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# Quasi-periodic gaits in multi-legged robots

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## ABSTRACT

This paper studies quasi-periodic gaits of multi-legged robot locomotion systems based on the analysis of the dynamic model. The purpose is to determine the system performance during walking and the best strategy to overcome an obstacle. For that objective the robot prescribed motion is characterized in terms of several locomotion and obstacle variables. In this perspective, we formulate three performance measures of the walking robot namely, the mean absolute power, the mean power lost in the joint actuators and the mean force of the interface body-legs per walking distance. A set of model-based experiments reveals the influence of the obstacle position and dimensions in the proposed indices.

## 1 INTRODUCTION

Walking machines allow locomotion in terrain inaccessible to other type of vehicles, since they do not need a continuous support surface (1). On the other hand, the requirements for leg coordination and control impose difficulties beyond those encountered in wheeled robots (2). Gait selection is a research area requiring an appreciable modeling effort for the improvement of mobility with legs in unstructured environments (3,4). Previous studies mainly focused in the structure and selection of locomotion modes (5 – 9). Nevertheless, there are different optimization criteria such as energy efficiency (10), stability (2), velocity (11,12), comfort, mobility (13) and environmental impact. With these facts in mind, a simulation model for multi-leg locomotion systems was developed, for several periodic gaits. This study intends to generalize previous work (14 – 16) through the formulation of several dynamic indices measuring the average power during different walking trajectories, the power lost in the joint actuators and the mean force acting on the hips along the space-time walking cycle.

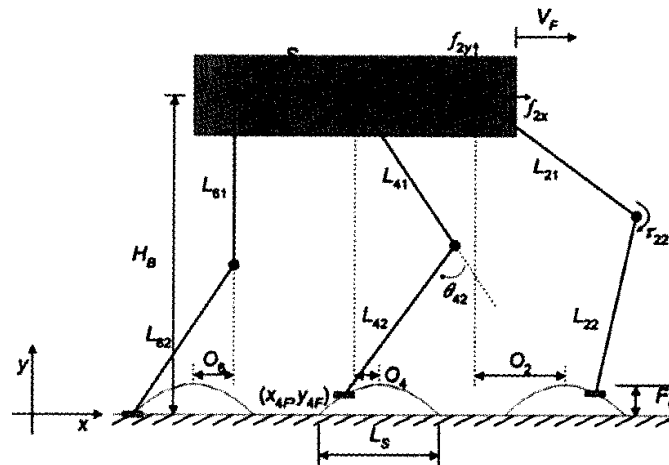
The foot and body trajectories are analyzed in what concerns its variation with the obstacle position, height and width. Several simulation experiments reveal the system configuration and the type of the movements that lead to a better mechanical implementation, for a given locomotion mode, from the viewpoint of the proposed indices.

Bearing these facts in mind, the paper is organized as follows. Section two introduces the model for a multi-legged robot and the motion planning algorithms. Section three formulates

the optimizing indices and section four presents the strategies for crossing over obstacles, respectively. Section five develops a set of experiments that reveal the influence of the system parameters in the quasi-periodic gaits, and, finally, section six presents the main conclusions and directions towards future developments.

## 2 A MODEL FOR MULTI-LEGGED LOCOMOTION

We consider a longitudinal walking system with  $n$  identical legs ( $n \geq 2$  and  $n$  even), with the legs equally distributed along both sides of the robot body, having each one two rotational joints (Figure 1).



**Fig. 1** Coordinate system and variables that characterize the motion trajectories of the multi-legged robot

Motion is described by means of a world coordinate system. The kinematic model comprises the cycle time  $T$ , the duty factor  $\beta$  (the time fraction of a cycle time in which the leg is in the support phase), the transference time  $t_T = (1-\beta)T$ , the support time  $t_S = \beta T$ , the step length  $L_S$ , the stroke pitch  $S_P$ , the body height  $H_B$ , the maximum foot clearance  $F_C$ , the  $i^{\text{th}}$  leg lengths  $L_{i1}$  and  $L_{i2}$  and the foot trajectory offset  $O_i$  ( $i=1, \dots, n$ ). Moreover, we consider a periodic trajectory for each foot, with body velocity  $V_F = L_S / T$ .

The algorithm for the forward motion planning accepts the body and  $i^{\text{th}}$  feet cartesian trajectories  $\mathbf{p}_F(t) = [x_{iF}(t), y_{iF}(t)]^T$  as inputs and, by means of an inverse kinematics algorithm, generates the related joint trajectories  $\boldsymbol{\theta}(t) = [\theta_{i1}(t), \theta_{i2}(t)]^T$ , selecting the solution corresponding to a forward knee.

The body of the robot, and by consequence the legs hips, are assumed to have a horizontal movement with a constant forward speed  $V_F$ . Therefore, for leg  $i$  the cartesian coordinates of the hip of the legs are given by:

$$\mathbf{p}_H(t) = \begin{bmatrix} x_{iH}(t) \\ y_{iH}(t) \end{bmatrix} = \begin{bmatrix} V_F \cdot t \\ H_B \end{bmatrix} \quad (1)$$

Given a particular gait and duty factor  $\beta$ , it is calculated, for leg  $i$ , the corresponding phase  $\phi_i$  and the time instant where each leg leaves and returns to contact with the ground (2). From these results, and knowing  $T$ ,  $\beta$  and  $t_s$ , the cartesian trajectories of the tip of the foot must be completed during  $t_T$ .

For each cycle the trajectory of the tip of the swing leg is computed through a cycloid function given by (considering, for example, that the transfer phase starts at  $t = 0$  sec for leg 1), with  $f = 1/T$ :

- during the transfer phase:

$$x_{1F}(t) = V_F \left[ t - \frac{1}{2\pi f} \sin(2\pi f t) \right] \quad (2a)$$

$$y_{1F}(t) = \frac{F_C}{2} [1 - \cos(2\pi f t)] \quad (2b)$$

- during the stance phase:

$$x_{1F}(t) = V_F \left[ T - \frac{1}{2\pi f} \sin(2\pi f T) \right] = V_F \cdot T \quad (3a)$$

$$y_{1F}(t) = 0 \quad (3b)$$

From the coordinates of the hip and feet of the robot it is possible to obtain the leg joint positions and velocities using the inverse kinematics:

$$\mathbf{p}(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} = \mathbf{p}_H(t) - \mathbf{p}_F(t) \quad (4a)$$

$$\boldsymbol{\theta}(t) = f[\mathbf{p}(t)] \quad (4b)$$

$$\dot{\boldsymbol{\theta}}(t) = \mathbf{J}^{-1}[\dot{\mathbf{p}}(t)] \quad (4c)$$

Based on this data, the trajectory generator is responsible for producing a motion that synchronises and co-ordinates the legs. In order to avoid the impact and friction effects we impose null velocities of the feet in the instants of landing and taking off, assuring also the velocity continuity.

After planning the joint trajectories we calculate the inverse dynamics in order to 'map' the kinematics into power consumption. The robot inverse dynamic model is of the form:

$$\boldsymbol{\tau} = \mathbf{H}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{g}(\boldsymbol{\theta}) \quad (5)$$

where  $\boldsymbol{\tau} = [f_{ix}, f_{iy}, \tau_{i1}, \tau_{i2}]^T$  ( $i=1, \dots, n$ ) is the vector of forces/torques,  $\boldsymbol{\theta} = [x_i, y_i, \theta_{i1}, \theta_{i2}]^T$  is the vector of position coordinates,  $\mathbf{H}(\boldsymbol{\theta})$  is the inertia matrix and  $\mathbf{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$  and  $\mathbf{g}(\boldsymbol{\theta})$  are the vectors of centrifugal/Coriolis and gravitational forces/torques, respectively.

### 3 MEASURES FOR PERFORMANCE EVALUATION

In mathematical terms, we provide three global measures of the overall performance of the mechanism in an average sense.

#### 3.1 Mean Absolute Power

The key measure in this analysis is the mean absolute power per travelling distance. It is computed assuming that power regeneration is not available by actuators doing negative work, that is, by taking the absolute value of the power. At a given joint  $j$  (each leg has  $m = 2$  joints) and leg  $i$  (since we are adopting an hexapod it yields  $n = 6$  legs), the mechanical power is the product of the motor torque and angular velocity. The global index is obtained by averaging the mechanical absolute power delivered over a period  $T$  and travelled distance  $L$ :

$$P_{av} = \frac{1}{L} \cdot \frac{1}{T} \sum_{i=1}^n \sum_{j=1}^m \int_0^T |\tau_{ij}(t) \cdot \dot{\theta}_{ij}(t)| dt \quad (6)$$

The average of the absolute power consumption, per travelling distance,  $P_{av}$ , should be minimised.

#### 3.2 Mean Power Lost

Another optimisation strategy for an actuated system considers the power lost in the joint actuators per cycle  $T$  and travelled distance  $L$ . From this point of view, the index mean power lost per meter can be defined as:

$$P_L = \frac{1}{L} \cdot \frac{1}{T} \sum_{i=1}^n \sum_{j=1}^m \int_0^T [\tau_{ij}(t)]^2 dt \quad (7)$$

The most suitable trajectory is the one that minimizes  $P_L$ .

#### 3.3 Mean Force at the Interface Body-Legs

A third possible optimisation strategy considers the forces that occur on the hips of the robot per cycle  $T$  and travelled distance  $L$ . The index mean force on the hips per meter is defined as:

$$F_L = \frac{1}{L} \cdot \frac{1}{T} \sum_{i=1}^n \sum_{j=1}^m \int_0^T \left\{ [f_{ix}(t)]^2 + [f_{iy}(t)]^2 \right\} dt \quad (8)$$

The best trajectory is the one that minimizes  $F_L$ .

### 4 STRATEGIES TO OVERCOME OBSTACLES

In the absence of obstacles, the robot is assumed to adopt the Wave Gait (*WG*) (2). When the robot finds an obstacle, it changes the gait according to one of two predefined strategies. After crossing the obstacle, the robot returns to the Wave Gait.

The obstacle is fully characterised by its position along the  $x$ -axis  $X_o$ , its height  $H_o$  and its width  $L_o$ . Moreover, for simplicity, it is assumed that the obstacle only affects the trajectories of foot 1.

Both obstacle avoidance strategies (Figure 2) assume that during the swing phase the trajectory of the tip of the foot 1 is computed through a cycloid function.

#### 4.1 Crossing one obstacle by variation of the foot clearance – FC

In the first strategy, the robot lifts the foot above the top of the obstacle and crosses over it, taking only one step of length  $L_S$ . For this strategy to be applicable one must have  $L_o < L_S$ .

#### 4.2 Crossing one obstacle by stepping over it – SO

The second strategy allows the robot to cross obstacles larger than one step length. In this case, during the first step the robot lifts the foot above the top of the obstacle and steps over it. Afterwards, in the second step, the robot steps again over the ground. For this strategy to be applicable it is required that  $L_o < 2L_S$ .

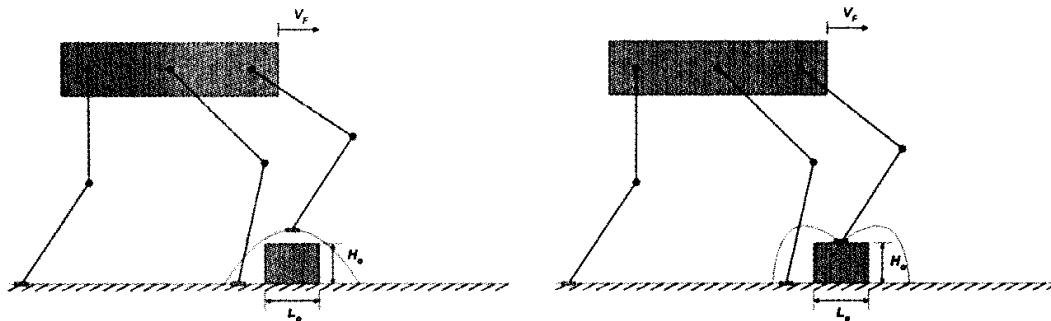


Fig. 2 Crossing one obstacle by using strategy FC and SO

As stated previously, after crossing over the obstacle the robot returns to the periodic gait.

## 5 SIMULATION RESULTS

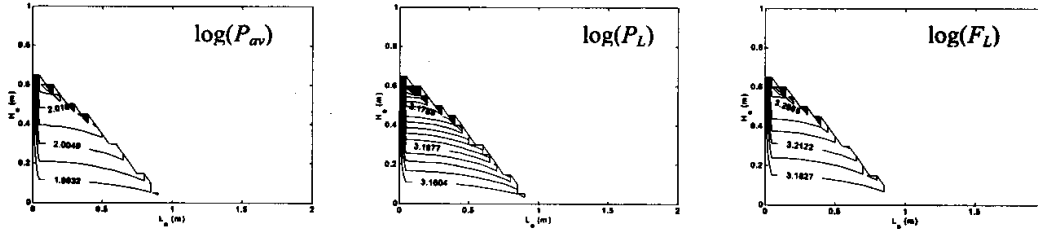
To illustrate the adoption of the preceding concepts, this section develops a set of simulations to estimate the influence of the obstacle position ( $X_o$ ) and its dimensions ( $L_o$ ,  $H_o$ ) during quasi-periodic gaits. In a first phase we study strategies for overcoming obstacles placed in particular coordinates and afterwards, in a second phase, we analyse the case of any possible location.

### 5.1 Strategy comparison for crossing obstacles at special places

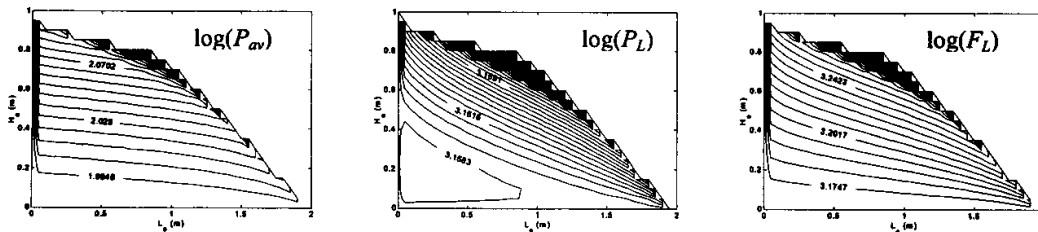
The multi-legged locomotion is simulated during four steps in order to examine the role of the walking gait with  $\beta = 50\%$ ,  $L_S = 1$  m,  $H_B = 1,8$  m,  $F_C = 0.001$  m,  $V_F = 1$  m/s,  $S_P = 1$  m,  $L_{i1} = L_{i2} = 1$  m,  $O_i = 0$  m,  $M_{i1} = M_{i2} = 1$  Kg,  $M_b = 36$  Kg and  $M_{if} = 0$  Kg. Moreover, are considered the cases of obstacles placed at  $X_o = 1.75$  m (middle of step) and  $X_o = 2.25$  m (end/begin of the step), that require a transient from periodic to quasi-periodic gait of step 2 or steps 2 and 3 in the FC and SO strategies, respectively.

- **Strategy FC:** Figure 3 presents the charts of  $P_{av}(L_o, H_o)$ ,  $P_L(L_o, H_o)$  and  $F_L(L_o, H_o)$  considering that the obstacle is in the centre of the robot second step. These charts show that the three indices present the same type of variation, namely they increase with  $L_o$  and  $H_o$ . From these charts it is also seen that the maximum dimensions of the obstacle that the robot can cross are  $L_o = 0.95$  m and  $H_o = 0.65$  m.

- Strategy SO:** Figure 4 presents  $P_{av}(L_o, H_o)$ ,  $P_L(L_o, H_o)$  and  $F_L(L_o, H_o)$  considering that the obstacle is in the position where the robot should conclude the second step. It can be seen that  $P_{av}$  increases slightly with  $L_o$  and sharply with  $H_o$ . We can also conclude that the maximum dimensions of the obstacle that the robot can cross are  $L_o = 1.95$  m and  $H_o = 0.95$  m. Comparing with the FC strategy, we verify that the SO allows to cross obstacles with higher and larger dimensions. Furthermore, the charts of  $P_L(L_o, H_o)$  and  $F_L(L_o, H_o)$  show the same type of variation with  $L_o$  and  $H_o$ .



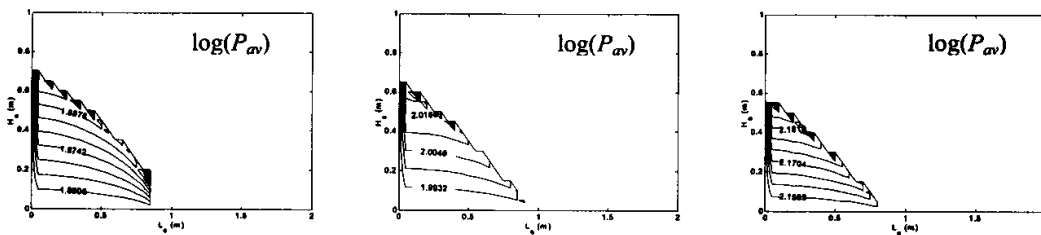
**Fig. 3** Plots of  $P_{av}$ ,  $P_L$  and  $F_L$  vs.  $(L_o, H_o)$ ,  $X_o = 1.75$  m, for  $\beta = 50\%$ ,  $V_f = 1$  m/s with strategy FC



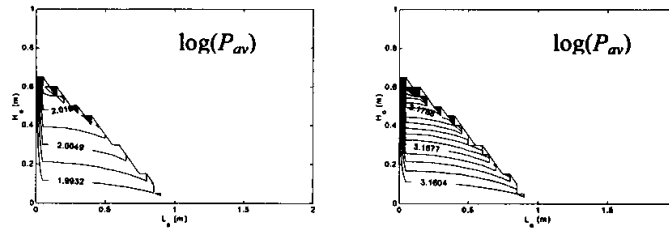
**Fig. 4** Plots of  $P_{av}$ ,  $P_L$  and  $F_L$  vs.  $(L_o, H_o)$ ,  $X_o = 2.25$  m, for  $\beta = 50\%$ ,  $V_f = 1$  m/s with strategy SO

Figure 5 presents the charts of  $P_{av}(L_o, H_o)$ . It can be seen that the size of the obstacles that the robot can negotiate decreases with  $\beta$ . The same can be concluded analyzing the indices  $P_L(L_o, H_o)$  and  $F_L(L_o, H_o)$ .

Concerning the variation with  $V_f$  (figure 6), as expected, the size of the (static) obstacles do not affect  $P_{av}(L_o, H_o)$  that just increases with  $V_f$ . The indices  $P_L(L_o, H_o)$  and  $F_L(L_o, H_o)$  show the same type of variation with  $L_o$  and  $H_o$  versus  $V_f$ .



**Fig. 5** Plots of  $P_{av}$  vs.  $(L_o, H_o)$ ,  $X_o = 1.75$  m for  $\beta = \{40\%, 50\%, 60\%\}$  with  $V_f = 1$  m/s and strategy FC, respectively



Moreover, we will also address the effects of the foot – ground interaction and a model describing the ground characteristics. The contact and reaction forces at the robot feet will enable further insight towards the development of efficient multi-legged locomotion robots.

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