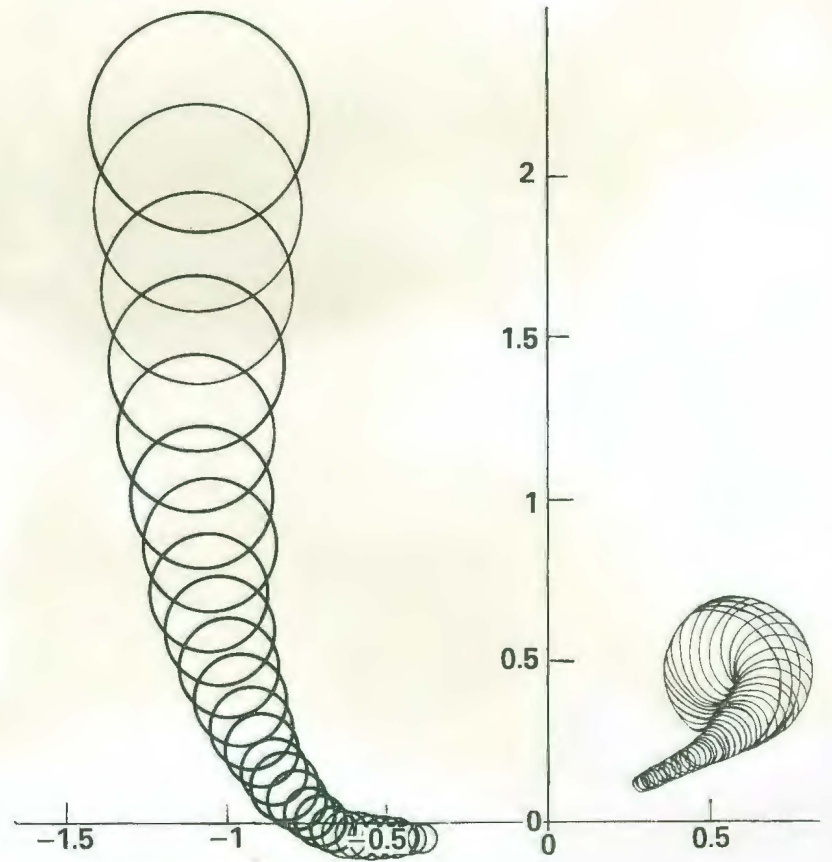


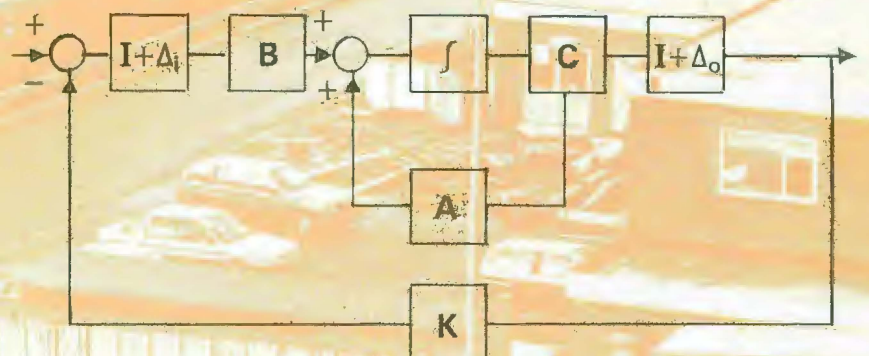


International Conference



CONTROL 88

Conference Publication No 285



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INTRODUCTION

In the last decades robot control theory has been a major topic of research and development. Linear control was proved to be inadequate for rapid and accurate trajectory tracking, and consequently several other control schemes have been proposed. Control strategies based on nonlinear compensation and adaptive control are two of the kind. Nevertheless, they impose high computational burden, using present day microprocessors (Chang-Huan et al. (1)).

An approach that overcomes this drawback is the use of variable structure (VSS) controllers (Utkin (2)). Both theoretical or simulation studies (Young (3) (9), Morgan et al (5), Slotine et al (4) (6) (7), Kasuge et al (8), Ozguner et al (10)), and practical implementations (Klein et al (11), Hiroi et al (12), Staszulonek et al (13), Hashimoto et al (14) (15)), have demonstrated its feasibility.

In the VSS controllers proposed so far, the robot manipulator is induced to match a first order linear decoupled system. The robot state trajectories can be decomposed in a reaching phase where the robot manipulator evolves towards the linear law, and a sliding phase where the robot manipulator follows the linear characteristic. Nevertheless two problems arise:

- In the reaching phase, the robot/controller system is sensitive to parameter variations, thus convergence is not assured.
- In the sliding phase, chattering about the switching linear law may excite unmodelled high frequency modes of the mechanical structure.

In order to eliminate these drawbacks, research lead to VSS control laws using feedforward (3) (12), a simplified robot model (5), "smooth" switching algorithms (4) (6) (7) (14), and more sophisticated sliding trajectories (10) (11) (15).

In this paper a new VSS controller which is a development of those techniques is proposed. This controller is composed of two blocks: the first defines an appropriate reference model and the second implements a smooth control law. With respect to the selection of the reference model two requirements were taken into account: sufficient low order, compatible with the robot manipulator dynamics, and linearity in order to simplify the mathematical treatment. Second order linear models were found to obey these requirements.

The control law is an adaption of the standard PI controller to the VSS philosophy. This structure lead to an easily and intuitively adjustable controller; moreover, the resulting position/velocity trajectories and corresponding torques are "ripple free", with negligible coupling between axis.

THE SLIDING MODE CONTROL ALGORITHM

The VSS controller is now described. In the first block an adequate sliding law, that stems from the properties of the robot manipulator dynamics is implemented. The sliding mode controllers proposed so far, use a first order linear system trajectory

$$s_i = c_i x_i + \dot{x}_i = 0 \quad ; \quad i=1, \dots, n \quad (1)$$

where n denotes the number of degrees of freedom of the manipulator, x and \dot{x} the vectors of position and velocity coordinates, respectively.

Based on equation (1), the switching control law

$$u = u[\text{sgn}(s)] \quad (2)$$

is implemented where u is the control vector and $\text{sgn}()$ is the sign function. Asymptotic convergence is guaranteed if the control law (2) ensures that the condition

$$s_i \dot{s}_i < 0 \quad ; \quad i=1, \dots, n \quad (3)$$

is satisfied.

In contrast with first order systems, which can have discontinuous phase plane trajectories, robot manipulators do not allow such dynamics; they have moving inertias thus implying continuous position and velocity trajectories in the phase plane. Therefore, when the robot controller tries to mimic a first order linear system, it is confronted with conflicting requirements. In practice, because torque can not be infinite, the phase plane trajectory is continuous. Nevertheless, high torque requirements saturate the robot actuators giving, consequently, a longer (in time) reaching phase which, as previously stated, is highly sensitive to parameter variations; at the same time (undesirable) integral mode wind-up may also take place. Because the least system order compatible with robot manipulator dynamics is two, a second-order reference model is an obvious choice. Furthermore, for any point in the phase plane there is always a continuous and smooth trajectory containing the point and satisfying

$$s_i = \ddot{x}_i + 2e_i \dot{x}_i + w_{n1} x_i = 0 \quad ; \quad i=1, \dots, n \quad (4)$$

for a given e_i and w_{n1} . Therefore, two problems are avoided at once: the undesirable reaching phase is obviously eliminated, and the chatter usually present in the sliding phase is also attenuated. In fact, when some perturbation arises, the actual robot trajectory moves away from the desired one. If a first order linear system sliding curve is used, the controller reacts, providing opposite phase plane trajectories towards the desired trajectory. As some delay is unavoidable in digital implementations a

"switching" between those curves arises, giving the well known chatter. The use of second order curves attenuates this problem. Because there is always a trajectory of the type defined in (4) containing a given point (x, \dot{x}) , after a perturbation the system will not be forced to follow the initial characteristic; instead it will follow a new trajectory (having the same e_i and $w_{n,i}$) with initial conditions corresponding to the present values of x and \dot{x} , and which is "almost" parallel to the previous one.

We now describe the control law implementation. This is motivated by the discussion of both the VSS switching law structure and the robot manipulator dynamics. As pointed by Young (3) the weight of the corrective sliding torque can be alleviated if some form of feedforward is envisaged. The reduction of the sliding torque gives a reduction in the chatter amplitude. Another cause for the chatter problem is the difference between the applied torque, computed by a crude law, and the torque actually required, which is given by a complex matrix equation. Morgan et al (5) tried a compromise law, which had some insight from the robot dynamic equations. An applied torque more suited to the robot requirements, obviously reduces the chattering. Several studies (4) (6) (7) (14), demonstrated that chattering could be alleviated if the on-off like sliding control law was converted to a continuous one, with a "proportional band" in the neighbourhood of the origin.

Because the inertial, Coriolis/centripetal and gravitational torques are sine and cosine complex matrix functions, and the variables x , \dot{x} are continuous in time, a closer look at the robot manipulator equations

$$T = J(x)\ddot{x} + C(x, \dot{x}) + G(x) \quad (5)$$

suggests that a "good guess" for the total torque will be a conservative and smooth curve. To accomplish such an estimation, a smooth torque component T_s , is computed by the equation

$$T_s(j) = T_s(j-1) + KT_{vss}(j) \quad (6)$$

where $j-1$ and j are consecutive sampling instants, K is a gain factor and T_{vss} is an estimated adjustment torque.

Nevertheless, the total torque may have discontinuities due to the inertial component $J(x)\ddot{x}$. Therefore some form of quick estimation is necessary. These "fast acting" requirements, both for the adjustment torque in (6) and the inertial one, make necessary the computation of a "quick" corrective torque. Such a torque can be given by a continuous VSS condition ($i=1, \dots, n$)

$$(T_{vss})_i = \begin{cases} -D_i \text{sgn}(s_i) & \text{if } \text{abs}(s_i) > \delta \\ -D_i s_i / \delta & \text{if } \text{abs}(s_i) < \delta \end{cases} \quad (7)$$

where $(T_{vss})_i$ denotes the i th component of vector T_{vss} , and δ , and D_i define the parameters of a proportional/saturation like characteristic, namely D_i/δ is the gain of the proportional part and D_i corresponds to the value of the saturation part of the characteristic. Equations (6) and (7) may also be viewed as a standard PI controller with a saturation on the proportional block. As a result, the total control torque

vector, at time j , is given by

$$T(j) = T_s(j) + T_{vss}(j) \quad (8)$$

The overall VSS controller configuration is depicted in Fig. 1.

The calculation of $s = (s_1, \dots, s_n)$ in (7) using equation (4) requires \ddot{x} . Usually, only the vectors x and \dot{x} are available from sensor measurement; similarly to Morgan (5) method, \ddot{x} is computed by the finite differentiation formula

$$\ddot{x}(j) = [\dot{x}(j) - \dot{x}(j-1)]f \quad (9)$$

where f is the controller frequency in Hz. The use of acceleration either from sensor measurement (Futami et al (16), Luo et al (17)) or from the velocity finite differentiation formula, has been shown to be a feasible procedure, namely without noise problems as long as appropriate standard filtering techniques are considered. In the proposed controller the finite differentiation (9) is computed at high sampling rates, and no special filtering was required, as will be seen in section 3.

SIMULATION RESULTS

Based on the previously defined controller structure, several position control simulations were performed, as follows:

- first an empirical choice of controller parameters was made;
- interactively, the parameters were varied and the results compared with expected ones;
- finally, conclusions were drawn regarding robustness, axis decoupling, computational burden and controller parameter adjustment.

Although formulae for the determination of the controller parameters are available (Machado et al (18)), this "black box" or empirical controller adjustment, seems more efficient when thinking on its implementation in an industrial environment.

In the simulations a 2R robot manipulator (Fig. 2) model was used, which is described by the equations

$$T_1 = [(m_1 + m_2)r_1^2 + m_2 r_2^2 + 2r_1 r_2 m_2 C_3 + J_1] \ddot{x}_2 + (m_2 r_2^2 + r_1 r_2 m_2 C_3) \ddot{x}_4 - r_1 r_2 m_2 S_3 x_4^2 - 2r_1 r_2 m_2 S_3 x_2 x_4 + m_1 g r_1 C_1 + m_2 g (r_1 C_1 + r_2 C_{13}) \quad (10)$$

$$T_2 = (m_2 r_2^2 + r_1 r_2 m_2 C_3) \ddot{x}_2 + (m_2 r_2^2 + J_2) \ddot{x}_4 - m_2 r_1 r_2 S_3 x_2^2 + m_2 g r_2 C_{13} \quad (11)$$

with

$$x_2 = x_1; \quad x_4 = x_3; \quad C_1 = \cos(x_1); \quad C_{13} = \cos(x_1 + x_3) \\ C_3 = \cos(x_3); \quad S_3 = \sin(x_3) \quad (12)$$

Similarly to (3) and (6) studies, the manipulator parameters were set to:

$$m_1 = 0.5 \text{ Kg}; \quad m_2 = 6.25 \text{ Kg}; \quad r_1 = 1 \text{ m}; \quad r_2 = 0.8 \text{ m} \\ J_1 = 5 \text{ Kg m}^2; \quad J_2 = 5 \text{ Kg m}^2 \quad (13)$$

and the position control experiments were required to move the manipulator from the initial state

$$x_1 = -2.784 \text{ rad}; \quad x_2 = 0 \text{ rad/sec} \\ x_3 = -1.204 \text{ rad}; \quad x_4 = 0 \text{ rad/sec} \quad (14)$$

to the final state

$$\begin{aligned} x_1 &= 0 \text{ rad}; x_2 = 0 \text{ rad/sec} \\ x_3 &= 0 \text{ rad}; x_4 = 0 \text{ rad/sec} \end{aligned} \quad (15)$$

The first choice of controller parameters was:

$$\begin{aligned} e_1 = e_2 = 2; w_{n1} = w_{n2} = 10; K_1 = K_2 = 0.1 \\ D_1 = D_2 = 100; \delta a_1 = \delta a_2 = 100 \end{aligned} \quad (16)$$

The simulations (Fig. 3) showed rather oscillating convergent trajectories in the phase plane. Two attempts were made to correct that behaviour. The damping coefficient was doubled and the natural frequencies were decreased one decade. Both situations correspond to slower systems, but only the second situation showed improvement. This is a natural result as it is well known that the reference model should have a damping ratio near, but greater than the critical one, to prevent overshoot; the major model restriction lies in the natural frequency due to the limitations on the speed of response of the robot.

Intuitively, one might expect that the second link should have larger bandwidth performances. To test this hypothesis a faster second order linear system model was tried, as a sliding curve for link two, showing an improvement in the resulting performances. Interactive simulations revealed that higher gains in the switching law (7) could achieve more accurate tracking. Nevertheless, for very high gains oscillation once more resulted, showing that a middle range of possible values was the best choice.

Time plots of the position, velocity and torque variables showed two undesirable factors:

-A high frequency oscillation at the torque curves. This oscillation is filtered, thus negligible in position or velocity curves. Nevertheless it may excite unmodelled high frequency resonant modes in the mechanical structure of the manipulator.
-During the transient phase, high torques were demanded. More conservative values should be attained. Comparing the ideal torque curves with the actual ones, it is shown that the "filtering action" of the control law (6) is responsible for a slower but more conservative torque curve.

Both undesirable performances can be eradicated with a lower gain in the control law (7). As pointed out previously, this implies worse phase plane sliding curve tracking. A compromise between these criteria is necessary. Also, a somewhat slower second link curve was tried; the simulated phase plane results (Fig. 4), and time plotted results (Figs. 5 and 6) show the excellent performance achieved. To test the new VSS controller robustness, several simulations were performed with different loads; the phase plane results (Fig. 7) show the remarkable insensitivity to those perturbations. Finally, experiments were made with critical damping ratios i.e. $e=1$; as expected, a more oscillating behaviour arose showing that those values should be avoided.

An important aspect that should be highlighted, is the torque versus time results, which are frequently overlooked in the literature. The PWM like robot demanded torques, that correspond to the on-off sliding control law, impose either mechanical stress on the robot manipulator structure or actuator stress as the current (pressure) of the electrical

(hydraulic) actuator tries to follow the controller torque reference. The proposed algorithm eliminates these problems, as the torque curves (Fig. 6) is continuous with negligible chattering.

PATH CONTROL

This algorithm can be easily generalised to the path control problem. In this case the sliding curve becomes

$$\begin{aligned} s_1 = x_1 + 2e_1 w_{n1} x_1 + w_{n1}^2 x_1 - \\ - (\dot{x}_{1d} + 2e_1 w_{n1} \dot{x}_{1d} + w_{n1}^2 x_{1d}) = 0 \end{aligned} \quad (17)$$

with x_{1d} , \dot{x}_{1d} and \ddot{x}_{1d} as the desired values for position, velocity and acceleration variables, as given by the trajectory planning block.

CONCLUSION

A new sliding controller was proposed. Second order linear system sliding curves eliminate the reaching phase and consequently the associated problems of load sensitivity and high demanded torque. Moreover, the chattering usually present in the sliding phase also disappears with the new control law.

Simulation results show a negligible coupling between axis, thus enabling an easier and more efficient controller adjustment. It is also shown that the controller parameter set can be easily adjusted, and there is a large set of possible quasi optimal values. This is of utmost importance, when one thinks of the industrial applications of this type of algorithms.

The experiments were developed based on the position control problem. Nevertheless, generalization to the trajectory tracking problem is trivial.

In either case the controller computational requirements are low, thus well adapted to today's microprocessor based digital control technology.

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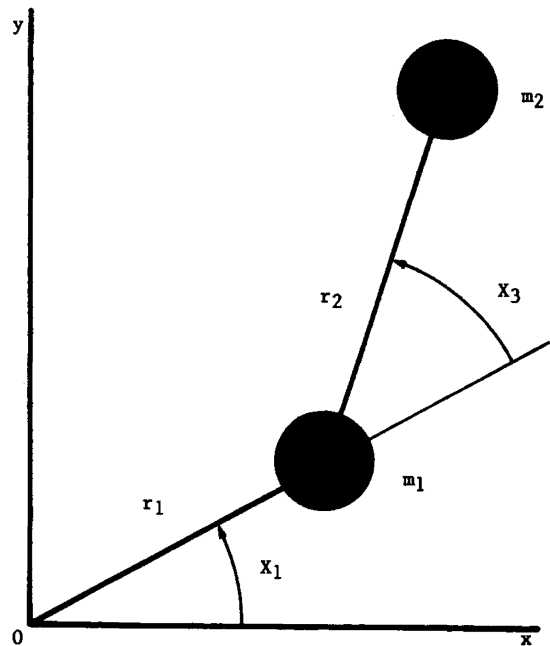
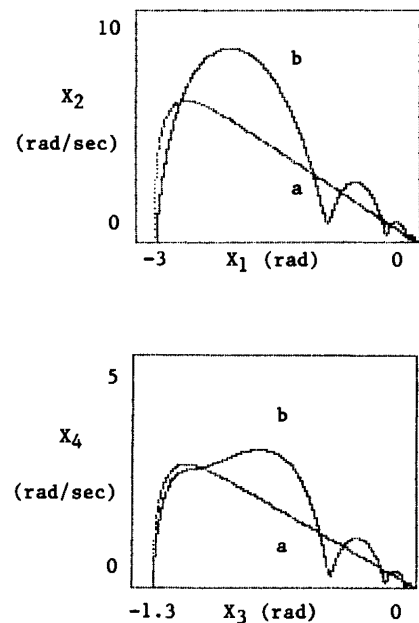


Figure 2 2R robot manipulator



a- ideal curve; b- actual curve

$e_1=2$; $e_2=2$; $W_{n1}=10$; $W_{n2}=10$

$D_1=100$; $D_2=100$; $\delta_1=100$; $\delta_2=100$

Freq. of the robot simulation=10 KHz

Freq. of the controller=2 KHz

Figure 3 Phase plane trajectories for position control

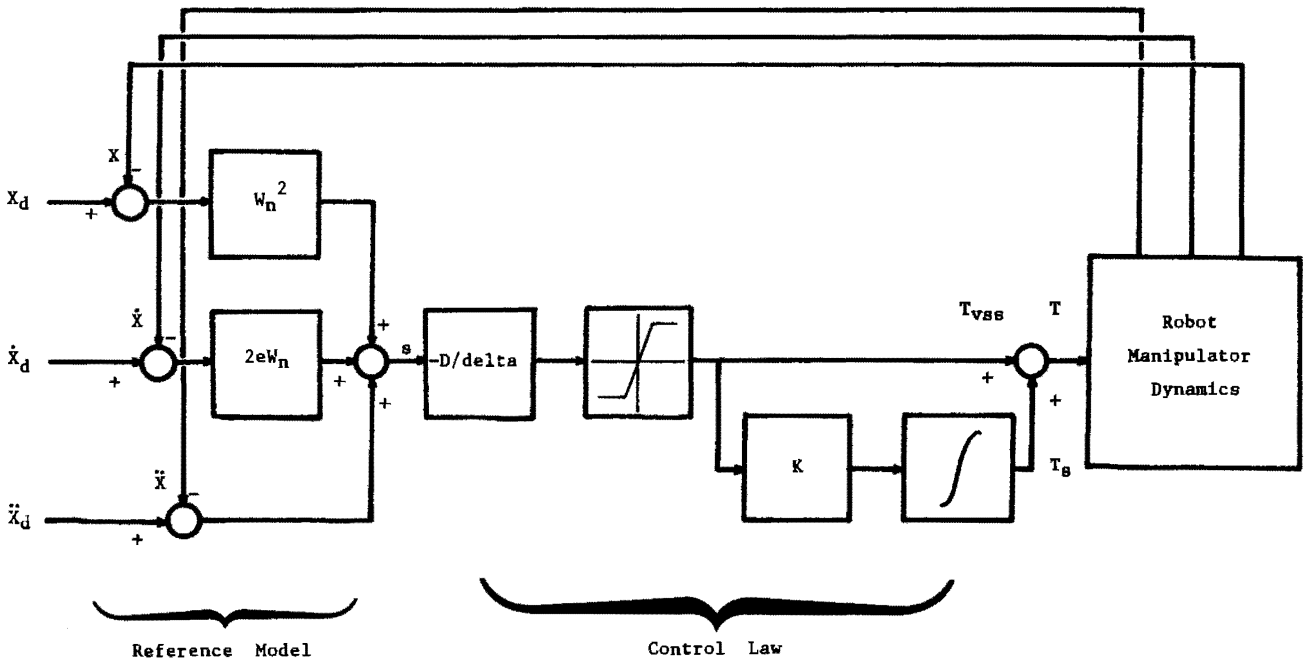
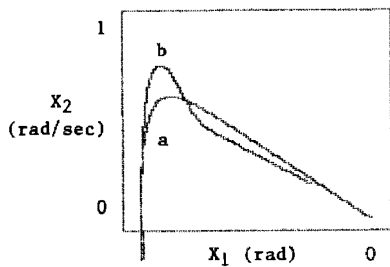
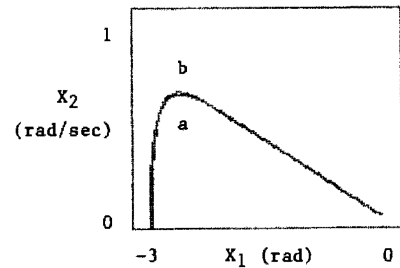


Figure 1 The new VSS controller



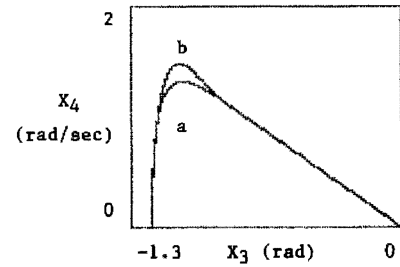
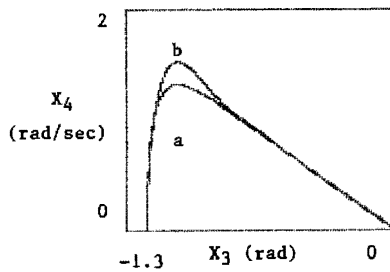
a- ideal curve; b- actual curve
 $e_1=2; e_2=2; W_{n1}=1; W_{n2}=5$
 $D_1=100; D_2=100; \delta_1=100; \delta_2=100$
 Freq. of the robot simulation=10 KHz
 Freq. of the controller=2 KHz

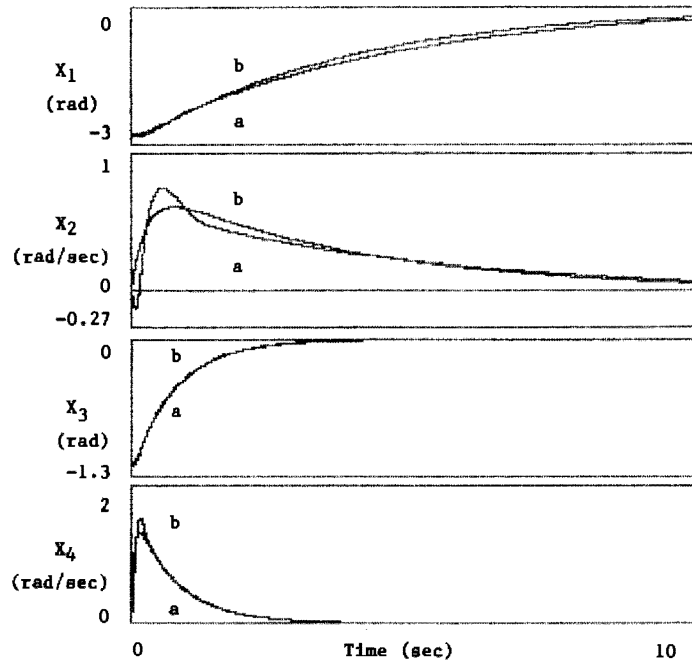
Figure 4 Phase plane trajectories for position control



a- ideal curve; b- actual curve
 $e_1=2; e_2=2; W_{n1}=1; W_{n2}=5$
 $D_1=100; D_2=100; \delta_1=100; \delta_2=100$
 Freq. of the robot simulation=10 KHz
 Freq. of the controller=2 KHz

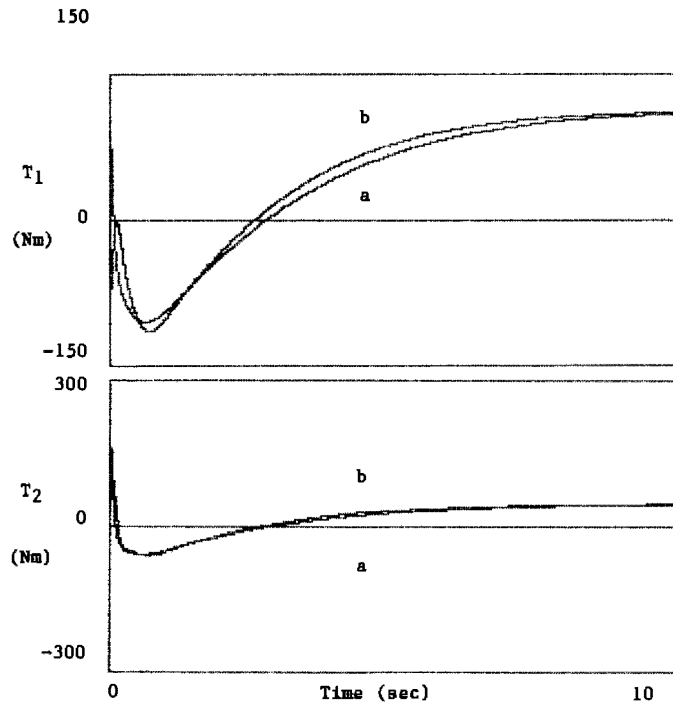
Figure 7 Phase plane trajectories for position control ($m_2=0.5$ Kg)





a- ideal curve; b- actual curve
 $e_1=2$; $e_2=2$; $\omega_{n1}=1$; $\omega_{n2}=5$; $D_1=100$; $D_2=100$; $\delta_1=100$; $\delta_2=100$

Figure 5 Position and velocity transients



a- ideal curve; b- actual curve
 $e_1=2$; $e_2=2$; $\omega_{n1}=1$; $\omega_{n2}=5$; $D_1=100$; $D_2=100$; $\delta_1=100$; $\delta_2=100$

Figure 6 Torque transients