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## ADAPTIVE REDUCTION OF THE ORDER OF DERIVATION IN THE CONTROL OF A HYDRAULIC DIFFERENTIAL CYLINDER

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**ABSTRACT-** Servo valve controlled hydraulic differential cylinders are non-linear, strongly coupled multivariable electromechanical tools applicable for driving e.g. manipulators. When the piston has finite but considerable velocity with respect to the cylinder the system's behavior can be "linearized" because the viscous friction i.e. the main source of disturbance is smooth function of this velocity and causes linear damping. When this velocity is in the vicinity of zero the effect of adhesion is the dominating disturbance force that abruptly changes direction depending on the sign of this velocity. Furthermore, at zero relative velocity adhesion can compensate arbitrary forces within certain limits keeping the piston fixed. In the paper a concise application of the Stribeck model of friction and adhesion is reported in an adaptive control in which varying fractional order derivatives are used to reduce the hectic behavior of friction in the case of "critical" trajectories that asymptotically converge to a fixed position and zero velocity. Simulation results made by INRIA's Scilab are presented. It is concluded that the combined application of the two adaptive techniques reported accurate control can be achieved without knowing the accurate model of the piston-cylinder system.

**Keywords:** Adaptive Control; Fractional Order Derivatives; Nonlinear System's Control; Positive Displacement Hydraulic Systems.

### INTRODUCTION

Hydraulic servo valve controlled differential cylinders are strongly coupled non-linear electromechanical devices of multiple parameters which are difficult to be kept under perfect control. The viscosity of the oil in the pipe system is very sensitive to the temperature that normally increases due to the friction in the circulation. Oil compressibility depends on the amount of air or other gases solved in it. Adhesion of the piston at the cylinder introduces rough non-linearity into the behavior of the system. The combination of these effects with the not always measurable external disturbance forces can make a quite complex control task arise.

The problem of driving a flexible robot arm under external disturbances by a hydraulic servo valve controlled differential cylinder was studied and solved in two alternative manners by Bröcker and Lemmen in [1]. Their first approach was based on the "disturbance rejection principle", the other one on the "partial flatness principle", respectively. In each case it was necessary to measure the disturbance force and its time-derivative as well as to

know the exact model of the hydraulic cylinder they developed in details and identified for a particular robot arm-drive system. However, the identification of such a system needs a lot of laboratory work the result of which may also be temporal. A serious problem is the need for measuring the external disturbance forces, too.

In general it seems to be expedient to apply adaptive control instead of trying to measure the ample set of unknown and time-varying parameters. This adaptive control need not to be too intricate, actually should not be much more complicated than an industrial PID controller. For this purpose Soft Computing (SC) based approaches would be more attracting than detailed analytical modeling. Unfortunately traditional SC (both fuzzy systems, and neural networks) suffer from bad scalability properties: the number of either the network nodes or the fuzzy rules is drastically increasing function of the degree of freedom of the system to be controlled.

In order to get rid of the scalability problems of the classical Soft Computing a novel approach was initiated that is based on a compromise between the

need of generality and scalability in [2]. It was shown by the use of perturbation calculus that this method can be applied for a quite wide class of physical systems, e.g. in the case of Classical Mechanical Systems, too [3]. This approach uses far simpler and far more lucid uniform structures and procedures than the classical ones: various algebraic blocks originating from different Lie groups can be incorporated into the “model”.

Hydraulic cylinders also have a very important property: due to the compressibility of the working fluid and elastic deformation of the pipe system the pressures in its chambers cannot abruptly be changed. It is the time-derivative of the oil pressure related to the 3<sup>rd</sup> time-derivative of the piston’s displacement can abruptly be prescribed. The hectic behavior (drastic time-derivative) of friction forces also are related to this 3<sup>rd</sup> derivative, that is the control has to be developed for ab ovo noisy signals.

In order to reduce noise-sensitivity the approach described in this paper allows a PID<sup>var</sup> control for the piston’s trajectory, in which the order of derivation depends on the past fluctuation of the piston’s velocity that generates harsh modification in the friction forces. In the sequel the main point of the scalable soft computing is very briefly outlined. Following that the analytical model of the differential hydraulic servo cylinder is presented together with the new control approach applied. The paper is closed by the simulation results and the conclusions.

#### ON THE ADAPTIVE CONTROL APPLIED

The concept of Complete Stability is often used as a practical criterion for the controlled system. It means that for a constant input excitation the system’s output asymptotically converges to a constant response. If the variation of the input is far slower than the system’s dynamics, with a good accuracy, it provides us with a continuous response corresponding to some mapping of the time-varying input [5]. From purely mathematical point of view the here presented learning adaptive control can be formulated as follows. There is given some imperfect model of the system on the basis of which some excitation is calculated to obtain a desired system response  $\mathbf{i}^d$  as  $\mathbf{e}=\boldsymbol{\varphi}(\mathbf{i}^d)$ . The system has its inverse dynamics described by the unknown function  $\mathbf{i}^f=\boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d))=f(\mathbf{i}^d)$  and resulting in a realized response  $\mathbf{i}^f$  instead of the desired one,  $\mathbf{i}^d$ . Normally one can obtain information via observation only on the function  $f()$  considerably varying in time, and no any possibility exists to directly “manipulate” the nature of this function: only  $\mathbf{i}^d$  as the input of  $f()$  can be “deformed” to  $\mathbf{i}^{d*}$  to achieve and maintain the  $\mathbf{i}^d=f(\mathbf{i}^{d*})$  state. On the basis of the modified Renormalization Transformation the following iteration was suggested for finding the proper deforma-

tion. Let the  $\mathbf{S}_n$  matrices be certain linear transformations. These matrices map the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control as follows:

$$\mathbf{i}_0; \mathbf{S}_1 \mathbf{f}(\mathbf{i}_0) = \mathbf{i}_0; \mathbf{i}_1 = \mathbf{S}_1 \mathbf{i}_0; \dots; \mathbf{S}_n \mathbf{f}(\mathbf{i}_{n-1}) = \mathbf{i}_0; \quad (1)$$

$$\mathbf{i}_{n+1} = \mathbf{S}_{n+1} \mathbf{i}_n; \mathbf{S}_n \xrightarrow{n \rightarrow \infty} \mathbf{I}$$

It is evident that if this series converges to the identity operator just the proper deformation is approached, therefore the controller „learns” the behavior of the observed system by step-by-step amendment and maintenance of the initial model.

For making the problem mathematically unambiguous (1) can be transformed into a matrix equation by putting the values of  $\mathbf{f}$  and  $\mathbf{i}$  into well-defined blocks of bigger matrices as e.g.

$$\mathbf{S}_n \begin{bmatrix} \mathbf{f}_{n-1} \dots \\ d \dots \end{bmatrix} = \begin{bmatrix} \mathbf{i}^d \dots \\ d \dots \end{bmatrix} \Rightarrow \mathbf{S}_n = \begin{bmatrix} \mathbf{i}^d \dots \\ d \dots \end{bmatrix}^{-1} \times \begin{bmatrix} \mathbf{f}_{n-1} \dots \\ d \dots \end{bmatrix} \quad (2)$$

in which the dots ... denote the other columns of the matrices that contain the arbitrary parameters of this ambiguous task, and  $d$  is a “dummy”, that is physically not interpreted constant value in order to evade the occurrence of the mathematically dubious  $0 \rightarrow 0$ ,  $0 \rightarrow \text{finite}$ ,  $\text{finite} \rightarrow 0$  transformations. If the “columns of the arbitrary parameters” are well defined continuous functions of the first column and a set of linearly independent initial set of vectors, then the  $\mathbf{f}=\mathbf{i}^d=\mathbf{0}$  case evidently results in  $\mathbf{S}_n=\mathbf{I}$  that cannot cause computational problems. Via computing the inverse of the matrix containing  $\mathbf{f}$  the problem becomes mathematically well-defined. For this purpose it is expedient to choose special matrices of fast and easy invertibility. There are various algebraic possibilities to meet this requirement. Let  $\mathbf{G}$  be nonsingular, quadratic, otherwise arbitrary constant matrix! The set of the  $\mathbf{V}$  matrices for which

$$\det \mathbf{V} = 1, \mathbf{V}^T \mathbf{G} \mathbf{V} = \mathbf{G} \Rightarrow \mathbf{V}^{-1} = \mathbf{G}^{-1} \mathbf{V}^T \mathbf{G} \quad (3)$$

trivially forms a Lie group that contains elements in arbitrary close vicinity of the unit matrix. The satisfactory condition for the convergence of (1) can be determined on the basis of the classic Perturbation Theory. Suppose that there is given an unknown, differentiable, invertible function  $\mathbf{f}(\mathbf{x})$  for which there exists an inverse of  $\mathbf{x}^d$  as  $\hat{\mathbf{x}} = \mathbf{f}^{-1}(\mathbf{x}^d) \neq 0$ . Let the Jacobian of  $\mathbf{f}$  that is  $\mathbf{f}'(\hat{\mathbf{x}}) \equiv \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$  be positive definite and of norm considerably smaller than 1. Furthermore, let us suppose that the actual estimation of the deformed input  $\mathbf{x}$  is quite close to  $\mathbf{f}^{-1}(\mathbf{x}^d)$ . Consequently there must exist two near-identity linear transformations in the group for which

$\mathbf{T}\hat{\mathbf{x}} = \mathbf{x}$ ,  $\mathbf{S}\mathbf{f}(\mathbf{x}) = \mathbf{x}^d$ . If  $\xi$  is chosen as the “small variable” of perturbation calculation the above operators can be written as  $\mathbf{T} = \mathbf{I} + \xi \mathbf{G}$ ,  $\mathbf{S} = \mathbf{I} + \xi \mathbf{H}$  in which  $\mathbf{G}$  and  $\mathbf{H}$  are certain generators of the group. Taking into account only the 0<sup>th</sup> and the 1<sup>st</sup> order terms in  $\xi$  the following estimations can be obtained:

$$\begin{aligned} \mathbf{f}(\mathbf{x}) &\equiv \mathbf{f}(\mathbf{I} + \xi \mathbf{G} \hat{\mathbf{x}}) \equiv \mathbf{x}^d + \xi \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T \mathbf{G} \hat{\mathbf{x}} \\ \mathbf{S}\mathbf{f}(\mathbf{x}) &\equiv (\mathbf{I} + \xi \mathbf{H}) \left( \mathbf{x}^d + \xi \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T \mathbf{G} \hat{\mathbf{x}} \right) \equiv \\ &\equiv \mathbf{x}^d + \xi \left( \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T \mathbf{G} \hat{\mathbf{x}} + \mathbf{H} \mathbf{x}^d \right) + O(\xi^2) = \mathbf{x}^d \\ \mathbf{f}(\mathbf{S}\mathbf{x}) &= \mathbf{f}(\mathbf{S}\mathbf{T}\hat{\mathbf{x}}) \equiv \mathbf{f}(\mathbf{I} + \xi \mathbf{H}) \times (\mathbf{I} + \xi \mathbf{G}) \hat{\mathbf{x}} \equiv \\ &\equiv \mathbf{x}^d + \xi \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T (\mathbf{G} + \mathbf{H}) \hat{\mathbf{x}} \end{aligned} \quad (4)$$

For the convergence decreasing error is needed, that is the restriction  $\|\mathbf{f}(\mathbf{S}\mathbf{x}) - \mathbf{x}^d\| \leq K \|\mathbf{f}(\mathbf{x}) - \mathbf{x}^d\|$  has to be satisfied with  $0 \leq K < 1$ , which means that according to (4) and (5)

$$\left\| \xi \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T (\mathbf{G} + \mathbf{H}) \hat{\mathbf{x}} \right\| \leq K \left\| \xi \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)^T \mathbf{G} \hat{\mathbf{x}} \right\|, \text{ i.e.} \quad (6)$$

$$\left\| -\mathbf{H} \mathbf{x}^d + \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \mathbf{H} \hat{\mathbf{x}} \right\| \leq K \|\mathbf{H} \mathbf{x}^d\| \quad (7)$$

Now take it into account that in the case of a perfect dynamic model  $\hat{\mathbf{x}} = \mathbf{x}^d$ . For an approximate one at least an acute angle can be expected between these vectors. For a finite generator  $\mathbf{H}$   $\mathbf{H} \hat{\mathbf{x}}$  and  $\mathbf{H} \mathbf{x}^d$  must be approximately of the same norm and direction that is the angle between them is acute, too. Due to the positive definite nature and small norm of  $\mathbf{f}' := \partial \mathbf{f} / \partial \mathbf{x}$  multiplication with it also results in acute angle between  $\mathbf{H} \mathbf{x}^d$  and  $-(\partial \mathbf{f} / \partial \mathbf{x}) \mathbf{H} \hat{\mathbf{x}}$ . This leads to a simple geometric interpretation: these vectors have approximately opposite directions, and the 2<sup>nd</sup> term serves as a little correction of the 1<sup>st</sup> one. Therefore the requirement expressed by (7) can be met. Consider now the Euler-Lagrange equation of motion of the Classical Mechanical systems and its time-derivative in a wider context!

$$\begin{aligned} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) &= \mathbf{Q}, \\ \mathbf{M}(\mathbf{q})\dot{\ddot{\mathbf{q}}} + \dot{\mathbf{M}}(\mathbf{q}, \dot{\mathbf{q}})\ddot{\mathbf{q}} + \dot{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}}) &= \dot{\mathbf{Q}} \end{aligned} \quad (8)$$

in which  $\mathbf{M}$  is the positive definite inertia matrix,  $\mathbf{C}$  describes the Coriolis and gravitational terms, and  $\mathbf{Q}$  denotes the generalized forces. For the servo valve controlled hydraulic cylinder in the roles of the “excitation” and the “response” the time-derivative of  $\mathbf{Q}$ , and the third time-derivative of  $\mathbf{q}$  stand as abruptly modifiable variables, therefore the above described conditions of convergence possibly can be satisfied. In this paper Special Symplectic Transformations detailed in [4] are applied.

#### THE MODEL OF THE HYDRAULIC CYLINDER

The operation of the differential hydraulic cylinder was described in details e.g. in [1]. Let  $x$  denote the linear position of the piston in  $m$  units. The acceleration of the piston is described by (1) as

$$\ddot{x} = \frac{1}{m} \left[ \left( p_A - \frac{1}{\varphi} p_B \right) A_A - F_f(\dot{x}) - F_d \right] \quad (9)$$

in which  $p_A$  and  $p_B$  denotes the pressures in chamber A and B of the piston in *bar*,  $\varphi = A_A/A_B$ , that is the ratio of the “active” surfaces of the appropriate sides,  $m$  is the mass of the piston in *kg*,  $F_f$  denotes the internal friction acting between the piston and the cylinder,  $F_d$  denotes the external disturbance force.

For practically acceptable modeling of friction partly the *Stribeck model* is used as in [1], that states, that there is a functional relationship between the relative piston-cylinder velocity  $v$  and the friction forces as

$$F_f = f_{vi} v \cdot \text{sign}(v) (F_c + F_s \exp(-\text{abs}(v)/c_s)) \quad (10)$$

in which  $f_{vi}$  describes the viscosity coefficient,  $0 < (F_c + F_s)$  is the force that is necessary for bringing the piston at rest into motion, and  $F_c < (F_c + F_s)$  means the stabilized value of the adhesion forces when the velocity is considerable. Eq. (10) describes the transition of the sticking forces between the very small and the considerable values of the relative velocity if  $v \neq 0$ . When  $v = 0$  the piston sticks in the cylinder and friction/adhesion can compensate any external force in the  $[-(F_c + F_s), (F_c + F_s)]$  region without letting it be accelerated. Numerical modeling of this behavior is quite difficult because the “exact” value of zero seldom occurs in a numerical finite element calculation. In the present paper we supposed that within the  $v \in [-kc_s, kc_s]$  region ( $0 < k = 0.1 < 1$ ) the piston has zero acceleration if the external forces acting on it are within the interval  $[-(F_c + F_s), (F_c + F_s)]$ . For a velocity value  $v \notin [-kc_s, kc_s]$  simply (10) was applied. This model qualitative well describes the everyday experience we have about the behavior of friction forces. The pressure of the oil in the chambers also

depends on the piston position and velocity as

$$\dot{p}_A = \frac{E_{oil}}{V_A(x)} (-A_A \dot{x} + B_v K_v a_1(p_A, \text{sign}(U))U) \quad (11)$$

$$\dot{p}_B = \frac{E_{oil}}{V_B(x)} \left( \frac{A_A}{\varphi} \dot{x} - B_v K_v a_2(p_B, \text{sign}(U))U \right) \quad (12)$$

where  $B_v$  denotes the flow resistance,  $K_v$  is the valve amplification,  $U$  is the *normalized valve voltage*. The oil volume in the pipes and the chambers can be expressed as

$$\begin{aligned} V_A(x) &= V_{pipeA} + A_A x, \\ V_B(x) &= V_{pipeB} + A_B (H - x) \end{aligned} \quad (13)$$

( $H$  is the cylinder stroke.) The hydraulic drive has two stabilized pressure values, the *pump pressure*  $p_0$ , and the *tank pressure*  $p_t$ . Under normal operating conditions (that is when no shock waves travel in the pipeline) these pressures set the upper and the lower bound to  $p_A$  and  $p_B$ . The functions  $a_1$  and  $a_2$  are defined in (14). Under “normal conditions”  $\text{sign}(a_1) \geq 0$ , and  $\text{sign}(a_2) \geq 0$ , too, according to the limiting role of the pump and tank pressures.

$$\begin{aligned} a_1(p_A, \text{sign}(U)) &= \begin{cases} \text{sign}(p_0 - p_A) \sqrt{|p_0 - p_A|} & \text{if } U \geq 0, \\ \text{sign}(p_A - p_t) \sqrt{|p_A - p_t|} & \text{if } U < 0 \end{cases} \\ a_2(p_B, \text{sign}(U)) &= \begin{cases} \text{sign}(p_B - p_t) \sqrt{|p_B - p_t|} & \text{if } U \geq 0, \\ \text{sign}(p_0 - p_B) \sqrt{|p_0 - p_B|} & \text{if } U < 0 \end{cases} \end{aligned} \quad (14)$$

In the sequel the control of this complex system is considered.

#### THE PARTICULAR CONTROL TASK

For the tracking error  $e := (x^R - x^{Nom}) - p_A$  a simple PID controller was constructed in the following manner:

$$\ddot{e} = -Pe - D\dot{e} - I \int_0^t e dt \quad (15)$$

The appropriate  $P$ ,  $D$ , and  $I$  coefficients were determined simply by substituting an expected  $e = \exp(\alpha t)$  type relaxation into the time-derivative of (15) that results in a third order polynomial for  $\alpha$ . For this polynomial three, slightly different negative real roots were prescribed in the form of  $(\alpha - \alpha_1)(\alpha - \alpha_2)(\alpha - \alpha_3)$ . Substituting this into (15)  $P$ ,  $D$ , and  $I$  can conveniently be determined. The time-derivative of (15) therefore leads to the *desired third time-derivative of the piston's trajectory* as

$$\ddot{x}^d = \ddot{x}^{Nom} - P\dot{e} - D\ddot{e} - Ie \quad (16)$$

The very rough approximate model of the cylinder was obtained by omitting the friction forces and the external disturbance forces in (9) as

$$\ddot{x}^d = \frac{A_A}{m} \left( \dot{p}_A - \frac{1}{\varphi} \dot{p}_B \right) \quad (17)$$

into which the desired time-derivative of the piston's acceleration was substituted and the desired value for  $dp_A/dt$  was set to 0. Eq. (17) thus immediately yields an “expected” value for  $d(p_A - p_B)/\varphi/dt$ . Via computing [(11)-(12)/ $\varphi$ ] this determines the proposed control signal  $U$ , and from the known current state of the system and (11) and (12) the actually obtained  $dp_A/dt$ , and  $dp_B/dt$  values can be computed. This can be substituted into the time-derivative of (9) yielding the “actual” third time-derivative of the piston's displacement. Here special attention has to be paid to the problem of observing  $d^3x/dt^3$ , which, in the case of the presence of friction forces, may be critical. For filtering out the noisy part of this signal Caputo's definition of the fractional order derivatives can be applied. It *re-integrates* the integer order derivative with a kernel function of long tail acting as a frequency filter. According to that (16) can be modified as

$$\begin{aligned} x^{(2+\beta)^d} &= \int_0^t d\tau [\ddot{x}^{Nom}(\tau) - P\dot{e}(\tau) - D\ddot{e}(\tau) - Ie(\tau)] \times \\ &\times \frac{(t-\tau)^{\beta}}{\Gamma(1-\beta)}, \quad \beta \in (0,1) \end{aligned} \quad (18)$$

In the practical realization of that the lower limit of the integration is replaced by a finite memory  $t-T$ . In the numerical approximation of the integral with singular integrand the full interval of the integration of length  $T$  is divided into small ones of length  $\delta$  during which the reintegrated derivative is supposed to be approximately constant (details are given in e.g. in [6]).

The next essential point is setting the order of derivation. Since according to (10) changing sign of the velocity generates drastic changes in the friction forces, due to the controller's feedback this force can oscillate whenever zero-transmission happens in the velocity. That is,  $\beta \approx 1$  is needed for non-zero velocities, and  $\beta < 1$  whenever the velocity is in the vicinity of zero. In the present paper the following adaptive formula was applied, in which instead of the velocity, the observed 3<sup>rd</sup> time-derivatives are used, because this signal is directly related to the controller's feedback:

$$0 < \beta = \frac{A + \left| \sum_{s=1}^{\lceil T/\delta \rceil} \text{sign}(\ddot{x}(t-s\delta)) \right|^{\gamma}}{A + (T/\delta)^{\gamma}} \leq 1 \quad (19)$$

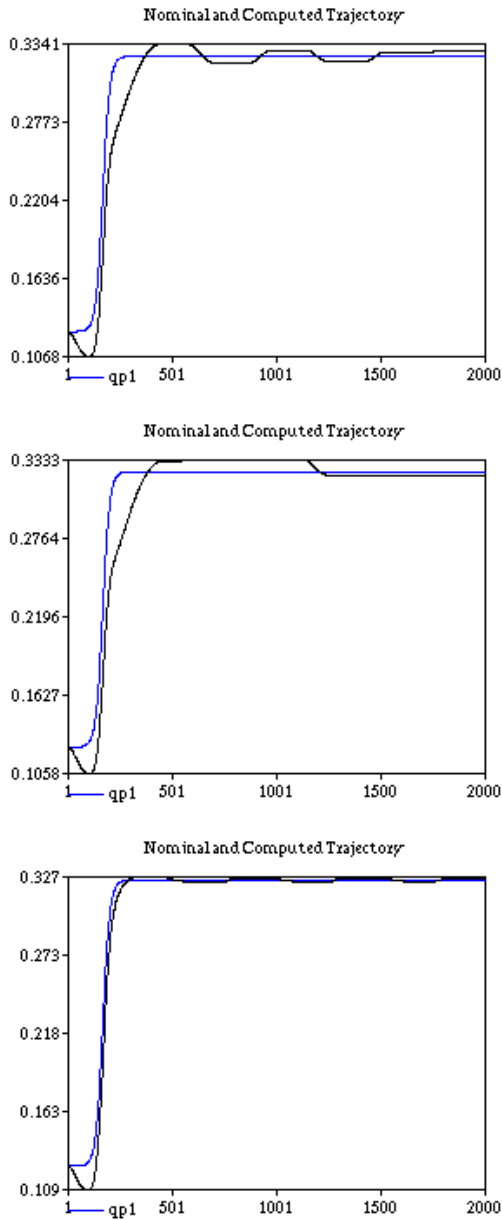


Figure 1: Trajectory tracking without any adaptivity (top), with adaptive fractional order control only (middle), and fully adaptive case (bottom), displacement and time in  $m$  and  $ms$  units, respectively

In (19) there are various parameters as  $A$ ,  $T$ ,  $\delta$ , and  $\gamma$  the actual values of which numerically concern the quality of control.

In the simulations  $\delta=1 ms$  was chosen as a fixed value. The value of  $A$  varied between 0.1 and 10,  $T$  was investigated between 2 to 20  $ms$ , and  $\gamma$  was investigated between  $1 \times 10^{-4}$  and  $4 \times 10^{-4}$ . It was found that it is expedient to choose very sharp reduction of the order of derivation, i.e.  $\gamma=1 \times 10^{-4}$  was found to be optimal. The actual value of  $A$  was not very

important, eventually it was set to  $A=1$ . The “optimal length of memory” of the fractional order derivation was found to be equal to  $T=6 ms$ . Due to the shortage of free room only certain comparative figures belonging to this optimum are presented in the sequel.

## SIMULATION RESULTS

With the exception of the parameter  $E_{oil}$  all the other parameters given by Bröcker in [1] were used. For  $E_{oil}$  Bröcker used  $1800 \times 10^6 Pa$ , which is a huge value representing the approximate incompressibility of liquids. However, in a pipe system, due to the elasticity of the pipe walls, or due to complementary components intentionally built into the system to reduce this huge stiffness (e.g. via using hydraulic accumulators, flexible hoses) this value can be considerably smaller. In this paper  $18 \times 10^6 Pa$  was used in the simulations. In the top of Fig. 1 the non-adaptive control's results are described. As it can be expected the dynamic parts of the trajectory are badly tracked, but the static tail is well approximated, apart from the sticking of the piston. However, as the integrating term yields enough contribution to bring the piston into motion it gets enough momentum to produce some overshoot, etc., therefore some oscillation is formed in the trajectory tracking. According to the graph in the middle of this figure application of the adaptive fractional order derivatives makes this tail smoother but to some extent “conserves” the tracking errors for a long time. Turning on full adaptivity “amends” both the “dynamic part” and the “tail” of trajectory tracking.

Fig. 2 reveals the friction forces that are very “hectic” in the case of the non-adaptive control, become relatively “calm” due to the application of the fractional order derivatives, and obtain again hectic parts due to the zero transition of the velocity of the fully adaptive control. (The periodic part is the consequence of the sinusoidal external force of 200  $N$  amplitude also applied in the simulations.)

## CONCLUSIONS

In this paper a possible improvement of an adaptive control developed for electrical servo valve operated differential hydraulic cylinders was considered. In the case of trajectories having asymptotically zero velocity the piston's friction is a considerable nonlinear disturbing factor. It was found that the proposed adaptive  $PID^{var}$  control can efficiently defy this difficulty.

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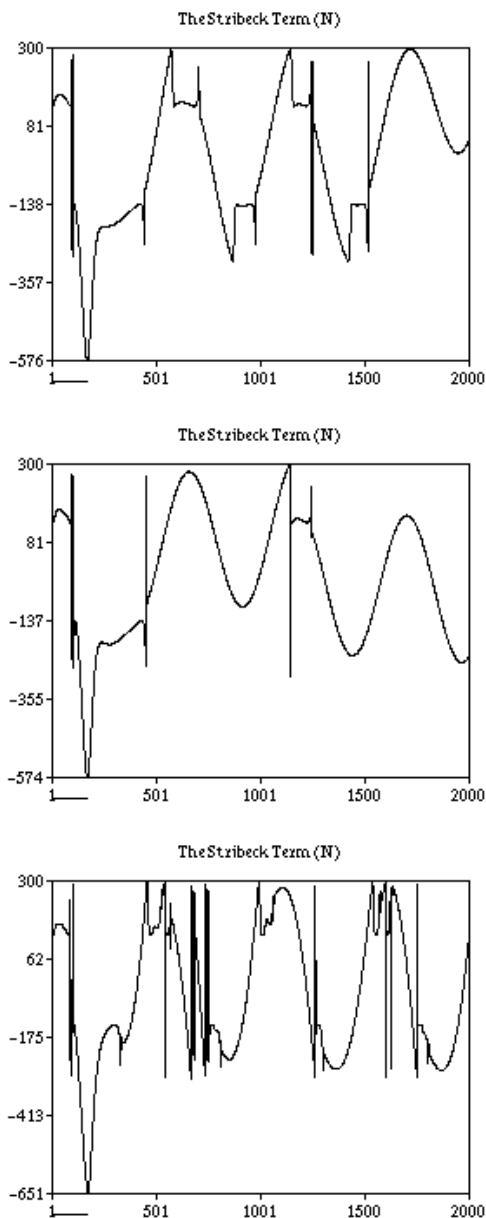


Figure 2: Friction forces without any adaptivity (top), with adaptive fractional order control only (middle), and fully adaptive case (bottom), force and time in  $N$  and  $ms$  units, respectively

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