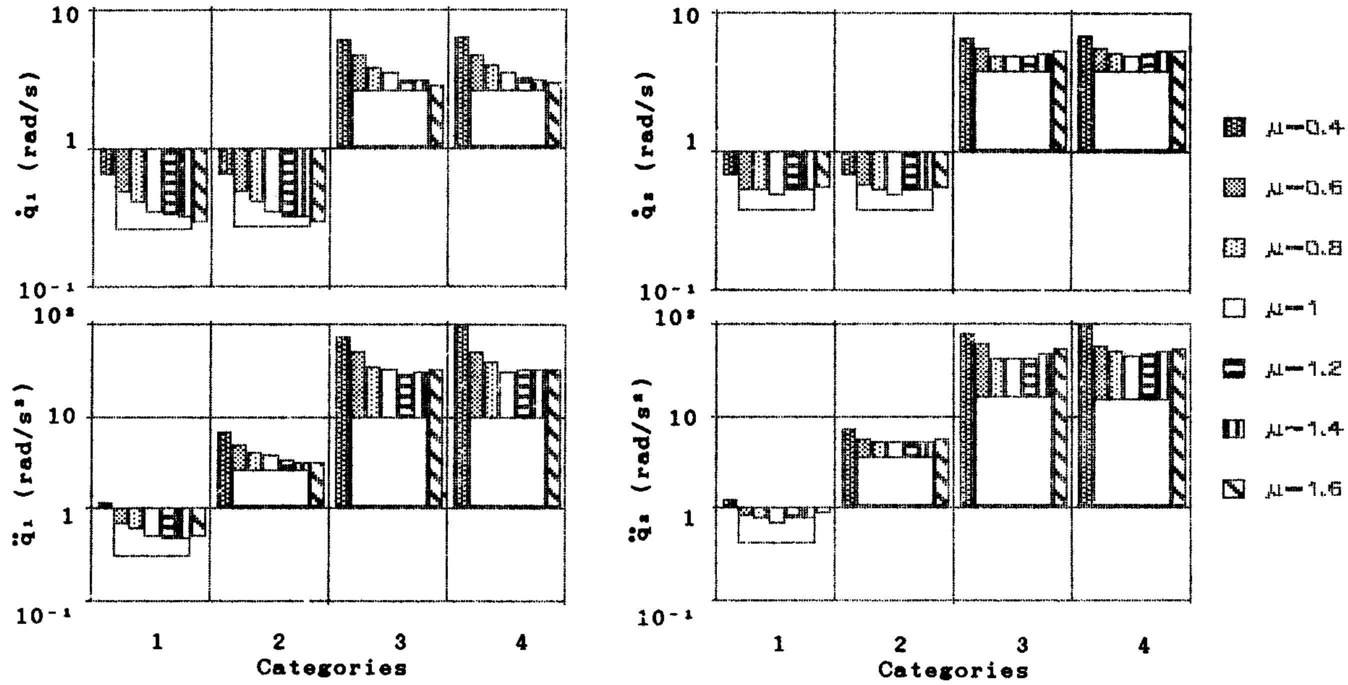

Fig. 2. Comparison charts for the 2R robot kinematic performances.
The narrow columns correspond to seven geometric configurations "excited" with p.d.f.'s (4), (5) and (6).
The wider columns correspond to the optimum geometric configuration $\mu=1$ "excited" with the enhanced p.d.f. (12), and p.d.f.'s (5)-(6).

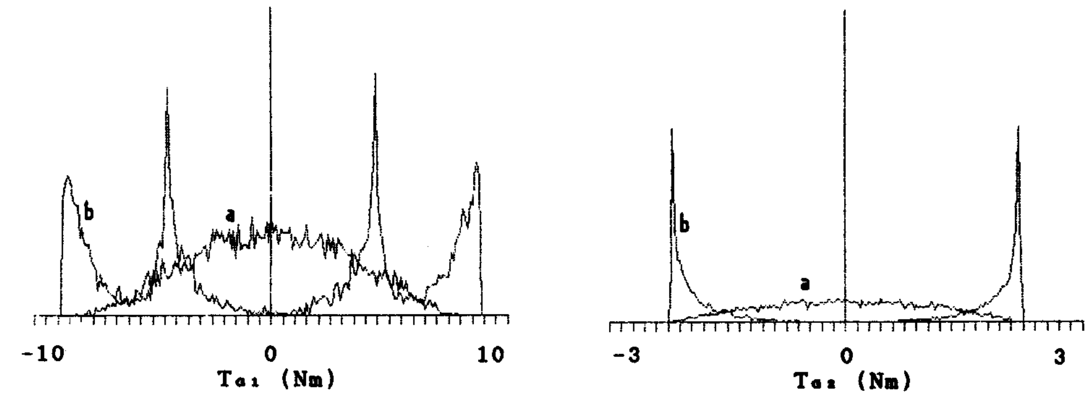

Fig. 3. Histograms of the gravitational torques T_G for "excitation" p.d.f.'s:
a. $f_q(q_1,q_2)=constant*(S_1S_{12})^2$, b. $f_q(q_1,q_2)=constant*(C_1C_{12})^2$

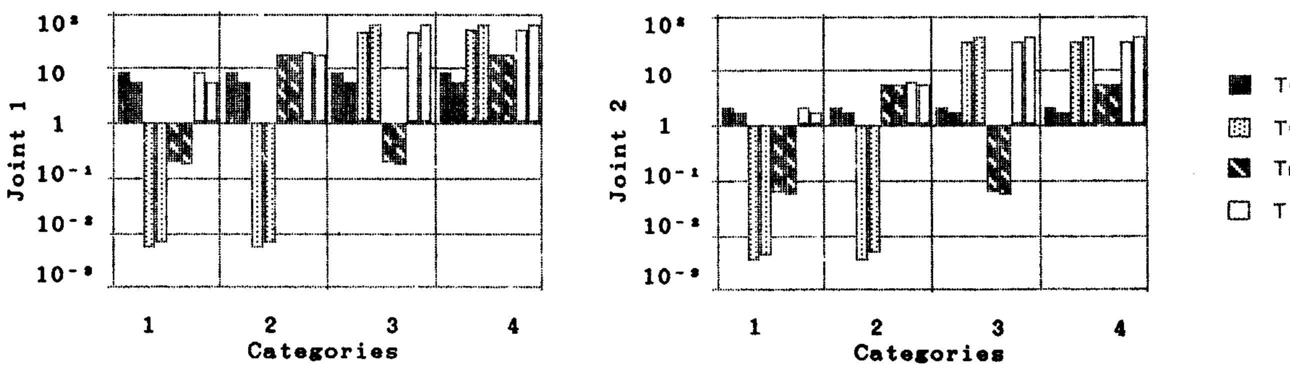

Fig. 4. Comparison charts of T_G, T_C, T_I and T for the 2R robot with $\mu=1$, when subjected to "excitation" p.d.f.'s:
First column: $f_q(q_1,q_2)=constant*C_2^2$
Second column: $f_q(q_1,q_2)=constant*(S_1S_{12})^2$