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ON THE ANALYSIS AND DESIGN OF ROBOT MANIPULATORS: A STATISTICAL PERSPECTIVE

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Abstract A new approach to the analysis and design of robot manipulators is presented. The novel feature resides on a non-standard formulation to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, in general, require laborious computations. This motivates the need of alternative models based on other mathematical concepts. The proposed statistical method is a step in this direction which gives clear guidelines towards the robot kinematic and dynamic optimization. Furthermore, the proposed method points out new concepts well adapted to the study of biological-like muscle-actuated robotic arms.

Introduction

Mathematical modelling of physical phenomena is an essential step towards the study of the real world. Fundamental sciences like mathematics and physics provide a framework where basic concepts are formulated and general laws are derived. Applied sciences are embedded in such concepts and, therefore, they assume the use of mathematical strategies adapted to these laws. This blend of mathematical and physical considerations forms a model and, as such, it constitutes an appropriate tool to investigate system's behaviour. The study of robot manipulators is a typical example where we may find as fundamental sciences both the differential and matrix calculus, and the classical Newtonian physics, while the model corresponds to the standard kinematic and dynamic descriptions. The close relationship between the fundamental sciences and the system model is, by consequence, the suppot of a research strategy; the analysis of this methodology may provide further insight towards new modelling perspectives.

It is well known that, for example, (different) classes of physical phenomena may be treated either by means of classical physics, or quantum physics or thermodynamics. While in the first case differential calculus is the inherent mathematical tool, quantum physics requires the adoption of statistical concepts. On the other hand, thermodynamics may be studied both through classical or statistical concepts. These facts suggest that, for a given problem, we may develop different models, each with its own merits and drawbacks. Therefore, the classical approach to robot manipulators may be not the unique modelling perspective. Furthermore, if there are alternative modelling concepts one may ask to what extent they are fertile in new problem solving schemes. These considerations are further stressed when we consider not only the study of (pure) mechanical manipulators but also biological systems such as the human arm.

This paper presents a modelling strategy which is an alternative to the standard methods. The new model, based on statistical concepts, provides a complementary perspective of manipulator properties and highlights their influence on muscle-like actuated arms. In order to develop this new formulation the paper is organized as follows. In section 2 the new strategy is presented. Section 3 shows the application of this method to the kinematics and dynamics of mechanical joint-actuated manipulators, while, in section 4, we analyse the corresponding properties of muscle-actuated arms. Finally, in section 5 future developments of this work are discussed and some conclusions are drawn.

On the Statistical Modelling of Robot Manipulators

The classical modelling of robot manipulators is well known. For the kinematics a set of equations relating the joint space and the operational space, can be found to be of the form

$$q=\alpha(p) \qquad (1a)$$
$$\dot{q}=\beta(p,\dot{p}) \qquad (1b)$$
$$\ddot{q}=\Phi(p,\dot{p},\ddot{p}) \qquad (1c)$$

where p, $\dot{p}$ and $\ddot{p}$ (q, $\dot{q}$, and $\ddot{q}$) are the position, velocity and acceleration in the operational (joint) space. Associated with the kinematic model we have the statics model, that relates the operational space forces Γ with the joint actuator torques T:

$$T=J(q)^T\Gamma \qquad (2)$$

where $J(q)$ is the jacobian matrix corresponding to the differential relationship $\dot{p}=J(q)\dot{q}$. The dynamics is described by a nonlinear matrix differential equation

$$T=I(q)\ddot{q}+C(q,\dot{q})+G(q) \qquad (3)$$

where $T_I=I(q)\ddot{q}$, $T_C=C(q,\dot{q})$ and $T_G=G(q)$ are the n-dimensional vectors of the inertial, Coriolis/centripetal and gravitational torques.

Based on these equations considerable research has been done on issues such as manipulator structure optimization[1-8] and path planning algorithms[9,10]. However, a more sound consideration of the whole theme reveals that these methods are far from achieving a comprehensive formulation. This observation motivates the re-evaluation of the tools in use. In fact, expressions (1)-(3) show that the large number of parameters involved, gives rise to a cumbersome work either in a design or in an analysis stage. On the other hand, an alternative model, based on a statistical description of the manipulator system may prove to be more fertile on the generation of appropriate concepts and conclusions. If with such a strategy, we lose the "certainty" of the deterministic model, we gain a simpler and more intuitive viewpoint. This approach has already been used by other researchers[11,12] in some restricted classes of problems. In the sequel we refer to the new approach, as the statistical model[13,14] to stress the contrast with the standard method. Our modelling procedure comprises:

- The statistical description of a set of input variables, that is variables that are free to change independently.
- The statistical description of a set of output variables, that is, variables that are functions of the the previous ones.
- A set of parameters which are to be optimized in the design stage.

The above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is established as follows:

- $\{p, \dot{p}, \ddot{p}\}$ act as input variables of the kinematic system. This option enables a definition of the required kinematic performances on the operational space which are more natural to the designer.
- $\{q, \dot{q}, \ddot{q}\}$ act as output variables of the kinematic system, but play the role of input variables set in the dynamic model. In this way we arrive at a relationship between kinematics and dynamics in a form amenable to performance optimization criteria as defined in the sequel.
- The set of dynamic output variables consists of the required joint torques $\{T\}$.
- The parameter set consists of link lengths, masses and inertias.

In other words, we are stating that in the kinematics (dynamics), p, $\dot{p}$ and $\ddot{p}$ (q, $\dot{q}$ and $\ddot{q}$) are considered as independent random variables, its probability density functions (p.d.f.'s) being similar to the histograms of a long run sampling, while q, $\dot{q}$, $\ddot{q}$ (T) are the corresponding random dependent variables. The statistical description of the involved variables, does not consider the (implicit) time variable. In this way, variables that are related through the time derivative operator - $\{p, \dot{p}, \ddot{p}\}$ and $\{q, \dot{q}, \ddot{q}\}$ - are considered independent of each other.

Let us now adopt the 2R robot manipulator (Fig. 1) as the support for the development and implementation of the new modelling concepts. In the next sub-section we begin by introducing our approach in the kinematic case. In the second sub-section we shall analyse the dynamic case and in the third sub-section we investigate the properties of the overall (i.e. kinematics + dynamics) system.

A Statistical Model for the Kinematics of the 2R Joint-Actuated Manipulator

The set of kinematic input variables consists of position, velocity and acceleration

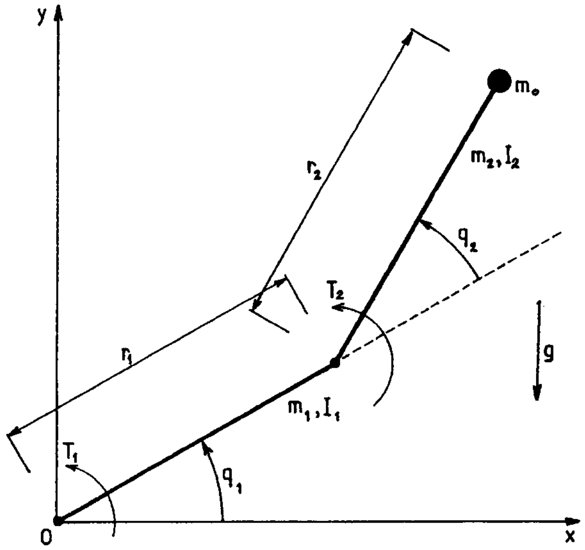

Fig. 1 The 2R joint-actuated robot manipulator

that our prototype manipulator is required to perform in the operational space. Therefore, it is necessary to characterize them in statistical terms, namely by defining appropriate p.d.f.'s for each variable. As there is no a priori knowledge about the typical behaviour we start with some reasonable assumptions namely, for the position variable $p=[x,y]^T$ we consider a bidimensional uniform p.d.f.

$$f_P(p)=\begin{cases} C & \text{if } (r_1-r_2)^2 \le x^2+y^2 \le (r_1+r_2)^2 \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

with $C=1/\{\pi[(r_1+r_2)^2-(r_1-r_2)^2]\}$.

In the sequel we will see how to modify the input p.d.f. in order that the kinematic performances are optimized. It is also necessary to define the p.d.f.'s for velocity and acceleration. By the same above arguments, we decided to use bidimensional Gaussian p.d.f.'s with zero mean

$$f_{\dot{P}}(\dot{p})=1/(2\pi\sigma_{\dot{P}}^2)*\exp[-(\dot{x}^2+\dot{y}^2)/(2\sigma_{\dot{P}}^2)] \qquad (5)$$
$$f_{\ddot{P}}(\ddot{p})=1/(2\pi\sigma_{\ddot{P}}^2)*\exp[-(\ddot{x}^2+\ddot{y}^2)/(2\sigma_{\ddot{P}}^2)] \qquad (6)$$

Moreover, using these p.d.f.'s we impose some useful properties, such as:

- The random variables position, velocity and acceleration in the operational space are independent of each other.
- The velocity and acceleration vectors are made of two independent components, that is $\dot{x}$ ($\ddot{x}$) is independent of $\dot{y}$ ($\ddot{y}$).

The "excitation" of the (inverse) kinematic system produces output random variables q, $\dot{q}$ and $\ddot{q}$, with p.d.f.'s which are related to the previous ones by:

$$f_Q(q)=J_P f_P(p) \qquad (7a)$$
$$f_{Q\dot{Q}}(q,\dot{q})=J_V f_{P\dot{P}}(p,\dot{p}) \qquad (7b)$$
$$f_{Q\dot{Q}\ddot{Q}}(q,\dot{q},\ddot{q})=J_A f_{P\dot{P}\ddot{P}}(p,\dot{p},\ddot{p}) \qquad (7c)$$

where the jacobians J_P, J_V, J_A are

$$J_P = \frac{\partial(p)}{\partial(q)} = r_1 r_2 S_2 \quad (8a)$$

$$J_V = \frac{\partial(p,\dot{p})}{\partial(q,\dot{q})} = J_P(r_1 r_2 S_2) \quad (8b)$$

$$J_A = \frac{\partial(p,\dot{p},\ddot{p})}{\partial(q,\dot{q},\ddot{q})} = J_V(r_1 r_2 S_2) \quad (8c)$$

Each of the expressions (7) is made of two distinct factors:

- Weighting factors - J_P, J_V and J_A - which depend solely on the system kinematic properties
- The "excitation" p.d.f.'s - $f_P(p)$, $f_{P\dot{P}}(p,\dot{p})$ and $f_{P\dot{P}\ddot{P}}(p,\dot{p},\ddot{p})$ - which are a measure of the task requirements.

These factors can be interpreted in a system theoretic framework. The jacobians characterize the system intrinsic properties, while the excitation p.d.f.'s correspond to the system response to the input variables.

Bearing these facts in mind, several experiments were performed, having:

- The total link length constant, L=0.6.
- Several ratios $\mu=r_1/r_2$, namely 0.4, 0.6, 0.8, 1, 1.2, 1.4 and 1.6.
- Operational space categories corresponding to nine distinct requirements:

1. $\sigma_{\dot{P}}=0.1$, $\sigma_{\ddot{P}}=0.1$	6. $\sigma_{\dot{P}}=1$, $\sigma_{\ddot{P}}=10$
2. $\sigma_{\dot{P}}=0.1$, $\sigma_{\ddot{P}}=1$	7. $\sigma_{\dot{P}}=10$, $\sigma_{\ddot{P}}=0.1$
3. $\sigma_{\dot{P}}=0.1$, $\sigma_{\ddot{P}}=10$	8. $\sigma_{\dot{P}}=10$, $\sigma_{\ddot{P}}=1$
4. $\sigma_{\dot{P}}=1$, $\sigma_{\ddot{P}}=0.1$	9. $\sigma_{\dot{P}}=10$, $\sigma_{\ddot{P}}=10$
5. $\sigma_{\dot{P}}=1$, $\sigma_{\ddot{P}}=1$	

- Excitation of the kinematic system with a numerical random sample of 4000 operational space variables obeying the p.d.f.'s (4)-(6).
- Analysis of the resulting histograms of the output variables amplitude. In order to simplify matters, only marginal p.d.f.'s were considered.

After a large number of experiments using the numerical set of parameters depicted in Table 1, we concluded that the shape of the resulting p.d.f.'s varied significantly from variable to variable, but all of them showed symmetry around zero. For this reason, and in order to characterize the resulting histograms by a scalar index, we decided to adopt for this index the upper limit of the integral which corresponds to 95% of probability. The resulting histograms are condensed through this index and depicted in Fig. 2. We can observe in the majority of the charts a minimum about $\mu=1$; nevertheless, this conclusion can be easily inferred from (7). In fact, for symmetrical histograms about zero on the x-axis, a larger value of the jacobian corresponds to a smaller dispersion of the random variable, which in turn means average smaller amplitude requirements posed to that variable. Therefore, we found an algorithm, based on statistical modelling concepts, that leads naturally to an optimization criterion.

As the maximization of J_P, J_V and J_A requires the same steps, we have for:

$$r_1+r_2=L,\ r_1/r_2=\mu \quad (9)$$

that a maximum occurs when

TABLE 1 Numerical values of the 2R robot

$R_1=0.05$ m, $R_2=0.0389$ m
$m_1=2.16$ kg, $m_2=1.68$ kg, $m_0=0$ kg
$r_i=0.3$ m, $I_i=m_i(r_i^2/12+R_i^2/4)$, i=1,2

$$\mu=1,\ q_2=\pi/2 \quad (10)$$

which coincide with the results obtained (using the classical approach) in previous studies[1,2]. Furthermore, our optimization criteria enables additional conclusions and procedures:

- Because J_P, J_V and J_A are consecutive powers of $r_1r_2S_2$, we see that for a given deviation from the optimal values (10) we have an increasingly degradation of the cost function with the powers of $r_1r_2S_2$. In other words this means that for a given deviation, we have, by increasing order of sensitivity, position, velocity and acceleration.
- Due to (2) a kinematic optimization is equivalent to a static optimization.
- If further optimization is desired, then the next step will be the selection of an optimum "excitation" p.d.f.. This second step of optimization will define, in a statistical sense, an optimum kinematic class for the manipulator trajectories. Obviously, we can find a multitude of different p.d.f's obeying (10); nevertheless, for the subsequent study a particular choice is of minor importance. Consequently we decided to adopt the following family of position p.d.f.'s in the operational space (with $K\geq 1$):

$$f_P(x,y)=\text{constant}*\{1-[(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2)]^2\}^{(K-1)/2} \quad (11)$$

which, in the joint space, corresponds to:

$$f_Q(q_1,q_2)=\text{constant}*S_2^K \quad (12)$$

As extreme cases, we have that for K=1 it becomes the uniform p.d.f. (4), while for $K\to\infty$ we get Dirac type p.d.f. ($\delta(.)$) optimum in the sense of (10):

$$f_P(x,y)=\delta[x^2+y^2-(r_1^2+r_2^2)] \quad (13a)$$

$$f_Q(q_1,q_2)=1/2[\delta(q_2+\pi/2)+\delta(q_2-\pi/2)] \quad (13b)$$

As far as velocity and acceleration are concerned we can see that the kinematic study does not point out any special class of p.d.f.'s. Nevertheless, these variables are affected negatively by the position deviation from the optimum configuration $q_2=\pi/2$. Therefore, we decided to study the system behaviour both for performance requirements described by p.d.f.'s (12), (5) and (6) and for the alternative situation corresponding to p.d.f. (12) associated with the "enhanced" q_2-dependent velocity and acceleration p.d.f.'s:

$$f_{\dot{P}}(\dot{p},q_2)=\exp\{-(\dot{x}^2+\dot{y}^2)/[2\sigma_{\dot{P}}^2(q_2)]\}/[2\pi\sigma_{\dot{P}}^2(q_2)] \quad (14a)$$

$$\sigma_{\dot{P}}(q_2)=\begin{cases}2\sigma_{\dot{P}}|q_2|/\pi & \text{if } 0<|q_2|\leq\pi/2\\ 2\sigma_{\dot{P}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\leq\pi\end{cases} \quad (14b)$$

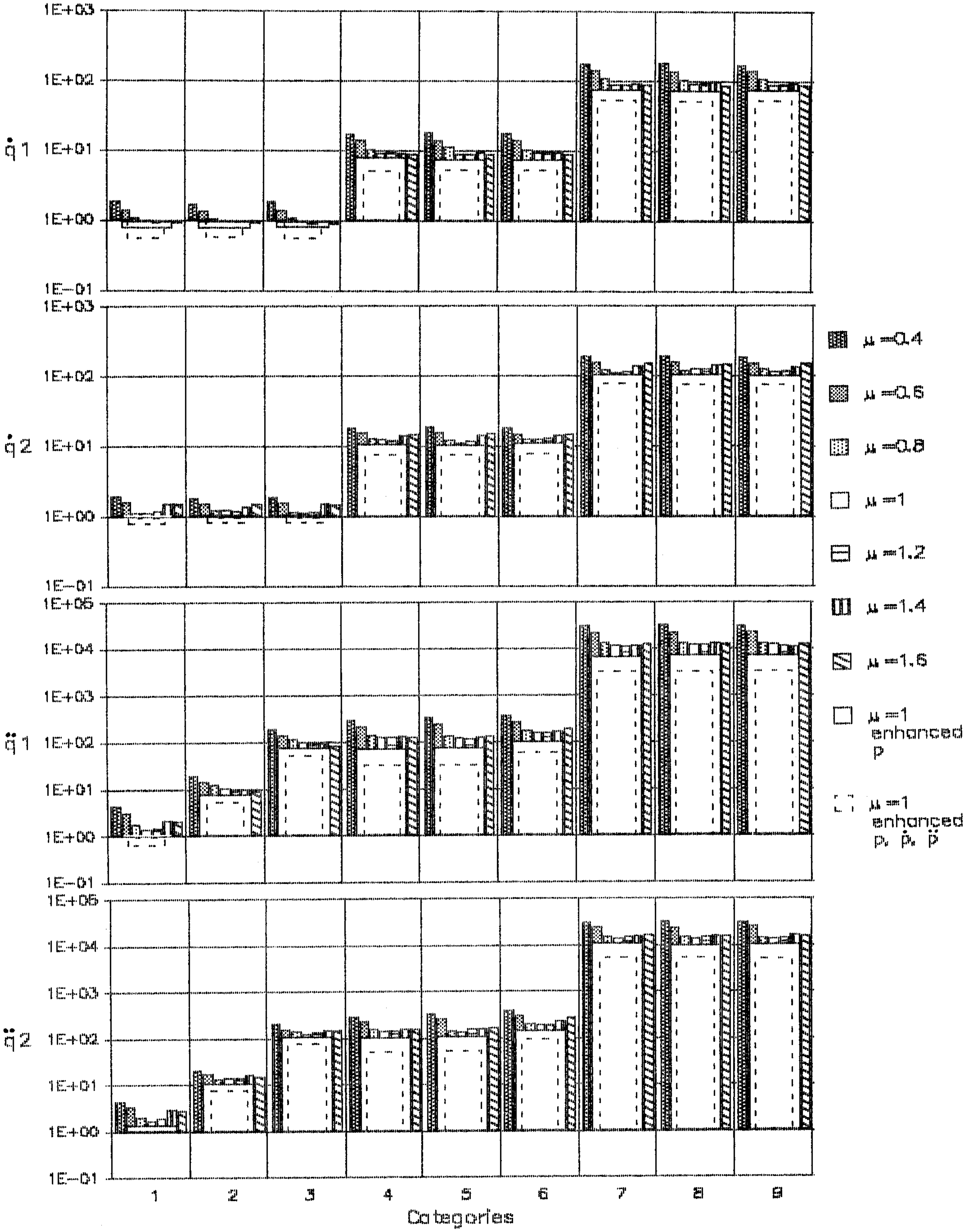

Fig. 2 Comparison chart for the 2R joint-actuated robot kinematic performances. The narrow columns correspond to seven geometric configurations "excited" with p.d.f.'s (4), (5) and (6). The wider columns correspond to the optimum geometric μ=1 "excited" with the enhanced p.d.f. (12) and (5)-(6) for the solid borders and enhanced p.d.f.'s (12), (14) and (15) for the dotted borders.

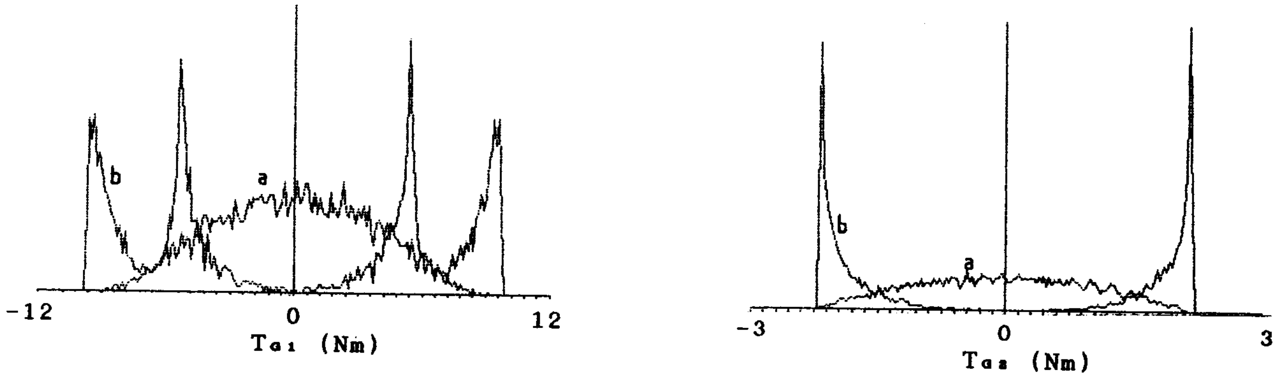

Fig. 3 Histograms of the gravitational torques T_G for "excitation" p.d.f.'s:
a. $f_q(q1,q2)=constant*(S_1S_{12})^2$, b. $f_q(q_1,q_2)=constant*(C_1C_{12})^2$.

$$f_{\ddot p}(\ddot p,q_2)=\exp[-(\ddot x^2+\ddot y^2)/[2\sigma_{\ddot p}^2(q_2)]\}/[2\pi\sigma_{\ddot p}^2(q_2)] \tag{15a}$$

$$\sigma_{\ddot p}(q_2)=\begin{cases} 2\sigma_{\ddot p}|q_2|/\pi & \text{if } 0<|q_2|\le\pi/2 \\ 2\sigma_{\ddot p}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\le\pi \end{cases} \tag{15b}$$

To test numerically the above conjectures, the previous results for $\mu=1$ are compared with a new case using $\mu=1$ and K=3 in (11)-(12). This has revealed a remarkable performance improvement as shown in Fig. 2, particularly for velocity-dependent requirements.

A Statistical Model for the Dynamics of the 2R Joint-Actuated Manipulator

The statistical description of the dynamics requires steps similar to those adopted in the kinematics, namely:

- Characterisation of the input variables ($q,\dot q$ and $\ddot q$) through appropriate p.d.f.'s.
- "Stimulation" of the system behaviour through numerical experiments.
- Analysis of the histograms of the output variables (T).

However, a preliminary observation shows that the dynamic study is much more complex than the kinematic one. Due to this, and in order to gain a deeper insight for the subsequent study we decided to consider, in a first stage, as dynamic output variables, the components of the joint torques, that is the gravitational, Coriolis/centripetal and inertial torques. Based on this preliminary analysis then, in a second stage, we consider the total joint torques. In the first stage we have:

$$f_G(q)=J_Gf_q(q) \tag{16a}$$
$$f_C(q,\dot q)=J_Cf_{q\dot q}(q,\dot q) \tag{16b}$$
$$f_I(q,\dot q,\ddot q)=J_If_{q\dot q\ddot q}(q,\dot q,\ddot q) \tag{16c}$$

where

and $f_G(q)$, $f_C(q,\dot q)$, $f_I(q,\dot q,\ddot q)$ represent the p.d.f.'s of the gravitational, Coriolis/centripetal and inertial torques, respectively. Unlike the kinematic situation, where the optimization was similar for all the jacobians, now their effects differ according to each dynamic term. Analysing the jacobians (17) we conclude that:

- The maximizing of J_G stipulates that q_1 and q_{12} should have p.d.f.'s with maxima at 0 or π. The observation of histograms resulting from "excitation" p.d.f.'s obeying these conditions showed an interesting result. As expected the (symmetrical) histograms resembled Dirac pulses; however, those peaks were located at non-zero values. In fact, the plots showed sharp symmetrical peaks located at the maxima (positive and negative) values attained by the gravitational torques. This means that, for this case, the optimization procedure must adopt an inverse strategy, that is to say we must minimize J_G (Fig. 3).
- The maximizing of J_C implies that q_2 must have a p.d.f. with a maximum on 0 or π. Numerical experiments showed that in this case the resulting histograms of the Coriolis/centripetal terms tended, as desired, towards a Dirac on zero.
- The analytical expression of J_I is more complex. Nevertheless, its analysis revealed a maximizing condition similar to the previous one (i.e. q_2 should have a p.d.f. with maxima at 0 or π).
- J_G defines a "rest region" while J_C and J_I define an "active region".

These partial results were confirmed experimentaly[14] and, therefore, we can proceed to the second stage, that is, the study of the (total) dynamics. The direct application of our optimizing method to the dynamics would require the mathematical and numerical treatment of 3n-dimensional p.d.f.'s. In order to avoid this intricate analysis, we decided to integrate the

$$J_G=\frac{\partial(q)}{\partial(T_G)}=[(m_1/2+m_2+m_0)(m_2/2+m_0)g^2r_1r_2S_1S_{12}]^{-1} \tag{17a}$$

$$J_C=\frac{\partial(q,\dot q)}{\partial(T_G,T_C)}=J_G\{[2(m_2/2+m_0)r_1r_2S_2]^2\dot q_1\dot q_{12}\}^{-1} \tag{17b}$$

$$J_I=\frac{\partial(q,\dot q,\ddot q)}{\partial(T_G,T_C,T_I)}=J_C\{[(m_1/4+m_2+m_0)r_1^2+I_1+(m_2/2+m_0)r_1r_2C_2][(m_2/4+m_0)r_2^2+I_2]-[(m_2/4+m_0)r_2^2+I_2+(m_2/2+m_0)r_1r_2C_2](m_2/2+m_0)r_1r_2C_2\}^{-1} \tag{17c}$$

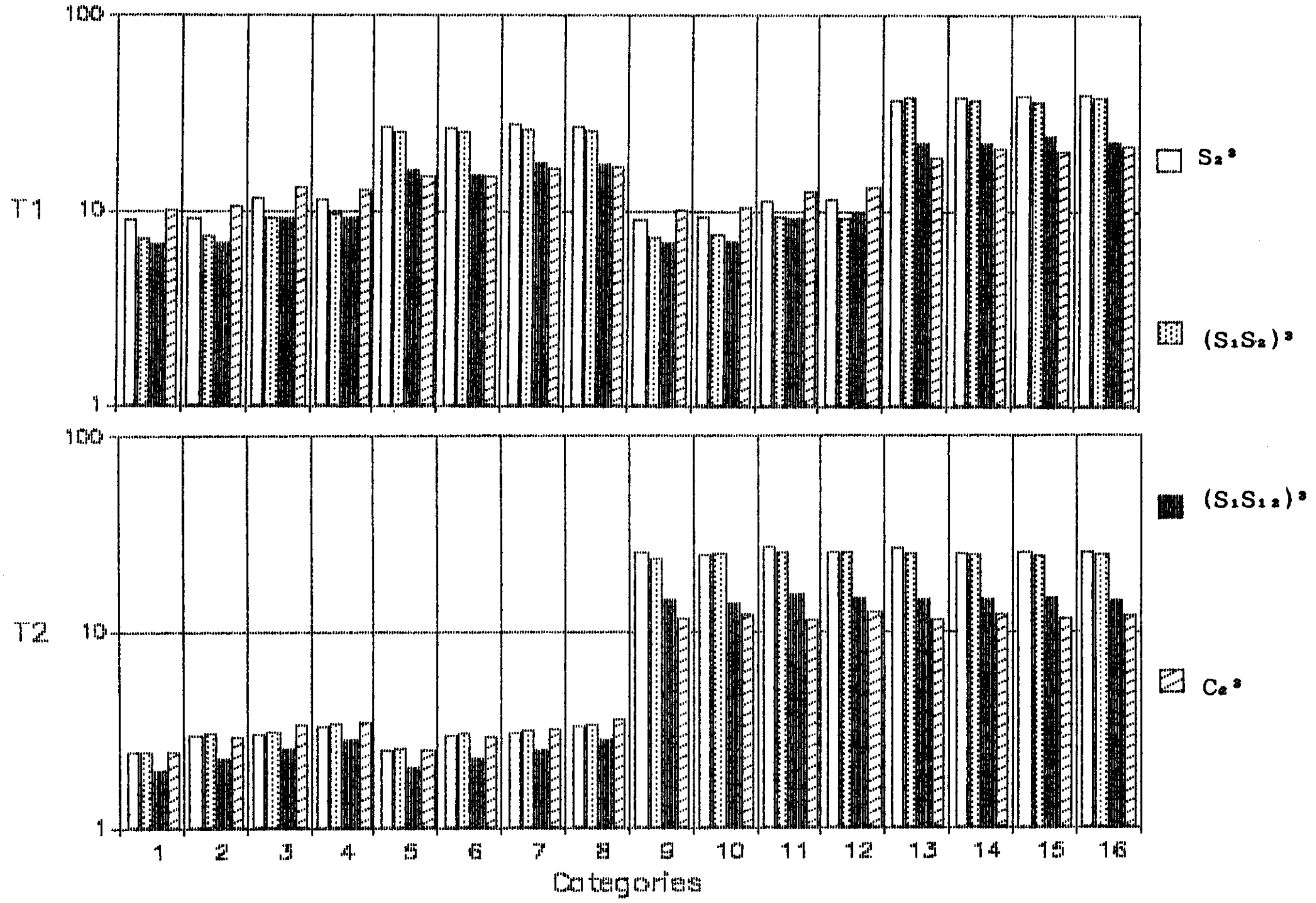

Fig. 4 Comparison charts for the 2R joint-actuated robot dynamic performances with $\mu=1$ when subjected to "excitation" p.d.f.'s:
1st column: $f_Q(q_1,q_2)=\text{constant}*S_2^3$; 2nd column: $f_Q(q_1,q_2)=\text{constant}*(S_1S_2)^3$
3rd column: $f_Q(q_1,q_2)=\text{constant}*(S_1S_{12})^3$; 4th column: $f_Q(q_1,q_2)=\text{constant}*C_2^3$

(partial) conclusions pointed out in the first stage (i.e. the guidelines resulting from the study of T_a, T_c and T_I) on the formulation of our present investigation. Furthermore, this approach will show that the new statistical concepts give considerable freedom on the research strategy and, consequently, are a robust modelling perspective. In this sense we decided to "excite" the dynamics with four different position p.d.f.'s:

$$f_Q(q_1,q_2)=\text{constant}*S_2^x \quad (18)$$
$$f_Q(q_1,q_2)=\text{constant}*(S_1S_2)^x \quad (19)$$
$$f_Q(q_1,q_2)=\text{constant}*(S_1S_{12})^x \quad (20)$$
$$f_Q(q_1,q_2)=\text{constant}*C_2^x \quad (21)$$

which are suggested by the optimization of the kinematics, a compromise between kinematics and gravitational torques, the gravitational torques, and the Coriolis/centripetal and inertial torques, respectively. Due to the non-existence of optimization guidelines on $\dot{q}$ and $\ddot{q}$, we decided to consider two alternative "excitation" p.d.f.'s (i=1,2):

$$f_{\dot{q}_i}(\dot{q}_i)=\exp[-\dot{q}_i^2/(2\sigma_{\dot{q}_i}^2)]/(2\pi\sigma_{\dot{q}_i}^2) \quad (22)$$
$$f_{\ddot{q}_i}(\ddot{q}_i)=\exp[-\ddot{q}_i^2/(2\sigma_{\ddot{q}_i}^2)]/(2\pi\sigma_{\ddot{q}_i}^2) \quad (23)$$

and the following sixteen different categories:

1. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=0.1$
2. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=10$
3. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=0.1$
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6. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=10$
7. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=0.1$
8. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=10$
9. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=0.1$
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15. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=0.1$
16. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=10$

Figure 4 depicts the results of T_1 and T_2 when the 95% index is applied to the corresponding histograms. These charts revealed several important properties:

- T_1 (T_2) depends strongly on $\dot{q}_2$ ($\dot{q}_1$).
- The joint torques (T) have low sensitivity, in a statistical sense, to acceleration ($\ddot{q}$) requirements.

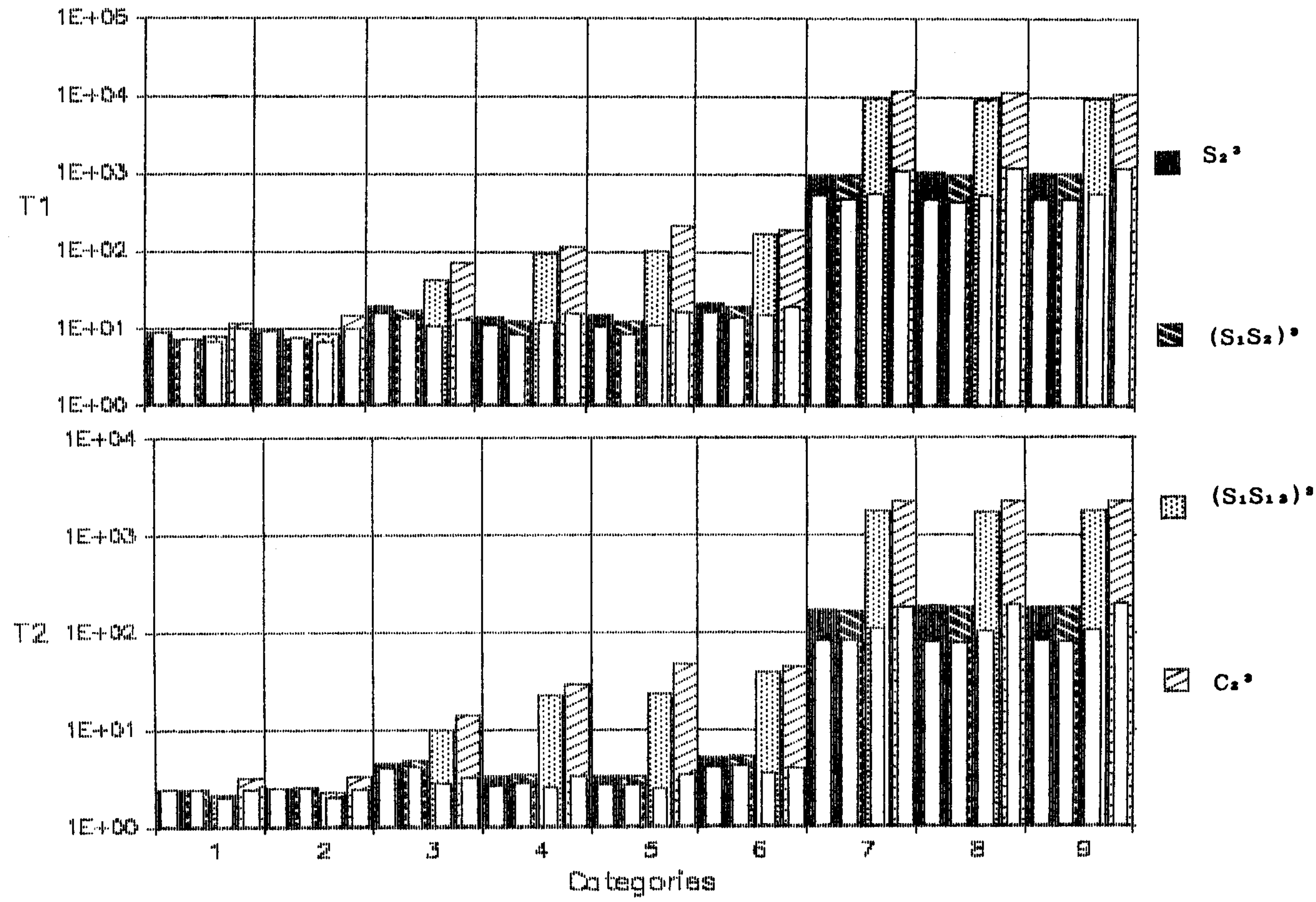

Fig. 5 Comparison chart for the 2R joint-actuated robot overall (kinematic + dynamic) performances with $\mu=1$ when subjected to "excitation" p.d.f.'s:
1st column: $f_Q(q_1,q_2)=constant*S_2^*$; 2nd column: $f_Q(q_1,q_2)=constant*(S_1S_2)^*$
3rd column: $f_Q(q_1,q_2)=constant*(S_1S_{12})^*$; 4th column: $f_Q(q_1,q_2)=constant*C_2^*$
The back columns correspond to p.d.f.'s (5)-(6) and the front white columns correspond to the enhanced p.d.f.'s (14)-(15).

• The suggestions pointed out by the first stage are compatible with the new results. In fact, for "rest" (or "non-active") requirements, p.d.f. (20) is the more appropriate, while for the "active" (or "non-rest") situation p.d.f. (21) is the optimal.

The Statistical Description of the Total System

Up to now we discussed the kinematics and dynamics separately. However, in the real manipulator these systems can not be divided. In other words, the study of a robot manipulator, must consider both systems simultaneously. Therefore, the statistical description of the total system will have cross-coupling effects and its influence must be evaluated. To test these effects both the kinematics and dynamics were numerically "excited" through random samples according to p.d.f.'s (18)-(21). These position p.d.f.'s combined with the two alternative velocity and acceleration p.d.f.'s (5)-(6) or (14)-(15) (more precisely we are using their equivalent p.d.f.'s defined on the operational space), reveal (Fig. 5) that:

• For low velocity/acceleration requirements (category 1), the 95% index gives almost similar results for all p.d.f.'s, because the gravitational torques predominate.
• Velocity requirements ($\dot{p}$) have a much stronger influence than acceleration requirements ($\ddot{p}$).
• Kinematic effects prevail over the dynamic ones and, therefore, the best results come from p.d.f.'s (18) and (19).

The Statistical Model of the 2R Muscle-Actuated Arm

As shown in the previous section, a mechanical manipulator - which is a mimic of the human arm - is very sensitive to velocity requirements. On the other hand, acceleration requirements are less stringent. These facts indicate that we are dealing with "position/ acceleration machines" and not "velocity machines". Although obvious, this aspect has been somewhat overlooked. Moreover, it points out that the usual robot actuators, which are developments of standard "velocity machines" are not well adapted to

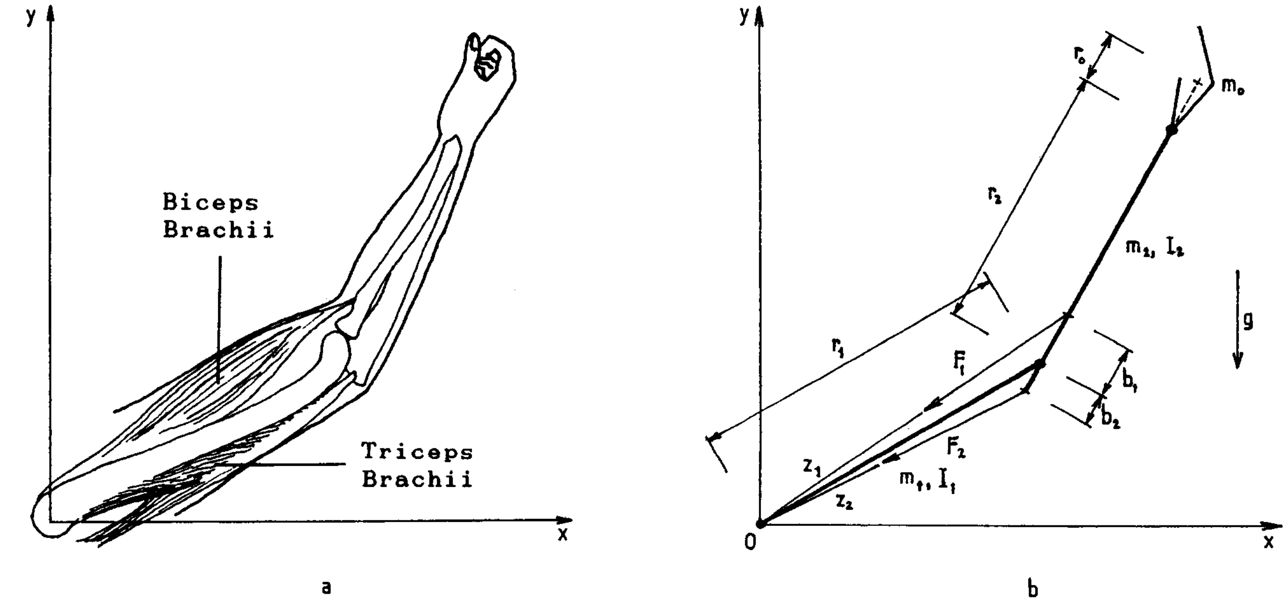

Fig. 6 The human arm
a) Biological aspect.
b) Engineering characterisation.

robotic applications. Alternative solutions, such as muscle like "position/acceleration" actuators[15,16] will allow more efficient robot structures. This section shows the benefits of using muscle actuator structures over joint actuator ones. The first sub-section introduces some base biological concepts associated with its simplified engineering formulation. The second sub-section treats one biological-like manipulator structure through the statistical model.

Some Human Arm Characteristics and its Engineering Formulation

The human arm may be considered as the optimal manipulator and, therefore, it constitutes our reference. Extensive biological studies[17-21] have been carried out on this subject. Unfortunately, precise conclusions on all of the involved phenomena are still lacking. Due to this reasons we will focus our attention mainly on the human elbow, retaining the shoulder investigation[22] to future work. Biological studies show that the elbow movement involves a plethora of arm muscles[23]. Nevertheless, biceps brachii and triceps brachii are the more influential[24], consequently, in order to simplify matters, they will be the only muscles under consideration. Figure 6 shows a biological picture of these muscles, as well as its engineering characterisation. The kinematics and dynamics of the new structure are related to (1) and (3) through the formulae (i=1,2)

$$z_i=(r_1^2+b_i^2+2r_1b_iC_2)^{1/2} \qquad (24)$$
$$\dot{z}_i=-r_1b_iS_2\dot{q}_2/z_i \qquad (25)$$
$$\ddot{z}_i=-r_1b_i[C_2+r_1b_i(S_2\dot{q}_2/z_i)^2+S_2\ddot{q}_2] \qquad (26)$$
$$F_i=(r_1^2+b_i^2+2r_1b_iC_2)^{1/2}T_2/(r_1b_iS_2) \qquad (27)$$

TABLE 2 Numerical characteristics of the human arm

R_1=0.05 m, R_2=0.0389 m, b_1=0.05m, b_2=-0.02 m
m_1=2.16 kg, m_2=1.2 kg, m_0=0.48 kg
r_1=0.3 m, r_2=0.25 m, r_0=0.05 m
I_1=0.01755 kgm^2, I_2=0.0067 kgm^2, I_0=0.00028 kgm^2

These expressions indicate that we may describe the system "muscle-space" which is an alternative to the standard "operational space" and "joint space". Moreover, the analysis of (25) reveals that the $1/S_2$ degrading factor, that affects q_2, is now compensated.

Table 2 depicts "average" data[17,18] that will be used in the sequel. One should note that this data may vary significantly with several factors and, therefore, is better adapted to a statistical treatment.

On the Statistical Performances of Muscle-Actuated Arms

In this sub-section we repeat the kinematics and kinematics + dynamics studies for the new manipulator structure. For the sake of completness we decided to limit the range of movement to

$$-\pi/2\leq q_1\leq\pi/2 \text{ and } 0\leq q_2\leq\pi \qquad (28)$$

although the human arm can reach a somewhat larger area in the sagittal plane. Figures 7 and 8 show two typical charts of the 95% index results. We conclude that:

- As expected, requirements have small kinematic effects in the muscle actuators.
- In a statistical sense, q_2 - optimized velo-

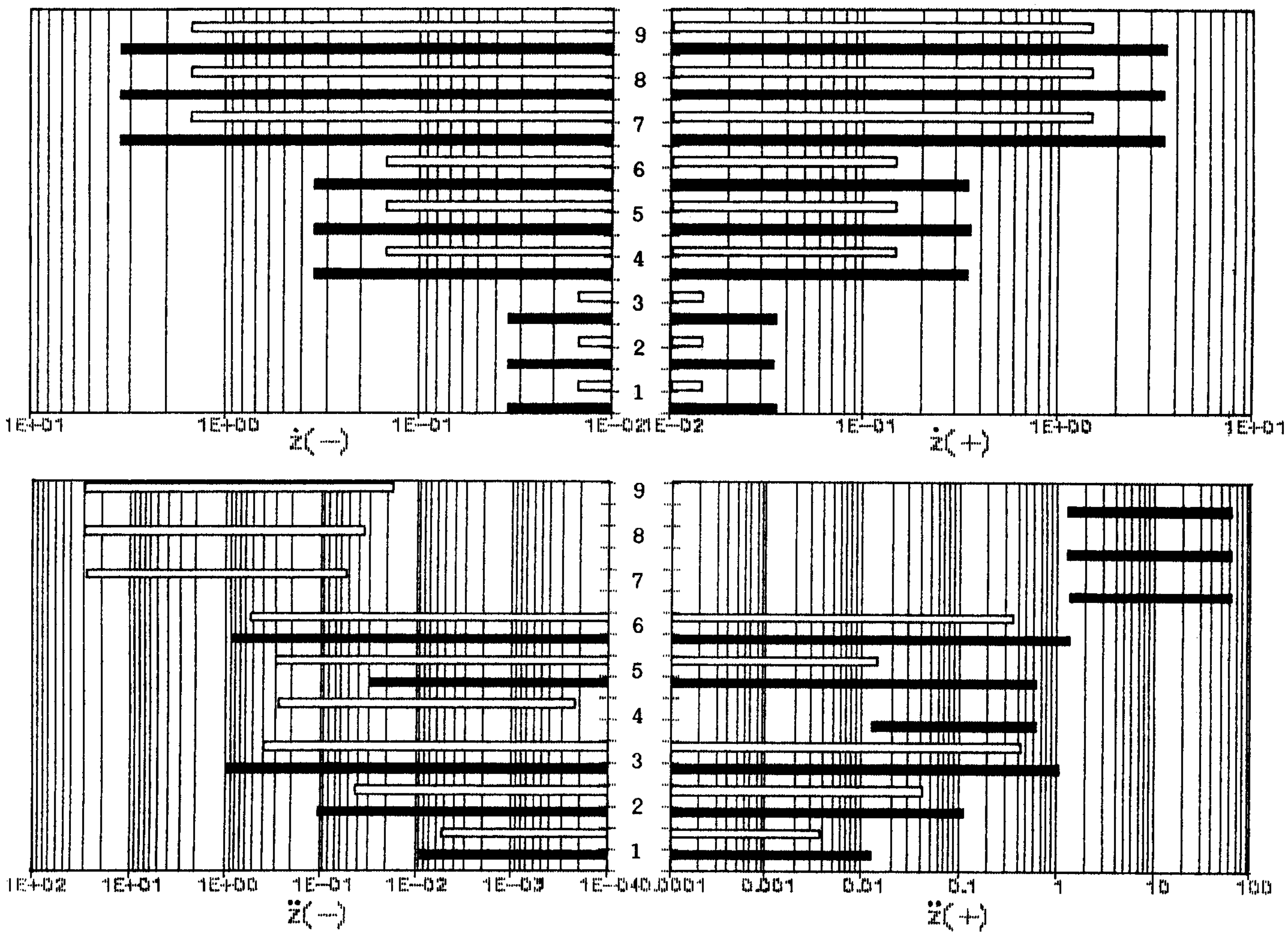

Fig. 7 Comparison charts for the 2R muscle-actuated arm kinematic performances when "excited" with p.d.f.'s (19) and (14)-(15) for the categories:

1. $\sigma_{\dot{p}}=0.1$, $\sigma_{\ddot{p}}=0.1$; 4. $\sigma_{\dot{p}}=1$, $\sigma_{\ddot{p}}=0.1$; 7. $\sigma_{\dot{p}}=10$, $\sigma_{\ddot{p}}=0.1$
2. $\sigma_{\dot{p}}=0.1$, $\sigma_{\ddot{p}}=1$; 5. $\sigma_{\dot{p}}=1$, $\sigma_{\ddot{p}}=1$; 8. $\sigma_{\dot{p}}=10$, $\sigma_{\ddot{p}}=1$
3. $\sigma_{\dot{p}}=0.1$, $\sigma_{\ddot{p}}=10$; 6. $\sigma_{\dot{p}}=1$, $\sigma_{\ddot{p}}=10$; 9. $\sigma_{\dot{p}}=10$, $\sigma_{\ddot{p}}=10$

■ biceps, □ triceps

city and acceleration p.d.f.'s (14)-(15) produce better results.

• The "amplification" between operational requirements $\dot{p}(\ddot{p})$ and $\dot{z}(\ddot{z})$ is much smaller than previously (i.e. between $\dot{p}(\ddot{p})$ and $\dot{q}(\ddot{q})$).

• Muscles may be required to develop larger forces.

• The muscle forces are dominated by gravitational effects for low and medium velocity requirements. For high velocities forces become higher.

• Acceleration requirements have no effect upon the muscle forces.

Conclusions

A new approach to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, on the other hand, can be very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The proposed statistical scheme is a step in that direction which has been shown to present interesting properties and to enable new procedures. It provides a framework giving clear guidelines towards the robot structure optimization. As a result, the manipulator design procedure, both kinematic and dynamic, leads to simple and intuitive conclusions. Furthermore, it should be highlighted the results pointing out some characteristics of the trajectory planning block, defining optimal rest and active regions, and ideal-actuator properties, as "position/acceleration" devices instead of "velocity" machines. This observation is of utmost importance as it gives a clear basis to the design of new mechanical robot manipulator structures, with performances close to the muscle-actuated biological systems.

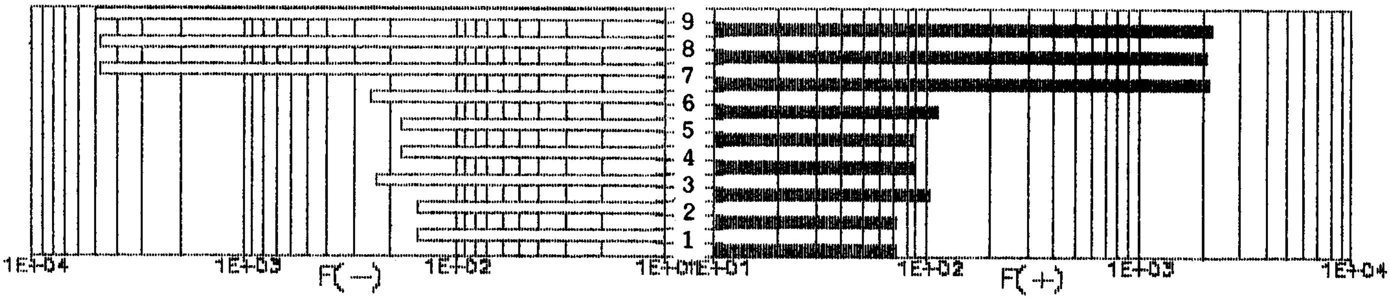

Fig. 8 Comparison charts for the 2R muscle-actuated arm dynamic performances when "excited" with p.d.f.'s (19) and (14)-(15) for the nine categories under study.
■ biceps, □ triceps

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