Railway critical speed assessment: an in-situ experimental-analytical approach

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Abstract

When constructing a new railway line, its linear nature means there are significant financial implications associated with determining the geodynamic ground properties. Therefore this paper presents recommendations to optimize the efficiency and depth of such a geotechnical site investigation. Firstly a numerical analysis is performed to investigate the effect of soil layering, soil stiffness and track bending stiffness on critical velocity. It is shown that each of these variables play an important role, however for most practical cases, only the top 8m of soil is influential. Track dynamics are rarely affected by soil properties at depths below this, meaning this is the maximum required depth of soil investigation. Using this knowledge, a hybrid experimental-numerical ground investigation methodology is presented, based on a SASW experimental setup to compute the ground dispersion curve and an analytical model to compute the track dispersion curve. The experimental and analytical results are combined directly, to accurately compute the critical velocity. This approach is attractive because: 1) SASW tests are typically accurate to ≈8m (when using a mobile exciter) thus matching the required depth needed for critical velocity computation, 2) soil property uncertainties are inherently accounted for, 3) the uncertainties associated with SASW inversion are avoided. The approach is attractive when constructing new railway lines and upgrading the speed of existing lines because it can potentially yield site investigation cost savings.

Keywords (6): railway critical speed; railroad vibration; geotechnical site investigation;; wave propagation; railway track dynamics

1. Introduction

The global railway network is undergoing rapid expansion, with vehicles becoming faster, heavier, and having reduced headway between successive passages. A challenge with higher speeds is that they can induce elevated levels of track displacement. This issue occurs when train speeds approach or exceed the wave propagation speed of the track-ground system. If this happens, sudden dynamic amplification occurs, characterized by large ground strains, to large depths, which can cause permanent ground deformation [1-3]. Therefore when dealing with high-speed railway lines, deep geotechnical investigation is required to determine how the ground properties will affect dynamic track amplification and critical speed.

The issue of railway critical speed has been studied over the past 20 years and the understanding of the problem is well established [4]. The state of art about the topic benefited from the experimental data recorded at Ledsgaard and the numerical techniques developed by several authors to predict the critical speed of high-speed railway tracks [5-10]. There are two distinct prediction approaches that have been proposed: i) the 'direct' method; ii) the 'simplified' method. The direct requires the computation of the track displacements due to a moving load passing at increasing (discrete) speeds. The speed that yields maximum amplification is usually assumed to be the critical speed. Although this method is robust, it is demanding from computational point of view, requiring the development, and multiple executions of 3D or 2.5D numerical models [7, 10-12]. In contrast, the simplified approach is based on the analysis of the dispersion characteristics of the waves propagating along the track-ground system. This approach is efficient from a computational point of view because it doesn't require the computation of the dynamic track response due to moving loads [4, 13, 14]. More recently Norén-Cosgrif et al. [15] proposed a method for critical speed prediction based on experimental data. However, application of this method is only possible for existing railway lines.

The studies developed on the basis of the prediction approaches presented above showed that the critical speed value is influenced by following properties: i) elastic wave velocity in the ground; ii) ground layering, namely the stiffness contrast between adjacent layers and layer thickness; iii) track bending stiffness. The influence of the track stiffness on the critical speed is only noticeable in layered grounds where there is an increase in stiffness (and elastic wave velocity) with depth. Therefore, in homogeneous ground or in such cases where the thickness of the upper homogenous layer is large, track stiffness doesn't affect critical velocity [4, 13, 16].

Considering the potential challenges caused by running trains near the critical speed, railway administrations have begun to mobilize significant resources during geotechnical investigation, in order to understand the ground conditions. This is true for both new lines and lines with planned speed increases. Due to the large depth of wave propagation during train passage close to critical speed, the soil properties must be known at significant depth, thus requiring deep geotechnical investigation. This information is used to assess the critical speed and dynamic amplification of the track-ground system. Without this information, it is difficult to determine whether the critical speed is comparable to the expected train speed, or whether mitigation is required (e.g. ground improvement [17] or increasing track bending stiffness).

The linear nature of railway infrastructure means that large numbers of geotechnical surveys are required for a new line. Therefore, minimizing the depth of each ground investigation can result in significant savings for railway administrations. This can be achieved by determining the maximum depth of soil that will influence the critical velocity at a particular site.

The objectives of this work are therefore to:

1. Present straightforward and easy to use guidelines on the recommended depth of ground investigation to permit critical velocity analysis. This information is useful for practical engineering purposes because it can reduce the cost of geotechnical surveys required during the design/upgrade of rail lines.
2. Present a hybrid experimental-analytical approach for critical speed assessment. This method has the advantage of incorporating experimental data from in situ tests, and, therefore, it can incorporate several aspects that are not feasible to introduce in terms of numerical modelling (e.g. soil property undertainties).

2. Methodology

Prediction of railway track critical speed is a challenging task, requiring thorough simulation of the track-ground system. A variety of approaches can be used, the most popular of which is the 'direct method', where the dynamic response of the track-ground system is computed, for a wide range of moving load speeds. Then the critical speed is simply the (single wheel) load speed that results in maximum amplification of track displacement (see Figure 1a). This method typically requires a detailed numerical model, capable of simulating the dynamic response of the track ground system. Currently the preferred numerical approaches to achieve this is via 2.5D methods, where the track-ground system is considered invariant and infinite in the direction of train passage [18]. These 2.5D formulations can be implemented using a variety of numerical tools, including BEM (Boundary Elements Method), MFS (Method of Fundamental Solution), FEM (Finite Elements Method) [10, 18-25]. In this work, all 'direct' results are obtained using the 2.5D FEM-PML (Perfectly Matched Layers) model previously presented by the 1st author (Figure 1b). 2.5D approaches are advantageous because they balance the need for computational efficiency with the need to model complex geometry cross-sections.

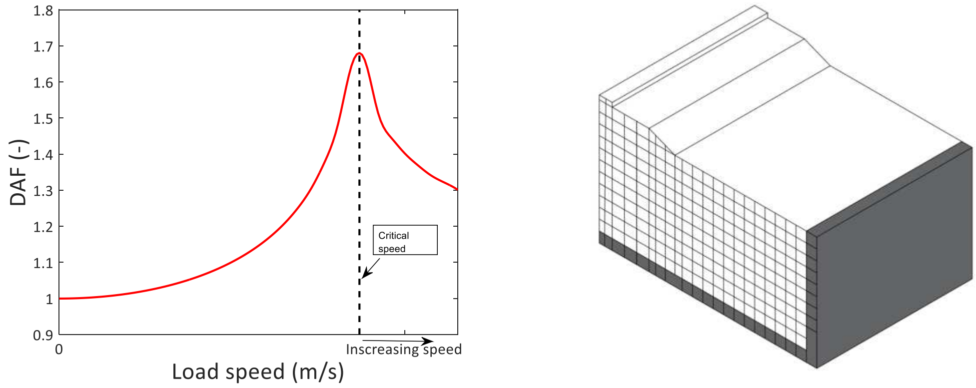

Figure 1 – Critical speed assessment based on the direct method: a) amplification curve as function of load speed; b) Model visualization of the 2.5D FEM-PML model used to compute amplification curves (adapted from [1])

A challenge with the direct approach is that it is demanding from a computational point of view. This is because the construction of dynamic amplification curves require many model executions (i.e. >20) at discrete train speeds. Therefore an alternative approach is to simply consider the interaction between the propagation of P-SV waves in the ground and the bending waves in the track. This 'dispersion approach' is based upon the fact that the critical speed can be defined as the speed for which there is a match between the wavelength of the waves that propagate in both the ground and track, as depicted in Figure **2** [26]. The velocity at which the dispersion curves of the first ground P-SV mode and bending wave propagation on the track intersect is the critical speed. Theoretical details about this approach and its accuracy are described in [4, 13]. It is seen in Figures 2 a and b that the error is negligible - the "dispersion approach" yields a solution that is within 1% of the2.5D model (V_{cr}=f/k=181 m/s). However, the simplified approach has the advantages of being straightforward to apply and requiring low computational effort, making it useful for scoping-type analysis. In this current research, both direct and dispersion approaches are used, with the direct method being used to validate the dispersion method.

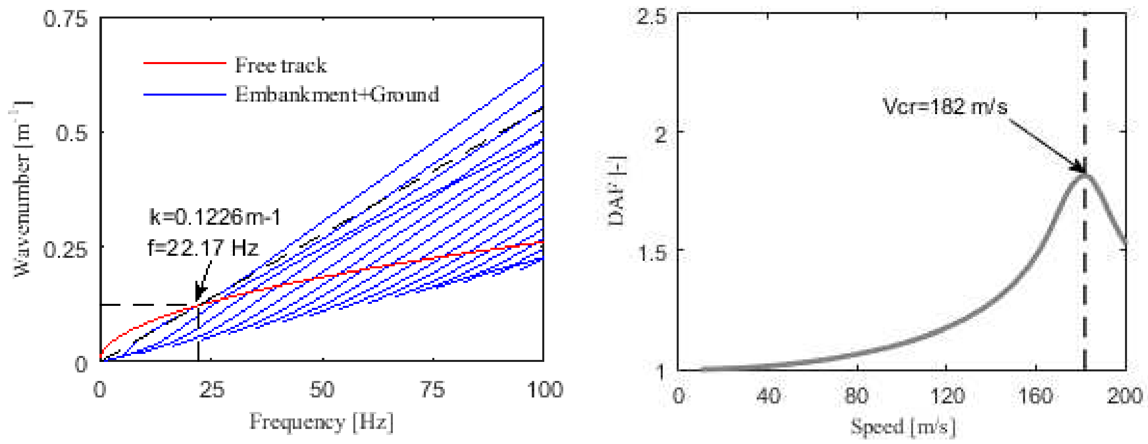

Figure 2 – Critical speed assessment: a) dispersion method analysis; b) comparison between the dispersion methodology and the direct approach [26]

3. To what depth must geotechnical survey be performed?

3.1 The influence of uppermost ground layer thickness

The interaction between the track and the ground means the thickness of the uppermost ground layers play a dominant role in dynamic track amplification. In the absence of the track, (i.e., loading applied directly to the ground surface), the critical velocity is equal to the Rayleigh wave speed of a homogenous ground with the same properties of the shallow layer [4]. However, taking into account the track-ground coupling, the critical speed of the track-ground system can be significantly larger than that value, particularly if the ground stiffness increases with depth, giving rise to normally dispersive P-SV waves. Therefore it is important to know the maximum depth of soil that affects the critical speed. Knowing this allows railway administrations to minimize their depth of geotechnical survey, thus avoiding the costs associated with unnecessary additional survey depths. The proposed method is confined to scenarios where the soil stiffness increases with depth. Such situations are the most common because soil stiffness increases with confining pressure. However it should be noted that it is not always true, for instance when soft clay layers are *sandwiched* between sandy layers. Nevertheless, such cases are out of the scope of the present study and the conclusions here presented are only valid for normally dispersive mediums.

To better understand this effect, consider a slab track overlying a 1m high embankment, as depicted in Figure **3** (embankment properties also presented in same figure). The track properties are shown in Table 1. Since the track bending stiffness influences the critical speed, three distinct concrete slab thicknesses are considered. Regarding the ground, it is composed of a shallow upper layer with a shear wave velocity of 200 m/s, overlying a stiffer halfspace. The thickness of the shallow upper layer (H) is variable as shown in Figure **3** and the halfspace stiffness contrast is defined via the shear wave velocity ratio, r:

$$\mathrm{r} = \frac{Cs_1}{Cs_2} \qquad (1)$$

where C_{s1} and C_{s2} are the shear wave velocities of the upper layer and of the bottom halfspace, respectively.

A reduced embankment height was selected, since when embankment stiffness is larger than natural soil stiffness (which is usual when dealing with soft to medium stiff natural soils) there is a trend for the increase of the critical speed with the increase of the embankment height, as already discussed by Colaço and Alves Costa [26]. Therefore, for the problem addressed in the present paper, the most critical situation happens when the embankment height is reduced or even in the absence of embankment.

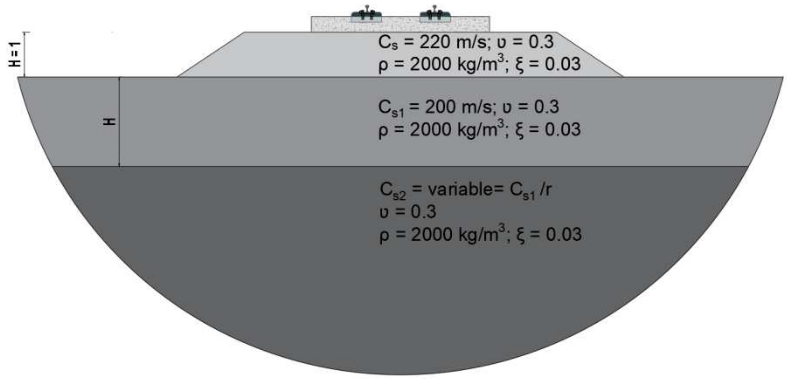

Figure 3 – Railway track-ground cross-section.

Table 1 – Track properties

		V1	V2	V3
Rail	EI_r [Nm^2]	6.12×10^6		
	m_r [kg/m]	60		
	d_v [m]	1.4		
Raipad	k_p [N/m]	5×10^8		
	c_p [Ns/m]	2.5×10^5		
Slab	h_l [m]	0.3	0.4	0.5
	2b [m]	2.5		
	EI_l [$Pa.m^4$]	1.9×10^8	4.4×10^8	8.6×10^8
	ρ [kg/m^3]	2400		

The variables in the table have the following meaning: EI_r is the rail bending stiffness; m_r is the rail mass; d_v is the track gauge; k_p is the railpad stiffness; c_p is the rail pad damping; h_s is the slab track thickness; EI_s is the slab track bending stiffness; 2b is the slab width and ρ is the volumetric mass of the concrete slab.

Figure 4a shows the dynamic amplification factors of the track vertical displacement for different thicknesses of the uppermost soil layer. The ratio between the shear wave velocities in the upper and lower layers is equal to 0.5, and the track is type V2 (see Table 1). As expected, the critical speed increases as the upper layer gets thinner, i.e., the influence of the lower halfspace is increasingly evident as the thickness of the softer shallow layer is reduced. This is also shown in Figure 4b, where the black line represents the evolution of the critical speed with upper layer thickness, computed using dispersion curve analysis (GDC). The lower limit of the critical speed is equivalent to the Rayleigh wave velocity of a halfspace with the properties assumed for the upper layer, i.e., assuming that the upper layer is so thick that ground can be assumed as an homogenous halfspace. This effect becomes noticeable when the thickness of the upper layer is greater than 8m. In this case, the lower (stiffer) layer plays a less dominant role on the critical speed.

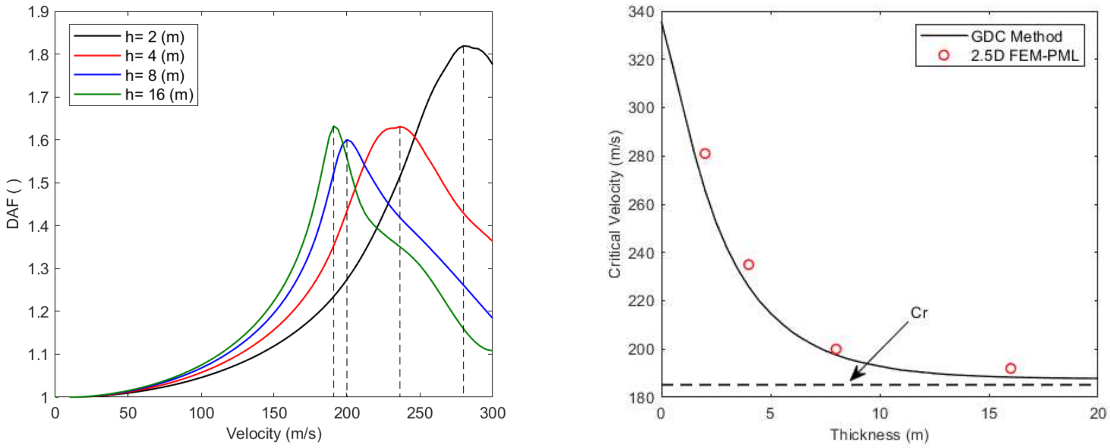

Figure 4 – Critical speed calculation for different layer thicknesses, consideringr=0.5 (see equation [1]): a) direct approach; b) comparing direct and simplified approaches

The same conclusions are also be inferred from Figure 5, which shows a similar case study, but considering the ratio of the shear wave velocities of both layers to be 0.7. In both situations it is possible to observe that changes to the upper layer thickness (from 8 m to 16 m) only influence the critical velocity by <3%. As expected, when the upper layer thickness increases the critical speed tends towards the Rayleigh wave velocity of a homogeneous ground with the properties of the uppermost layer.

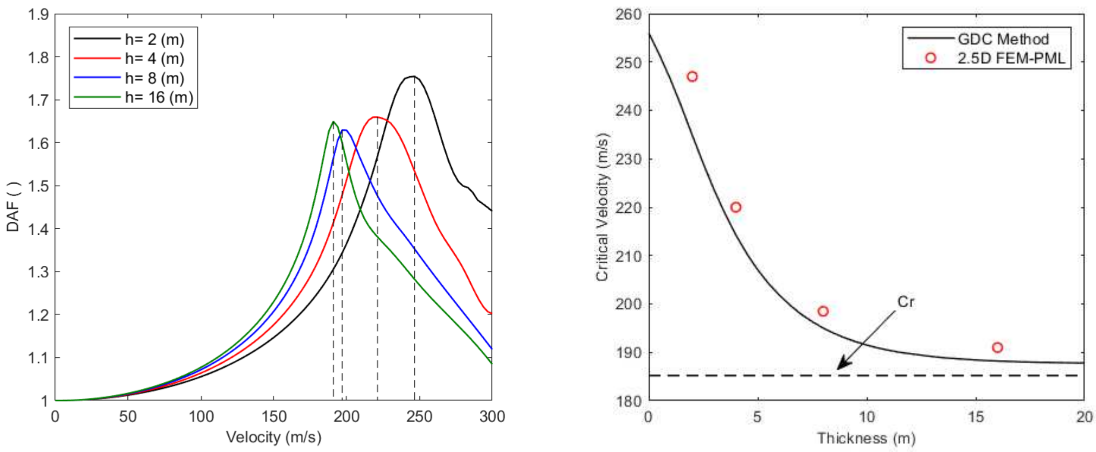

Figure 5 – Critical speed calculation for different layer thicknesses, considering r=0.7 (see equation [1]): a) direct approach; b) comparing direct and simplified approaches

Regarding the accuracy of the simplified approach, the circular markers depicted in Figure 4b and Figure 5b correspond to the results computed using the direct method, while the continuous line represents the results calculated using the simplified method. There is a strong match between results, thus validating the accuracy of the "dispersion approach”."

For the purpose of extending the simplified results, Figure 6 shows a 2D contour plot of the critical speed, for different configurations of layer thickness and shear wave velocities ratios between the upper and lower soil layers. Therefore fig4 and fig5 are sub-sets of fig 6, defined by r=0.5 and r=0.7. The results are non-dimensional, being the critical speed divided by the critical speed calculated assuming the ground as a homogenous half-space with the shallow layer material properties. It is seen that as the stiffness contrast of the layers increases, the influence of the bottom half-space increases. However stabilization occurs when the thickness of the shallow layer is greater than 6 m, meaning that even when the half-space stiffness is much greater than the upper layer stiffness (i.e. r≈0.5, meaning the stiffness of the lower layer is 4 times larger than the stiffness of the upper layer), it does not influence the critical speed value.

In addition to the above, the critical speed of a layered ground is not only governed by the P-SV wave dispersion characteristics, but also by the track's wave propagation characteristics, and the interaction between the waves that are able to propagate in both domains. The following sections analyze these.

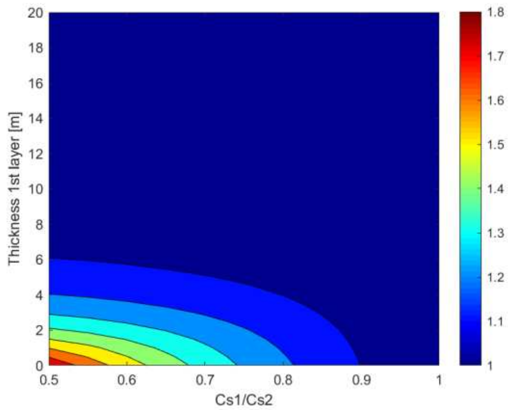

Figure 6 –Critical speed contour for different layer thickness and shear wave ratios: Cs1=200 m/s; track V2

3.2 The influence of uppermost ground layer stiffness

To better discern the maximum soil depth that affects critical velocity, computations similar to the previous section are repeated for varying values of shear wave velocity of the uppermost layer. These are summarized in Figure 7 for the following values of Cs1: 100 m/s; 150 m/s; 250 m/s; 300 m/s.

From the inspection of the four sub-figures it is seen that although there is a similar trend, there is a distinct variation in the maximum depth up to which the results are affected by the presence of the lowest layer. For a constant shear wave speed ratio, the influence of the upper layer thickness changes the critical speed more greatly when the layer is soft. In fact, if the shear wave ratio is 0.5, the critical speed is affected by the thickness of the layer up to approximately 13 m when considering a layer wave speed of Cs=100 m/s. Alternatively, the influence of the uppermost layer is only noticeable for thicknesses of approximately 4m, when its shear wave speed is 300 m/s.

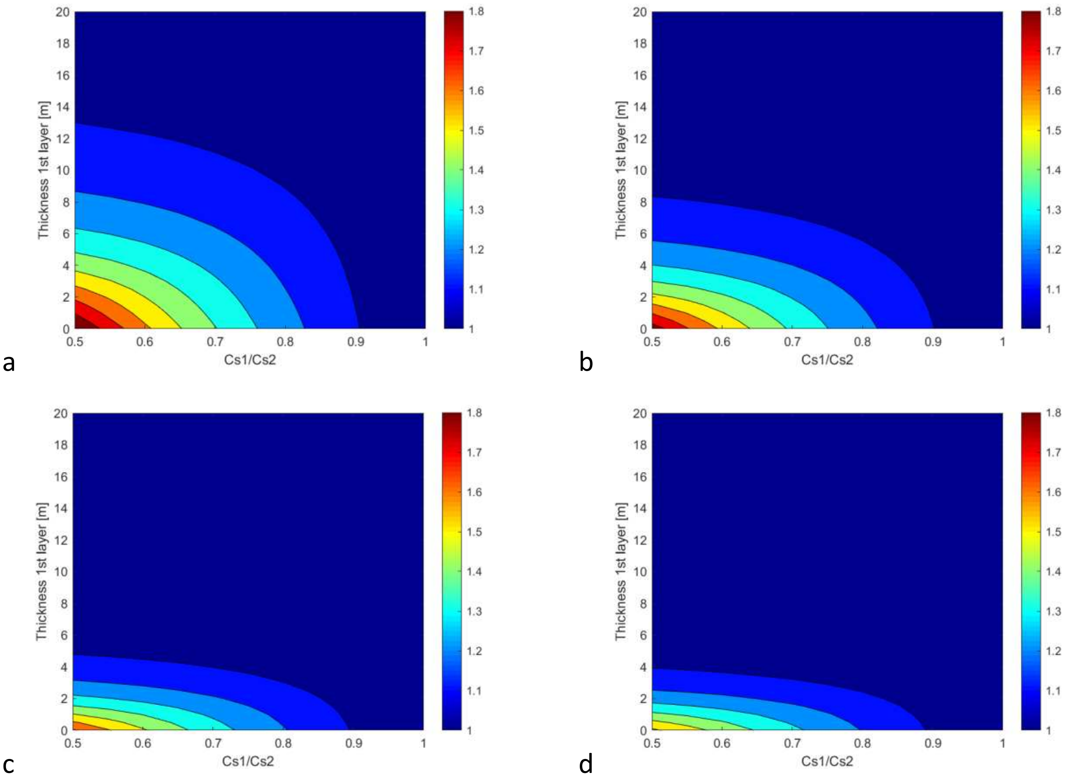

Figure 7 – Critical speed contour for different shear wave velocities of the upper layer (track V2): a) 100 m/s; b) 150 m/s; c) 250 m/s; d) 300 m/s.

These results show that for the examples considered, softer soils require deeper geotechnical investigation to obtain the properties required for accurate critical speed prediction. When dealing with stiffer soils (i.e. when the shallow layer shear velocity is larger than 200 m/s), it is not necessary to perform geotechnical investigation to depths greater than 6 m, because the critical speed will be almost fully governed by the dynamic properties of the shallow layer. This rule is applicable when dealing with layered grounds where stiffness increases with depth. However, as will be seen in the following section, track bending stiffness is important because a greater stiffness results in a higher critical speed.

3.3 The influence of track bending stiffness

Previous studies by the authors [13, 27] show that when dealing with layered ground, the critical speed increases with increasing slab track bending stiffness. This is because increasing track stiffness limits the propagation of short wavelength waves in the ground. Therefore, the critical speed increases because the presence of a stiff track only allows the propagation of longer wavelength waves in the track-ground system. These penetrate to greater depth, and therefore are more affected by the stiffer soils that typical exist deeper in the stratum. This is illustrated in Figure 8 where the critical speed is presented as function of the upper layer thickness and shear wave speed ratio (track properties shown in Table 1). As expected, for the same shear wave ratio and layer thickness, the critical speed increases with track stiffness. Comparing the results from Figure 8a and Figure 8c, it is possible to infer that changes to the critical speed occur when the shear wave ratio is low and are negligible when the shear wave velocity ratio is larger than 0.9. This is because when the ratio is large, the ground is similar to a homogeneous half-space and the critical speed is no longer governed by the wave propagation characteristics within the track. However, Figure 8c shows that even for a very stiff slab track and for a shear wave-speed ratio (r) equal to 0.5, ground properties at depths larger than 9.0 m do not affect the prediction of the critical speed. This value decreases to lower than 8.0 m when the shear wave-speed ratio (r) becomes larger than 0.65.

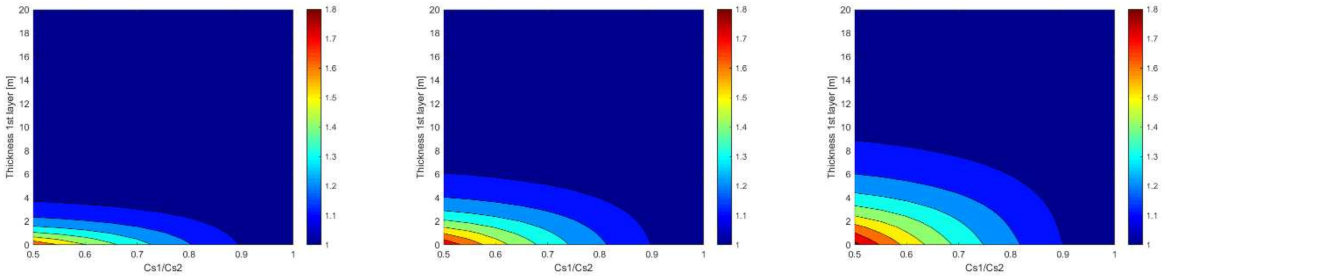

Figure 8 –Critical speed contours for tracks with different bending stiffness (C_{s1}=200 m/s): a) Track V1; b) Track V2; c) Track V3;

A similar trend is also found when dealing with scenarios of a very soft (C_{s1}=100 m/s) or very stiff (C_{s1}=300 m/s) shallow layer, as can be observed in Figure 9 and Figure 10 respectively. However, for stiff tracks overlying very soft soils (Figure 9), the knowledge of the soil layering and material properties up to large depths must be determined in order to obtain a reasonable prediction of the critical speed. The opposite is true for stiff shallow soil layers, where even tracks with high bending stiffness are considered. In this case, if the shallow soil layer is thicker than 6 m, the problem is fully governed by this layer and assuming the ground is a halfspace with the properties of the shallow layer gives accurate results. In other words, the properties of the soil deeper than 6m are not relevant for critical speed assessment, because the contribution of any additional soil stratum is negligible.

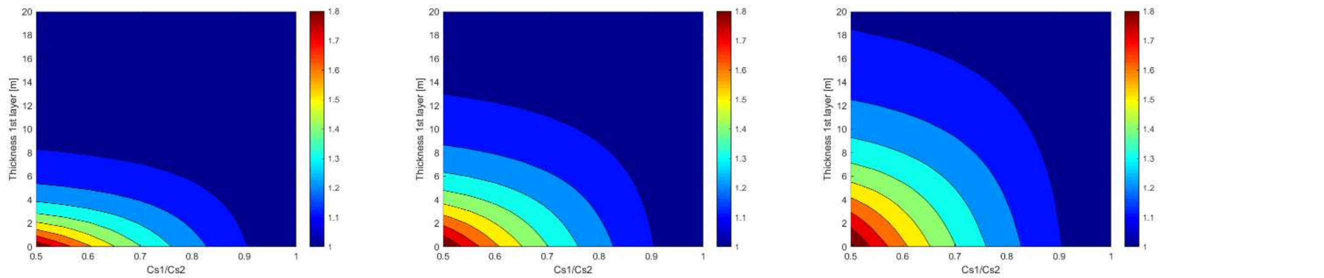

Figure 9 – Critical speed contours for tracks with different bending stiffness (C_{s1}=100 m/s): a) Track V1; b) Track V2; c) Track V3;;

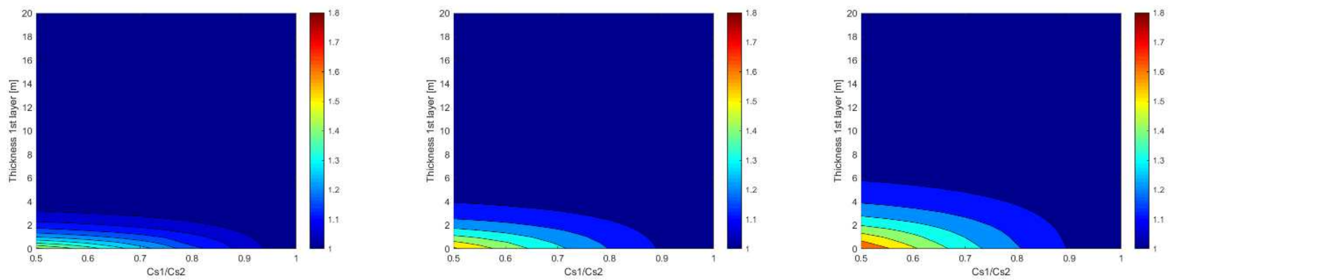

Figure 10 – Critical speed contours for tracks with different bending stiffness (C_{s1}=300 m/s): a) Track V1; b) Track V2; c) Track V3;

4. Implications for geotechnical survey

The prediction of critical speed requires knowledge of the ground properties in the immediate vicinity of the track, which should be obtained by suitable geotechnical testing. The most suitable geotechnical test that should be performed is dependent on the intended engineering analysis. Although trains running at critical speed can cause strain-induced non-linear behavior (as shown by Alves Costa et al. [10] and more recently by Dong et al. [28]), the computation of the critical speed must always be based on the low strain properties of the ground profile. Therefore, geophysical testing is the most suitable technique to obtain the required properties, namely the low-strain wave propagation velocities in the ground. Several kinds of tests can be used to obtain the required information, including cross-hole tests, down hole tests, up hole tests, SCPTU (Seismic cone penetration test) tests and SASW (spectral analysis of surface waves) tests. It should be highlighted that the geotechnical surveys required for the design and construction of a high-speed railway line are not limited to investigating dynamic behavior. Instead, a comprehensive range of laboratory and field tests should be used, depending upon the analyses required (e.g. additional tests would be required to determine ground-water fluctuation). However, this research is aimed solely at investigating low-strain dynamic ground behavior.

The selection of the most suitable test should consider the required depth of investigation. From the numerical results shown above, the ground depth up to which the critical speed is governed is dependent on the track stiffness and on the shear wave speed contrast between the upper and lower layers. Nevertheless, for typical in-situ conditions, (i.e., if the track bending stiffness is within the typical range found in practise), it is possible to conclude that if shear wave data is obtained with respect to the upper 8 meters of the ground, reasonable accuracy can be achieved during critical speed computation. In this case significant error will not occur if the soil properties below this depth are inaccurately estimated.

Although a variety of tests can be adopted for the geotechnical characterization of the surface layers, the most suitable is often the SASW test. This test has the advantage of avoiding boreholes, making it cheap and straightforward to perform. The SASW method is a non-intrusive geophysical technique used for the dynamic characterization of the ground and the procedure can be summarized in three main steps: i) field testing; ii) signal processing and dispersion curve evaluation; iii) inverse processing [29]. Despite its advantages, it has 2 main drawbacks:

i) the energy source is usually unable to excite the low frequency range, and therefore, the information that can be extracted by the test is confined to reduced depths (usually less than 10 m). This can be improved by dropping a large mass instead of using an instrumented hammer.

ii) the soil properties obtained are done so via an inversion procedure of the P-SV dispersion relationships, which can give rise to a non-unique solution.

However, when applied to the prediction of railway critical speed, both drawbacks are overcome because:

i) As shown in the previous numerical analysis, the problem is almost exclusively governed by the properties of the upper ground layers. Therefore having accurate information about the layers below 8m is unimportant

ii) taking into account the previously discussed simplified method for critical speed prediction, experimental P-SV dispersion relationships can be used directly without the need of inversion. Therefore the next section presents an example of this approach.

5. A hybrid methodology for critical speed assessment

As shown by Alves Costa et al. [4], the critical speed of the track-ground system can be approximated by the interpretation of the characteristics of wave propagation in the ground, assuming that both systems are decoupled. In this previous work, it is proposed that the ground dispersion relationships can be computed from the knowledge of the ground wave velocity profile. In geophysics this is performed via inversion, however, for critical speed analysis, it can be extracted directly from the experimental data obtained during the SASW procedure.

First an impulse force is applied on the ground surface and the transient signal is recorded using accelerometers placed in a straight line starting from the impulse location (Figure 11). In order maximize accuracy/resolution of the transfer function in the wavenumber domain, it is advised to acquire the response along a length as long as possible (e.g. ≈100 m) with a sensor spacing of 1 m. Considering that many commercial data acquisition units consist of 24 in-built channels, four 23m sensor setups can be performed separately and the results appended during post-processing.

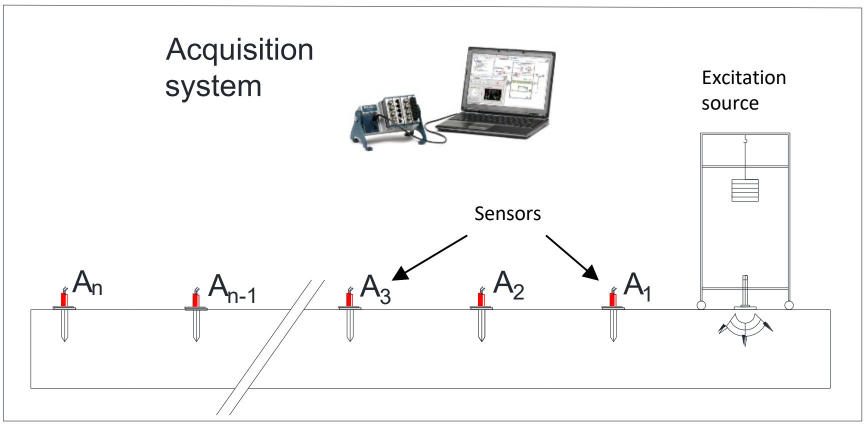

Figure 11 –Setup for the experimental assessment of the P-SV dispersion relationship.

Using the results, a transfer function between the free-field response (particle velocity or acceleration)(channel *n*) and the impulse applied (channel *m*) can be computed in frequency-space domain (where ω is the Fourier image of time). To minimize noise errors, several impacts (N) should be considered and the average transfer function $\widehat{\mathrm{H}}_{\mathrm{mn}}(\omega,\mathrm{r})$ computed using:

$$\widehat{\mathrm{H}}_{mn}(\omega,\mathrm{r}) = \frac{\hat{\mathrm{S}}_{nm}}{\hat{\mathrm{S}}_{mm}} \tag{2}$$

Where:

$$\hat{\mathrm{S}}_{nm}(\omega) = \frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}} \hat{\mathrm{x}}_{m}^{\mathrm{i}}(\omega)\, \hat{\mathrm{x}}_{\mathrm{n}}^{\mathrm{i}*}(\omega) \tag{3}$$

where r is the distance between receiver-source, $\hat{\mathrm{x}}_{\mathrm{m}}^{\mathrm{i}}$ is the frequency content of the signal recorded in channel m for impact i, and $\hat{\mathrm{x}}_{\mathrm{n}}^{\mathrm{i}*}$ is the complex conjugate of $\hat{\mathrm{x}}_{\mathrm{n}}^{\mathrm{i}}$.

When obtaining the P-SV ground dispersion relationship, the transfer functions must be defined in the wavenumber-frequency domain. Therefore a new transform operation, related to the change from the space to the wavenumber is required. This can be achieved using the procedure proposed by Forbriger for sparse data transformation operations [30]. This approach results in a frequency-wavenumber spectra, with the transfer function given by:

$$\widetilde{H}(k_r, \omega) = \int_0^{\infty} \widetilde{H}(r, \omega) J_0(k_r r) r \, dr \tag{4}$$

where J_0 is a Bessel function of order zero and k_r is the Fourier image of r.

Since the number of receivers is finite, with the farthest located at $r = r_M$, the integration domain can be truncated. Taking this into account and replacing the Bessel function from equation (4) with a Hankel function of order zero ($\frac{H_0^{(1)}}{2}$.), equation (4) takes following format:

$$\widetilde{H}(k_r, \omega) = \frac{1}{2} \int_0^{r_M} \widetilde{H}(r, \omega) H_0^{(1)}(k_r r) r \, dr \tag{5}$$

Finally, to improve the visualization of the spectra given by the function $\widetilde{H}(k_R, \omega)$, it is normalized:

$$\widetilde{H}^{Norm}(k_R, \omega) = \frac{\widetilde{H}(k_R, \omega)}{\max_{k_r} \left| \widetilde{H}(k_R, \omega) \right|} \tag{6}$$

The experimental P-SV ground dispersion curve is then given by the regions of the spectra with highest energy, i.e., for a fixed frequency ω, the dispersion relationship is given by the wavenumber k_r with greatest energy.

The spectra of the transfer function can also be also represented as function of wave propagation velocity, taking into account the following relationship:

$$C_R = \frac{\omega}{k_r} \tag{7}$$

As an example, Figure 12 shows the experimental spectra of the transfer function obtained from in-situ testing on a railway line in Portugal in terms of wave propagation velocity (data available from [31]). In this case the excitation source is a 130 kg mass from a height of 1 m. The excitation impulse produced a signal with strong coherence in the frequency range between 6 Hz and 120 Hz.

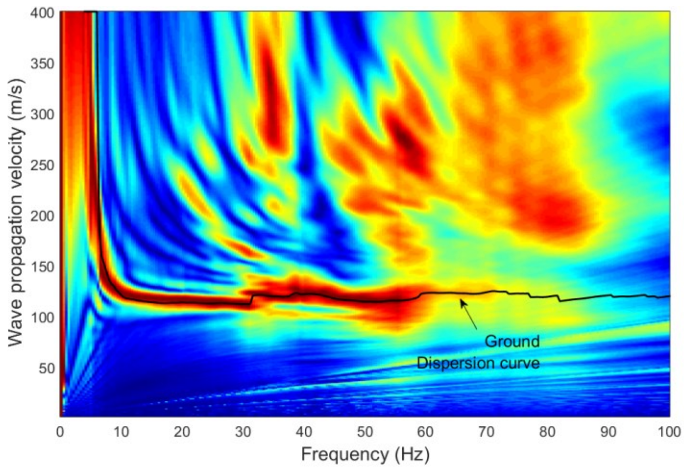

Figure 12 –Experimental P-SV ground dispersion relationship (data available from [31]).

In order to obtain the critical velocity, the dispersion relationship of the bending waves that propagate along the track must be computed. Since during the design stage the track is not yet constructed, this part of the problem can be solved analytically or numerally. This is a straightforward step where only the eigenvalues of the system of equation that govern the dynamic track response in free conditions require computation. This is achieved by finding, for each frequency ω, the wavenumbers that satisfy the following constraint:

$$\det\left(\left[K_{dyn}\left(k_r,\omega\right)\right]\right)=0 \tag{8}$$

where K_{dyn} is the dynamic stiffness matrix of the track.

For a slab track, this matrix can be written as follows:

$$\left[K_{dyn}\left(k_r,\omega\right)\right]=\begin{bmatrix} EI_r k_r^4 + k_p^* - \omega^2 m_r & -k_p^* \\ -k_p^* & k_p^* + EI_s k_r^4 - \omega^2 m_s \end{bmatrix} \tag{9}$$

where kp* is the complex stiffness of the railpads (kp*= kp +iωc).

The analytical dispersion curve of the bending waves in the track for tracks V1 and V2 (Table 1) are plotted in Figure 13. The results are presented in the frequency-wave propagation velocity domain and overlaid on the spectra of the ground transfer function.

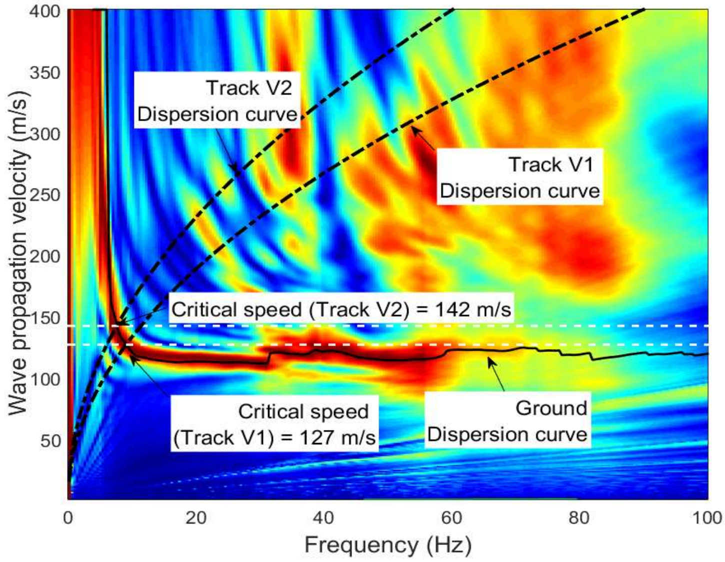

Figure 13 – Prediction of the critical speed based in the hybrid approach.

As discussed, the critical speed is given by the wave propagation velocity where both dispersion curves intersect. For this ground-track combination the critical velocity is 142 m/s for track V2 while it is 127 m/s for track V1. Since the assessment is performed directly using on-site experimental data, the uncertainties and potential errors associated with the inversion of the experimental data are avoided. Moreover, the method can be easily applied for the assessment of the impact of the track properties on the track-ground critical speed.

Regarding practical application, the method can be directly applied to situations with low height embankments, i.e., up to 2m in agreement with the studies performed by Colaço and Alves Costa [26]. However, if the embankment is tall and thus comparable to the thickness of the ground layers, it is recommended to assess the SASW test at the upper embankment surface to take into account its influence. The method can also be very useful when renewing existing tracks, with the aim of increasing traffic speed. In such cases, the experimental tests can be performed on the existing embankment in order to estimate the critical speed.

A possible shortcoming of the proposed method is when dealing with inversely dispersive P-SV waves in the ground (i.e., when ground stiffness decreases over ground depth). As shown by Alves Costa et al. [4] the accuracy of the simplified method is lower when dealing with these type of situations. Nevertheless, the majority of soils increase in stiffness with increasing of confining pressure (i.e. with depth from the ground surface). Therefore, the proposed approach is applicable for the most common scenarios where high track dynamics are issue.

6. Conclusions

In this paper, analysis was performed to investigate the maximum depth to which geotechnical surveys should be performed, in order to accurately predict railway track critical speed. The study was conducted using a numerical and an analytical approach. These were the 'direct' method, based on a 2.5D FEM-PML approach, and a 'simplified' method, where the critical speed was estimated through the interpretation of track-ground dispersion relationships. Firstly, a comparison between model outputs was performed and it was concluded that both yielded similar results. Therefore, the simplified approach was used to develop interpretation charts related to the impact of the thickness of the ground's uppermost layer on the critical speed.

It was found that the required depth of ground investigation is dependent on the stiffness of the upper soil layers and on the track bending stiffness. Assuming ground stiffness increases with depth, it was shown that deeper geotechnical investigation is required when the ground is soft. Similarly, when the track bending stiffness is high, wave propagation is deeper meaning deeper geotechnical investigation is required. Nevertheless, it was found that, for the mostly commonly encountered field situations, the critical speed is most greatly affected by the soil properties within the top 8m. Therefore, geotechnical investigation should be designed to give accurate soil properties to this depth. If the ground properties below this limit are known only with low confidence, a reasonable estimation of the critical speed is still achievable. This conclusion is useful for the design of new railway lines, allowing time/financial savings regarding geotechnical survey.

Based on the finding that the impact of soil properties on the critical speed is dominated by the properties of the top 8m, a hybrid experimental-analytical approach was proposed for critical speed assessment. This procedure can be summarized in the following step: i) the experimental P-SV dispersion curves are assessed using an experimental procedure. This test is based on the interpretation of the wavenumber-frequency domain transfer function of the vertical ground surface response induced by a point load; ii) the bending wave propagation dispersion curve of the track is computed analytically; iii) the critical speed is obtained by finding the interception point of both curves in a frequency-wave propagation velocity spectra. The hybrid approach can be very useful in the design stage of new high speed railway lines, since the procedure is straightforward to conduct and the experimental data can be obtained directly from SASW tests without inversion.

ACKNOLEDGMENTS

This research was financially supported by: Project POCI-01-0145-FEDER-007457–CONSTRUCT–Institute of R&D InStructures and Construction funded by FEDER funds through COMPETE2020–Programa Operacional Competitividade e Internacionalização (POCI)–and by national funds through FCT–Fundação para a Ciência e a Tecnologia;Project POCI-01-0145-FEDER-029577–funded by FEDER funds through COMPETE2020 – Programa Operacional Competitividade e Internacionalização (POCI), national funds (PIDDAC) through FCT and the Leverhulme Trust (PLP-2016-270)

The financial support provided by University of Leeds Cheney Award Scheme is also appreciated.

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