

On the Statistical/Harmonic Modelling of Mechanical Manipulators

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Abstract

A new approach to the analysis and design of robot manipulators is presented. The novel feature resides on a non-standard formulation to the modelling problem. Usually, system descriptions are based on a set of differential equations which, in general, require laborious computations and may be difficult to analyze. These facts motivate the need of alternative models based on different mathematical concepts. The proposed statistical approach to the Fourier modelling gives clear guidelines towards the optimization of the robot kinematics and point out structural characteristics of the trajectory planning algorithms.

Keywords: Modelling, Robots, Kinematics, Fourier Analysis, Statistics.

Introduction

Mechanical manipulators are developed according to engineering and scientific principles which are based on fundamental concepts such as those arising from mathematics and physics. Based on these formulations, the first step on the study of a physical phenomena is the development of an adequate model. Usually the fundamental concepts are the differential and matrix calculus and the classical newtonian physics, while the model consists on a set of differential equations. Nevertheless, several phenomena, such as quantum physics and thermodynamics, may be studied through different mathematical tools, namely using statistical methods. These facts suggest that, for a given problem, we may adopt different mathematical models, each with its own merits and drawbacks. The second step on the study of the physical phenomena is the analysis of the properties revealed by the model. For a model consisting on a set of linear differential equations we can adopt simple strategies (*e.g.* the Fourier analysis) but, for a non-linear model these tools are not adequate and the analysis becomes complex and difficult to generalize. In fact, experience demonstrates that for a large number of cases, such as the kinematic and dynamic models arising in robotics, efficient tools capable of rendering clear results are still lacking.

This paper presents a framework where it is developed a new modelling formalism based on the embedding of statistical and Fourier transform concepts. These concepts are then illustrated on several experiments. The examples reveal not only the capabilities of the new method but also the limitations of standard robot structures and path planning algorithms. Consequently, in order to develop the new formalism the paper is organized as follows. Next section starts by presenting the fundamental modelling concepts. Based on the new concepts, the following section illustrates its application on the kinematic and trajectory planning analysis of mechanical manipulators. Finally, the last section outlines the main conclusions.

Embedding Statistics and Fourier Transform Towards the Modelling of Robot Manipulators

For a robot having n degrees of freedom (*dof*) the classical direct kinematic model is described by a set of equations $\mathbf{p} = \psi(\mathbf{q})$ relating the operational and the joint spaces, where $\mathbf{p} = [p_1, \dots, p_n]^T$ and $\mathbf{q} = [q_1, \dots, q_n]^T$ are the $n \times 1$ vectors of position in the operational and joint spaces, respectively. Based on these equations considerable research has been done on issues such as the optimization of the manipulator structure (Tsai et al., 1981; Yoshikawa, 1985) and the development of efficient path planning algorithms (Sahar et al., 1986; Scheinman et al., 1984). However, the kinematic equations usually are non-linear and reveal a plethora of variables and parameters that give rise to a cumbersome

work, both in the analysis and design stages. Therefore, in order to overcome these problems alternative concepts are required. Statistics is a mathematical tool well adapted to handle a large volume of data that has already been used in some restricted classes of robotic problems (Mooring et al., 1987; Singh et al., 1992; Galhano et al., 1990). Nevertheless, for the kinematic modelling, statistics is not capable of dealing with time-dependent relations. Therefore, to overcome the limitations of statistics (Galhano et al., 1990; 1991; Machado et al., 1993), the new method (Machado et al., 1997) will also take advantage of the Fourier transform by embedding both tools in a broader formalism. In this line of thought, the first stage of the new modelling formalism starts by comprising:

- A set of input variables (*ivs*), that is, variables that are free to change independently;
- A set of output variables (*ovs*), that is, variables that depend on the *ivs*, and
- A set of parameters which are to be optimized in the design stage.

As usually, in the direct kinematics the *ivs* and *ovs* are established by the relation $\psi : \{q, \dot{q}, \ddot{q}\} \rightarrow \{p, \dot{p}, \ddot{p}\}$, while for the inverse kinematics we get the reverse relation $\psi^{-1} : \{p, \dot{p}, \ddot{p}\} \rightarrow \{q, \dot{q}, \ddot{q}\}$. In both cases, the set of parameters depend on the manipulator structure and the time/space evolution of the trajectories.

The second stage of the formalism consists on the embedding of the statistical analysis into the Fourier transform through the algorithm:

- i) A statistical sample for the variables is obtained by driving the manipulator through a large number of trajectories (generated according with adequate statistics) having appropriate time/space evolutions. All the variables (*i.e.* the *ivs* and the *ovs*) are calculated, sampled in the time domain, and the resulting numerical values are stored in arrays.
- ii) For each of the previous arrays, the Fourier transform is computed (numerically) and the corresponding frequency spectrum is stored in a second class of arrays.
- iii) After concluding the statistical sample of trajectories, for all the variables and for each frequency within the spectrum range under study, several statistical indices (*e.g.* percentiles) of the amplitudes and/or phases of the arrays obtained in *ii*) are calculated. The statistics of the frequency spectrum is stored in a third class of arrays.
- iv) For all the variables and for each frequency, the values of the statistical indices calculated in *iii*) are collected on a 'composite' frequency spectrum and stored in a fourth class of arrays.
- v) The procedure *i*) to *iv*) is repeated for different numerical values of the link lengths. The numerical results for the fourth class of arrays obtained in *iv*) are compared and analytical expressions, that fit the numerical data, are extrapolated.
- vi) The algorithm *i*) to *v*) is repeated for different time/space trajectories.
- vii) The algorithm *i*) to *vi*) is repeated for different robot structures.
- viii) The partial conclusions drawn by the analytical expressions obtained in *v*), *vi*) and *vii*) are integrated and final conclusions are drawn.

In order to illustrate the new method in the next section we analyze the kinematics of planar manipulators and we compare the results with the classical direct and inverse kinematic equations.

A New Model for the Kinematics of Planar Robots

The application of the formalism defined in the previous section requires the development of numerical calculations for the statistics. Therefore, before proceeding, we need to establish the different experiments according with the following guidelines:

- i) Modelling case:
 - $IK = \{\text{Inverse Kinematics}\}$,
 - $DK = \{\text{Direct Kinematics}\}$
- ii) Type of trajectory
 - ii.1) Time acceleration profile:
 - $O = \{\text{On/Off acceleration}\}$,
 - $T = \{\text{Triangular acceleration}\}$,
 - $P = \{\text{Parabolic acceleration}\}$,
 - $S = \{\text{Sinusoidal acceleration}\}$

(Continuation)

Experiment	$F\{\ddot{p}_1\}, F\{\ddot{p}_2\}$	$F\{\ddot{q}_1\}$	$F\{\ddot{q}_2\}$	
	$0.888\sqrt{l_1 l_2}$	$1.623 - 0.287 \frac{l_1 - l_2}{l_1 + l_2} - 1.158 \left \frac{l_1 - l_2}{l_1 + l_2} \right $	0.758	Gain
IK-RR T, MAL, SL	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.445 \pm j1.660),$ $\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} 0.583$	$\sqrt{A_{max}} \left(\frac{0.469}{\sqrt[4]{l_1 l_2}} \pm j \frac{2.001}{\sqrt[6]{l_1^2 l_2}} \right),$ $\frac{\sqrt{A_{max}}}{\sqrt[6]{l_1^2 l_2}} 0.794$	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.419 \pm j2.149),$ $\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} 1.261$	Poles
	$0.883\sqrt{l_1 l_2}$	$1.611 - 0.290 \frac{l_1 - l_2}{l_1 + l_2} - 1.154 \left \frac{l_1 - l_2}{l_1 + l_2} \right $	0.759	Gain
IK-RR P, MAL, SL	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.566 \pm j0.824),$ $\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.938 \pm j3.176)$	$\sqrt{A_{max}} \left(\frac{0.612}{\sqrt[4]{l_1 l_2}} \pm j \frac{1.072}{\sqrt[6]{l_1^2 l_2}} \right),$ $\sqrt{A_{max}} \left(\frac{1.069}{\sqrt[4]{l_1 l_2}} \pm j \frac{4.133}{\sqrt[6]{l_1^2 l_2}} \right)$	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.575 \pm j1.406),$ $\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (1.194 \pm j3.941)$	Poles
	$0.806\sqrt{l_1 l_2}$	$1.456 - 0.237 \frac{l_1 - l_2}{l_1 + l_2} - 1.068 \left \frac{l_1 - l_2}{l_1 + l_2} \right $	0.691	Gain
IK-RR O, MVL, SL	$\frac{V_{max}}{\sqrt{l_1 l_2}} (0.525 \pm j0.843)$	$V_{max} \left(\frac{0.614}{\sqrt{l_1 l_2}} \pm j \frac{1.061}{\sqrt[3]{l_1^2 l_2}} \right)$	$\frac{V_{max}}{\sqrt{l_1 l_2}} (0.522 \pm j1.431)$	Poles
	$0.795\sqrt{l_1 l_2}$	$1.465 - 0.221 \frac{l_1 - l_2}{l_1 + l_2} - 1.082 \left \frac{l_1 - l_2}{l_1 + l_2} \right $	0.709	Gain
IK-RR O, RAL, SL	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.338 \pm j0.600)$	$\sqrt{A_{max}} \left(\frac{0.472}{\sqrt[4]{l_1 l_2}} \pm j \frac{0.855}{\sqrt[6]{l_1^2 l_2}} \right)$	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.441 \pm j1.161)$	Poles
	$0.741\sqrt{l_1 l_2}$	$1.444 - 0.244 \frac{l_1 - l_2}{l_1 + l_2} - 1.062 \left \frac{l_1 - l_2}{l_1 + l_2} \right $	0.713	Gain
DK-RR O, MAL, SL	$\sqrt{A_{max}} (0.388 \pm j1.459)$	$\sqrt{A_{max}} (0.353 \pm j0.841)$	$\sqrt{A_{max}} (0.337 \pm j0.830)$	Poles
	$\sqrt{l_1 l_2} \left(0.924 + 2.093 \left \frac{l_1 - l_2}{l_1 + l_2} \right ^3 \right)$	1.665	0.827	Gain
IK-RR O, MAL, DP	$\ddot{p}_1 \sim \frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.495 \pm j0.884)$ $\ddot{p}_2 \sim \frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.348 \pm j1.373)$	$\sqrt{A_{max}} \left(\frac{0.632}{\sqrt[4]{l_1 l_2}} \pm j \frac{1.147}{\sqrt[6]{l_1^2 l_2}} \right)$	$\frac{\sqrt{A_{max}}}{\sqrt[4]{l_1 l_2}} (0.479 \pm j2.065)$	Poles
	$\ddot{p}_1 \sim 2.24\sqrt{l_1 l_2}$ $\ddot{p}_2 \sim 1.373\sqrt{l_1 l_2}$	$2.469 - 1.586 \left \frac{l_1 - l_2}{l_1 + l_2} \right $	0.749	Gain

- Numerical convergence - After repeating a large number of numerical experiments the charts with the *SHC* of the variables do not change significantly.
- Derivative/integral sensitivity - Although being composite curves, the *SHC* still obey the 'standard' $j\omega$ operator for variables that are related by the derivative operator in the time domain.
- Analytical coherence - The numerical data that results from the experiments 'fits' the analytical expressions that lead to clear conclusions. For example, the *DK*, that calculates the trajectories in the joint space, leads to expressions for the poles that do not depend on the length of the workspace, while the *IK*, that calculates the trajectories in the operational space, leads to poles that are sensitive to the length of the workspace. Therefore, trajectory planning algorithms on the joint space are more 'robust', that is, lead to a constant bandwidth.

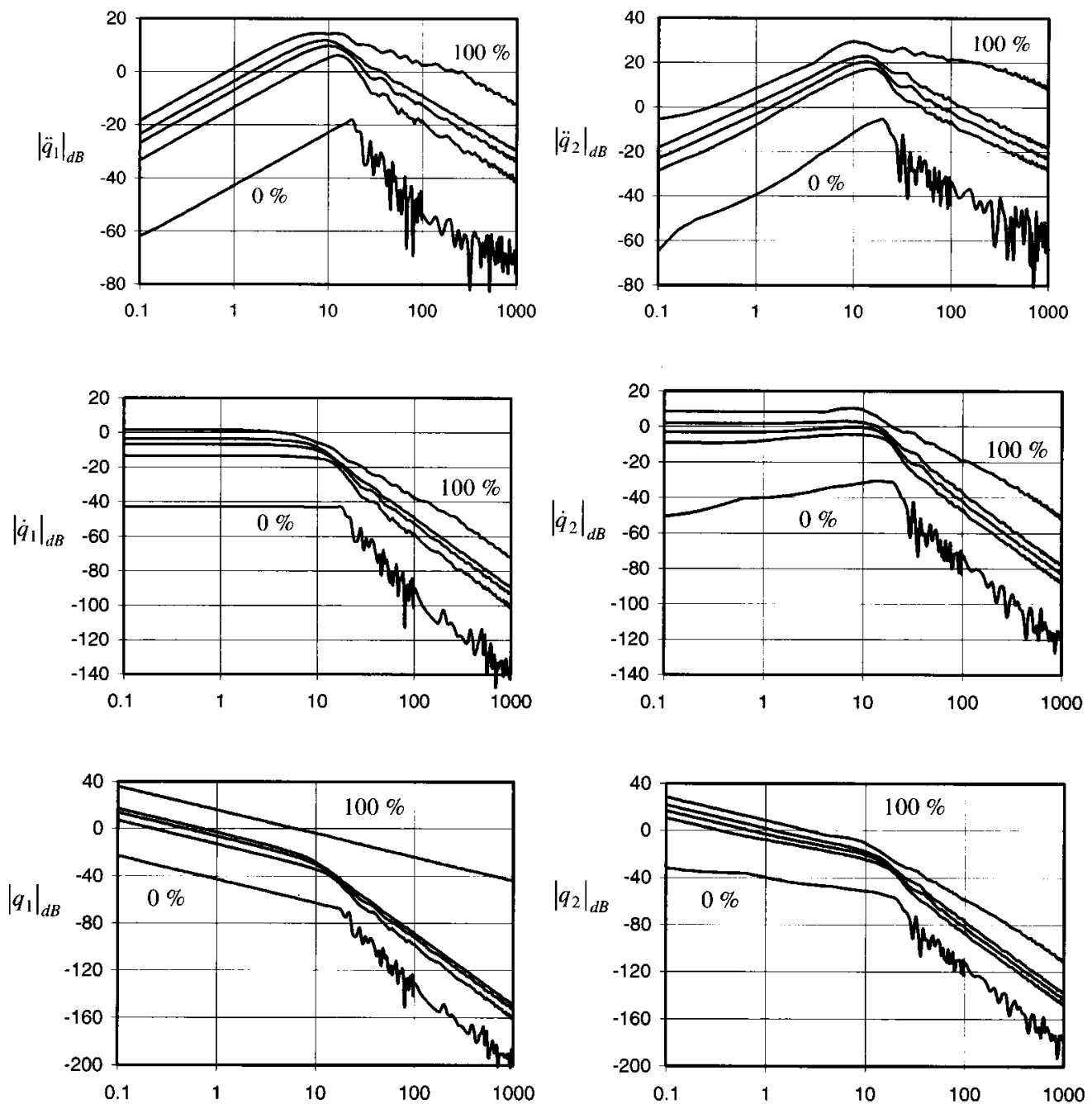


Fig. 2 Percentiles of the *SHC* of the ovs in the experiment $\{IK-RR, O, MAL, SL\}$ with $l_1=1$, $l_2=0.1$, $A=10$.

- **Generality** - While for the classical deterministic models we can not find clear relations between different robot structures, with the *SHC* general characteristics are highlighted. For example, joint 1 of the *RR* robot reveals a *SHC* that follows closely the *SHC* of joint 1 of the *RP* structure which reflects that, in both cases, we have a rotational joint. On the other hand, for joint 2, we have different mechanical articulations in the two cases and, therefore, the experiments lead to distinct results (see Fig. 3).
- **Compatibility** - The conclusions based on the analysis of the *SHC* are coherent with the results of previous studies using different mathematical tools (Tsai *et al.*, 1981; Yoshikawa, 1985; Machado *et al.*, 1993). For example, if $l_1 + l_2 = \text{constant}$ we verify that the maximum gain and bandwidth of the *SHC* occurs for $l_1 = l_2$. Also, the lower and upper elbow solutions for the *IK* where analyzed revealing, as expected, similar properties in both cases.

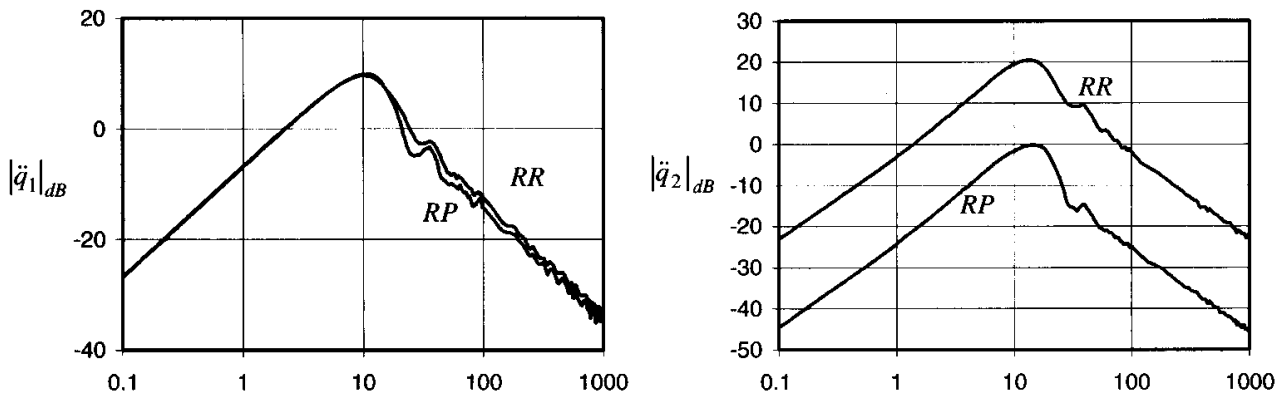


Fig. 3 Comparison of the 50%-percentiles of the *SHC* of the *ovs* in the experiments $\{IK-RR, O, MAL, SL\}$ with $l_1 = 1, l_2 = 0.1, A = 10$ and $\{IK-RP, O, MAL, SL\}$ with $l_{max} = 1.1, l_{min} = 0.9, A = 10$.

Another aspects of interest is that the *SHC* of *ovs* depends, not only on the system, but also on the excitations that, for the kinematics, is related with the type of trajectory planning algorithm. For example, Fig. 4 shows that the *SHC* of the *ivs* are identical for the *SL* trajectories while the symmetry is not preserved for the *DP* experiment. Moreover, the *SL* trajectories ‘avoid’ the singular points near the boundary of the robot workspace in contrast with the *DP* case where we may get very high amplitudes for the *ovs*, as can be observed in the 100% percentile in Fig. 5.

The aforementioned experiments were performed with the kinematics of planar robots. Therefore, two possible developments are the study of the dynamics and the application of the new method to robots with more *dofs*. In both cases, the major point to be further investigated consists on the complexity of the heuristic equations of the *SHC*. On the other hand, the analysis of the histogram for each *iv* or *ov* remains almost immediate.

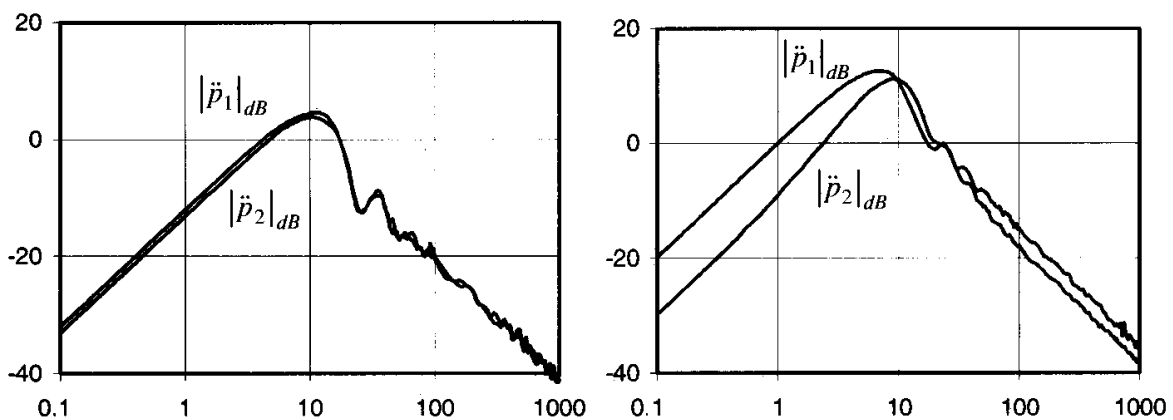


Fig. 4 Comparison of the 50%-percentiles of the *SHC* of the *ivs* in the experiments $\{IK-RR, O, MAL, SL\}$ and $\{IK-RR, O, MAL, DP\}$ with $l_1 = 1, l_2 = 0.1, A = 10$.

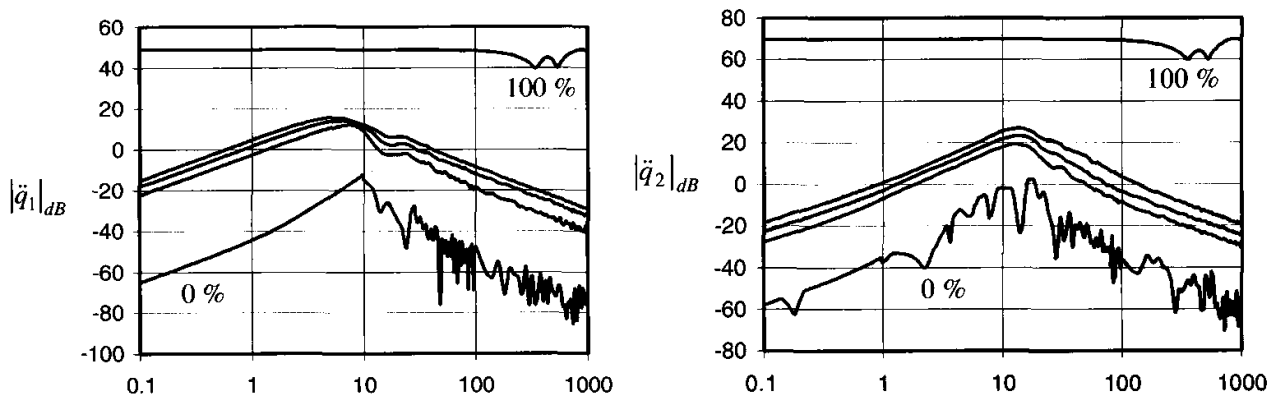


Fig. 5 Percentiles of the SHC of the ovs in the experiment $\{IK-RR, O, MAL, DP\}$ with $l_1 = 1, l_2 = 0.1, A = 10$.

Conclusions

A new method to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which can be very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The proposed method, by embedding the statistical analysis into the Fourier transform, provides a framework giving clear guidelines towards the optimization both of the path planning algorithm and the robot structure and a deeper understanding of the actuator requirements.

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