IO2019

XX Congresso da Associação Portuguesa
de Investigação Operacional

22nd – 24th July 2019, Tomar, Portugal

Sectorization for managing maintenance technicians

Cristina Lopes
CEOS.PP / ISCAP / P.PORTO - Portugal
(joint work with Ana Maria Rodrigues, Cristina Oliveira,
José Soeiro Ferreira, Maria João Cortinhal)

Sectorization = Cut the map into small pieces?

Sectorize…

- **…** to group small geographical areas into larger regions.
- …to divide a large territory in smaller elements, also known as **sectors**, **districts** or responsibility areas.

Sectorize…

- **…** to group small geographical areas into larger regions
- …to divide a large territory in smaller elements, also known as **sectors**, **districts** or responsibility areas
- according to a relevant amount of **criteria** and **constraints**.

Sectorize…

- This division usually aims at:
- better organizing,
- or simplifying a large problem into smaller sub-problems,
- or promoting groups with similar characteristics

Applications

Managing municipal waste collection

Assigning neighborhoods to schools

Defining sales territories

Designing political districts

Locating health care services

Electrical power distribution

Forestry and Harvesting

Sectorization problems

- **Basic areas**: points, lines, or small areas (represented by the coordinates of its center)
- **Activity measures**: number of inhabitants, workload, sales potential, frequency of visits, service times
- **Number of sectors**: p (usually fixed in advance by sales force size)
- ***Objective***: Define a **partition** of the set of basic areas into a number of p sectors which satisfy the specified planning **criteria** like balance, compactness and contiguity.

Sectorization Criteria

Criteria

Balance

- sectors should be identical portions of the whole regarding activity measure

Compactness

- regular forms like circles are preferred, avoiding “tentacle” shapes

Contiguity

- avoid sectors divided into portions

- Depending on the application, other criteria can be considered: **Accessibility, Capacity, Boundaries, Desirability …**
- These criteria are often weighted differently by decision makers, and therefore multicriteria approaches should be used.

How to measure the criteria?

Rodrigues, Ferreira (2015) *Measures in Sectorization Problems*

Measures

- Balance
- Compactness
- Contiguity

Balance

sectors should be identical portions of the whole regarding some activity measure

- Evenly distribuition of population, workload, or travel times among service staff
- Fairness of potential profit
- Bounds for size of sectors: max travel times, min number of customers

coefficient of variation: standard deviation of total activity in sector divided by average activity

$$CV_q = \frac{s'_q}{\overline{q}}$$

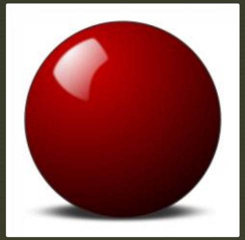
Measures

- Balance
- Compactness
- Contiguity

How to measure the criteria?

Rodrigues, Ferreira (2015) *Measures in Sectorization Problems*

Compactness



Rounded shapes, avoiding *boomerangs*.

- Geographically concentrated activity: less travel, more sales or service time
- Higher concentration should avoid sparse sectors

Concentration of each sector can be computed as a total activity in the sector weighted by the maximum distance between basic areas and district centres

$$d_i = \frac{\sum_j q_{ij}}{dist(o_i, p_i)}$$

qij = quantity assigned to the point *j* in sector *i*
dist(oi,pi) = distance (Euclidean) between the centroid of sector *i* and the farthest point on the same sector.

How to measure the criteria?

Rodrigues, Ferreira (2015) *Measures in Sectorization Problems*

Measures

- Balance
- Compactness
- Contiguity

Contiguity (weak)

If there is a walk between any pair of elements of the sector without leaving it

- Each sector is formed by one whole body and is geographically connected.

$$m_{wj}^{i} = \begin{cases} 1 & \text{if in sector i exists a walk between w and j} \\ 0 & \text{otherwise} \end{cases}$$

$$c_i = \frac{\sum_{j=1}^{n_i} \left(\sum_{w=1}^{n_i} m_{wj}^{i} \right)}{n_i (n_i - 1)}$$

contiguity can be measured by relative number of walks, and then weighted by number of vertices

$$\bar{c} = \frac{\sum_{i=1}^{k} c_i n_i}{N}$$

n_i =number of vertices of sector i

Elevator maintenance sectorization

Elevator maintenance

- We face a specific sectorization problem that arises in an elevator company, which involves **designating technicians for predetermined maintenance zones**.

- The definition of these zones has a great impact on the company costs and therefore needs to be optimized.

- **Regular** preventive maintenance but also **on call maintenance** must be considered, in order to define the best sectorization and to obtain a better maintenance plan.

Elevator maintenance

Define sectors of clients and assign a technician to each sector

- Fixed number of technicians
- Each technician is responsible for all existing clients in the sector
  - and also for any new clients
  - transparent responsibilities
- Each client is assigned to only one technician
  - long-term customer relations

Elevator maintenance

- The technician residence shouldn’t be far from its sector
- Once defined, zones are fixed for a long time.
- Daily scheduling of visits and routing are tackled separately from sectorization

Elevator maintenance

- **Regular visits** to each client: once a month, but no specific date
- Estimated service time is known for regular scheduled maintenance
- One client can have multiple elevators, escalators, automatic building doors, etc.

Elevator maintenance

- **On call maintenance** must be serviced by the technician responsible for that sector
- Several levels of urgency: hospitals
- Also, if a technician finds any failures during a scheduled maintenance, he cannot leave until the equipment is fixed.
- Potential unexpected maintenance calls can be computed based on the equipment, usage, past calls,…

Mathematical Models

Allocation Model

Kalcsics, Nickel, Schröder (2005) *Towards a Unified Territorial Design Approach-Applications, Algorithms and GIS Integration*

- **Indices and sets**

$V =$ set of basic areas (clients with equipments)

$C =$ set of territory centers (technicians)

- **Parameters**

$p = \#C =$ number of technicians

$d_{iv} =$ distance from center i to basic area v

$w_v =$ activity measure of basic area v

$\mu = \sum_{v \in V} w_v \big/ p =$ average activity level per sector

$\tau =$ tolerance

- **Decision variables**

$$x_{iv} = \begin{cases} 1 & \text{if basic area v is assigned to center i} \\ 0 & \text{otherwise} \end{cases}$$

Model

$$\min \sum_{v \in V} \sum_{i \in C} w_v d_{iv} x_{iv}$$

Minimize distances to center, weighted by activity measure

$$s.t. \quad \sum_{i \in C} x_{iv} = 1 \qquad \forall v \in V$$

Each basic area is assigned to one technician

$$\sum_{v \in V} w_v x_{iv} \le (1+\tau)\mu \qquad \forall i \in C$$

$$\sum_{v \in V} w_v x_{iv} \ge (1-\tau)\mu \qquad \forall i \in C$$

Deviation from average activity level is within given tolerance

$$x_{iv} \in \{0,1\} \qquad \forall v \in V, \forall i \in C$$

Bi-objective Allocation Model

Salazar-Aguilar, Ríos-Mercado, González-Velarde (2011) *A bi-objective programming model for designing compact and balanced territories in commercial districting*

- **Indices and sets**

  $V =$ set of basic areas (clients and technicians)

  $N^v =$ set of adjacent nodes to v

- **Parameters**

  $p = \#C =$ number of technicians

  $d_{iv} =$ distance from basic area i to basic area v

  $w_v^a =$ activity measure of type a of basic area v

  $\mu^a = \sum_{v \in V} w_v^{\,a} \Big/ p =$ average activity level per sector

  $\tau =$ tolerance

- **Decision variables**

  $$x_{iv} = \begin{cases} 1 & \text{if basic area v is assigned to center i} \\ 0 & \text{otherwise} \end{cases}$$

  $\Delta W_i^+, \Delta W_i^- \geq 0$ to represent imbalance in number of clients

$$\min \sum_{v \in V} \sum_{i \in V} d_{iv} x_{iv}$$

Minimize distances to center

$$\min \gamma$$

$$\text{s.t.} \quad \Delta W_i^+ + \Delta W_i^- \leq \gamma \qquad \forall i \in V$$

Minimize relative imbalance in number of clients

$$\Delta W_i^+ - \Delta W_i^- = \left(\sum_{v \in V} w_v^{\,1} x_{iv} - \mu^1 x_{ii} \right) \Big/ \mu^1 \qquad \forall i \in V$$

$$\sum_{i \in V} x_{ii} = p$$

Exactly p sectors

$$\sum_{i \in V} x_{iv} = 1 \qquad \forall v \in V$$

Each basic area is assigned to one sector

$$\sum_{v \in V} w_v^{\,2} x_{iv} \leq (1+\tau) \mu^2 x_{ii} \qquad \forall i \in V$$

$$\sum_{v \in V} w_v^{\,2} x_{iv} \geq (1-\tau) \mu^2 x_{ii} \qquad \forall i \in V$$

Deviation from average sales volume is within given tolerance

$$\sum_{v \in \cup_{j \in S} \left(N^j \setminus S \right)} x_{iv} - \sum_{v \in S} x_{iv} \geq 1 - |S| \qquad \forall i \in V, S \subset V \setminus (N^i \cup \{i\})$$

Connectivity of sectors

$$x_{iv} \in \{0,1\} \qquad \forall i, v \in V$$

$$\Delta W_i^+, \Delta W_i^- \geq 0 \qquad \forall i \in V$$

Bi-objective Allocation Model

Salazar-Aguilar, Ríos-Mercado, González-Velarde (2011) *A bi-objective programming model for designing compact and balanced territories in commercial districting*

- **Indices and sets**

  $V =$ set of basic areas (clients and technicians)

  $N^v =$ set of adjacent nodes to v

- **Parameters**

  $p = \#C =$ number of technicians

  $d_{iv} =$ distance from basic area i to basic area v

  $w_v^a =$ activity measure of type a of basic area v

  $\mu^a = \sum_{v\in V} w_v{}^a \big/ p =$ average activity level per sector

  $\tau =$ tolerance

  $\gamma =$ maximum deviation

- **Decision variables**

  $$x_{iv} = \begin{cases} 1 & \text{if basic area v is assigned to center i} \\ 0 & \text{otherwise} \end{cases}$$

  $\Delta W_i^+, \Delta W_i^- \geq 0$ to represent imbalance in number of clients

$$\min \sum_{v\in V}\sum_{i\in V} d_{iv} x_{iv} - \lambda s$$

Minimize distances to center

$$\text{s.t.} \quad \gamma + s \leq \varepsilon$$

$$\Delta W_i^+ + \Delta W_i^- \leq \gamma \qquad \forall i \in V$$

Control the imbalance in number of clients

$$\Delta W_i^+ - \Delta W_i^- = \left(\sum_{v\in V} w_v{}^1 x_{iv} - \mu^1 x_{ii} \right) \Big/ \mu^1 \qquad \forall i \in V$$

$$\sum_{i\in V} x_{ii} = p$$

Exactly p sectors

$$\sum_{i\in V} x_{iv} = 1 \qquad \forall v \in V$$

Each basic area is assigned to one sector

$$\sum_{v\in V} w_v{}^2 x_{iv} \leq (1+\tau)\mu^2 x_{ii} \qquad \forall i \in V$$

$$\sum_{v\in V} w_v{}^2 x_{iv} \geq (1-\tau)\mu^2 x_{ii} \qquad \forall i \in V$$

Deviation from average sales volume is within given tolerance

$$\sum_{v \in \cup_{j\in S}\left(N^j \setminus S\right)} x_{iv} - \sum_{v\in S} x_{iv} \geq 1 - |S| \qquad \forall i \in V, S \subset V \setminus (N^i \cup \{i\})$$

Connectivity of sectors

$$x_{iv} \in \{0,1\} \qquad \forall i, v \in V$$

$$\Delta W_i^+, \Delta W_i^- \geq 0 \qquad \forall i \in V$$

$$s \geq 0$$

Activity measures in elevator maintenance

Some considerations

- Activity measures
  - Number of equipments in area unit
  - Frequency of scheduled visits
  - Estimated service times
  - Potential for breakdowns
- Network distances between area units are measured by estimated travel times (Google maps)
- The technicians residence (or the main office) plays as the center of the sector

Some considerations

- 1500 equipments
- 9 technicians
- Per day, each technician makes an average of 9 planned visits + 1 unplanned
- Registered service times = base time + imponderables
- Sometimes, acessibility is an issue:
  - time windows in hotels
  - private residencial visits must be scheduled with owner
  - boats in river navigation

References

- Kalcsics, Jörg, Stefan Nickel, e Michael Schröder. «Towards a Unified Territorial Design Approach - Applications, Algorithms and GIS Integration». *Top* 13, n. 1 (1 de Junho de 2005): 1–56. https://doi.org/10.1007/BF02578982.
- Lei, Hongtao, Gilbert Laporte, Yajie Liu, e Tao Zhang. «Dynamic design of sales territories». *Computers & Operations Research* 56 (1 de Abril de 2015): 84–92. https://doi.org/10.1016/j.cor.2014.11.008.
- Maggioni, Mario A. *Clustering Dynamics and the Location of High-Tech-Firms*. Contributions to Economics. Physica-Verlag Heidelberg, 2002. https://www.springer.com/gb/book/9783790814316.
- Ríos-Mercado, R. Z., & Escalante, H. J. (2016). «GRASP with path relinking for commercial districting». *Expert Systems with Applications*, *44*, 102–113. https://doi.org/10.1016/j.eswa.2015.09.019
- Rodrigues, Ana Maria, e José Soeiro Ferreira. «Measures in Sectorization Problems». Em *Operations Research and Big Data: IO2015-XVII Congress of Portuguese Association of Operational Research (APDIO)*, editado por Ana Paula Ferreira Dias Barbosa Póvoa e Joao Luis de Miranda, 203–11. Studies in Big Data. Cham: Springer International Publishing, 2015. https://doi.org/10.1007/978-3-319-24154-8_24.
- Salazar-Aguilar, M. Angélica, Roger Z. Ríos-Mercado, e José Luis González-Velarde. «A bi-objective programming model for designing compact and balanced territories in commercial districting». *Transportation Research Part C: Emerging Technologies*, Freight Transportation and Logistics (selected papers from ODYSSEUS 2009 - the 4th International Workshop on Freight Transportation and Logistics), 19, n. 5 (1 de Agosto de 2011): 885–95. https://doi.org/10.1016/j.trc.2010.09.011.

THANK YOU!

cristinalopes@iscap.ipp.pt

This work is financed by the ERDF - European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation - COMPETE 2020 Programme and by National Funds through the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia within project POCI-01-0145-FEDER-031671.