

INSTITUTO POLITÉCNICO DO PORTO

Trabalho de natureza profissional para a atribuição do título de especialista

Os perfis de investimento dos clientes de um banco suíço e os seus comportamentos: um estudo comparativo

Resumo:

As finanças de mercados tradicionais baseiam-se na hipótese dos mercados eficientes. É suposto os investidores serem racionais, e tomarem as decisões de investimento maximizando o binómio risco e retorno. Não é suposto sofrerem de enviesamentos comportamentais tais como aversão a perdas ou falácia dos custos afundados. Neste trabalho mediram-se as atitudes face ao risco de uma centena de investidores financeiros em dois momentos do tempo, bem como as suas decisões de investimento. Concluiu-se que não se pode rejeitar a hipótese dos investidores sofrerem de enviesamentos comportamentais, e como tal, de terem racionalidade limitada. Trata-se de uma conclusão que reforça a importância que as finanças comportamentais têm vindo ganhar nas ciências económicas e empresariais.

Por:

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Motivação.

Entre 2004 e 2009 trabalhei como Subdiretor num banco suíço de Private Banking (gestão de carteiras de investimento financeiro para particulares, normalmente com valores acima do 1 milhão de euros).

Tendo formação de base em Gestão e Mercados Financeiros, fiquei surpreendido pelos comportamentos aparentemente irracionais dos cerca de 100 clientes que acompanhei nesse período. Verifiquei que muitas das suas decisões de investimento eram tomadas com base em emoções, e não nas análises financeiras que eu lhes apresentava.

A teoria financeira tradicional baseia-se na hipótese dos mercados eficientes, a qual por sua vez postula que os investidores são racionais e escolhem ativos com base na maximização do binómio risco e retorno, para um determinado nível de tolerância ao risco. A isto chama-se o perfil de investimento de cada investidor.

Seguindo esses princípios, cada cliente tinha de preencher um questionário de perfil de investimento (suitability test) no momento da abertura da conta, e actualizá-lo a cada dois anos. Era com base nesses questionários que eu apresentava as propostas de investimento aos clientes, tendo por base os cânones da moderna teoria financeira, sugerindo carteiras de investimento teóricas adequadas para os clientes investirem.

O que notei foi que com o tempo as carteiras de investimento reais (após as instruções de compra e venda dadas pelos clientes) afastavam-se das carteiras teóricas que eram propostas tendo por base a teoria financeira. Os investidores pareciam não ter uma consciência do seu perfil de investimento, tomando posições nas suas carteiras desadequadas ao seu nível de tolerância de risco.

Chegado a 2009, e com o início da grande recessão, resolvi realizar um trabalho profissional de análise das carteiras dos meus clientes à luz dos pressupostos dos mercados eficientes. Recolhi e comparei os questionários dos clientes feitos em 2007 e 2009, bem como as suas reais carteiras de investimento.

Para realizar esse estudo (para o qual tive o apoio da administração do banco em termos de horas de trabalho no próprio banco) efectuei uma revisão da literatura sobre finanças, e especificamente finanças comportamentais, e recorri a métodos e *software* estatístico para que as comparações entre 2007 e 2009 tivessem validade.

As conclusões a que cheguei reforçam o que a escola das finanças comportamentais tem vindo a defender: o ser humano tem racionalidade limitada, e toma decisões com base em emoções. Os profissionais de bancos e sociedades gestoras de patrimónios ganhariam em compreender esta realidade, e poderiam maximizar o seu valor acrescentado ao incorporar estas características comportamentais humanas nos seus modelos de investimento.

O estudo foi feito em inglês (fi-lo na Suíça, no contexto de trabalho de um banco multinacional), e está apresentado nas próximas páginas.

“Successful investing is anticipating the anticipations of others”.

“The market can stay irrational longer than you can stay solvent.”

“It is generally agreed that casinos should, in the public interest, be inaccessible and expensive. And perhaps the same is true of stock exchanges.”

John Maynard Keynes, Economist, 1883-1946

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1. INTRODUCTION

1.1. Financial Markets and Behavioural Finance

One of the most studied areas of Economics is the way financial markets work. It is surprising that such a small part of the economic world, where only a minority of the world citizens has a direct intervention (through the purchase of stocks or bonds, mutual funds, etc.) creates such an interest in the academic world.

Several theories were stated and models were developed in order to correctly price the instruments traded in financial markets. Some of them are still in use today, and are generally accepted by the financial community, but almost all of them are based on some propositions, or assumptions, being the most known of all the Efficient Market Hypothesis. This hypothesis assumes that financial markets are efficient and that the players participating in the market are rational, making the right investment decisions at any given time.

Unfortunately (or fortunately for this study) empirical evidence has shown that this is not the case, and that in fact there are many specific situations on which at least some investors do behave irrationally. Examples of these situations can be seen in the stock market ‘bubbles’ (when there is frenzy for some type of financial instruments) or ‘crashes’ (when all investors decide to sell all financial instruments at the same time).

These kind of situations, that some researchers call ‘herd behaviour’ puzzles the academic community and lays doubts on the so called Efficient Market Hypothesis, thus putting in stake most of the currently asset pricing models used nowadays by financial markets professionals. According to some researchers this behaviour (not rational) is mostly observed in individual small investors, but recent studies have also observed it in professional institutional investors, but in a more contained way and with some understandable causes (informal pressures from external institutions or individuals to behave in pre-determined ways, independent of rationality).

Researchers have put a lot of efforts into the theoretical and empirical investigation of the influence of irrational investors on asset prices. However academic research also suggests that this apparent irrational phenomenon can also arise naturally in settings with rational investors.

People may seem to be acting independently when making investment decisions on financial markets, but usually they are not. In situations with uncertainty or risk, people feel the need for advice and support from others. Investment decisions are decisions with great uncertainty, and these are situations where social influence is largest, when people are assumed to posse knowledge useful in finding the best or optimal investment decision.

According to Festinger (1954), people have a number of reasons why such influences occur. For instance, people have an innate tendency to compare themselves with others.

In particular when there are no objectives and non-social objects of comparison, then people evaluate their own attitude and capability against those of others.

Of all players in the market, individual investors are probably the ones whose behaviour is least studied and the ones that academic researchers in the economics field understand worse. Probably due to human bounded rationality, which cannot be put in a model (up until now), standard finance theory, rooted in economics, has skipped investor behaviour when trying to explain how financial markets work and in the pricing of financial instruments. In fact, it catalogues individual investors as ‘Noise Traders’, a kind of ‘dumb’ investor that acts erroneously, but as a group is neutral to the market because all ‘Noise Traders’ actions neutralize each other. The fact is that the importance of these ‘Noise Traders’ (individual investors) has increased as a percentage of all transactions in the financial markets, due to the increase of wealth of Western world families.

Recently (since last century 80’s) some studies have started trying to address this issue by merging some psychology approaches to the study of financial markets. Sociology and Consumer Behaviour (from Marketing) have also served as sources for new developments in the financial theory, and a new branch of finance studies was born: Behavioural Finance.

1.2. Private Banks and individual investors

For centuries the access to financial markets was restricted to a few players (Governments, Banks, Insurance companies), making it a mature and stable industry. Several types of banks exist (Commercial, Wholesale, Consumer, Mutual, etc.), but in what matters to financial markets only Investment Banks and Private Banks really have a regular presence in the industry.

Private Banks are banks specialized in financial assets management (or wealth management) for individuals. They differ from other banks by not lending money to individual people for any other purpose (mortgage, consumer) than to invest in securities. Most of the activity of a Private Bank is about managing an individual client securities portfolio in a financial market, under a mandate given by the client or advising him to purchase or sell specific securities. The bank’s income is generated through commissions, based on the assets under management the individual has mandated the bank to invest.

In the current study, I accessed the data regarding individual investors through a Private Bank. These kind of specialized banks provide financial services to wealthy individuals (minimum of EUR/USD/CHF 1 million to be invested). Individual investors are not professional investors (like pension fund managers, etc.), they have limited cognitive capabilities in what regards securities’ investment, so they tend to trust institutions with a good track record of service (meaning above average returns for a given amount of risk).

For a Private Bank to grow, regardless of the forces shaping the industry (substitute players, Internet, slowing demand) the secret for success remains in their core competence, which is a function of the quality of the people working in the institution. When a Private Bank loses its best professionals to a competing bank, it should worry as clients may also defect. Studies from the Private Banker International have proved that a Private Banker may shift to a new institution up to 20% of the assets he was responsible for. On the other hand, investment teams are the ones responsible for the clients' portfolios performance, which means that if they move to a competitor, most probably the investment service will suffer a change (for better or for worse).

More and more Private Banks are starting to think in new approaches to handle client relationships, namely in a new branch of finance – the Behavioural Finance.

1.3. Private Banking, individual investor and Behavioural Finance.

According to a study by Michael Pompian (2007) one of the toughest tasks for a private banker adviser is to understand the basic psychological drivers of clients, beyond their apparent rational personal investment and planning requirements. Often decisions are taken by the client that are irrational, leading to tensions with the adviser who fails to 'get inside the head' of the customer.

One classic example is that clients often go to great lengths to justify previous investment decisions, especially losing ones, as many wealth management professionals will have noted. Clients often delay unloading assets that are not generating adequate returns because they don't want to admit having made a mistake.

Indeed, financial advisers lose clients for host of reasons – and, contrary to popular belief, the primary reason is not poor investment results. The most common reason why financial advisers lose clients is the failure to effectively understand their clients, and their personality types, to build a solid personal and financial relationship.

Sometimes building that relationship is easy – the client being advised is rational in approach, has reasonable expectations and a good understanding of asset allocation. For these clients, the typical method for arriving at an asset allocation is to administer a risk tolerance questionnaire and use financial planning software to create a mean-variance optimised asset allocation.

At other times, financial advisors encounter irrational behaviour from clients. Irrational clients might overestimate their risk tolerance, be unrealistic in their return expectations, or generally behave in a way that appears resistant to rational investment principles.

Advisers can get frustrated and impatient when confronted with an irrational client. To avoid this situation, and according to a report from Private Banker International (2007) advisers are increasingly using behavioural finance with their clients. Experienced financial advisers know that defining financial goals is critical to creating an appropriate investment programme for the client. To best define financial goals, it is helpful to

understand the psychology and emotions underlying the decisions behind creating the goals.

The key result of behavioural finance enhanced relationship will be a portfolio to which the adviser can comfortably adhere while fulfilling the client's long-term goals. This result has obvious advantages – advantages that suggest that behavioural finance will continue to play an increasing role in portfolio structure.

2. FINANCE: A LITERATURE REVIEW

2.1. Introduction

Finance is the branch of economics that focuses on the capital markets. In the early life of the subject, emphasis was placed on describing the market environment and valuing individual securities. In more recent years, attention has switched towards broader aspects of valuation (Fundamental Analysis¹ versus Technical Analysis²) and there were developments in new methodologies for valuing a wide variety of assets whose characteristics extend across time, and which impose intricate and complex risk on investors.

Modern financial theory is founded on three main assumptions: markets are highly efficient, investors exploit potential arbitrage opportunities, and investors are rational (Dimson et al, 2000). In finance, the efficient-market hypothesis (EMH) asserts that financial markets are "informational efficient", or that prices on traded assets, e.g., stocks, bonds, or property, already reflect all known information. The efficient-market hypothesis states that it is impossible to consistently outperform the market by using any information that the market already knows, except through luck. Information or news in the EMH is defined as anything that may affect prices that is unknowable in the present and thus appears randomly in the future.

The efficient-market hypothesis was first expressed by Louis Bachelier (1900), a French mathematician, in his 1900 dissertation, "The Theory of Speculation". His work was largely ignored until the 1950s; however beginning in the 30s scattered, independent work corroborated his thesis. A small number of studies indicated that US stock prices and related financial series followed a random walk model (Cowles and Jones, 1937). Also, work by Alfred Cowles in the '30s and '40s showed that professional investors were in general unable to outperform the market.

The efficient-market hypothesis emerged as a prominent theoretic position in the mid-1960s. Paul Samuelson³ had begun to circulate Bachelier's work among economists. In 1965 Eugene Fama published his dissertation (Fama, 1965) arguing for the random walk hypothesis and Samuelson published a proof for a version of the efficient-market hypothesis (Samuelson, 1965). In 1970 Fama published a review of both the theory and the evidence for the hypothesis (Fama, 1970). The paper extended and refined the

¹ Fundamental analysis of a business involves analyzing its financial statements and health, its management and competitive advantages, and its competitors and markets. Fundamental analysis is performed on historical and present data, but with the goal of making financial forecasts, or attributing the expected rate of return – $E(r)$ – to a security.

² Technical analysis is a financial markets technique that claims the ability to forecast the future direction of security prices through the study of past market data, primarily price and volume. In its purest form, technical analysis considers only the actual price behavior of the market or instrument, on the assumption that price reflects all relevant factors before an investor becomes aware of them through other channels.

³ Nobel Prize of Economy in 1970.

theory, included the definitions for three forms of market efficiency: weak, semi-strong and strong.

Behavioural finance explains that when entering positions market participants are not driven primarily by whether prices are cheap or expensive, but by whether they expect them to rise or fall. To ignore this can be hazardous: Alan Greenspan⁴ warned of "irrational exuberance" in the markets in 1996, but some traders who sold short new economy stocks that seemed to be greatly overpriced around this time had to accept serious losses as prices reached even more extraordinary levels. As John Maynard Keynes⁵ succinctly commented, "Markets can remain irrational longer than you can remain solvent".

Prior to the efficient-market hypothesis, the prevailing view was that markets were inefficient. Inefficiency was commonly believed to exist e.g., in the United States and United Kingdom stock markets. However, earlier work by Kendall (1953) suggested that changes in UK stock market prices were random. Other empirical evidence in support of the EMH comes from studies showing that the return of market averages exceeds the return of actively managed mutual funds (Bogle, 1993). Thus, to the extent that markets are inefficient, the benefits realized by seizing upon the inefficiencies are outweighed by the internal fund costs involved in finding them, acting upon them, advertising, etc. These findings gave inspiration to the formation of passively managed index funds⁶.

Adherents to a stronger form of the EMH argue that the hypothesis does not preclude - indeed it predicts - the existence of unusually successful investors or funds occurring through chance. In addition, supporters of the EMH point out that the success of Warren Buffett and George Soros may come as a result of their business management skill rather than their stock picking ability.

The efficient-market hypothesis also appears to be inconsistent with many events in stock market history. For example, the stock market crash of 1987 saw the S&P 500 drop more than 20% in the Month of October despite the fact that no major news or events occurred prior to the Monday of the crash, the decline seeming to have come from nowhere. This would tend to indicate that rather irrational behaviour can sweep stock markets at random.

⁴ Alan Greenspan (born March 6, 1926 in New York City) is an American economist and was from 1987 to 2006 Chairman of the Board of Governors of the Federal Reserve of the United States.

⁵ John Maynard Keynes (5 June 1883 – 21 April 1946) was a British economist whose ideas, called Keynesian economics, had a major impact on modern economic and political theory as well as on many governments' fiscal policies. He advocated interventionist government policy, by which the government would use fiscal and monetary measures to mitigate the adverse effects of economic recessions, depressions and booms. He is one of the fathers of modern theoretical macroeconomics.

⁶ An index fund or index tracker is a collective investment scheme (usually a mutual fund or exchange-traded fund) that aims to replicate the movements of an index of a specific financial market, or a set of rules of ownership that are held constant, regardless of market conditions.

Behavioural finance focuses upon how investors interpret and act on information to make informed investment decisions. As investors do not always behave in the rational, predictable and unbiased manner predicted by quantitative models, behavioural finance places an emphasis on how investor behaviour leads to various market anomalies, namely the incorrect pricing of financial assets (either pricing them above or below their fundamental value⁷) that breach the theoretical asset pricing models now generally accepted.

In the following paragraphs we will review some literature on the historical background of standard finance to understand how it evolved and why behavioural finance was born.

2.2. The roots of standard finance

We can trace many of the issues addressed in modern finance back to the remarkable paper presented to the Imperial Academy of Sciences in St. Petersburg by Daniel Bernoulli (1738). This paper, originally published in Latin and translated into German in 1896, was referenced widely in the fields of mathematics, logic and, subsequently, economics, but was not available in English until the 1950s. Nevertheless, Bernoulli addressed a series of issues that are at the core of modern financial economics.

Bernoulli examines the proposition that “*expected values are computed by multiplying each possible gain by the number of ways in which it can occur, and the dividing the sum of these products by the total number of possible cases*”. He rejects this approach because it fails to consider the range of possible outcomes that might occur. Instead he argues that “*the determination of the value of an item must not be based on its price, but rather on the utility it yields*”. Bernoulli suggests that increases in wealth will result in an increase in utility which is inversely related to the quantity of goods in an individual’s possession, and this enables him to demonstrate the trade-off between expected changes in wealth, and the risk associated with such an opportunity.

During the eighteenth and nineteenth centuries, Bernoulli’s concept of utility was regarded as the province of mathematicians rather than economists. By that time, of course, economists such as Bentham had independently developed utility theory as a central element of economics. From the late nineteenth century onwards, Bernoulli’s idea of decreasing marginal utility became central to economics, notably in the works of Jevons, Menger, Walras and Marshall.

Bernoulli also introduced the concept of maximization of expected utility. However, despite endorsement by Laplace and others, Bernoulli’s approach had little impact in the economics of decision making under risk until the development of expected-utility theory by Von Neumann and Morgenstern (1944, 1947) and Savage (1954).

The Expected-utility theory by von Neumann and Morgenstern- Rational decisions are based on the axioms of von Neumann und Morgenstern which lay the foundations of

⁷ Where required return (k) is less or greater than expected return, $E(r)$ or μ .

expected utility theory. A rational investor determines his portfolio allocation by optimizing his risk preference on the background of his risk ability hence he has neither emotions nor biases with respect to his risk perception. In an efficient market, a rational investor with constant relative risk aversion will rebalance his portfolio as long as his risk ability allows him to do so.

However there is an alternative model for decision making – the Prospect Theory - developed by Kahneman and Tversky (1979). Prospect theory is not meant to be a normative decision theory but being based on extensive evidence found in laboratory experiments it is a descriptive model. According to prospect theory investors have systematic biases concerning risk perceptions and their risk preference depends on whether one considers lotteries with gains or with losses. Investors react stronger to losses than to gains (loss aversion), concerning lotteries over gains they are risk averse and instead of facing sure losses investors prefer to take risks. In essence, prospect theory describes how individuals evaluate gains and losses. Prospect theory has probably done more to bring psychology into the heart of economic analysis than any other approach.

Prospect theory bears more than a passing rebalance to Expected-utility theory. Expected utility theory says that the expected utility is the sum of the probability weighted outcomes measured in terms of utility:

$$\Rightarrow \sum P_1 U(x_1)$$

Prospect theory assumes people maximize a weighted sum of utilities', although the weights are not the same as the true probabilities, and the utilities are determined by a function rather than a utility function:

$$\Rightarrow \sum \pi(p_1) v(x_1 - r)$$

Where π is a non-linear function, $v(x - r)$ is the value function evaluated with respect to the reference point.

Non-linear weights:

Kahneman and Tversky noted that people tended to give zero weight to relatively unlikely outcomes (but not impossible), and tended to give a weight of one to relatively certain outcomes (but not guaranteed). That is people behave as if extremely unlikely events are impossible and extremely like events are certain. In general it has been found that people tend to exaggerate the true probability (over-confidence at work).

The concept of risk is now pervasive in economics, and especially in financial economics. However, this has not always been the case. Knight (1921) makes a distinction between risk and uncertainty. When the randomness facing an individual can be expressed in terms of numerical probabilities, whether these are objective or reflect the individual's subjective beliefs, the situation is said to involve risk. When probabilities cannot be assigned to alternative outcomes, then the situation is said to involve uncertainty.

In a setting that embraces both risk and uncertainty, Arrow and Debreu developed a model of general equilibrium that has been fundamental to economics and finance. Their work starts with a series of papers, notably Arrow (1951), Debreu (1951) and Arrow-Debreu (1954). Arrow and Debreu assume that markets are complete – that is, there are as many markets as goods – and this provides a framework for analyzing general equilibrium. Since each good is defined by attributes such as its physical characteristics, its location, the date that it becomes available and the state of nature when it is available, the Arrow-Debreu model might be seen as one in which Pareto efficient outcomes could only occur with an almost infinite number of markets.

However, Arrow was not satisfied with a framework that could be applied only when markets are perfectly complete. Arrow's theory of general equilibrium with incomplete asset markets is presented in 1953 (Arrow, 1953). He shows that, by using the temporal structure of the economy, equilibrium can be attained with a more limited number of markets. He explains how one can achieve markets that are almost complete by setting up a series of contingent claims that follow the resolution of uncertainty. This has provided a conceptual framework to underpin the theory of asset pricing.

Arrow's concept of a complete market is one in which it is possible to insure against any loss that is of concern to any individual. In an economy where it is possible to insure against the range of possible future outcomes, individuals are more likely to be willing to bear risk. He provides encouragement for investors to hold diversified portfolios, rather than putting all their eggs in one basket. Arrow's framework set the scene for a more structured analysis of investor's portfolio decisions.

These decisions (investment of portfolio) must be done under a pre-determined risk profile, which can be defined basically through the degree of a person risk aversion. This is a concept in economics, finance, and psychology related to the behaviour of consumers and investors under uncertainty. Risk aversion is the reluctance of a person to accept a bargain with an uncertain payoff rather than another bargain with a more certain payoff, but possibly lower, expected payoff. The inverse of a person's risk aversion is sometimes called their risk tolerance

For example, a person is given the choice between two scenarios, one certain and one not. In the uncertain scenario, the person is to make a gamble with an equal probability between receiving €100 or nothing. The alternative scenario is to receive a specific euro amount with certainty (probability of 1). Investors have different risk attitudes. A person is:

- Risk-averse if he or she would accept a certain payoff of less than €50 (for example, €40) rather than the gamble;
- Risk-neutral if he or she is indifferent between the bet and a certain €50 payment;
- Risk-seeking (or risk-loving) if the certain payment must be more than €50 (for example, €60) to induce him or her to take the certain option over the gamble.

The average payoff of the gamble, known as its expected value, is €50. The euro amount accepted instead of the bet is called the certainty equivalent, and the difference between it and the expected value is called the risk premium.

In utility theory, a consumer has a utility function $U(x_i)$ where x_i are amounts of goods with index i . From this, it is possible to derive a function $u(c)$, of utility of consumption c as a whole. Here, consumption c is equivalent to money in real terms, i.e. without inflation. The utility function $u(c)$ is defined only modulo linear transformation.

The utility of the bet, $E(u) = (u(0) + u(100)) / 2$, is as big as that of the Certainty Equivalence (CE). The risk premium is $(€50-€40) / €40$, or 25%.

Arrow and Pratt developed two measures of Risk aversion: Absolute Risk Aversion (ARA) and Relative Risk Aversion (RRA).

Absolute risk aversion.

The higher the curvature of $u(c)$, the higher the risk aversion. However, since expected utility functions are not uniquely defined, a measure that stays constant is needed. This measure is the Arrow-Pratt measure of Absolute Risk-Aversion (ARA), after the economists Kenneth Arrow (1964) and John W. Pratt (1965), or coefficient of absolute risk aversion, defined as:

$$r_u(c) = -\frac{u''(c)}{u'(c)}.$$

Experimental and empirical evidence is mostly consistent with decreasing absolute risk aversion. Contrary to what several empirical studies have assumed, wealth is not a good proxy for risk aversion when studying risk sharing. In other words, tests of contractual risk sharing relying on wealth as a proxy for risk aversion remain unidentified.

Relative risk aversion.

The Arrow-Pratt measure of relative risk-aversion (RRA) or coefficient of relative risk aversion is defined as

$$R_u(c) = cr_u(c) = \frac{-cu''(c)}{u'(c)}.$$

This measure has the advantage that it is still a valid measure of risk aversion, even if it changes from risk-averse to risk-loving, i.e. it is not strictly convex/concave over all c .

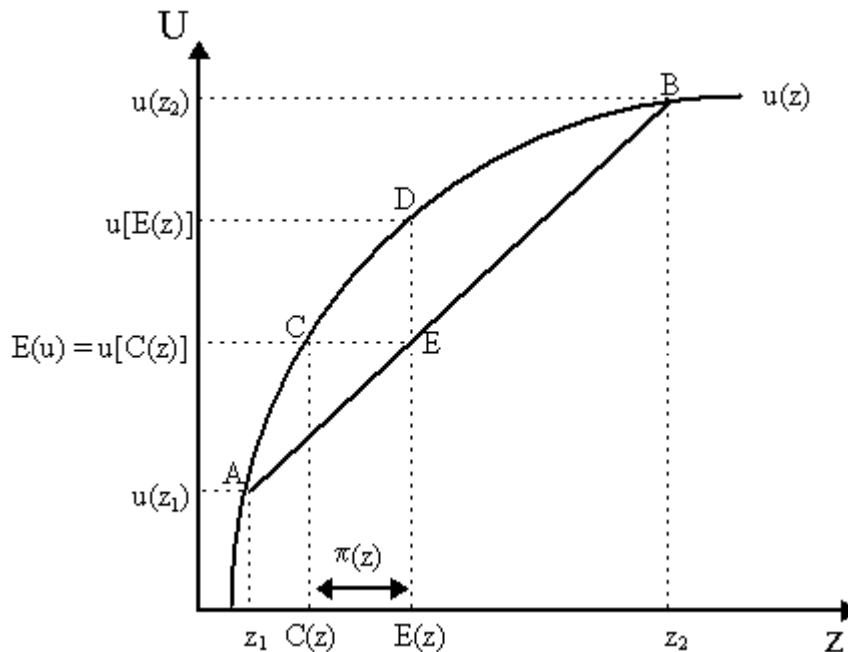


Figure 1- Risk-Aversion and Certainty

Application: Portfolio allocation.

In modern portfolio theory, risk aversion is measured as the additional marginal reward an investor requires to accept additional risk. Risk is being measured as standard deviation of the return on investment, i.e. the square root of its variance.

$$A = \frac{dE(r)}{d\sigma}$$

Kenneth J. Arrow (1965) hypothesized that individuals ought to display decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA) with respect to wealth, i.e. $dr_u(x)/dx \leq 0$ and $dR_u(x)/dx \geq 0$. The reasoning for DARA is, recall, arguing that wealthy individuals are not more risk-averse than poorer ones with regard to the *same* risk.

Thus, as Arrow notes, DARA is necessary if risky assets are to be "normal goods", i.e. a rise in wealth leads to an increase in demand for them - whereas IARA implies they are an inferior good. The reasoning for hypothesizing IRRA is that as wealth increases *and* the size of the risk increases, then the willingness to accept the risk should decline. Alternatively stated, IRRA implies that the wealth elasticity of demand for risky assets is less than unity.

How might risk-aversion affect portfolio allocation decisions? Recall that the Arrow-Pratt measure of risk-aversion implies there is a relationship between the degree of concavity of the utility function and the degree of risk-aversion. In the space of random

variables, this implies there is a relationship between the degree of *convexity* of indifference curve and the degree of risk-aversion - with *more* risk-averse agents having *more* convex indifference curves.

Thus, the more risk-averse agent holds a smaller proportion of his wealth in a risky asset. And if we have decreasing absolute risk aversion (DARA) and increasing relative risk aversion (IRRA) with respect to wealth, then optimal holdings of risky assets increase with wealth but the proportion of wealth invested in assets declines. And finally, as wealth increases, the proportion of wealth held in risky assets declines.

There are some limitations to this theory: The notion of (constant) risk aversion has come under criticism from behavioural economics. According to Matthew Rabin (2000), a consumer who, from any initial wealth level turns down gambles where she loses \$100 or gains \$110, each with 50% probability will turn down 50-50 bets of losing \$1,000 or gaining any sum of money.

The point is that if we calculate the constant relative risk aversion (CRRA) from the first small-stakes gamble it will be so great that the same CRRA, applied to gambles with larger stakes, will lead to absurd predictions. The bottom line is that we cannot infer a CRRA from one gamble and expect it to scale up to larger gambles.

It is noteworthy that Rabin's article has often been wrongly quoted as a justification for assuming risk neutral behaviour of people in small stake gambles.

One solution to the problem observed by Rabin is that proposed by prospect theory and cumulative prospect theory, where outcomes are considered relative to a reference point (usually the status quo), rather than to consider only the final wealth.

2.3. Portfolio Theory and Risk Measurement

Finance was transformed with the publication of the Markowitz (1952) article on Portfolio Selection. Ever since the days of Bernoulli, it was clear that individuals would prefer to increase their wealth, and also to minimize the risk associated with any potential gain. But could these two criteria be combined? Markowitz considers and rejects the idea that there might be a portfolio, which gives both the maximum expected return and the minimum variance. He explains that, "*The portfolio with maximum expected return is not necessarily the one with minimum variance. There is a rate at which the investor can gain expected return by taking on variance, or reduce variance by giving up expected return*".

The most important contribution made by Markowitz is his distinction between the variability of returns from an individual security and its contribution to the risk of a portfolio. He notes: "*In trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high co variances among themselves*".

Markowitz shows that provided we have the appropriate input data and computing power, than we can identify a set of portfolios that provide the highest possible expected return for a given level of risk, while at the same time giving the lowest level of risk for each level of expected return. These portfolios form the efficient frontier, and Markowitz shows that for any investor who only cares for the trade-off between expected return and risk, it is economically efficient to limit choice to portfolios that fall on this frontier.

Another approach, which shares many similarities to Markowitz's framework, was the "Safety first" model developed independently by Roy (1952). Roy addresses the question of how individuals can ensure a suitable small probability that their wealth will fall below some disaster level, which is essentially the same problem as the addressed by Markowitz. Roy's representation of his problem, with risk as the independent variable and expected return as the dependant variable, was adopted as standard by the finance profession. Perhaps because the Portfolio Selection article was published a few months early, however, it is Markowitz who is generally regarded as the 'Godfather' of Portfolio Theory.

Tobin (1958) takes Markowitz's analysis one-step further by showing how to identify which efficient portfolio should be held by an individual investor. He considers how an investor should divide his or her funds between a safe liquid asset such as cash (or treasury bills) and a risky asset (a bond or equity portfolio). He shows that *"the proportionate composition of the non-cash assets is independent of their aggregate share of the investment balance. This fact makes it possible to describe the investor's decisions as if there was a single non-cash asset, a composite formed by combining the multitude of actual non-cash assets in fixed proportions"*.

Tobin therefore proposes a framework for asset allocation that is intuitively appealing. He proposes *"breaking down the portfolio selection problem into stages at different levels of aggregation – allocation first among, and then within, asset categories"*. The asset mix, namely the allocation to cash (or treasury bills), should reflect the degree of risk aversion or risk tolerance of the investor. The optimal portfolio of risky assets, however, should be independent of the risk preferences of the investor. This proposition, which is known as the Separation Theorem, provides a basis for identifying the efficient portfolio.

Tobin's separation theorem clarified the task of portfolio selection. But even with Tobin's contribution it was still necessary to use Markowitz's full covariance model. The data and computational requirements of this approach were onerous, particularly for applications that embrace individual securities. There are thousands of shares traded only in the United States of America, but even if we consider a limited number of two thousand shares (for instances, the Russell 2.000 shares index, an USA stock index compounded by two thousand shares daily prices), using the Markowitz model requires estimates of over two million risk and return characteristics. This task was beyond the computation capabilities of the most powerful computers available in the 1950's and 60's.

These difficulties were addressed when Sharpe (1963) devised his Simplified Model for Portfolio Analysis. Sharpe draws on an insight of Markowitz (1959) that stocks are likely to co-move with the market. His model assumes that security returns are linearly related to fluctuations in a market-wide index, with a known degree of sensitivity; and that additionally, security-specific returns are generated with a known mean and variance. With only three parameters per security, the tasks of risk measurement and portfolio optimization are greatly simplified. Sharpe's approach is readily extended to embrace richer and more complex factor models of asset pricing.

Inherent in asset allocation is the idea that the best-performing asset varies from year to year and is not easily predictable. Therefore having a mixture of asset classes is more likely to meet the investor's goals. A more fundamental justification for asset allocation is the notion that different asset classes offer non-correlated returns, hence portfolio diversification reduces the overall financial risk in terms of the variability of returns for a given level of expected return. In this respect diversification has been described as "the only free lunch you will find in the investment game."

This way, an optimal portfolio displays the lowest possible level of risk for its level of return. Additionally, since each additional asset introduced into a portfolio further diversifies the portfolio, the optimal portfolio must comprise every asset, (assuming no trading costs) with each asset value-weighted to achieve the above (assuming that any asset is infinitely divisible). All such optimal portfolios, i.e., one for each level of return, comprise the efficient frontier.

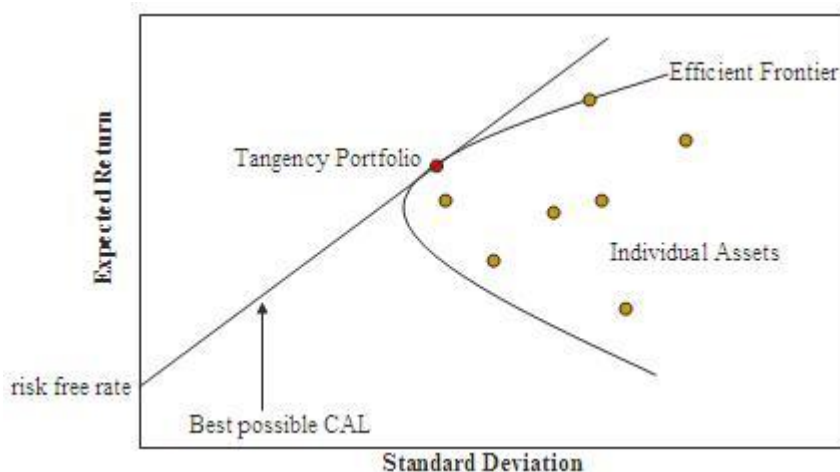


Figure 2 - The (Markowitz) efficient frontier

The Markowitz full-covariance model and Sharpe's index model, together with Sharpe's development of the capital asset pricing model (to be reviewed in the forthcoming paragraphs) marked the end of the beginning of modern finance. Arrow and Tobin separately received Nobel prizes in Economics, in part for their respective contributions to the theory of finance. In 1991, the fundamental contributions of Markowitz and Sharpe were honoured by the first Nobel Prize to be awarded for research in financial economics (an award shared with Miller, primarily for his contributions to corporate finance). In turn, others were able to build on these early foundations, and in 1997 the theory of asset pricing was once again recognized through

the award of the Nobel prizes to Scholes and Merton for their work in valuing derivative securities.

Nevertheless, Robert C. Merton (1973) was the first to publish a paper expanding the mathematical understanding of the options pricing model and coined the term "Black-Scholes" options pricing model, by enhancing work that was published by Fischer Black and Myron Scholes. The paper was first published in 1973. The foundation for their research relied on work developed by scholars such as Louis Bachelier, A. James Boness, Sheen T. Kassouf, Edward O. Thorp, and Paul Samuelson. The fundamental insight of Black-Scholes is that the option is implicitly priced if the stock is traded.

The important assumptions of the Black–Scholes model are:

- The price of the underlying instrument S_t follows a geometric Brownian motion with constant drift μ and volatility σ , and the price changes are log-normally distributed:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

- It is possible to short sell the underlying stock.
- There are no arbitrage opportunities.
- Trading in the stock is continuous.
- There are no transaction costs or taxes.
- All securities are perfectly divisible (e.g. it is possible to buy any fraction of a share).
- It is possible to borrow and lend cash at a constant risk-free interest rate.
- The stock does not pay a dividend
- .

The above lead to the following formula for the price C of a European call option with exercise price K on a stock currently trading at price S , i.e., the right to buy a share of the stock at price K after T years. The constant interest rate is r , and the constant stock volatility is σ .

$$C(S, T) = S\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

Where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

Here Φ is the standard normal cumulative distribution function.

Regarding the derivative valuations, there is another model, the binomial options pricing model (BOPM), which provides a generic numerical method for the valuation of

options. The binomial model was first proposed by Cox, Ross and Rubinstein (1979). Essentially, the model uses a "discrete-time" model of the varying price over time of the underlying financial instrument.

The Binomial options pricing model approach is widely used as it is able to handle a variety of conditions for which other models cannot easily be applied. This is largely because the BOPM models the underlying instrument over time - as opposed to at a particular point. Although slower than the Black-Scholes model, it is considered more accurate, particularly for longer-dated options, and options on securities with dividend payments. For these reasons, various versions of the binomial model are widely used by practitioners in the options markets.

2.4. Asset Pricing Theory

In 1964 Sharpe notes that *“through diversification, some of the risk inherent in an asset can be avoided so that its total risk is obviously not the relevant influence on its price; unfortunately little has been said concerning the particular risk component which is relevant”*. Sharpe aims to use the theory of portfolio selection *“to construct a market equilibrium theory of asset prices under conditions of risk”* and notes that his model *“sheds considerable light on the relationship between the price of an asset and the various components of its overall risk”*. The resulting capital asset pricing model (CAPM) is the model of investors’ returns expectations that was to remain dominant as a research paradigm until the 1980’s.

It is worth notice that Sharpe submitted the initial version of his capital asset pricing model paper to the Journal of Finance in 1963, but the Journal’s editor rejected it on the grounds that *“his assumption that all investors made the same predictions was so preposterous as to make his conclusion uninteresting”*. Although the paper was published the following year, this editor’s objection would be of great importance in the future, as it is one of the key assumptions that drive the formulation of the model.

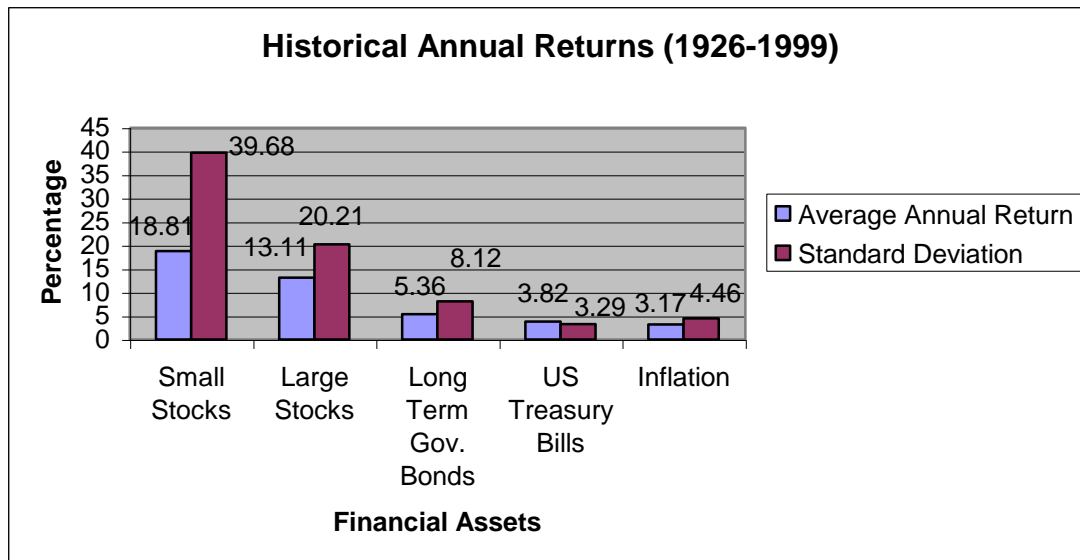
The concepts of portfolio theory and the development of risk measurement, taken together with the capital asset pricing model, have had a major impact on the theory and practice of investment management. It is now common to view a managed portfolio as a blend of a passive portfolio (such as index fund) and an active portfolio comprising a series of bets on the relative performance of individual securities. Treynor and Black (1973) explain when a manager should choose to run an almost perfectly diversified index fund, and how the portfolio’s diversification should vary with the prospects for the stocks in which the portfolio is invested; they also provide the first analysis to underpin the market-neutral hedge funds. Modern portfolio optimization and risk management systems are often extensions of the Treynor-Black model. As we expect all investors to make a trade-off between risk and return, then in equilibrium high risk assets must compensate investors by offering higher returns.

Nevertheless, in several empirical tests conducted in the 1980’s it was found evidence that the CAPM does not seem to hold, having rejected it. Apart from these tests there was mounting evidence that other risk factors also affect stock returns. The factors

include Price/Earnings ratio (Basu, 1977), company size (Banz, 1981), Book to Market Equity (Fama and French, 1992) and a variety of other systematic influences on security prices. The empirical evidence provided further motivation for research into other models of asset pricing that might more successfully explain returns, or at least indicate why in practice the CAPM did not seem to be hold.

2.5. Main concepts of Risk and Return

The General Concept: Higher expected returns require taking higher risks. Most investors are comfortable with the notion that taking higher levels of risk is necessary to expect to earn higher returns. Intuitively an investor would require a higher expected return in exchange for accepting greater risk. And in face this relationship is observed in historical long run returns of stocks, bonds and less risky securities, as shown in the following chart.



Source: Stocks, Bonds, Bills and Inflation 2002 Yearbook, 2002 Ibbotson Associates.

Figure 3 - Historical annual returns

Another concept: Volatility as a Proxy for Risk.

One widely accepted measure of risk is volatility, the amount an asset’s return series through successive time periods, and is most commonly quoted in terms of the standard deviation of returns. An asset whose return fluctuates dramatically is perceived to have greater risk because the asset’s value at the time when the investor wishes to sell it is less predictable. In addition, greater volatility means that, from a statistical perspective, the potential future values of more volatile assets span a much wider range.

Thus, the annualized volatility σ is the standard deviation σ of the instrument's logarithmic returns in a year. The generalized volatility σ_T for time horizon T in years is expressed as:

$$\sigma_T = \sigma\sqrt{T}.$$

For example, if the daily logarithmic returns of a stock have a standard deviation of 0.01 and there are 252 trading days in a year, then the time period of returns is 1/252 and annualized volatility is:

$$\sigma = \frac{\sigma_{SD}}{\sqrt{P}}$$
$$\sigma = \frac{0.01}{\sqrt{1/252}} = 0.1587$$

The monthly volatility (i.e., $T = 1 / 12$ of a year) would be:

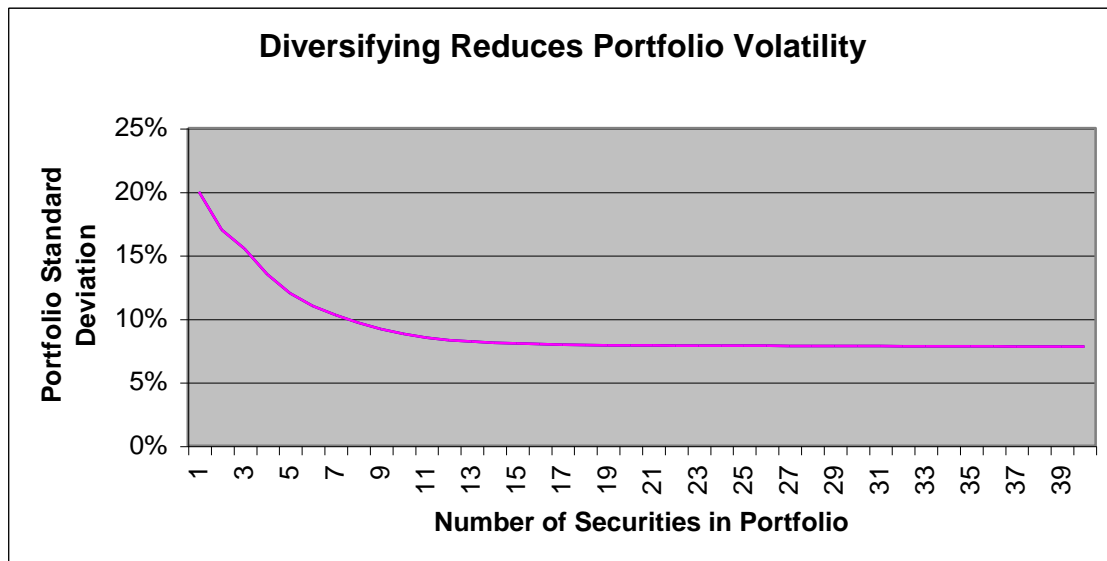
$$\sigma_{month} = 0.1587\sqrt{1/12} = 0.0458$$

Last concept: Diversification and Systematic Risk.

Systematic risk refers to the risk common to all securities - i.e. market risk. Unsystematic risk is the risk associated with individual assets. Unsystematic risk can be diversified away to smaller levels by including a greater number of assets in the portfolio (specific risks "average out"). The same is not possible for systematic risk within one market. Depending on the market, a portfolio of approximately 30-40 securities in developed markets such as UK or US will render the portfolio sufficiently diversified to limit exposure to systemic risk only. In developing markets a larger number is required, due to the higher asset volatilities.

Although somewhat counterintuitive, an individual stock's volatility in and of itself is not the most important consideration when assessing risk. Consider a situation in which an investor could, without incurring additional cost, reduce the volatility associated with her/his portfolio of assets. This is most commonly accomplished through diversification. So consider holding two stocks that have the same expected return, instead of one stock. Because stock returns will not be perfectly correlated with each other, it is unlikely that both stocks will experience extreme movements (positive or negative) simultaneously effectively reducing the volatility of the overall portfolio. As long as the assets do not move in lock step with one another (are less than perfectly positively correlated), overall volatility can be reduced, without lowering expected returns, by spreading the same amount of money across the multiple assets.

This concept of diversification is one of the main tenets of modern portfolio theory – volatility is reduced through the addition of more assets to a portfolio. It should be noted, however, that the rate of volatility reduction from adding assets decreases as the number of assets in the portfolio increases (Borchert et al, 2003). As the chart below demonstrates for one potential scenario (20% volatility in each asset and zero covariances between assets), the general rule of thumb is that a portfolio containing 30 or more assets is considered well diversified.



Source: Tuck School of Business at Dartmouth.

Figure 4 - Portfolio diversification and volatility

Volatility can be effectively reduced without significant cost by diversifying, so it makes sense that investors should not be compensated for that proportion of volatility which is merely stock specific and has no impact on a well diversified portfolio. This type of volatility is called *unsystematic risk* in the finance literature because it does not co vary with the market as a whole, but is merely the additional random ‘noise’ present in that specific asset’s returns. Since this random noise has an expected return of zero, it can be diversified away by adding more securities to the portfolio. Its mean will be zero, and its standard deviation will be reduced as more assets are added.

The logical extension of this argument is that with enough assets in a portfolio, the portfolio volatility matches that of the overall market. Thus, investors should only expect to be compensated for the risk that cannot be diversified away (*i.e., the systemic risk*).

An asset exhibits both systematic and unsystematic risk. The portion of its volatility considered systematic is measured by the degree to which its returns vary relative to those of the overall market. To quantify this relative volatility, a parameter called beta was conceived as a measure of the risk contribution of an individual security to a well-diversified portfolio:

$$\beta_A = \frac{COV(r_A, r_M)}{\sigma_M^2}$$

Where:

r_A is the return of the asset;

r_M is the return of the market;

σ_M^2 is the variance of the return of the market;

$COV(r_A, r_M)$ is co variance between the return of the market and the return of the asset.

In practice, beta is calculated using historical returns for both the asset and the market, with the market portfolio being represented by a broad index such as the S&P 500 or the Russell 2000. This type of data is widely available from financial databases and can be downloaded into software packages like Excel or SPSS for easy manipulation. To determine the beta of a portfolio, simply average the individual securities' betas, weighted by the market capitalization of each security.

We shall now analyze how the beta as measure of risk is used by the CAPM, through the relationship of systematic risk and expected return.

2.6. The Capital Asset Pricing Model into practice

The Capital Asset Pricing Model (CAPM) attempts to quantify the relationship between the beta of an asset and its corresponding return. The CAPM model makes a number of simplifying assumptions, of which the most relevant are about investor behaviour and the presence of a single common risk factor.

The first assumption is that investors care only about expected returns and volatility. Therefore, as rational consumers, they will always maximize expected return for any given level of expected volatility.

Second, all investors have homogeneous beliefs about the risk / reward tradeoffs in the market. This, as stated before, provoked some objections from the Journal of Finance editor, and may still be one of the fragilities of the market, as it is considering all investors as rational people, emotion free.

The third assumption is that only one risk factor is common to a broad-based market portfolio. This risk factor is the systematic market risk, which drives non-diversifiable volatility. Investors are assumed to hold diversified portfolios, as the market does not reward investors for the bearing of diversifiable risk. As a result, the CAPM states that if a security's beta is known, it is possible to calculate the corresponding expected returns.

To build the intuition for this model, let us first consider an asset that has no volatility, and thus, no risk; thus, its returns do not vary with the market. As a result, the asset has a beta equal to zero and expected return equal to the risk-free rate.

Next, let us consider an asset that moves in lock step with the market, or has a beta of one. As a result of this perfect correlation with the market, this asset, by definition, earns a return equal to that of the market, $E(r_A) = E(r_M)$.

Lastly, let us think about an asset that experiences greater swings in periodic returns than the market, or has a beta greater than one. We would expect this asset to earn returns superior to those of the market as compensation for this extra risk.

If we generalize this relationship between expected return on assets and their exposure to market risk, we are led to the CAPM equation:

$$E(r_A) = r_f + \beta_A \times (E(r_M) - r_f)$$

Where r_f is the risk-free rate, and $(E(r_M) - r_f)$ is the expected excess return of the market portfolio beyond the risk-free rate, often called equity risk premium.

Essentially, the CAPM states that an asset is expected to earn the risk-free rate plus a reward for bearing risk as measured by that asset's beta. The chart below demonstrates this predicted relationship between beta and expected return – this line is called the Security Market Line (SML).

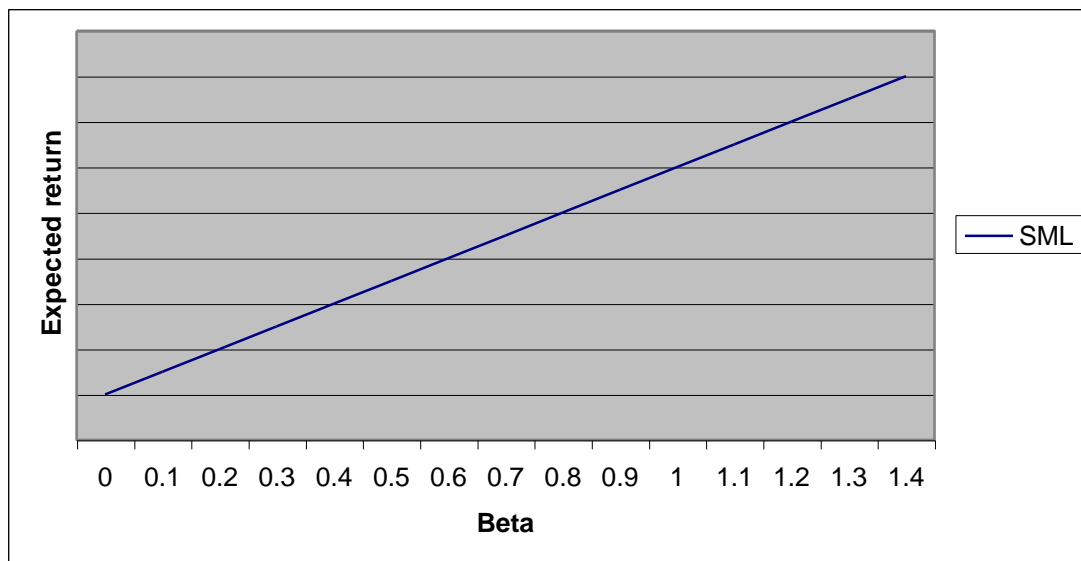


Figure 5 - Security Market Line

Beta is the ratio of the expected excess return of an asset relative to the overall market's excess return, where excess return is defined as the return of any given asset less the return on a risk-free asset. For example, a stock with beta of 1,5 would be expected to have an excess return of 15% in a time period where the overall market beat the risk-free asset by 10%. Effectively, beta is a numerical way to express the idea that expected returns are more sensitive to market swings for those assets that are highly covariant with the market.

Given that the CAPM predicts what a particular asset or portfolio's expected return should be relative to its risk and the market return, the CAPM can also be used to evaluate the performance of active fund managers.

Active fund managers try to outperform the market by selecting stocks in a portfolio based on research and informed opinions. One of the key questions surrounding realized returns is whether the manager of the fund is actually achieving a return higher than what would be predicted by the risk the manager took. The CAPM model gives an estimate of what the return should have been, given the beta risk if the portfolio. If the realized return is greater than the predicted return from the CAPM model, this points

towards ‘adding value’; if the manager has lower or equivalent returns, he or she might be ‘just collecting fees’ but adding no investment value.

Based on the previous discussion of risk / return analysis, we can see that one way for a manager to increase the expected return on a given fund is to invest in positions that embody greater systematic risk. In effect, by accepting more variance, the manager can increase the beta (and thus the portfolio risk) of the fund and thereby increase his or her expected returns.

While some investors may choose to accept greater risk to increase expected returns, real value comes from a mutual fund manager who is able to deliver higher returns at the same or reduced level of risk. Essentially, we are asking if the manager is able to create a portfolio which would have higher returns than those predicted by the CAPM. If we compare the realized return of a portfolio with its expected return predicted by CAPM, the difference is ‘excess return’, which is often referred to as ‘ α ’ (or, alpha).

Graphically, if α is greater than zero, the portfolio would lie above the Security Market Line. The presence or absence of a positive alpha can be used to evaluate a manager’s performance.

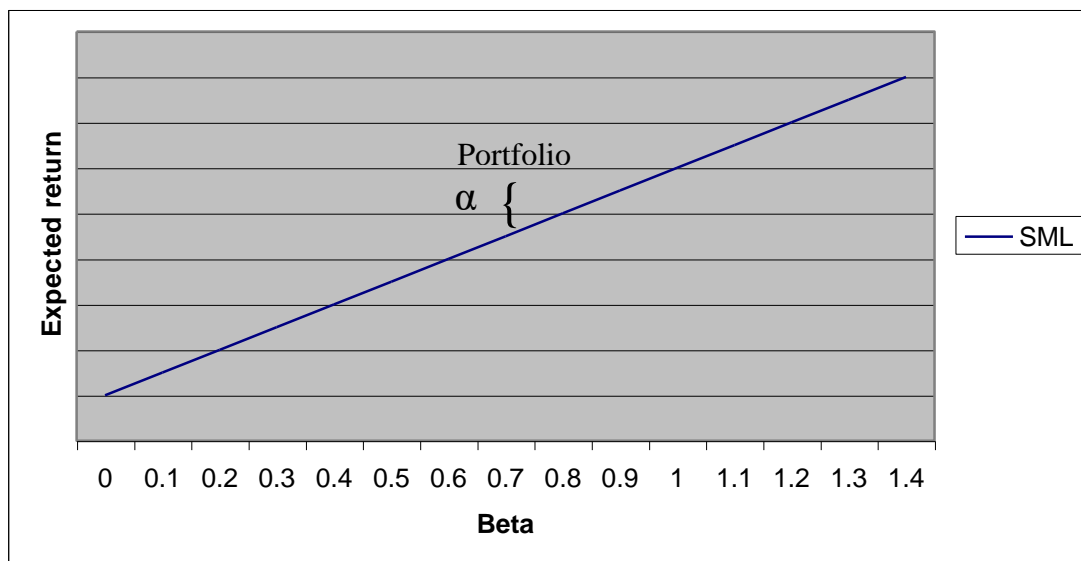


Figure 6 - Alpha

2.7. New developments in measuring risk and return – The three-factor model

The CAPM’s true predictive power is questionable. Tests made throughout the years prove that the CAPM usually achieves an R^2 measure of about 85%, which means that roughly 15% of the variation in observed returns remains unexplained. In addition, many researchers believe that other risk factors have significant impact on expected return in the market. As a result, the simplicity of the CAPM’s assumption of a single risk factor explaining expected returns has been called into question.

There are a myriad of risk factors facing companies today. Some of these factors are market risk, bankruptcy risk, currency risk, supplier risk, etc. And given that the CAPM uses a single factor to describe aggregate risk, it seems logical that a model including more sub-factors might provide a more descriptive and predictive model. Effectively, additional factors allow more specific attribution of the risks to which a company is exposed. The single risk factor can be decomposed along multiple dimensions.

Eugene Fama and Kenneth French (1992) found factors describing ‘value’ and ‘size’ to be the most important factors, outside of market risk, for explaining the realized returns of publicly traded stocks. To present these risks, they constructed two factors: SMB (Small Minus Big) to address size risks and HML (High Minus Low) to address value risk. The first is designed to measure the additional return investors have historically received by investing in stocks of companies with relatively small market capitalization. This additional return is often referred to as the ‘size premium’. The later has been constructed to measure the ‘value premium’ provided to investors for investing in companies with high book-to-market values (essentially, the value placed on the company by accountants as a ratio relative to the value the public markets placed on the company, commonly expressed as B/M).

Nowadays the SMB and HML factors are the most commonly used simply because they work: they have the greatest predictive power of any two additional factors that researchers have tested, often yielding an R^2 value of approximately 95%.

By combining the original market risk factor and the newly developed factors, we have the commonly used Fama French Three Factor Model. Analogous to the CAPM, this model describes the expected return on an asset as a result of its relationship to three risk factors: market risk, size risk and value risk.

$$r_A = r_f + \beta_A (r_M - r_f) + s_A \text{SMB} + h_A \text{HML}$$

The coefficients in this model have similar interpretations to beta in the CAPM above. β_A is a measure of the exposure an asset has to market risk (although this beta will have a different value from the beta in a CAPM model as a result of the added factors), s_A measures the level of exposure to size risk and h_A measures the level of exposure to value risk.

A primary implication of the Three Factor Model is that investors can choose to weight their portfolios such as that they have greater or lesser exposure to each of the specific risk factors, and therefore can target more precisely different levels of expected return. The model also provides a way to categorize mutual funds by the size and value risks to which its portfolio is exposed and thus the return premiums expected, as a result of the assets held. But it also raises a very important question that remains unanswered: Why are size and value important for a stock performance?

Although Fama and French aren't particular about why book/price measures risk, they and others have suggested some possible reasons. For example, high book/price could mean a stock is "distressed", temporarily selling low because future earnings look doubtful. Or, it could mean a stock is capital intensive, making it generally more

vulnerable to low earnings during slow economic times. Those both sound plausible; but they seem to be describing completely different situations (and what happens when a company that isn't capital intensive becomes "distressed"?). It may be that the success of this model at explaining past performance isn't due to the significance of any of the three factors taken separately, but in their being different enough that taken together they do an effective job of "spanning the dimensions" of the market.

There's actually another interpretation that's so much less cerebral that it's probably correct. The broad market index weights stocks according to their market capitalization, making it size-biased and valuation blind; so maybe the extra two factors in this model are just a couple of tweaks to adjust for these two problems. This also explains why momentum is sometimes used as yet another factor: market capitalization shows where the market has been putting its money for years, while momentum shows where it has been putting it lately; so if you want to take advantage of market efficiency you start with the index and then tweak it a little with momentum.

In comparison with a single factor model (CAPM), there are two additional messages that the Multi-factor models give. First, the three factors together account for practically all of a portfolio's behaviour; that's the strongest evidence yet that mutual funds can't beat indexes. Second, history indicates that small value "just happens" to deliver higher returns and higher volatility than the stock market as a whole. Assuming the trend holds, then that's the practical message for investors. In particular, it improves what felt like a flaw in the Tobin (1958) argument: where Tobin said high-risk investors should buy the total stock market index on margin (according to his research there's only one portfolio plus borrowing and lending, so it's got to be the market), Fama and French offer the saner alternative of just adding some small value to your portfolio.

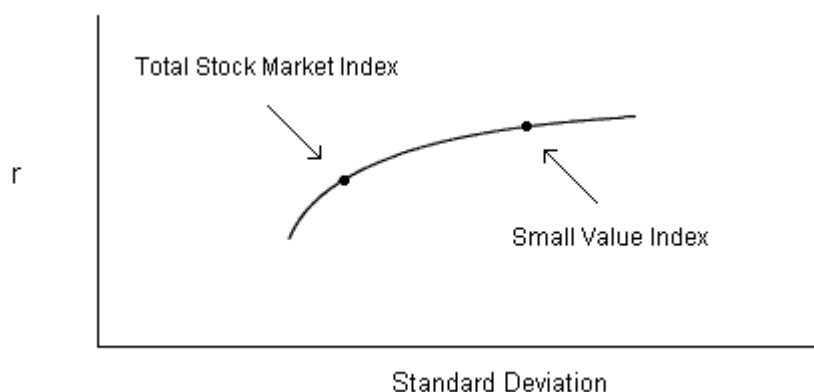


Figure 7 - Market returns and standard deviation

2.8. Brief History of Behavioural Finance

Foundations of EMH.

The efficient market hypothesis (EMH) was the dominant paradigm in finance from 1965 to 1999, thus being the dominating proposition in finance for nearly thirty years.

The EMH holds that prices reflect all available information. As such, security prices at any time should represent an unbiased estimate of the discounted value of all future returns.

Generally the theoretical case for the EMH rests on three arguments which rely on progressively weaker assumptions:

- Investors assumed to be rational thus value stocks rationally;
- To the extent that investors are rational, their trades are random hence cancel each other without affecting prices;
- To the extent that investors are irrational in similar ways, they are met in the market place by rational arbitrageurs who eliminate their influence on prices.

But remarkably the EMH does not live or die by the investor rationality. In many scenarios where some investors are not fully rational, markets are still predicated to be efficient. But arbitrage complicates this even further. To the extent that the securities that irrational investors are buying are overpriced and those they are getting rid of are under priced, such investors earn lower returns than either passive investors or arbitrageurs. In fact it has been argued that irrational investors lose more money than their peers. In practice however, practitioners and academics have come up with lots of evidence that challenge the EMH.

Theoretical Challenges to EMH.

To begin, it is difficult to sustain that the case that people in general and investors in particular, are fully rational. At the superficial level, many investors react to irrelevant information in forming their demand for securities in other words they trade on noise rather than information. Investors follow the advice of financial gurus, fail to diversify, actively trade stocks and churn their portfolios, sell winning stocks and hold on to losing stocks thereby increasing their tax liabilities, buy and sell actively and expensively managed mutual funds, follow stock patterns and other popular models. In short, investors hardly pursue the passive strategies expected by uninformed market participants by the efficient market theory (EMH). But this is just a tip of the iceberg. Investor's deviations from the maxims of economic rationality turn out to be highly pervasive and systematic.

As mentioned by Kahneman, Tversky and Amos (1979), people deviate from the standard decision making model in a number of fundamental areas which could simply be grouped as being:

- Attitudes towards risk;
- Non-Bayesian expectation formation;
- Sensitivity of decision making to framing of problems.

First, individuals do not assess risky gambles following the precepts of von Neumann-Morgenstern rationality. Rather, in assessing gambles, people look not at the levels of final wealth they can attain but at the gains and losses relative to some reference point, which may vary from situation to situation, and display loss aversion – a loss function that is steeper than a gain function.

Secondly, individuals systematically violate Bayes rule and other maxims of probability theory in their predications of uncertain outcomes. For example, people often predict future uncertain events by taking a short history of data and asking what broader picture this history is representative of. In focusing in such representativeness, they often do not pay enough attention to the possibility that the recent history is generated by chance rather than the model they constructed. Such heuristics are useful in many life situations – they help people to identify patterns in the data as well as to save on computation – but they may lead investors seriously astray. For example, investors may extrapolate short past histories of rapid earnings growth of some companies too far into the future and therefore overprice these glamorous companies without any slight recognition that, statistically speaking, trees do not grow to the sky. Such overreaction lowers future returns as past growth rates fail to repeat themselves and prices adjust to more plausible valuations. Perhaps most radically, individuals make different choices depending on how a given problem is presented to them, so that framing influences decisions. In choosing investments, for example, investors allocate more of their wealth to stocks rather than bonds when they see a very impressive history of long-term stock returns relative to those of bonds, than if they only see the volatile short-term stock returns.

The basic problem with the EMH is that it is a half truth. It is useful to present market efficiency as a concept to students and amateur investors lest they come to believe that it is easy to get rich quickly. It is not easy to get rich quickly by trading in speculative markets. The short run, day to day or month to month profit opportunities that many people imagine they have found are most probably not there.

But, one should not extrapolate from this simple notion of market efficiency to the idea that markets are also efficient in the long run. In fact, if one looks at data over long intervals of time, it appears that the stock market is anything but efficient. A study by Robert J. Shiller (2000) shows that ten year real return on the Standard & Poor Index have been substantially negatively correlated with price-earnings ratios at the beginning of the period. When the market gets high, it has tended to come down.

There are several other factors that empirical research proved to be fundamental causes of specific stocks higher returns: The January Effect (historically stock markets have higher positive performances in this month), The Reversion Effect (stocks that performed the worse in the past will reverse their performance in the future, and vice-versa), which is found in short term periods (one to three months), The Momentum Effect (stocks that performed better in the past will perform better in the future, and vice-versa), which is found in long term periods (one year). None of these factors effects in stock prices has been explained in a rational and scientific way.

All these factors have puzzled researchers, as they break the Efficient Market Hypothesis (EMH), the fundamental base of all standard finance (including the CAPM), unquestionable until the 1990's. The EMH holds that prices reflect all available information. As such, security prices at any time should represent an unbiased estimate of the discounted value of all future cash flows of a given security. Since market prices are supposedly unbiased and reflect all available information, a corollary to the EMH is that investors should not be able to outperform the market on a risk-adjusted basis.

In fact, early empirical tests supportive of the EMH have actually made the dominance of this hypothesis to be easily understandable. Moreover, researchers argue that investment professionals do not outperform the market at a rate exceeding that predicated by chance.

Nevertheless, critics of the theory contend that stocks do maintain price trends over time, and that it is possible to outperform the market by carefully selecting entry and exit points for equity investment. And empirical evidence supports their point of view, thus establishing some EMH anomalies (the above identified 'Effects').

Researchers have sought explanations for these anomalies. Regarding the HML (value premium) Fama and French argue that stocks with higher book-to-value tend to have higher fundamental risk, which is not incorporated in the CAPM as this model incorporates only systematic risk, so these stocks should have a higher performance.

But the main stream of researchers has admitted that investors do not behave in rational manner (De Bondt and Thaler, 1987), and sometimes, the rationality is limited due to professional imperatives: professional investors tend to stick to low book-to-market stocks due to the 'Glamour Effect' (Lakonishok et al. 1991) of having well known brands in the portfolios they are managing for clients, in order to easily justify any bad performance of the portfolio.

Other pattern found was the 'herd behaviour', when some fund managers tend to follow the investment choices of the previously well succeeded competitors. As explained by Scharfstein and Stein (1990) fund managers who are concerned with their reputation have an incentive to follow the well succeeded competitors, as the benefits of winning alone are outpaced by the risk of losing alone.

According to Fama (1998) "*Market efficiency can only be replaced by a better specific model of price formation, itself potentially rejectable by empirical tests*". The problem is that human behaviour, up to today, hasn't been possible to predict with mathematical models. Psychology and Sociology are the sciences specialized in human behaviour, and some researchers believe that they should be brought to the Finance science in order to solve these puzzles. This is why a new trend in finance flourished, and is growing exponentially: The Behavioural Finance.

As Adam Smith (1968) put it, "You are – face it – a bunch of emotions, prejudices, and twitches, and this is all very well as long as you know it. Successful speculators do not necessarily have a complete portrait of themselves, warts and all, in their own minds, but they do have the ability to stop abruptly when their own intuition and what is happening Out There are suddenly out of kilter. If you don't know who you are, this is an expensive place to find out".

As found by Slovic (1972), careful accumulation and skilled interpretation of this information is said to be the *sine qua non* of accurate evaluation of securities. The basic tenet of those in charge of helping the investor to make market decisions seems to be 'the more information, the better'.

But too much information seems to be raising a problem: as Graham, Dodd, Cottle and Tatham (1962) say, “After the analyst has learned what information he can get and where to get it, he faces the harder question: what use to make of it?”.

Being Behavioural Finance the study of investors’ psychology while making financial decisions, and knowing that investors fall prey to their own and sometimes others’ mistakes due to the use of emotions in financial decision making, it is worth developing further the research on this theme, as the knowledge of our own flaws may prevent us from doing bad investment decisions.

According to a study (Kennickel et al, 2000) almost 50% of USA families hold stocks, directly or through mutual funds or retirement plans. The future welfare of these households depends on their ability to make sound investment decisions. While it would be perfect if all investors could always made personally optimal decisions, probably all of them sometimes make mistakes and many of them would benefit from education and advice, and investment advisors could help individual investors avoid common errors by understanding the decision processes that lead to those errors.

People tend to react stronger to losses than to gains (loss aversion) (Kahneman et al, 1979), concerning lotteries over gains they are risk averse and instead of facing sure losses investors prefer to take risks. This implies that individual investors will tend to hold to their losing investments while selling their winners. Under Prospect-Theory, when faced with choices involving simple two and three outcomes, people behave as if maximizing an ‘S’ shaped value function. This value function is similar to a standard utility function, except it is defined on gains and losses rather than on levels of wealth.

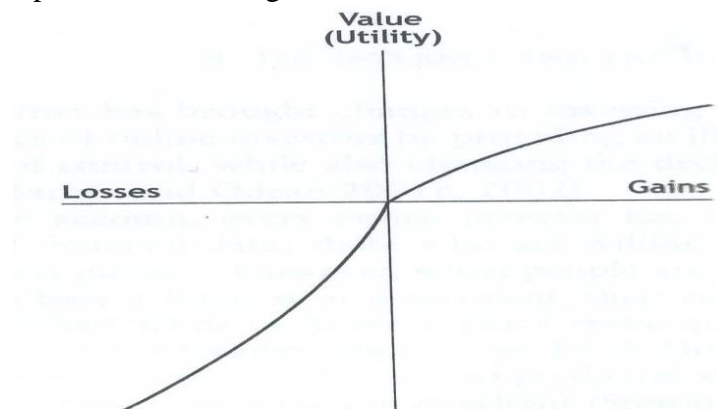


Figure 8 - Prospect Theory Value Function

The function is concave in the domain of gains and convex in the domain of losses. It is also steeper for losses than for gains, which implies that people are generally risk-averse. According to these researchers (Kahneman and Tversky 1979, p.287) “... a person who has not made peace with his losses is likely to accept gambles that would be unacceptable to him otherwise.”

The Internet has brought changes to investing that may bolster over confidence of online investors by providing an illusion of knowledge and control, while also changing the decision criteria to attention driven buying patterns (Barber et al, 2008).

In an empirical study of investors at a large discount brokerage firm that switched from phone based trading to personal computer based trading, there is evidence that after going online investors tend to trade both more actively and more speculatively (Barber and Odean, 2002). According to several research literatures, this has to do with the feeling of control investors have when trading online. The problem is that investors are likely to confuse the control over which investments they make with the control of the returns those investments realize. As a result, they are likely to act too speculatively.

The combination of these two types of behaviour (holding to losing investments and trading too much), synthesized in the above paragraphs, may in fact reduce the investors' welfare, so understanding these behaviours is important for investors and for those who advise them.

Although traditional models of financial markets give us insights into why people trade as much as they do, it is behavioural finance that gives us insights into why and how investors form heterogeneous beliefs. The disposition effect says that investors will generally trade less actively when their investments have lost money, and will trade more actively when their overconfidence is high. Psychologists have found that people tend to give themselves much credit for their own success and do not attribute enough of that success to chance or outside circumstances. Gervais and Odean (2001) found that this bias leads successful investors to become overconfident. They also found that in a market where most investors are successful (bull market), aggregate confidence and consequent trading rise. Statman (1999) found that over even short horizons, such as a month, current market returns predict subsequent trading volume.

For the last forty years, psychologists have amassed evidence that “the economic man is very unlikely a real man (Edwards, 1954, p. 382) and that reason - for now defined by the principles that underlie expected utility theory, Bayesian learning, and rational expectations - is not an adequate basis for a descriptive theory of decision making”. In response, behavioural decision theorists have introduced a series of new concepts under the general heading of ‘bounded rationality’, a term associated with Herbert Simon (1983).

Further into this notion of ‘bounded rationality’, research conducted by De Bondt (1998) showed that it is part of Wall Street folklore that small individual investors are ‘dumb’. Indeed, many financial experts use the sentiment of small traders as contrarian indicator, and De Bondt behavioural research also paints a negative picture, identifying four classes of anomalies. They have to do with (1) investors' perceptions of the stochastic process of asset prices - people tend to extrapolate past performances, to find patterns where they do not exist, (2) investors' perceptions of value – people are short-term oriented and ‘judge a book by its cover’, as most of them don't master valuation techniques, (3) the management of risk and return – households are poorly diversified and people tend to believe that they will have the presence of mind to sell quickly in a bear market, and (4) trading practices – discipline is difficult to maintain, and investors avoid realizing a loss, even if in economic terms it makes sense.

This type of behaviour of small individual investors may affect the performance of the market as a whole, because as stated by Graham and Dodd (1934), “The stock market is not a weighing machine, on which the value of each issue is recorded by an exact and impersonal mechanism – the market is rather a voting machine, whereon countless individuals register choices which are the product partly of reason and partly of emotion”. In fact, looking at the stock market we can see that share prices move up and down on a daily basis, many times without any change in the fundamental of the companies – this is crowd behaviour. People in stock markets move in herds, and this influences stock prices.

2.9. Conclusions

Behavioural finance research focused initially in professional investors (fund managers, etc.) behaviour to explain certain Efficient Market Hypothesis anomalies (studies of Fama and French), and only recently, especially after the 2000 dot.com crash, researchers have started to study the individual investor behaviour (studies of De Bondt).

One of the major problems in behavioural finance theories still open (Shleifer, 2000) is answering the questions “How do investors perceive risk?” and “How do investors evaluate risk?”. Probably if these questions had an answer we could have further knowledge in how securities’ prices move, as prices should reflect not only the expected return of a security but also its’ risk. If Economists can evaluate the future returns of a company, and by doing so also can evaluate the company stock’s returns, they cannot evaluate the different investors’ appetite for risk, both static and dynamic.

Standard finance has given us quite resourceful information about risk and return, but for behaviour finance there are some propositions, already tested, more important than others, as they are not the subject of behavioural finance research and thus may be used to develop further the discipline. First of all, it is now proved that stocks have higher risk than bonds, and these have higher risk than cash. Second, when allocating assets in a closed investment, the investor will chose an optimum allocation of cash, bonds and stocks according to his risk profile. Afterwards, the security market line (SML) will find the optimal allocation for each type of securities.

With these propositions, behavioural finance could concentrate on checking if different investors have different risk profiles, and how do they translate their own profile into practice. This would give us a help on understanding how investors perceive risk and how they evaluate it. To do this, we may concentrate first in their perceived risk profile (how they perceive risk) by just asking them that question through a structured approach, and how they evaluate risk by analysing the asset allocation they put in practice in their personal portfolios, checking if their risk exposure is in line with their declared risk perception.

3. RESEARCH METHODOLOGY

Behavioural finance research has been approached from many different directions. I believe that a survey and data collection through questionnaires is a well known scientific methodology that will need only minor explanations in what regards the effective application to the current study.

3.1. Survey

Statistical surveys are used to collect quantitative information about items in a population. Surveys of human populations and institutions are common in political polling and government, health, social science and marketing research – in Finance they haven't been so much used, but with the growing interest in Behavioural Finance, this methodology started to be more used.

3.2. Sampling

When studying a small population, a census would be the more appropriate tool to conduct the study. A census is a survey applied to all respondents in a population. In the case of the present study (studying the behaviour of High Net Worth Individuals, clients of Private Banks) the respondents of the population are estimated to be around forty million worldwide (Source: Private Banker International), and thus virtually impossible to approach. In such a population a sample will be the most appropriate tool for analysis. Moreover, the target population of this study is relatively homogeneous (clients of Private Banking) so sampling is even more valid.

In this study I used a nonprobability sampling method (convenience) as most of the members of the target population are not willing to participate in the survey, for discretion or secrecy reasons, and so it will not be possible to draw purely random sample from the target population. Nevertheless, as the target population is homogeneous (clients of Private Banking) it is considered to be acceptable a nonprobability sample, even for statistical testing (Aaker et al, 1998).

I will use questionnaires of individual investors from twelve different countries, three continents, 8 different languages, 8 different national currencies, so this sample, although not completely random, gives us a fairly good diversification of respondents in terms of age, gender, cultural origin, stage of their respective countries Gross Domestic Product, currency strength or weakness, thus allowing us to exclude these factors as conditionings to the survey final results.

Moreover, in the present case, the bias introduced by this methodology could be of little or negligible effect in the study as the biases could arouse just of the openness and

friendliness profiles of the respondents, which in turn would have little effect of their investment attitudes. On the other hand this type of sampling will allow us to avoid other problems, like nonresponse or the false answers that many people give when confronted with a question they don't want to answer – in this case they will be volunteers.

Regarding the sample size, we will assume that the variation of the sample average will assume a normal distribution (bell-shaped probability distribution), where the population mean equals the median and the mode, and where there is a 99% probability of all frequencies falling within a maximum distance of three standard errors from the population mean (Chebyshev's Theorem).

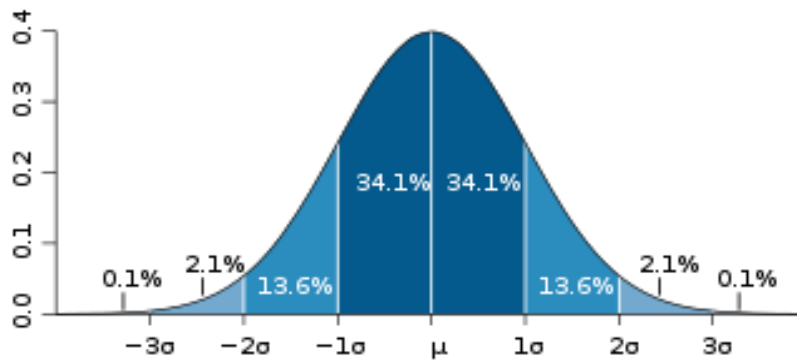


Figure 9 - Normal distribution⁸.

According to the Central-Limit Theorem if the population distribution is fairly symmetrical in appearance (normal distribution), a sample with more than 30 respondents is enough to consider that the sample follows a normal distribution. Nevertheless, we will always have a sampling error, whose magnitude depends of the interval estimate containing the unknown population mean, and reflects our judgement on the extent of this sampling error. The size of the interval will depend on the defined confidence level (usually 90%, 95% or 99%), on the population standard deviation (the lower, the smaller the interval), usually unknown, and the sample size (the larger, the smaller the interval).

As I do not know the population standard deviation, in order to decrease the standard error I decided to increase the number of respondents to 100, and work with a 95% confidence level for the interval estimate around the sample average, so we can expect to have the population mean estimate within two standard deviations from the sample average. Being:

⁸ Dark blue is less than one standard deviation from the mean. For the normal distribution, this accounts for about 68% of the set (dark blue) while two standard deviations from the mean (medium and dark blue) account for about 95% and three standard deviations (light, medium, and dark blue) account for about 99.7%.

\bar{X} – the sample average;
 μ – the true population mean;
 $\sigma_{\bar{X}}$ – the sample standard error;
 σ_X – the population standard deviation.

$\bar{X} \pm$ sampling error = the interval estimate of μ ;

If we chose a 95% confidence level for the interval estimate containing the true population mean, we will have:

$$\bar{X} \pm 2 \sigma_{\bar{X}}$$

If for example we have a sample average of 2,5 and a sample standard error of 1,5, for 95% confidence level we will have an interval estimate of the population mean of]2,5 – 1,5 ; 2,5 + 1,5 [, or an interval between 1 and 4 for the population true mean.

Nevertheless, although it is reasonable to consider that the target population follows a normal distribution, we will need to have metric data (in a numerical scale) from the survey in order to process it under the above parameters. If we have nominal data (not quantifiable, as for example the preference for colours) we will not be able to process it under the usual hypothesis testing associated with normal distributions.

This 100 respondents sample will include individual investors of different genders, different ages, 12 different countries, 3 different languages, 8 different national currencies.

3.3. Questionnaire

Questionnaire construction is properly regarded as a very imperfect art. There are no established procedures that will lead consistently to a ‘good’ questionnaire. One consequence is that the range of potential error contributed by ambiguous questions may be as much as 20 or 30 percentage points (Payne, 1951). Fortunately, such extreme errors can be reduced sharply by common sense and insights from the experience of other researchers.

I conducted the survey on one hundred Private Banking clients, through the administration of a structured questionnaire with closed questions that will allow us to compare the responses and cross tabulate them. The survey was conducted twice over the same respondents, with two years difference (2007 and 2009), so the use of open-ended questions was not considered, as I wished to measure the change of attitudes of the respondents over time, by posing the same questions after two years.

In order to make sure that the major dangers of questionnaire building are avoided, I opted for using the bank’s official questionnaire, developed by IBBOTSON (Morningstar group) specialized company in conducting surveys over financial markets’ players.

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The questionnaire has an algorithm to translate the answers of each respondent into a numerical value, which in turn rates each participant in a risk aversion scale, and suggests what profile of investment that respondent should have in his/her own portfolio. Afterwards we will develop some tables that will allow us to cross tabulate these self assessed risk profiles.

At the end of the risk profiling the participant were asked his/her current allocation of assets currently in the Private Bank's account, in percentage (avoid talking about the amounts held in the accounts, for privacy reasons), divided by 4 categories: Cash, Bonds, Stocks, Others. These categories are widely known in the finance industry as the main asset class in which to invest, being Cash the less risky and Stocks the more risky. Others are usually used for residual investments, like commodities (Oil, Gold) or real estate, not having significant impact on the average, well diversified, portfolio.

After the processing of the questionnaire through the algorithm, each participant was classified in one of five categories of risk profile: Preservation, Income, Income & Growth, Growth, and Aggressive Growth. According to Morningstar (and as it is financial industry practice), a portfolio with over 50% of the assets in cash is considered to be Preservation, with over 50% of the assets in bonds is consider to be Income. A portfolio with bonds and equity representing (each of these asset classes) between 30% and 50% of all assets is classified as an Income & Growth profile portfolio, while a portfolio with stocks representing more than 50% but no more than 70% of all assets is considered to be a Growth profile, and finally a portfolio with over 70% in stocks is considered to be an Aggressive Growth profile.

To recapitulate:

1. Preservation – Cash represents more than 50% of portfolio;
2. Income – Bonds represent more than 50% of portfolio;
3. Income & Growth – Bonds and Stocks represent each between 30% and 50% of portfolio;
4. Growth – Stocks represent more than 50% but no more than 70% of portfolio;
5. Aggressive Growth – Stocks represent more than 70% of portfolio.

Finally I matched the client profile outputted by the algorithm of the questionnaire (which is based in the participant's attitudes and beliefs regarding risk and return and his/her self knowledge and rationality) with the profile derived from his/her declared and current asset allocation, in order to detect patterns or incongruence. This procedure was repeated for the two years, with the same participants, to verify what changes occurred and how did those changes affect the participant attitudes and beliefs, as well as his/her actions.

The questionnaire and the algorithm are part of the Appendix of this work, but I shall explain them in detail in the next chapter. Here is an example of the final version of the questionnaire.

Participant n.º _____/100.

Please mark the correct answer in each of the following questions.

Section 1 – To match the portfolio with the client’s financial situation.

1.1. In relative terms, revenue from the:

1.1.1. Client’s profession is:

- a) very important;
- b) not important.

1.1.2. Client’s financial portfolio is:

- a) very important;
- b) not important.

1.2. The evolution of the professional revenue of the client during the next five years can be described the following way:

He/She expects his/her global revenue to:

- a) increase clearly above inflation (promotion, growth of own business, etc.);
- b) increase at a pace similar to inflation;
- c) decrease (due to retirement or other reason).

1.3. Approximately how much of the client’s total assets does he have invested in this bank?

- a) Less than 25%;
- b) Between 25% and 50%;
- c) Between 51% and 75%;
- d) More than 75%.

1.4. Does the client use credit regularly?

- a) Yes;
- b) No.

1.5. If he/she does, for what reason?

- a) Tax purposes;
- b) Financial leverage;
- c) Other reason.

Section 2 – To asses the client’s attitude and experience in financial assets.

2.1. What is the client’s reference currency?

- a) Euro;
- b) US dollar.

2.2. Does the client have financial assets denominated in other currencies?

- a) Yes;
- b) No.

2.3. Which of the following best describes the client’s level of investment experience?

- a) Novice / Beginner / Little Experienced – His first investment in the financial markets was in the past 12 to 18 months.
- b) Somewhat Experienced – The client understands the basics of investing but he/she is less confident about how the markets actually work and interact.
- c) Experienced – The client has been actively investing for several years or more and he is reasonably confident of his knowledge of the financial markets.
- d) Very Experienced – The client is quite knowledgeable about the financial markets and feels very comfortable making investment decisions.

2.4. Which statement best describes the client's attitude toward investing?

- a) He/She is extremely safety conscious and doesn't want the value of his investment portfolio to decline at all;
- b) He/She realizes that there are risks in investing, but tries to reduce them as much as possible;
- c) He/She is willing to assume some investment risk to enhance the potential return of the portfolio;
- d) He/She is willing to assume significant risk for a portion of the portfolio to increase the potential for higher overall returns;
- e) He/She is comfortable assuming significant risk for the overall portfolio in order to maximize the possibility of higher returns.

2.5. In order to increase the expected return of his/her investment the client would be willing to:

- a) Add quite a lot of risk in his total investment;
- b) Add quite a lot of risk in part of his investment;
- c) Add a little more risk in his total investment;
- d) Add a little more risk in part of his investment;
- e) Not increase the risk of his investment.

Section 3 – To determine the client's time horizon.

3.1. With the amount invested the client seeks a return on the investment:

- a) on the short term;
- b) on the medium term;
- c) on the long run.

3.2. The client expects to need the present amount in the time frame of:

- a) one year;
- b) between 2 to 3 years;
- c) between 4 to 5 years;
- d) more than 5 years.

3.3. Does the client expect to need more than half of the amount before that time frame?

a) No.

If yes, when does the client expects to withdraw that amount?

- b) Up to 1 year;
- c) Up to 2 years or more.

3.4. Is the client aware that the value of his assets under management can change during time and eventually be worth less than its present value?

- a) Yes;
- b) The client didn't know that but he has acquired this notion.

Section 4 – Current asset allocation.

Please input the current percentage of each of the following asset classes in the client's portfolio (under management in this bank):

Cash - _____%

Bonds - _____%

Stocks - _____%

Other - _____%

Total – 100%

Date: __/__/_____

Thank you...

4. EMPIRICAL STUDY

4.1. Survey

I had the access to the suitability tests of one hundred individual investors, clients of a Geneva based Private Bank. Under Swiss bank secrecy laws we are not allowed to publish their names or even socio-demographic data (as gender, age or even country of residence). Nevertheless, we can provide the following information:

Sample – 100 individual investors, with over one million euros (or equivalent in other currency) of invested financial assets each (high net worth individuals). Their countries of residence are: Switzerland, France, Italy, Spain, Portugal, Greece, Turkey, Romania, Poland, Brazil, Angola and Mozambique. These origins represent twelve different countries, three continents, 8 different languages, 8 different national currencies. We were not allowed to specify percentages of each origin.

This sample, although not completely random, gives us a fairly good diversification of respondents in terms of age, gender, cultural origin, stage of their respective countries Gross Domestic Product, currency strength or weakness, thus allowing us to exclude these factors as conditionings to the survey final results.

The interviews were carried out by the author of this research work in the first three months of 2007, and again in the first three months of 2009. The questionnaire was the one presented in the previous chapter.

After the survey was repeated in the first three months of 2009, all the questionnaire's answers were processed under the cross-tabulation tables given in this study's Appendix (Tables 1, 2, 3 and 4) and summarized in Table 5, as detailed in the following section.

4.2. Processing data

I will only go through the process for respondent number 1, as presenting the process for the 100 respondents would occupy too much space in this text and would be redundant as the for the other 99 we just repeated the process. At the end, through Table 5 we will analyse and cross-tabulate the aggregate results.

Applying the algorithm developed by Ibbotson, to each questions answered there are a certain number of points attributed, meaning that the higher the absolute value of the points attributed to the answer the higher the risk tolerance of the respondent. Let us process the answers for respondent number one (in red ink) in the following tables, for the two years of the survey.

Table 1 – Answers quantification. For each of the following answers attribute the indicated points, and sum up the total of points at the end of the table.

Participant	n.º <u>1</u>	In year 2007	In year 2009
QUESTION	VALUE POINTS	ATTRIBUTED POINTS	ATTRIBUTED POINTS
1.1.1. a)	8	8	8
1.1.1. b)	5		
1.1.2. a)	4		4
1.1.2. b)	8	8	
1.2. a)	5	5	
1.2. b)	2		2
1.2. c)	1		
1.3. a)	10		
1.3. b)	9		
1.3. c)	5	5	5
1.3. d)	3		
1.4. a)	7		
1.4. b)	4	4	4
1.5. a)	3		
1.5. b)	9		
1.5. c)	1		
2.1. a)	3	3	3
2.1. b)	4		
2.2. a)	10		
2.2. b)	6	6	6
2.3. a)	2		
2.3. b)	5	5	5
2.3. c)	24		
2.3. d)	26		
2.4. a)	2		2
2.4. b)	21	21	
2.4. c)	47		
2.4. d)	68		
2.4. e)	95		
2.5. a)	25		
2.5. b)	20		
2.5. c)	10		
2.5. d)	5	5	
2.5. e)	2		2
3.1. a)	0		0
3.1. b)	25		
3.1. c)	30	30	
3.2. a)	0		
3.2. b)	5		
3.2. c)	25		
3.2. d)	30	30	30

3.3. a)	30	30	30
3.3. b)	5		
3.3. c)	30		
3.4. a)	10	10	10
3.4. b)	0		
Total		170	111
Asset Allocation 2007			
Cash	Bonds	Stocks	Other
5%	35%	55%	5%
Asset Allocation 2009			
Cash	Bonds	Stocks	Other
30%	60%	5%	5%

As explained previously, and according to the algorithm developed by Ibbotson, the higher the number of points obtained in the answer processing (Table 1) the higher the risk tolerance of the participant, and thus the higher the risk profile he/she self declared in the survey. For the above questionnaire, the translation of points into the 5 standard generic risk profiles that we have considered was processed as explained in the following table.

Table 2 – Participant declared risk profile. Please tick the box in the second line of the below table that corresponds to the participant total points.

Participant	n.º <u>1</u>				
Total points	<134	[134 – 174[[174 – 224[[224 – 275]	> 275
Declared risk profile 2007	Preservation	Income X	Income & Growth	Growth	Aggressive Growth
Declared risk profile 2009	Preservation X	Income	Income & Growth	Growth	Aggressive Growth

Afterwards I had to fill in a new table the effective (or real) risk profile that the same participant was holding in his/her portfolio at the time of each survey, by translating his/her asset allocation into the same 5 standard generic risk profiles, as explained in Table 3.

Table 3 – Participant real risk profile. Please tick the box in the second line of the below table that corresponds to the participant declared asset allocation.

Participant	n.º <u>1</u>				
Real asset allocation	Cash > 50%	Bonds > 50%	Bonds and Stocks between 30% and 50% each	Stocks > 50% but no more than 70%	Stocks more than 70%

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Real risk profile 2007	Preservation	Income	Income & Growth	Growth X	Aggressive Growth
Real risk profile 2009	Preservation	Income X	Income & Growth	Growth	Aggressive Growth

To facilitate the processing of the data, each of the 5 pre-defined risk profiles were attributed a number, allowing for future quantitative analysis:

- 1 – Preservation;
- 2 – Income;
- 3 – Income & Growth;
- 4 – Growth;
- 5 – Aggressive Growth.

In Table 4 each participant is classified in this ordinal scale and the numerical differences between each individual investor declared risk profile and his/her real risk profile are quantified.

Table 4 – Participant risk profile differences between declared and real risk profile. Please tick the boxes below to compound the matches or differences (numerical) between the declared risk profile and the real risk profile of each participant.

Participant	n.º <u>1</u>				
Risk profile	Preservation	Income	Income & Growth	Growth	Aggressive Growth
A) Declared risk profile 2007	1	2 X	3	4	5
B) Real risk profile 2007	1	2	3	4 X	5
Difference 2007: Numerical subtraction of A – B	-2				
A) Declared risk profile 2009	1 X	2	3	4	5
B) Real risk profile 2009	1	2 X	3	4	5
Difference 2009: Numerical subtraction of A – B	-1				

This scale, from 1 to 5, translates into an ordinal scale the different degrees of risk appetite of each individual investor. It is important noting that this scale does not allow for a quantitative analysis in detail (as it is in fact an ordinal scale, and not numerical) as

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someone with a number 2 risk profile does not necessary have the 2 times more risk appetite than someone with a number 1 risk profile. What we can affirm is that he/she has more risk appetite than someone with a number 1 risk profile, and less risk appetite than someone with a number 3 profile or higher.

Moreover we can group different individual investor in the same risk profile, and quantify how many of them have the same risk profile and how has it changed throughout time. Table 5 includes all the participants' risk profiles and differences, analysing the data with the above identified ordinal scale.

Table 5 – Participants risk profiles and respective differences.

Participant Number	Declared Risk Profile 2007	Real Risk Profile 2007	Difference 2007	Declared Risk Profile 2009	Real Risk Profile 2009	Difference 2009	Average Difference
1	2	4	-2	1	2	-1	-1,5
2	2	4	-2	1	2	-1	-1,5
3	3	4	-1	2	3	-1	-1
4	2	4	-2	1	3	-2	-2
5	2	4	-2	1	2	-1	-1,5
6	3	4	-1	1	2	-1	-1
7	1	3	-2	1	2	-1	-1,5
8	2	4	-2	1	3	-2	-2
9	3	4	-1	1	2	-1	-1
10	3	4	-1	2	2	0	-0,5
11	3	4	-1	2	3	-1	-1
12	3	4	-1	1	2	-1	-1
13	3	4	-1	1	2	-1	-1
14	1	3	-2	1	1	0	-1
15	4	5	-1	2	2	0	-0,5
16	5	5	0	2	2	0	0
17	3	5	-2	1	2	-1	-1,5
18	1	4	-3	1	2	-1	-2
19	3	5	-2	2	3	-1	-1,5
20	4	4	0	2	3	-1	-0,5
21	1	3	-2	1	1	0	-1
22	1	2	-1	1	1	0	-0,5
23	2	3	-1	1	1	0	-0,5
24	1	4	-3	1	2	-1	-2
25	2	4	-2	1	1	0	-1
26	3	5	-2	1	3	-2	-2
27	2	4	-2	1	2	-1	-1,5
28	2	5	-3	1	2	-1	-2
29	1	3	-2	1	1	0	-1
30	2	4	-2	1	2	-1	-1,5
31	3	5	-2	1	2	-1	-1,5

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32	2	4	-2	1	1	0	-1
33	2	3	-1	1	1	0	-0,5
34	5	4	1	2	2	0	0,5
35	4	4	0	2	3	-1	-0,5
36	1	3	-2	1	1	0	-1
37	2	4	-2	1	2	-1	-1,5
38	2	4	-2	1	2	-1	-1,5
39	3	4	-1	1	3	-2	-1,5
40	4	5	-1	1	2	-1	-1
41	4	5	-1	2	3	-1	-1
42	3	4	-1	2	3	-1	-1
43	2	4	-2	1	1	0	-1
44	3	5	-2	1	2	-1	-1,5
45	4	4	0	2	1	1	0,5
46	2	4	-2	1	1	0	-1
47	2	5	-3	1	2	-1	-2
48	3	4	-1	2	3	-1	-1
49	4	4	0	2	3	-1	-0,5
50	1	3	-2	1	2	-1	-1,5
51	2	3	-1	1	1	0	-0,5
52	3	4	-1	2	2	0	-0,5
53	3	4	-1	2	2	0	-0,5
54	5	5	0	2	3	-1	-0,5
55	2	4	-2	1	1	0	-1
56	5	5	0	2	3	-1	-0,5
57	3	5	-2	2	3	-1	-1,5
58	2	4	-2	1	1	0	-1
59	3	4	-1	1	1	0	-0,5
60	3	4	-1	1	1	0	-0,5
61	2	3	-1	1	1	0	-0,5
62	5	5	0	2	1	1	0,5
63	4	4	0	2	2	0	0
64	1	3	-2	1	2	-1	-1,5
65	4	4	0	2	2	0	0
66	4	4	0	2	2	0	0
67	3	5	-2	2	3	-1	-1,5
68	4	4	0	2	2	0	0
69	3	4	-1	1	3	-2	-1,5
70	4	4	0	2	2	0	0
71	3	3	0	2	2	0	0
72	4	4	0	2	2	0	0
73	3	3	0	1	2	-1	-0,5
74	2	4	-2	1	3	-2	-2
75	1	3	-2	1	2	-1	-1,5
76	1	3	-2	1	2	-1	-1,5
77	5	5	0	2	3	-1	-0,5

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78	1	3	-2	1	1	0	-1
79	1	3	-2	1	1	0	-1
80	2	4	-2	1	3	-2	-2
81	2	3	-1	1	1	0	-0,5
82	3	3	0	2	2	0	0
83	2	3	-1	1	2	-1	-1
84	2	3	-1	2	2	0	-0,5
85	3	3	0	1	3	-2	-1
86	4	4	0	2	2	0	0
87	2	3	-1	2	2	0	-0,5
88	2	3	-1	2	2	0	-0,5
89	1	3	-2	1	2	-1	-1,5
90	1	3	-2	1	1	0	-1
91	2	3	-1	2	2	0	-0,5
92	3	3	0	2	3	-1	-0,5
93	4	4	0	2	3	-1	-0,5
94	1	3	-2	1	2	-1	-1,5
95	2	3	-1	1	2	-1	-1
96	2	3	-1	1	2	-1	-1
97	3	3	0	1	2	-1	-0,5
98	2	3	-1	1	2	-1	-1
99	2	3	-1	1	1	0	-0,5
100	3	3	0	1	2	-1	-0,5
Averages	2,60	3,80	- 1,2	1,37	2,00	- 0,63	- 0,92
Std. Dev.	1,12	0,72	0,89	0,49	0,70	0,66	0,62

4.3. Descriptive statistic outputs

According to this table, the survey participants changed their self declared risk profile from 2007 to 2009, by reducing it from Income & Growth (what we could call as a Balanced risk profile) to a Preservation risk profile. Furthermore, their effective risk profile (asset allocation) also changed in this period, from a Growth risk profile to an Income risk profile, which is consistent with the change that occurred in the self declared risk profile.

Strangely, in both moments the participants under estimated their risk profile, by holding in their portfolio securities or assets that were riskier than what the individual investors declared as being their risk profile. On average, both in 2007 and 2009 the individual investors held portfolios with one level of risk (0,92) above the one they had declared, and this over estimation is very consistent within the sample individuals, with a standard deviation of 0,62 (last column of Table 5). Nevertheless, it is interesting noting that in 2009 this under estimation is just half of what it was in 2007, thus being more close to reality.

Moreover, according to the Private Bank's CEO explanations, the clients just don't have more Cash (or fewer Stocks) because they just don't want to sell the securities at a loss,

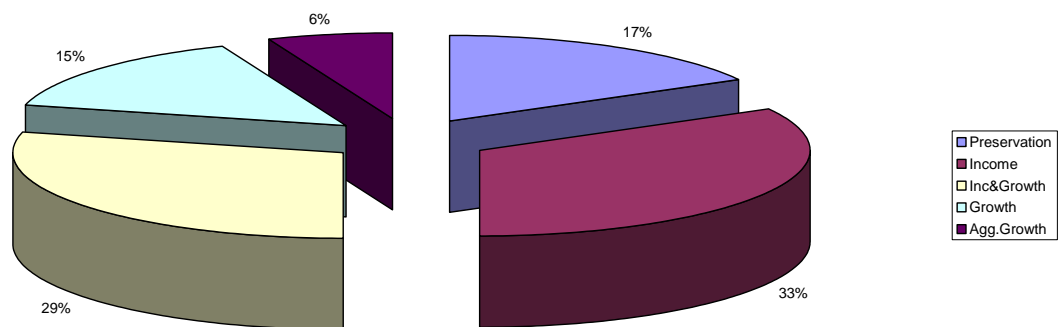
holding to bad investments on the hope that some day the invested amount will be recovered. This may explain why even in a bear market as the one being lived in 2009, the individual investors hold portfolios with an asset allocation that does not reflect their declared risk profile.

Also of some interest is the fact that the standard deviation of the self declared risk profile within the sample individuals fell sharply from 2007 (1,12) to 2009 (0,49), while the standard deviation of the real risk profiles within the same sample kept almost the same (0,7) during the two years. This might indicate some excess self confidence in 2007, while some more realistic and consistent market approach (through asset allocation) during time.

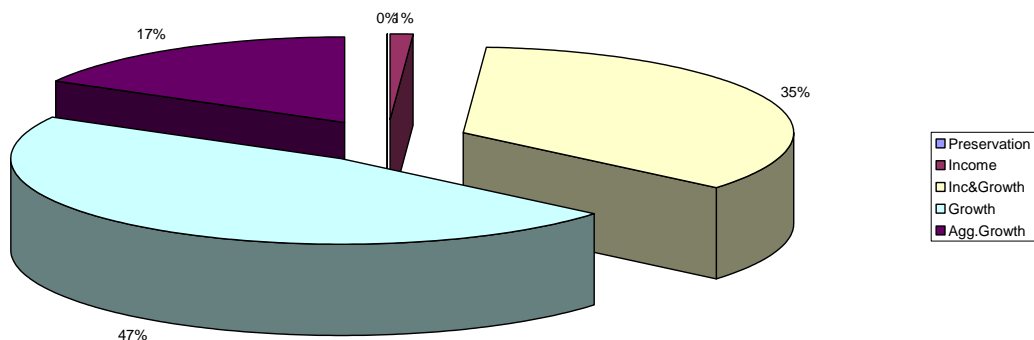
The standard deviation of the differences between self declared and real risk profile also fell in the period (from 0,89 to 0,66). This reduction in the volatility of differences within the sample individuals may indicate that some of them took a ‘cold shower’ (due to a financial market crash that caused them to bear some losses) between 2007 and 2009, tempering some excessive enthusiasm and optimism evidenced in 2007. Maybe some irrational exuberance could be seen that year, and in 2009 most of individual investors see their risk profile (imagined and real) more the same way.

Next we will look at some descriptive statistics to obtain a better picture of the results.

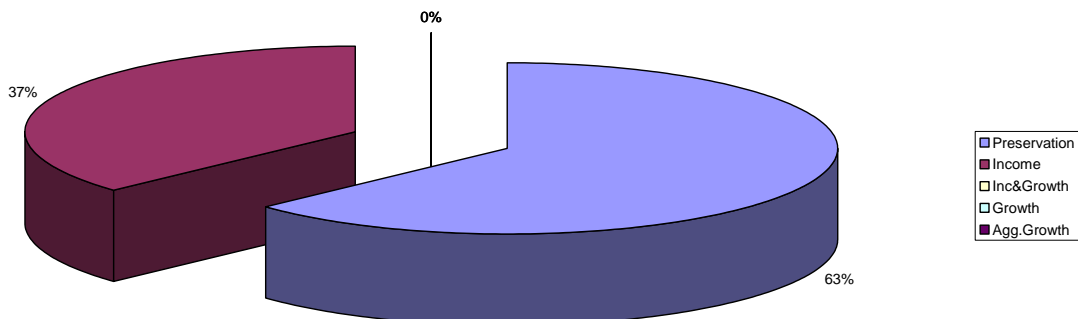
Graph 1 - Declared risk profile 2007



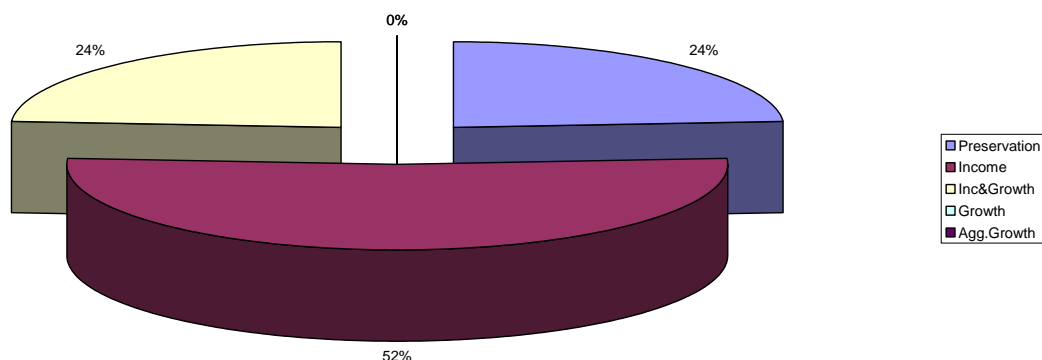
Graph 2 - Real risk profile 2007



Graph 3 - Declared risk profile 2009



Graph 4 - Real risk profile 2009



Although we may be surprised with the differences between Graph 1 and Graph 2 (50% of people declaring having a Preservation or Income risk profile, but only 1% of the participants had an asset allocation compatible with those profiles – all other 99% were incurring in higher market risks), it is worth noting that the changes of profiles between Graph 1 and 3 are overwhelming. If in 2007 50% of the respondents declared that they had a risk tolerance fitting in one of the three more risky profiles (Aggressive Growth, Growth, Income & Growth), in 2009 none of the respondents included him/her in one of those three profiles.

We must remember that these are same respondent answering to same questionnaire only after two years, which makes us think if the questionnaire has some kind of flaw that directs the respondents to questions related to their psychological mood towards financial markets. This would jeopardize the reliability of the questionnaire.

The fact is that the questions are absolutely neutral in what regards the growth stage of the financial markets (in fact it is because of this technical feature that this model of questionnaire was chosen), touching only aspects of the respondents stage in life and long term horizons. The most important questions are about the importance of the portfolio return for the client's day to day living versus his/her salary, the expected need of funds in the next five years, the amount of experience in investing in financial markets the client has, and the willingness to invest in short or long term investments.

What we verified is that respondents who affirmed having a long term horizon, no need for cash in the near future and so on, suddenly, as the financial markets crashed, completely changed their self perception or even self consciousness, as if these were different people answering the questionnaire.

Moreover, the changes in asset allocation did not mirror completely these changes of risk perception, as in 2009 there are still 24% of respondents with a real risk bearing profile above the two first levels (Preservation and Income).

This change of perceptions, attitudes, and even of self consciousness could be considered an indicator of irrational behaviour from the sample participants. Either they overestimated their own risk profile in 2007 or they underestimated it in 2009. Nevertheless, the changes can be of negligible significance (as they could be proceed from the sample error) or of statistical significance (in which case they represent a change of the population). To ascertain which of these is true is the final test of this dissertation, and will allow us to infer if the individual investors, clients of Private Banking, change so drastically their perceptions and self consciousness within a time range of two years that we could statistically prove that they are not behaving rationally.

These incongruences, detected through descriptive statistic, call for some statistical inference (through hypothesis testing), which will be carried out in the next sub-chapter.

4.4. Hypothesis testing

As seen before, the two samples present for each participant different perceptions of risk profile, but we still do not know if these differences are statistically relevant, or represent just sample error. The best way to ascertain this is to check if, statistically, the two samples were withdrawn from the same population (which we know for sure they were, as the individual of the samples are the same) or if the tests reject this hypothesis, inferring that the respondents should be from different populations (which would mean that they really changed their self awareness in this period of two years, thus proving their not so rational perception of risk).

So, the null hypothesis to test will be “the population of the two samples is the same”:

$$H_0: \mu_{2007} = \mu_{2009};$$

$$H_1: \mu_{2007} \neq \mu_{2009};$$

Where μ_{2007} represents the population on individual investors acting through a Private Bank in 2007 and μ_{2009} represents the population on individual investors acting through a Private Bank in 2009.

In order to conduct the statistical tests, we used a statistical software package, OriginPro8, freely available for download in the Internet (www.originlab.com). Other softwares are usually used (like SPSS), but this one is enough for the kind of hypothesis testing we will be conducting.

First of all we verified if there is some kind of correlation between the two samples of declared risk profiles and also between the two samples of real risk profiles:

- for the Declared Risk Profiles, being $N_{2007} = N_{2009} = 100$, the Pearson Correlation is 68% (2 tailed test of significance was used, with a significance close to zero);

- For the Real Risk Profiles, same sample, the Pearson Correlation is 40% (idem in test characteristics).

Here are the outputs of the OriginPro 8 software package:

User Name: Antonio Cunha

Pearson Correlations

```

-----
                A                B
-----
A Pearson Corr.    1                0,68448800089011
A Sig.             -                4,2188474935756E-15
B Pearson Corr.    0,68448800089011            1
B Sig.             4,2188474935756E-15        -
(2-tailed test of significance is used)
    
```

This means that the samples are lowly correlated, so we can proceed with hypothesis testing on both samples. Nevertheless, to proceed with further tests we need to ascertain if the population follows a normal distribution, so we can choose the right test for the statistic.

According the statistic literature, two tests yield the best results for normality testing:

- Shapiro-Wilk test for Normality – tests the probability of a sample coming from a normal distribution. The test statistic is:

$$W = \frac{\left(\sum_{i=1}^n a_i x_{(i)}\right)^2}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

And the null hypothesis is rejected if W is too small.

- Kolmogorov-Smirnov test for Normality – tests the probability of a sample coming from a pre-determined distribution. The test statistic is:

$$D_n = \sup_x |F_n(x) - F(x)|,$$

If the sample comes from F(x) distribution, then Dn converges to 0.

By inserting the data from Table 5 (profile classifications of the 100 respondents) in the OriginPro 8 software package and testing those two samples for the provenience from a population with a normal distribution (under a Shapiro-Wilk test for Normality and under a Kolmogorov-Smirnov test for Normality), we arrived at the following conclusions:

Shapiro-Wilk Test for Declared Profile samples with 95% confidence level:

Sample	Degrees of Freedom	Statistic	Probability < W
--------	--------------------	-----------	-----------------

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2007	100	0,905	0,00000024
2009	100	0,613	0,0000000000000001

Sample 2007: At the 0,05 level (95% confidence) the data was not significantly drawn from a normally distributed population.

Sample 2009: At the 0,05 level (95% confidence) the data was not significantly drawn from a normally distributed population.

Kolmogorov-Smirnov Test for Declared Profile samples with 95% confidence level:

Sample	Degrees of Freedom	Statistic	Probability > D
2007	100	0,2040	0,0003962
2009	100	0,4171	0,00000000000017683

Sample 2007: At the 0,05 level (95% confidence) the data was not significantly drawn from a normally distributed population.

Sample 2009: At the 0,05 level (95% confidence) the data was not significantly drawn from a normally distributed population.

Under both tests the population of the Private Banking clients does not follow a normal distribution in what regards their self perception of risk. This is a first major conclusion of this study, as the hypothesis testing will have to be carried out with nonparametric tests. Moreover, if this conclusion could be extrapolated to all financial markets investors (and not only the target population of this study – the individual investors acting through a Private Bank) it would imply that one of the assumptions of the Efficient Market Hypothesis could be questioned (the normal distribution is a base of most standard finance formulae developed upon the EMH).

As we do not have a normal distribution, we must use a nonparametric statistical test to verify if there are significant differences between the two samples that would lead us to infer that they represent in fact a statistically relevant change in the rational perceptions of the participants.

According to some statistical literature (Gibbons, 1976), the most appropriate test for this purpose is the Kolmogorov-Smirnov (K-S) test for two samples. The K-S test tries to determine if two datasets differ significantly (as most of statistical tests), but has the advantage of making no assumption about the distribution of the data. It is nonparametric and distribution free.

This kind of test (non dependent on the population frequencies distribution) will enhance the quality of this research as it will help minimize the statistical test usual errors:

- Type I error – reject the null hypothesis when in reality it is true;
- Type II error (also known as β) – not reject the null hypothesis when in reality it is false.

These errors are ‘the two faces of the same coin’ as if we increase the level of significance to, for example, 10%, although we are decreasing the probability of not rejecting the null hypothesis when it is false (β), on the other hand we are increasing the

probability of incurring in a Type I error. Thus, the power of a hypothesis test is measured by: $1 - \beta$. A high value of $1 - \beta$ indicates that the hypothesis test is working very well.

The power of a statistical test of a null hypothesis is the probability that it will lead to the rejection of the null hypothesis; that is, the probability that it will result in the conclusion that the phenomenon exists. This depends on three parameters: the significance level of the test (which we already defined as 5% - from the interval of confidence of 95%), the dimension of the effect (the degree to which the phenomenon exists) and the reliability of the sample results. If this later does not belong to a normal distributed population, and applied the hypothesis test is parametric (needing a normal population for validity and reliability), errors will probably happen.

The K-S test is a form of minimum distance estimation used as a nonparametric test of equality of one-dimensional probability distributions used to compare a sample with a reference probability distribution or to compare two samples (as we will do). The K-S statistics quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples. The null distribution of this statistic is calculated under the null hypothesis that the samples are drawn from the same distributions (in the two sample case). The test is considered to be the most useful and general nonparametric method for comparing two samples, as it is sensitive to differences in both location and shape of the empirical cumulative distribution functions of the two samples. The test statistic is found under the following formula:

$$D_{n,n'} = \sup_x |F_n(x) - F_{n'}(x)|.$$

And the null hypothesis is rejected at a level α if:

$$\sqrt{\frac{nn'}{n+n'}} D_{n,n'} > K_\alpha.$$

But the study of statistical inference is not the purpose of this dissertation, so we will make us of the OriginPro 8 software package to obtain the output results of the K-S test in the two samples. Here are the results:

Kolmogorov-Smirnov Test

User Name Antonio Cunha

Input Data

Data Range

Group Range [Book1]Sheet1!B [1*:100*]

Data Range [Book1]Sheet1!C1 [1*:100*]

Test Statistics

D	Z	Exact Prob> D
0,59422750424448	0,12242358879483	2,7887647302549E-8

Null Hypothesis: $F(x) = G(y)$

Alternative Hypothesis: $F(x) \neq G(y)$

(At the 0.05 level, the two distributions are significantly different.)

Thus, we reject the null hypothesis $H_0: \mu_{2007} = \mu_{2009}$.

4.5. Findings and Limitations

This is a very important result: at the 5% significance level, we reject the null hypothesis that the two samples were drawn from the same population (considering that the populations maintain the frequencies distribution throughout time). But we know for sure that in fact the two samples contain exactly the same individuals, belonging to the same population. What the test proves is that in two years (from 2007 to 2009) the frequencies distribution of this population changed, which is not compatible with the Efficient Market Hypothesis (that assumes a normal distribution, stable throughout time).

This means that the individuals changed their self-assessed risk profile in a relatively short period of time, and even contradicted themselves (in 2007 stated that they would not need funds in the next 5 years and wanted long term returns, and in 2009 stated the opposite), as if they were different individuals answering the same questions. If standard finance matches asset allocations under the Efficient Frontier (Markowitz) theorem according to individual investor risk tolerance (Expected Return versus Standard Deviation), it will be a very volatile portfolio, as the individual investor might be changing risk tolerance in very short periods of time.

We know that the individual investors' risk tolerance changes with time, but mostly due to socio demographic variables (age and income) such as retirement, unemployment, having children, and in periods of no less than five years (considered the long term). In this research we proved by statistical inference that a population of individual investors changed their risk tolerance in less than two years, for no other reason other than witnessing the crash of financial assets price.

Finally, the test was not carried out on the real risk profile sample (with the participants asset allocation) because we verified from the descriptive statistics that the changes occurred in these two samples were not as strong as in the samples related to the participants self declared risk profile. Moreover, as the Private Bank's CEO mentioned, the asset allocation is not really the one that clients would like to have, as it is conditioned by the existence of assets that the clients can't or are not willing to sell. We also believe that through the Private Bank's advisory service some of the assets that clients hold in their portfolios were not chosen entirely by them, but under counsel of

the bank's officers, thus not allowing this sample to be totally independent, and so being prone to errors in statistical testing.

We realize that the fact of the samples being non-random, obtained through convenience, is a limitation, as well as the non existence of a normal distribution of frequencies of the populations. Nevertheless, as these limitations were detected and taken into considerations, several potential errors of analysis were avoided and we were able to choose the most appropriate statistical test for these two samples, providing the highest validity and reliability possible.

5. CONCLUSIONS

This result leads us to conclude that we should doubt of the existence of a strong form of Efficient Market (Hypothesis), and put in perspective all the standard finance tools we have been using for the last 40 years, most of them developed upon this assumption.

The human behaviour is, and should kept being, an object of study of the social sciences, mainly sociology, psychology or even anthropology. Qualitative research might still be the best methodology to understand how and why individuals behave as they do, so the mathematical models developed by standard finance to explain the way financial markets work might fall short of the effect that the rational boundaries of the individual investor (one of the financial markets players) have in asset pricing.

Behavioural Finance can and should close this gap between mathematical models and sociological models, so we could obtain a better understanding of this fascinating world of Finance. The insights into Financial Markets and Behavioural Finance developed throughout this research allowed us to understand that mixing human behaviour with numbers is not a perfect match, as limited rationality cannot compete with computerized models of asset pricing and statistical tools for trend extrapolations based in theoretical populations with frequencies normally distributed. Neither can the standard finance, if maintaining these mathematical tools, predict or evaluate asset prices if these, at the end of the day, will be defined by human behaviour.

Stemming from neoclassical economics, *homo oeconomicus* is a simple model of human economic behaviour, which assumes that principles of perfect self-interest, perfect rationality, and perfect information govern economic decision by individuals. Like the Efficient Market Hypothesis, *homo oeconomicus* is a tenet that economists uphold with varying degrees of stringency. Some have adopted it in a semi strong form EMH; this version does not see rational economic behaviour as perfectly predominant but still assumes an abnormally high occurrence of rational economic traits. Other economists support a weak form (of EMH) of *homo oeconomicus*, in which corresponding traits exist but not strong. All these versions share the core assumption that humans are “rational maximizers” who are purely self-interested and make perfectly rational economic decisions.

Economists like to use the concept of rational economic man for two primary reasons:

- *Homo oeconomicus* makes economic analysis relatively simple. Naturally, one might question how useful such a simple model can be.
- *Homo oeconomicus* allows economists to quantify their findings, making their work more elegant and easier to digest. If humans are perfectly rational, possessing perfect information and perfect self-interest, then perhaps their behaviour can be quantified.

However there are criticisms to *homo oeconomicus* of which most of them proceed by challenging the basis of three underlying assumptions (perfect rationality, perfect self-interest and perfect information):

1. Perfect Rationality. When humans are rational, they have the ability to reason and make beneficial judgment. However rationality is not the sole driver of human behaviour. In fact, it may not even be the primary driver, as many psychologists believe that the human intellect is actually subservient to human emotion. They contend, therefore, that human behaviour is less the product of logic than of subjective impulses, such as fear, love, hate, pleasure and pain. Humans use their intellect only to achieve or to avoid these emotional outcomes.

2. Perfect Self-Interest. Many studies have shown that people are not perfectly self-interested. If they were, philanthropy would not exist. Religion prizing selflessness, sacrifice and kindness to strangers would also be unlikely to prevail as they have over centuries. Perfect self-interest would preclude people from performing such unselfish deeds as volunteering, helping the needy, or serving in the military. It would also rule out self-destructive behaviour, such as suicide, alcoholism, and substance abuse.

3. Perfect information. Some people may possess perfect or near-perfect information on certain subjects; a doctor or dentist; one would hope, is impeccably versed in his or her field. It is impossible, however, for everyone to enjoy perfect knowledge of every subject. In the world of investing, there is nearly an infinite amount of know and learn; even the most successful investors don't master all disciplines.

Many economic decisions are made in the absence of perfect information. For instance, some economic theories assume that people adjust buying habits based on the national central bank monetary policies. Naturally, some people know exactly where to find the Swiss National Bank or the Fed data, how to interpret it, and how to apply it; but many people don't know or care who or what the Federal Reserve or the Swiss national bank is. Considering that this inefficiency affects millions of people (or investors), the idea that all financial actors possess perfect information becomes implausible.

Again, as with market efficiency, human rationality rarely manifests in black or white absolutes. It is better modelled across a spectrum of grey. People are neither perfectly rational nor perfectly irrational; they possess diverse combinations of rational and irrational characteristics, and benefit from different degrees of enlightenment with respect to different issues. The question remains whether the concept of "investor rationality" is indeed realistic?

Psychological research has shown that the human brain often uses shortcuts to solve complex problems. These heuristics are rules or strategies for information processing, which help to find a quick, but not necessary optimal, solution. Once the information is simplified to manageable level, people use judgment heuristics. These shortcuts are needed to resolve the decision making as quickly as possible. Heuristics are also used to arrive at a quick judgment, they can, however, also systematically distort judgment in certain situations. The first step in reducing complexity is to simplify the decision. However it also adds the risk of arriving at a non-rational conclusion, unless one is careful.

In this study I managed to prove, for a defined confidence level, that in a short period of time a population of individual investors changed relevantly their judgement about their self risk tolerance, by influence of external factors like asset pricing. In fact, probably due to the panic effect, individual investors of the same sample even contradicted themselves, affirming having long term investment horizons in 2007 and changing this view to a short term horizon in 2009. Perfect rationality is less than evident in this population.

Most of behavioural finance research has been centred on the market effects of investor behaviour, as the ‘January effect’, the ‘Large Caps versus Small Caps’ or even attitudes of investors towards risk and loss aversion. I approached the subject in a slightly different way, by studying the self perception of risk tolerance of the individual investor, only to conclude that he/she is not aware of his/her own risk profile.

For future research, it would be interesting to approach this same theme (Behavioural Finance) from a different perspective. Behavioural Finance could help central banks in the detection of asset bubbles, by developing tools that could detect patterns of investor herd behaviour in determined assets, thus allowing for preventive actions to be implemented as a surgery (in the asset with inflated prices), avoiding the type of Medicine used today by central banks: in a recession lower rates to every sector of the economy, allowing for the creation of an asset price bubble in the sectors that are not affected by the recession, and increasing rates in a surge of inflation (or asset price bubble), destroying sectors of the economy that are not causing or being affected by inflation.

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7. APENDIX

7.1. Questionnaire

Participant n.º ____/100.

Please mark the correct answer in each of the following questions.

Section 1 – To match the portfolio with the client’s financial situation.

1.6. In relative terms, revenue from the:

1.1.1. Client’s profession is:

- a) very important;
- b) not important.

1.1.2. Client’s financial portfolio is:

- a) very important;
- b) not important.

1.7. The evolution of the professional revenue of the client during the next five years can be described the following way:

He/She expects his/her global revenue to:

- d) increase clearly above inflation (promotion, growth of own business, etc.);
- e) increase at a pace similar to inflation;
- f) decrease (due to retirement or other reason).

1.8. Approximately how much of the client’s total assets does he have invested in this bank?

- e) Less than 25%;
- f) Between 25% and 50%;
- g) Between 51% and 75%;
- h) More than 75%.

1.9. Does the client use credit regularly?

- c) Yes;
- d) No.

1.10. If he/she does, for what reason?

- d) Tax purposes;
- e) Financial leverage;
- f) Other reason.

Section 2 – To assess the client’s attitude and experience in financial assets.

2.1. What is the client's reference currency?

- a) Euro;
- b) US dollar.

2.2. Does the client have financial assets denominated in other currencies?

- a) Yes;
- b) No.

2.3. Which of the following best describes the client's level of investment experience?

- a) Novice / Beginner / Little Experienced – His first investment in the financial markets was in the past 12 to 18 months.
- b) Somewhat Experienced – The client understands the basics of investing but he/she is less confident about how the markets actually work and interact.
- c) Experienced – The client has been actively investing for several years or more and he is reasonably confident of his knowledge of the financial markets.
- d) Very Experienced – The client is quite knowledgeable about the financial markets and feels very comfortable making investment decisions.

2.4. Which statement best describes the client's attitude toward investing?

- a) He/She is extremely safety conscious and doesn't want the value of his investment portfolio to decline at all;
- b) He/She realizes that there are risks in investing, but tries to reduce them as much as possible;
- c) He/She is willing to assume some investment risk to enhance the potential return of the portfolio;
- d) He/She is willing to assume significant risk for a portion of the portfolio to increase the potential for higher overall returns;
- e) He/She is comfortable assuming significant risk for the overall portfolio in order to maximize the possibility of higher returns.

2.5. In order to increase the expected return of his/her investment the client would be willing to:

- a) Add quite a lot of risk in his total investment;
- b) Add quite a lot of risk in part of his investment;
- c) Add a little more risk in his total investment;
- d) Add a little more risk in part of his investment;
- e) Not increase the risk of his investment.

Section 3 – To determine the client's time horizon.

3.1. With the amount invested the client seeks a return on the investment:

- a) on the short term;
- b) on the medium term;
- c) on the long run.

3.2. The client expects to need the present amount in the time frame of:

- a) one year;
- b) between 2 to 3 years;

- c) between 4 to 5 years;
- d) more than 5 years.

3.3. Does the client expect to need more than half of the amount before that time frame?

a) No.

If yes, when does the client expects to withdraw that amount?

- b) Up to 1 year;
- c) Up to 2 years or more.

3.4. Is the client aware that the value of his assets under management can change during time and eventually be worth less than its present value?

- a) Yes;
- b) The client didn't know that but he has acquired this notion.

Section 4 – Current asset allocation.

Please input the current percentage of each of the following asset classes in the client's portfolio (under management in this bank):

Cash - _____%

Bonds - _____%

Stocks - _____%

Other - _____%

Total – 100%

Date: __/__/_____

Thank you...

7.2. Algorithm for responses processing.

Table 1 – Answers quantification. For each of the following answers attribute the indicated points, and sum up the total of points at the end of the table.

Participant	n.º _____	In year 2007	In year 2009
QUESTION	VALUE POINTS	ATTRIBUTED POINTS	ATTRIBUTED POINTS
1.1.1. a)	8		
1.1.1. b)	5		
1.1.2. a)	4		
1.1.2. b)	8		
1.2. a)	5		
1.2. b)	2		
1.2. c)	1		
1.3. a)	10		
1.3. b)	9		
1.3. c)	5		
1.3. d)	3		
1.4. a)	7		

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1.4. b)	4		
1.5. a)	3		
1.5. b)	9		
1.5. c)	1		
2.1. a)	3		
2.1. b)	4		
2.2. a)	10		
2.2. b)	6		
2.3. a)	2		
2.3. b)	5		
2.3. c)	24		
2.3. d)	26		
2.4. a)	2		
2.4. b)	21		
2.4. c)	47		
2.4. d)	68		
2.4. e)	95		
2.5. a)	25		
2.5. b)	20		
2.5. c)	10		
2.5. d)	5		
2.5. e)	2		
3.1. a)	0		
3.1. b)	25		
3.1. c)	30		
3.2. a)	0		
3.2. b)	5		
3.2. c)	25		
3.2. d)	30		
3.3. a)	30		
3.3. b)	5		
3.3. c)	30		
3.4. a)	10		
3.4. b)	0		
Total			
Asset Allocation 2007			
Cash	Bonds	Stocks	Other
Asset Allocation 2009			
Cash	Bonds	Stocks	Other

Table 2 – Participant declared risk profile. Please tick the box in the second line of the below table that corresponds to the participant total points.

Participant	n.º _____				
Total points	<134	[134 – 174[[174 – 224[[224 – 275]	> 275
Declared risk profile 2007	Preservation	Income	Income & Growth	Growth	Aggressive Growth
Declared risk profile 2009	Preservation	Income	Income & Growth	Growth	Aggressive Growth

Table 3 – Participant real risk profile. Please tick the box in the second line of the below table that corresponds to the participant declared asset allocation.

Participant	n.º _____				
Real asset allocation	Cash > 50%	Bonds > 50%	Bonds and Stocks between 30% and 50% each	Stocks > 50% but no more than 70%	Stocks more than 70%
Real risk profile 2007	Preservation	Income	Income & Growth	Growth	Aggressive Growth
Real risk profile 2009	Preservation	Income	Income & Growth	Growth	Aggressive Growth

Table 4 – Participant risk profile differences between declared and real risk profile. Please tick the boxes below to compound the matches or differences (numerical) between the declared risk profile and the real risk profile of each participant.

Participant	n.º _____	Date __/__/__			
Risk profile	Preservation	Income	Income & Growth	Growth	Aggressive Growth
A) Declared risk profile 2007	1	2	3	4	5
B) Real risk profile 2007	1	2	3	4	5
Difference 2007: Numerical subtraction of A – B					
A) Declared risk profile 2009	1	2	3	4	5
B) Real risk profile 2009	1	2	3	4	5
Difference 2009:					

Numerical subtraction of A – B					
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Table 5 – Participants risk profiles and respective differences. Please transpose data from tables 2, 3 and 4 for the following table.

Part.	Declared Risk Profile 2007	Real Risk Profile 2007	Differences 2007	Declared Risk Profile 2009	Real Risk Profile 2009	Differences 2009	Average Differences
1							
2							
3							
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95							
96							
97							
98							
99							
100							
Averages							
Standard Deviations							

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