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VARIABLE STRUCTURE CONTROL OF MANIPULATORS WITH JOINTS HAVING FLEXIBILITY AND BACKLASH

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This article studies the variable structure control of robot manipulators with joints having flexibility and backlash. A second order reference model and a smooth control law eliminate the reaching phase problems and the chattering usually present in the sliding mode. The results show a remarkable improvement of the stability over conventional variable structure controllers, while the algorithm maintains a low computational load that makes it well suited for microcomputer implementation.

KEY WORDS Robots, manipulators, control, variable structure systems, flexibility, backlash.

1. INTRODUCTION

Mechanical manipulators have complex dynamics that suggest the adoption of non-linear or adaptive control algorithms. Despite the research efforts most of today's industrial robots still use PID-like schemes [1]. This conflict between academic research and industrial practice has, however, several reasons to exist. The real-time calculation of model-based controllers imposes a very high computational burden to a microcomputer [2, 3] while, on the other hand, there is still insufficient knowledge about robustness and tuning issues. In essence, the problem arises from the necessity of devising a control strategy that not only avoids the referred drawbacks but also takes full profit of the available knowledge of control system theory. Variable structure system (VSS) theory [4] is a strategy that is currently under study. Both theoretical studies [5–8], and practical implementations [9–11] have demonstrated its feasibility, namely good robustness and low computational cost. In the variable structure controllers (VSC) proposed so far, each link of the manipulator is induced to match a first order linear decoupled law. The resulting trajectories reveal the reaching and sliding phases. In the reaching phase the robot manipulator evolves towards the reference law. During this phase the VSC does not guarantee convergence and, therefore, the system is sensitive to perturbations. In the sliding phase the system tries to follow the linear reference dynamics. Throughout this phase the actual trajectory presents a considerable chattering that may excite high frequency modes of the mechanical structure. Besides these problems, the design procedures of the proposed VSC do not take into account the dynamical characteristics of the system. Consequently, "brute force" implementations are often unavoidable.

This article analyses the performances of a VSS in the control of manipulators having joints with flexibility and backlash. Stemming from VSC notions, Section 2 outlines a control algorithm that consists of two blocks, namely a second order reference model and a smooth control law. Section 3 presents several simulation experiments for a robot having joints with flexibility and backlash and compares the results with the standard case of rigid joint tansmissions. Finally, at Section 4 conclusions are drawn.

2. THE VARIABLE STRUCTURE CONTROLLER

This section presents a new VSC [12–14] having two blocks: a reference model and a smooth control law.

2.1. The Reference Model

In the VSC's for robot manipulators proposed so far, each link is constrained to follow a first order model (FOM):

$$\sigma_i = \dot{e}_i + \zeta_i e_i = 0, \quad i = 1,\ldots,n \tag{1a}$$

$$e_i = q_{di} - q_i \tag{1b}$$

where n denotes the number of degrees of freedom, $q_{di}, \dot{q}_{di}$ and $q_i, \dot{q}_i$ are the desired and actual positions and velocities for the ith joint of the manipulator, respectively, ζ_i is the eigenvalue that determines the sliding phase, σ_i is the switching variable and e_i is the position error. Based on this model, the control algorithm implements a set of decision equations so that a control action $\mathbf{u}(t)$ forces the manipulator to match the reference model (1). Usually, the control vector obeys a law of the type:

$$\mathbf{u} = \mathbf{u}[\operatorname{sgn}(\sigma)] \tag{2}$$

where sgn() represents the sign function. If this control action satisfies the condition:

$$\sigma_i \dot{\sigma}_i < 0, \quad i = 1,\ldots,n \tag{3}$$

then it guarantees an asymptotic convergence. However, this system structure imposes conflicting requirements:

—First order systems can have discontinuous trajectories in the Phase Plane (PP)
—Robots have moving inertias that impose a time continuity to the positions and velocities.

Moreover, the first order discontinuous trajectories demand infinite joint driving torques which lead to actuator saturation. As a consequence, the actual reaching phase is not instantaneous and, due to its sensitivity to disturbances, convergence is not certain. Therefore, to match the dynamics of the reference model and dynamics of the manipulator the model must be, at least, of second order. Besides dynamic compatibility, this model should be simple enough to simplify the mathematical and computational treatment. In this line of thought, we must adopt for reference a second order model (SOM):

$$\sigma_i = \ddot{e}_i + 2\xi_i\omega_{ni}\dot{e}_i + \omega_{ni}^2 e_i = 0, \quad i = 1,\ldots,n \tag{4}$$

where ξ_i is the damping ratio and ω_{ni} is the undamped natural frequency. This option leads to a completely different approach concerning the PP trajectories. While for a FOM there is a unique trajectory in the PP defined by equation (1), now we have an infinite number of trajectories satisfying expression (4). If the characteristic equation of (4) has two different real roots ζ_{1i} and ζ_{2i}, (i.e. an overdamped response) then the second order trajectories obey a law of the type:

$$\frac{\left[\dfrac{\dot{q}_i(t) - \zeta_{1i}q_i(t)}{\dot{q}_i(0) - \zeta_{1i}q_i(0)}\right]^{\zeta_{1i}}}{\left[\dfrac{\dot{q}_i(t) - \zeta_{2i}\dot{q}_i(t)}{\dot{q}_i(0) - \zeta_{2i}q_i(0)}\right]^{\zeta_{2i}}} = 1, \quad i = 1,\ldots,n \tag{5}$$

where $q_i(0)$ and $\dot{q}_i(0)$ represent the initial conditions. In particular this means that there is always one trajectory passing through a given point $(q, \dot{q})$ of the PP. Therefore, we avoid two problems at once: we eliminate not only the undesirable reaching phase but also the chattering usually present in the sliding phase. Consequently, if the robot moves away from the desired trajectory FOM-VSC's and SOM-VSC's react differently. If the VSC uses a first order sliding curve the algorithm reacts providing opposite PP trajectories towards the reference trajectory (1). As some delay is inherent to the digital control a "switching" between those curves arises, giving the well-known chatter phenomenon. The use of a SOM eliminates this problem because there is always a reference trajectory (4) containing the present PP point $(q, \dot{q})$. After a perturbation, the control system, instead of forcing the robot towards the previous unique reference trajectory, it induces the manipulator to follow a new reference curve (4), with the same ξ_i and ω_{ni}. As a result, the controller uses a new curve, almost parallel to the previous one, passing through the present PP point. In other words, at the beginning of each sampling period j, the controller will discard the previous reference trajectory and will adopt a new one obeying equation (5) with $q(j)$ and $\dot{q}(j)$ as the new initial conditions. Therefore, for each link we will have a reference trajectory:

$$\frac{\left[\dfrac{\dot{q}_i(t) - \zeta_{1i}q_i(t)}{\dot{q}_i(j) - \zeta_{1i}q_i(j)}\right]^{\zeta_{1i}}}{\left[\dfrac{\dot{q}_i(t) - \zeta_{2i}\dot{q}_i(t)}{\dot{q}_i(j) - \zeta_{2i}q_i(j)}\right]^{\zeta_{2i}}} = 1, \quad i = 1,\ldots,n \tag{6a}$$

$$\frac{j}{f} \leqslant t \leqslant \frac{j+1}{f} \tag{6b}$$

where f denotes the controller sampling frequency. In this way, at the jth sampling instant there will be a new reference trajectory passing through the point $[\mathbf{q}(j), \dot{\mathbf{q}}(j)]$ in the PP.

2.2. The Control Law

Let us now look at the second block dealing with the control law. Young [5] suggests that the weight of the corrective sliding torque can be minimised through the introduction of a feedforward. Morgan and Özgüner [7] propose a VSC taking into consideration the robot dynamics. Several other researchers [6, 8, 10, 11] demonstrate that a reduction of the chattering can be achieved if the on-off type control law is converted to a continuous one. In this line of thought, we present the VSC in the sequel.

The robot manipulator dynamics is described by the equations

$$\mathbf{T} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \tag{7}$$

where $\mathbf{J}(\mathbf{q})$ is the inertial matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ represents the Coriolis/centripetal torques and $\mathbf{G}(\mathbf{q})$ are the gravitational torques. As the n-vector $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q})$ contains only continuous functions, and $\mathbf{q}(t), \dot{\mathbf{q}}(t)$ are continuous in time, the joint torque $\mathbf{T}$ must have a "smooth" component $\mathbf{T}_s$. Nevertheless, $\mathbf{T}$ may also have discontinuities due to the inertial component $\mathbf{J}(\mathbf{q})\ddot{\mathbf{q}}$. Therefore, the control action must provide both smooth and discontinuous components which can be achieved through the algorithm:

$$T_{VSS_i} = \begin{cases} D_i \operatorname{sgn}(\sigma_i) \text{ if } abs(\sigma_i) \geqslant \delta_i \\ (D_i/\delta_i)\sigma_i \text{ if } abs(\sigma_i) < \delta_i \end{cases} \quad i = 1, \ldots, n \tag{8a}$$

$$\mathbf{T}_s(j) = \mathbf{T}_s(j-1) + \mathbf{K}\mathbf{T}_{VSS}(j) \tag{8b}$$

$$\mathbf{T}(j) = \mathbf{T}_s(j) + \mathbf{T}_{VSS}(j) \tag{8c}$$

where $\mathbf{K}$ is a gain diagonal matrix. Moreover, δ_i and D_i define the proportional/saturation (PS) characteristic of joint i, namely D_i/δ_i is the gain of the proportional part and D_i is the output during saturation.

In essence, the controller corresponds to the integration of a VSS scheme into the PI algorithm. The VSS nonlinearity (8a) may be viewed as an amplitude dependent variant gain which is reduced in the presence of large amplitude input signals. This action is beneficial on the initial part of "unusual" transient by limiting the amplitude of the required driving torques. Nevertheless, this variant gain action demands a careful and appropriate tuning of the PS block, otherwise the action can have negative effects. The saturation must limit large amplitude signals that correspond to a negligent trajectory planning but should have a reduced action on normal amplitude signals arising from well planned trajectories. In this sense the use of the PS block is also in the line of several studies emphasising non-linear signal correction [15–16].

3. CONTROL OF MANIPULATORS WITH JOINTS HAVING FLEXIBILITY AND BACKLASH

In this section we analyse the performances of FOM-VSC and SOM-VSC in the control of manipulators. In order to compare the results, in the sequel we adopt the 2R manipulator as our prototype example and we consider three cases of increasing dynamic complexity (Fig. 1): rigid joints, flexible joints and joints having backlash.

In the first case, the robot dynamics is described by the set of equations:

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} (m_1 + m_2)r_1^2 + m_2 r_2^2 + & m_2 r_2^2 + m_2 r_1 r_2 C_2 \\ + 2m_2 r_1 r_2 C_2 + J_{1m} + J_{1g} & \\ m_2 r_2^2 + m_2 r_1 r_2 C_2 & m_2 r_2^2 + J_{2m} + J_{2g} \end{bmatrix} \tag{9a}$$

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -m_2 r_1 r_2 S_2 \dot{q}_2^2 - 2m_2 r_1 r_2 S_2 \dot{q}_1 \dot{q}_2 \\ m_2 r_1 r_2 S_2 \dot{q}_1^2 \end{bmatrix} \tag{9b}$$

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} g(m_1 r_1 C_1 + m_2 r_1 C_1 + m_2 r_2 C_{12}) \\ g m_2 r_2 C_{12} \end{bmatrix} \tag{9c}$$

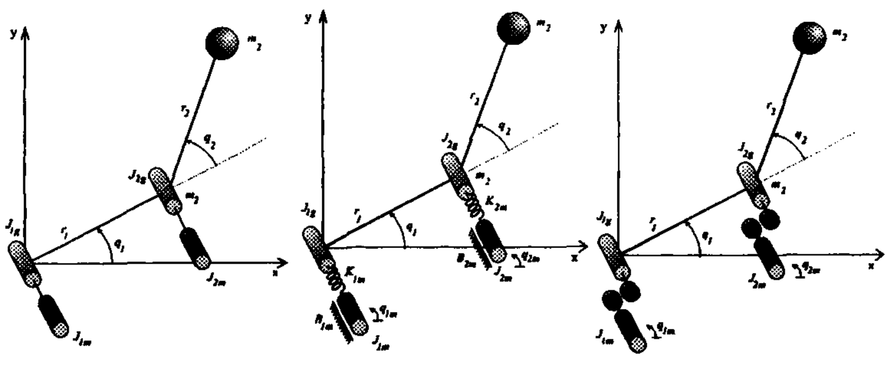
where $C_i = \cos(q_i)$, $C_{ij} = \cos(q_i + q_j)$ and $S_i = \sin(q_i)$.

For the case of compliant joints, the dynamic model corresponds to the previous expressions augmented by the equations [17]:

$$\mathbf{T} = \mathbf{J}_m \ddot{\mathbf{q}}_m + \mathbf{B}_m \dot{\mathbf{q}}_m + \mathbf{K}_m(\mathbf{q}_m - \mathbf{q}) \tag{10a}$$

$$\mathbf{K}_m(\mathbf{q}_m - \mathbf{q}) = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \tag{10b}$$

where $\mathbf{J}_m$, $\mathbf{B}_m$ and $\mathbf{K}_m$ are the $n \times n$ diagonal matrices of the motor and transmission inertias, damping and stiffness.



a) b) c)

Figure 1 The 2R robot having: a) Ideal (i.e., rigid) joints; b) Flexible joints; c) Joints with backlash.

For joints with backlash [18–20] (i.e. with gear clearance h_i at joint i), we have impact phenomena between the inertias which obey the principle of conservation of momentum according with the formulae:

$$\dot{q}'_i = \frac{\dot{q}_i(J_{ii} - \varepsilon J_{im}) + \dot{q}_{im} J_{im}(1+\varepsilon)}{J_{ii} + J_{im}} \tag{11a}$$

$$\dot{q}'_{im} = \frac{\dot{q}_i J_i(1+\varepsilon) + \dot{q}_{im}(J_{im} - \varepsilon J_{ii})}{J_{ii} + J_{im}} \tag{11b}$$

where $0 < \varepsilon < 1$ is the Newton constant that defines the elasticity of the impact ($\varepsilon = 0$ inelastic, $\varepsilon = 1$ elastic) and $\dot{q}'_i$ and $\dot{q}'_{im}$ are the velocities of the inertias of the joint and motor, respectively, after the collision.

In the experiments we assign numerical values for the parameters of the 2R robot (Tab. 1) identical to those adopted by Young [5] and Morgan and Özgüner [7]. In the same line of thought, the proposed VSC is tested for similar requirements and the manipulator is required to move from the initial state:

$$[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)]^T \equiv [-2.784, 0, -1.204, 0]^T. \tag{12}$$

to the final state:

$$[q_1(\infty), \dot{q}_1(\infty), q_2(\infty), \dot{q}_2(\infty)]^T \equiv [0, 0, 0, 0]^T \tag{13}$$

After a few experiments with the ideal 2R robot we set the controller parameters according with the numerical values presented in Table 2 (with $f = 2.10^3$ Hz). Figure 2 shows the (smooth) PP trajectories of the system.

Nevertheless, the SOM is redundant, in the sense that a FOM-VSC would be sufficient in the control of this manipulator. For comparison, Figure 3 shows the PP trajectories for a FOM-VSC where:

$$\zeta_i = \text{Dominant root}(\zeta_{1i}, \zeta_{2i}), \; i = 1, 2 \tag{14}$$

Table 1 Parameters of the 2R Robot.

	Rigid Joints				*Compliant joints*		*Joints with Backlash*	
i	m_i	r_i	J_{im}	J_{ig}	B_{im}	K_{im}	ε_i	h_i
1	0.5	1.0	1.0	4.0	100	$2 \cdot 10^4$	0.9	0.1
2	6.25	0.8	1.0	4.0	100	$2 \cdot 10^4$	0.9	0.1

Table 2 Controller Tuning.

	SOM-VSC					*FOM-VSC*			
i	ζ_{1i}	ζ_{2i}	K_i	D_i	δ_i	ζ_i	K_i	D_i	δ_i
1	0.26795	3.73205	0.1	100	100	0.26795	0.1	10^6	100
2	1.33975	18.6602	0.1	100	100	1.33975	0.1	10^6	100

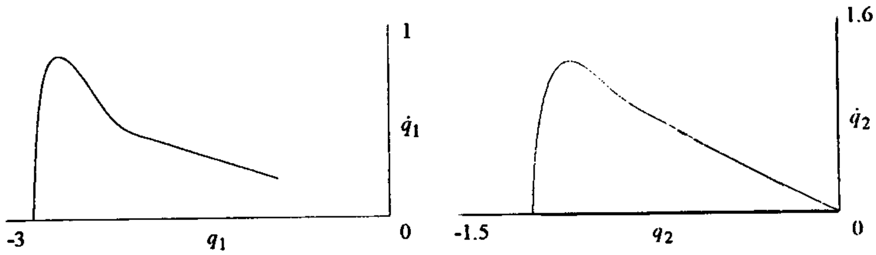


Figure 2 Phase plane trajectories for the ideal 2R robot with SOM-VSC.

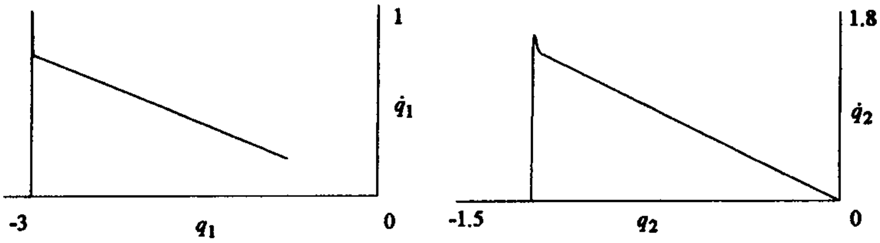


Figure 3 Phase plane trajectories for the ideal 2R robot with FOM-VSC.

and the gain is adjusted adequately (Tab. 2). In fact, the true value of the SOM only appears in the control of a manipulator with flexible joints. Figure 4 shows the PP trajectories of the SOM-VSC for this case, with the same controller tuning. However, the corresponding FOM-VSC is unstable and, furthermore, difficult to stabilise. For example, reducing the controller gain D_i/δ_i and setting $D_i = 10^4$, results in the (poor) performances depicted in Figure 5.

Repeating the experiments for a robot having backlash at the joints, with the numerical values presented at Table 2, we get the results depicted in Figures 6 and 7. It is clear that backlash phenomena present the strongest dynamic problems posed by the different experiments. In this case solely the SOM-VSC attains a reasonable performance. Therefore, we conclude that the SOM gives a larger stability margin than the FOM. This extra stability margin is not required in the control of an ideal robot. However, for the cases of a robot with joints having flexibility or backlash, the SOM-VSC is essential in order to have a stable and smooth response.

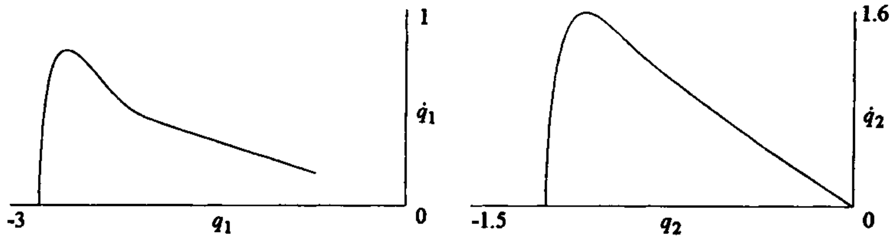


Figure 4 Phase plane trajectories for the compliant 2R robot with SOM-VSC.

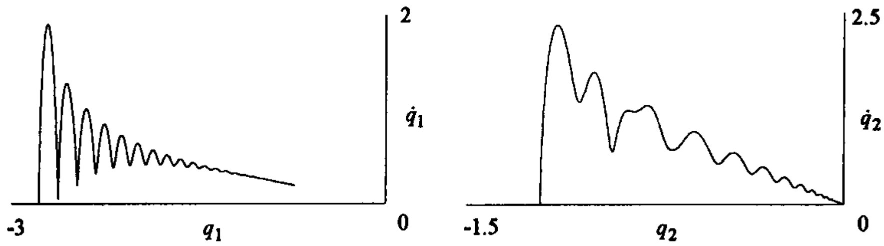


Figure 5 Phase plane trajectories for the compliant 2R robot with FOM-VSC.

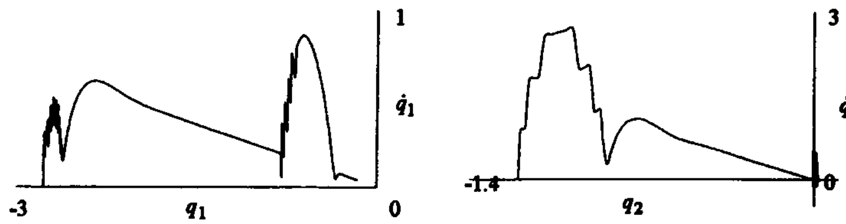


Figure 6 Phase plane trajectories for the 2R robot with joint backlash under SOM-VSC.

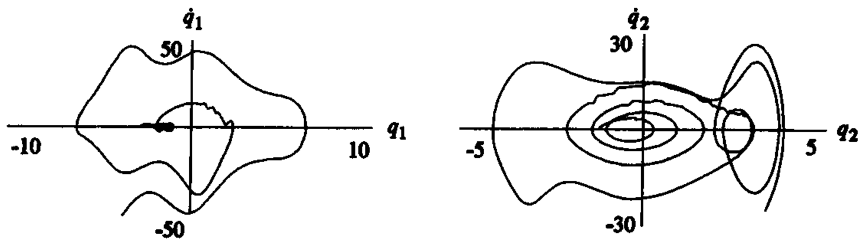


Figure 7 Phase plane trajectories for the 2R robot joint backlash under FOM-VSC.

4. CONCLUSIONS

This paper studied first and second order reference models for the VSC of manipulators with joints having flexibility and backlash. Second order models are not required for the control of robots having rigid transmissions. However, real mechanical manipulators have dynamic phenomena at the joints and its effect must be evaluated. In this case, first order models lead to poor performances due to the lack of stability and, consequently, second order models are a natural requirement. On the other hand, the proposed VSC corresponds to the integration of a decision equation, usual in VSS schemes, into a standard PI algorithm. Alternatively, the algorithm may be viewed, as a non-linear saturation-like controller that corrects "intelligently" the amplitude of the control action. This structure leads to smooth control actions that provides an high stability margin required by robots revealing complex dynamic phenomena.

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