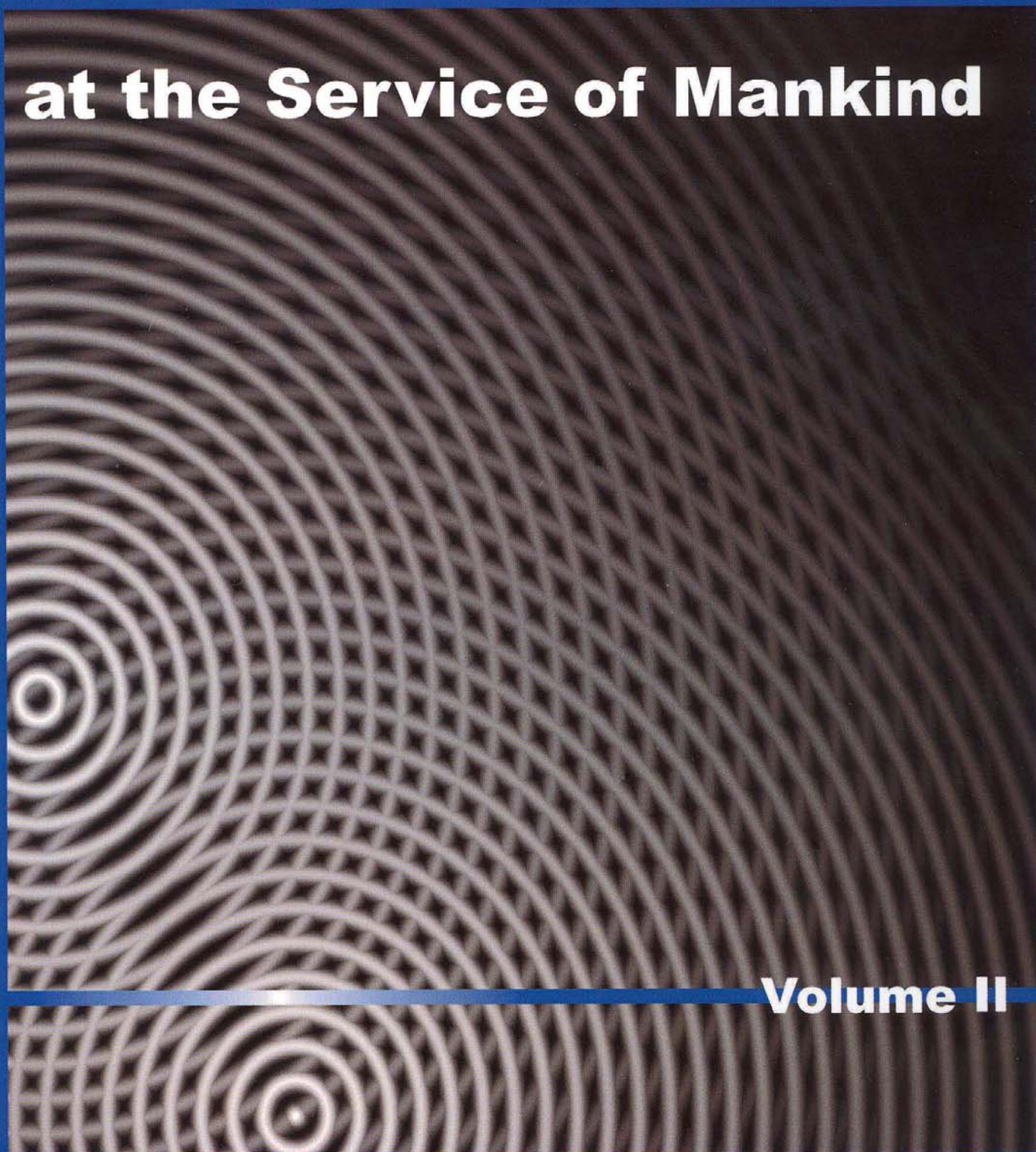


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at the Service of Mankind



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Modelling and Control of Freeway Traffic

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Abstract — *This paper presents the most recent developments of the Simulator of Intelligent Transportation Systems (SITS). The SITS is based on a microscopic simulation approach to reproduce real traffic conditions in an urban or non-urban network. In order to analyse the quality of the microscopic traffic simulator SITS a benchmark test was performed. A dynamical analysis of several traffic phenomena, applying a new modelling formalism based on the embedding of statistics and Laplace transform, is then addressed. The paper presents also a new traffic control concept applied to a freeway traffic system.*

1 Introduction

The difficulties concerned with the saturation of the transportation infrastructures due to the growing number of vehicles over the last five decades, motivated the research community to focus their attention in the area of ITS (Intelligent Transportation Systems). This research studies the technologies and the scientific aspects with the purpose of developing new systems capable of solving some of the most relevant problems, such as traffic congestion, accidents, transportation delays and large vehicle pollution emissions. ITS depend on results from research activities spread over many different areas such as electronics, control, communications, sensing, robotics, and information systems. This multidisciplinary nature increases the problem's complexity because it requires knowledge transfer and cooperation among different research areas [1] [2] [3].

Computer simulation has become a common tool in the evaluation and development of ITS. The advantages of this tool are obvious. The simulation models can satisfy a wide range of requirements, such as: evaluating alternative treatments, testing new designs, training personal and analyzing safety aspects [4].

The traffic simulation models can be classified according to various criteria, namely, the scale of independent variables, the representation of the processes and levels of detail [5]. Presently, most traffic system simulation applications are microscopic in nature and based on the simulation of vehicle-vehicle interactions [6].

The main modelling components of a microscopic traffic simulation model are: an accurate representation of the road network geometry, a detailed modelling of individual vehicles behaviour and an explicit reproduction of traffic control plans. The recent evolution of the microscopic simulators has taken advantages of the state-of-the-art in the development of object-oriented simulators and graphical user interfaces [7] [8].

Bearing these facts in mind, this paper is organized as follows. Section 2 describes the new developments of the microsimulation model SITS. Section 3 analyses the quality of the microscopic traffic simulator SITS. Section 4 and 5 present simulation results related with the dynamic behaviour and control of a traffic system. Finally, section 6 presents some conclusions and outlines the perspectives towards future research.

2 The SITS Simulation Package

SITS is a software tool based on a microscopic simulation approach, which reproduces real traffic conditions. The program provides a detailed modelling of the traffic network, distinguishing between different types of vehicles and drivers and considering a wide range of network geometries. SITS uses a flexible structure that allows the integration of simulation facilities for any of the ITS related areas. This new simulation package is an object-oriented implementation written in C++. The overall model structure is represented on Figure 1.

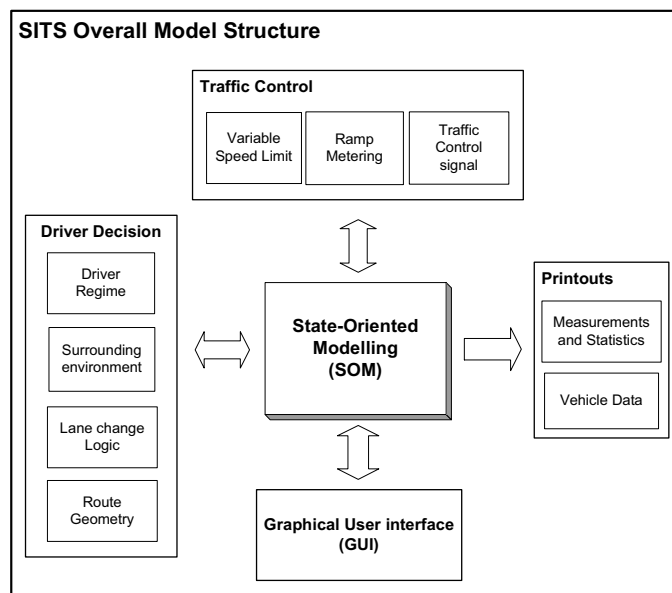


Figure 1: SITS overall Model Structure

SITS models each vehicle as a separate entity in the network according to the state diagram showing in Figure 2. Therefore, there are defined five states {1-aceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision} that represent the possible vehicle states in a traffic systems [9].

In this modelling structure, so called State-Oriented Modelling (*SOM*), every single vehicle in the network has one possible state for each sampling period. The transition between each state depends on the driver behaviour model and its surrounding environment.

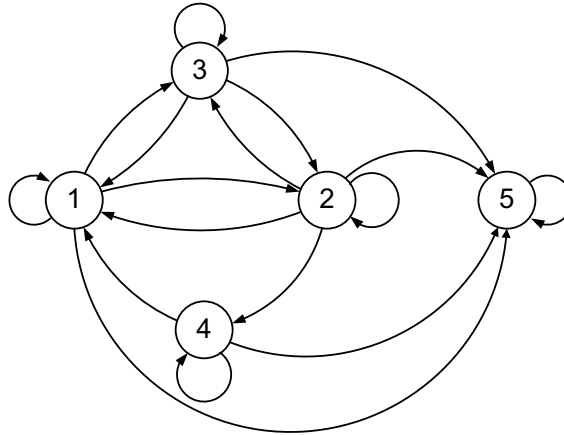


Figure 2: SITS state diagram: 1-acceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision

Some transitions are not possible; for instance, it is not possible to move from state #4 (stopped) to state #2 (braking), although it is possible to move from state #2 to state #4.

Included on the most important elements of SITS are the network components, travel demand, and driving decisions. Network components include the road network geometry, vehicles and the traffic control. To each driver is assigned a set of attributes that describe the drivers behaviour, including desired speed, and his profile (*e.g.*, from conservative to aggressive). Likewise, vehicles have their own specifications, including size and acceleration capabilities. Travel demand is simulated using origin destination matrices given as an input to the model.

At this stage of development the SITS considers different types of driver behaviour models, namely car following, free flow and lane changing logic. SITS considers each vehicle in the network to be in one of two driver regimes: free flow and car-following. The free flow regime prevails when there is either (*i*) no lead vehicle in front of the subject vehicle or (*ii*) the leading vehicle is sufficiently far ahead that it does not influence the subject vehicles behaviour. In the free flow case the driver travels at his desired maximum speed. Car-following regime dictates acceleration/deceleration decisions when a leading vehicle is near enough to the subject vehicle in order to maintain a safe following distance. Accelerations and decelerations are simulated using the Perception-Driver Model (*PDM*). According with the *PDM*, the driver decides to decelerate/accelerate depending on two factors: the difference between the distance to the leading vehicle and the critical distance, and his active state. The critical distance $d_{c,n}$ is defined as follows:

$$d_{c,n} = d_{sb,n} + d_{f,n} + L_{n+1} \quad (1)$$

where: $d_{sb,n}$ is the safety braking distance for the vehicle n , given by equation (2), $d_{f,n}$ is the following distance for the vehicle n , given by equation (5) and L_{n+1} is the length of the leading vehicle.

Figure 3 shows a schema of the critical distance for the n^{th} vehicle (assuming that the traffic conditions for both vehicles remain constant between time instants t_0 to t_1).

The safety braking distance $d_{sb,n}$ is given by:

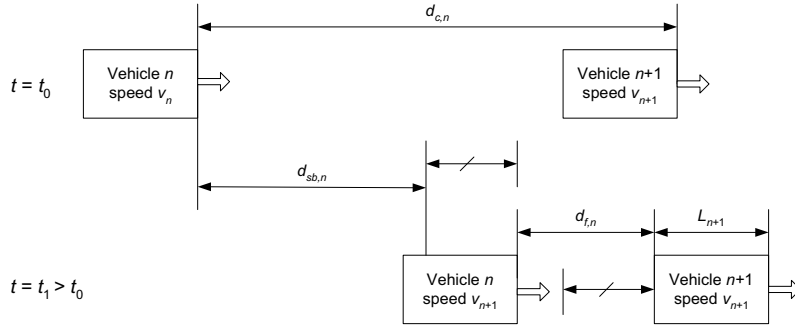


Figure 3: Critical distance scheme

$$d_{sb,n} = -\frac{(v_{n+1} - v_n)^2}{2(a'_n - s_{n+1})} \quad (2)$$

where: v_n is the current speed of vehicle n , v_{n+1} is the current speed of leading vehicle $n+1$, a'_n is the deceleration of vehicle n given by equation (3) and s_{n+1} is the deceleration/acceleration of the leading vehicle $n+1$, given by equation (3) or (4) depending on his current state.

The driver reduces the speed by applying a deceleration a'_n . The model relates the vehicle performances with the driver characteristics.

$$a'_n = a'_{\max,c} \gamma_d \quad (3)$$

where: $a'_{\max,c}$ is the maximum deceleration for a vehicle of type c and γ_d is a parameter for driver type d ($0.1 < \gamma_d < 1.0$).

The value of γ_d can be changed at any time in order to prevent a collision. This parameter defines the driver profile (e.g., from conservative $\gamma_d = 0.1$ up to aggressive $\gamma_d = 1.0$).

The value of the deceleration/acceleration s_{n+1} depends on the state of the leading vehicle. If the vehicle is in state #2 then s_{n+1} is given by equation (3); otherwise if it is in state #1, s_{n+1} is given by equation (4). Therefore, $s_{n+1} = 0$ only when the vehicle is in one of the other states.

$$s_{n+1} = a_{\max,c} \gamma_d \quad (4)$$

where: $a_{\max,c}$ is the maximum acceleration for a vehicle of type c .

The following distance d_f depends on the speed of vehicle n and the associated driver profile, yielding:

$$d_{f,n} = v_n^2 \gamma_d \quad (5)$$

The lane changing model in SITS uses a methodology that tries to mimic a driver behaviour when producing a lane change. This methodology was implemented in three steps: (i) decision to consider a lane change; (ii) selection of a desired lane; (iii) execution of the desired lane change if the gap distances are acceptable. A driver produces a lane change maneuver in order to increase speed, to overtake a slower vehicle or to avoid the lane connected to a ramp. After selecting a lane, the driver examines the lead g_b and lag g_a gaps

in the target lane in order to determine if the desired change can be executed, as shown in Figure 4.

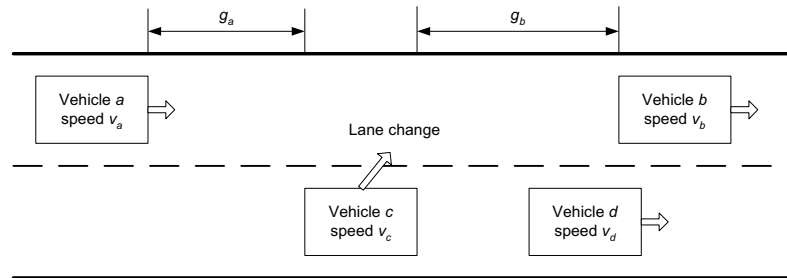


Figure 4: Lead g_b and lag g_a gaps for a lane change maneuver of vehicle n

If g_a and g_b are higher than the critical distances between vehicles a and c , and c and b , respectively, then the desired lane change is executed in a single simulation sampling interval Δt .

The simulation model adopted in the SITS is a stochastic one. Some of the processes include random variables such as, individual vehicle speed and input flow. These values are generated randomly according to a pre-defined amplitude interval.

The main types of input data to the simulator are the network description, the drivers and vehicles specifications and the traffic conditions. The output of SITS consists not only in a continuously animated graphical representation of the traffic network but also the data gathered by the detectors, originating different types of printouts.

3 Model Calibration and Testing

The research group of Robert Bosh GmbH developed in 1998 a benchmark to analyze the quality of microscopic simulators by checking their ability to reproduce a macroscopic behaviour [10]. More recently, this benchmark was also used to evaluate the performance of the AIMSUN simulator [11].

In [10] the authors test the macroscopic behaviour of a microscopic model by simulating the traffic on a cyclic one lane road with a length of $l = 1000$ m. A fixed number N of identical vehicles (4.5 m length) is set with a initial speed of $v = 0$ km/h, at randomly positions, having a limit free flow speed of $v = 54$ km/h. After elapsing the starting transient the steady-state traffic behaviour is recorded (the exact passing time and the speed value of each vehicle) at one measurement point during a period of 2 hours. The benchmark procedure consists on varying the number N of vehicles to accomplish different traffic densities Q and the corresponding traffic flow $\phi(Q)$.

Having this benchmark in mind, Figure 5 plots the traffic flow ϕ versus its density Q for the empirical (macroscopic), the SITS and the AIMSUN [11] simulators.

The output of SITS is clearly in accordance with the expected results. Moreover we have a maximum traffic flow of about 1800-2000 veh/km, which is known as a realistic value for long periods of measurement time.

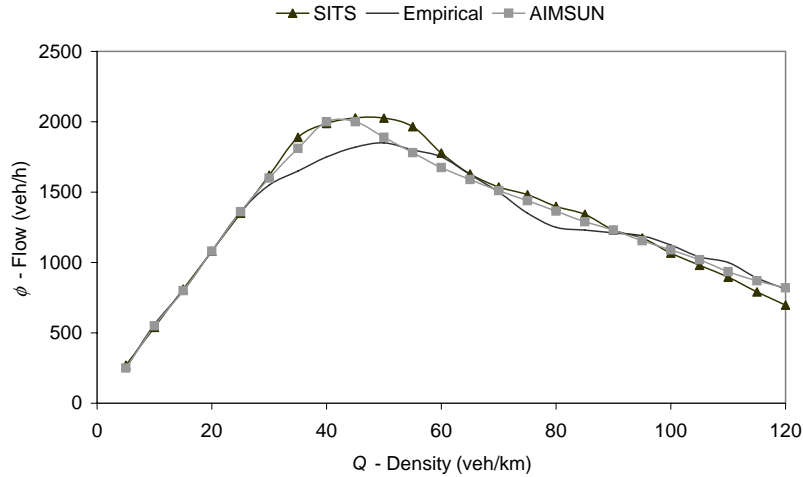


Figure 5: Empirical (macroscopic) versus SITS and AIMSUN simulated flow density curves

4 Dynamical Analysis

In the dynamical analysis, tools of systems theory are applied. In this line of thought, a set of simulation experiments are developed in order to estimate the influence of the vehicle speed $v(t;x)$, the road length l and the number of lanes n_l in the traffic flow $\phi(t;x)$ at time t and road coordinate x . For a road with n_l lanes the Transfer Function (*TF*) between the flow measured by two sensors is calculated by the expression:

$$G_{r,k}(s; x_j, x_i) = \Phi_r(s; x_j) / \Phi_k(s; x_i) \quad (6)$$

where $k, r = 1, 2, \dots, n_l$ define the lane number and, x_i and x_j represent the road coordinates ($0 \leq x_i \leq x_j \leq l$), respectively. The Laplace transform for each traffic flow is:

$$\Phi_r(s; x_j) = L\{\phi_r(t; x_j)\} \quad (7a)$$

$$\Phi_k(s; x_i) = L\{\phi_k(t; x_i)\} \quad (7b)$$

It should be noted that traffic flow is a time variant system but, in the sequel, it is shown that the Laplace transform can be used to analyse the system dynamics.

The first group of experiments considers a one-lane road (*i.e.*, $k = r = 1$) with length $l = 1000$ m. Across the road are placed n_s sensors equally spaced. The first sensor is placed at the beginning of the road (*i.e.*, at $x_i = 0$) and the last sensor at the end (*i.e.*, at $x_j = l$). Therefore, we calculate the *TF* between two traffic flows at the beginning and the end of the road such that, $\phi_1(t; 0) \in [0.12, 1]$ vehicles s^{-1} for vehicle speed $v_1(t; 0) \in [30, 70]$ km h^{-1} that is, for $v_1(t; 0) \in [v_{av} - \Delta v, v_{av} + \Delta v]$, where $v_{av} = 50$ km h^{-1} is the average vehicle speed and $\Delta v = 20$ km h^{-1} is the maximum speed variation. These values are generated according to a uniform probability distribution function.

The results obtained of the polar plot for the *TF* $G_{1,1}(s; 1000, 0) = \Phi_1(s; 1000) / \Phi_1(s; 0)$ between the traffic flow at the beginning and end of the one-lane road is distinct from those usual in systems theory revealing a large variability, as revealed by Figure 6a). Moreover,

due to the stochastic nature of the phenomena involved different experiments using the same input range parameters result in different TF s.

In fact traffic flow is a complex system but it was shown [12] that, by embedding statistics and Laplace transform (leading to the concept of Statistical Transfer Function (STF)) [13], we could analyse the system dynamics in the perspective of systems theory. To illustrate the proposed modelling concept (STF), the simulation was repeated for a sample of $n = 2000$ and it was observed the existence of a convergence of the STF , $T_{1,1}(s; 1000, 0)$, as show in Figure 6b), for a one-lane road with length $l = 1000$ m $\phi_1(t; 0) \in [0.12, 1]$ vehicles s^{-1} and $v_1(t; 0) \in [30, 70]$ km h^{-1} .

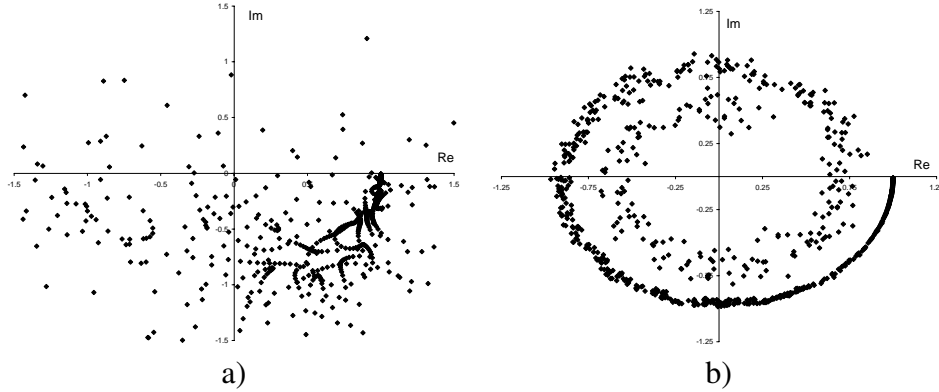


Figure 6: Polar diagram of a) TF for $n = 1$ experiment and b) STF for $n = 2000$ experiments, with $\phi_1(t; 0) \in [0.12, 1]$ vehicles s^{-1} and $v_1(t; 0) \in [30, 70]$ km h^{-1} ($v_{av} = 50$ km h^{-1} , $\Delta v = 20$ km h^{-1} , $l = 1000$ m and $n_l = 1$)

The chart has characteristics similar to those of a low-pass filter with time delay, common in systems involving transport phenomena. Nevertheless, in our case we need to include the capability of adjusting the description to the continuous variation of the system working conditions. This requirement precludes the adoption of the usual integer-order low-pass filter and points out the need for the adoption of a fractional-order TF . Therefore, in this case we adopt a fractional-order system [14] with time delay:

$$T_{1,1}(s; 1000, 0) = \frac{k_B e^{-\tau s}}{\left(\frac{s}{p} + 1\right)^\alpha} \quad (8)$$

With this description we get not only a superior adjustment of the numerical data, impossible with the discrete steps in the case of integer-order TF , but also a mathematical tool more adapted to the dynamical phenomena involved. For fitting (8) with the numerical data it is adopted a two-step method based on the minimization of the quadratic error. In the first phase (k_B , p , α) are obtained through error amplitude minimization of the Bode diagram. Once established (k_B , p , α), in a second phase, τ is estimated through the error minimization in the Polar diagram. For the numerical parameters of Figure 6b) we get $k_B = 1.0$, $\tau = 96.0$ sec, $p = 0.07$ and $\alpha = 1.5$.

The parameters (τ , p , α) vary with the average speed v_{av} and its range of variation Δv , the road length l and the input vehicle flow ϕ_1 . For example, Figure 7 shows (τ , p , α) versus Δv (with $v_{av} = 50$ km h^{-1}) and v_{av} , (with $\Delta v = 20$ km h^{-1}).

It is interesting to note in Figure 7a) that $(\tau, p) \rightarrow (\infty, 0)$, when $\Delta v \rightarrow v_{av}$, and $(\tau, p) \rightarrow (l v_{av}^{-1}, \infty)$, when $\Delta v \rightarrow 0$. These results are consistent with our experience that suggests a pure transport delay $T(s) \approx e^{\tau s}$ ($\tau = l v_{av}^{-1}$), $\Delta v \rightarrow 0$ and $T(s) \approx 0$, when $\Delta v \rightarrow v_{av}$ (because of the existence of a blocking cars, with zero speed, on the road). In the case of Figure 7b) we have $(\tau, p) \rightarrow (\infty, 0)$, when $v_{av} \rightarrow \Delta v$, and $(\tau, p) \rightarrow (0, \infty)$, when $v_{av} \rightarrow \infty$, which has a similar intuitive interpretation.

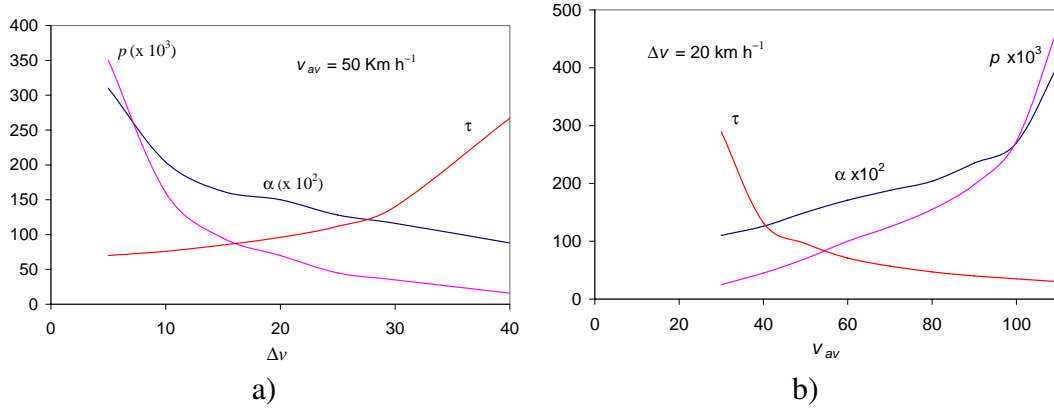


Figure 7: Parameters τ , p and α with $n_l = 1$, $l = 1000$ m and $\phi_1(t; 0) \in [0.12, 1]$ vehicles s^{-1} versus a) Δv (with $v_{av} = 50 \text{ km h}^{-1}$) and b) v_{av} (with $\Delta v = 20 \text{ km h}^{-1}$)

In a second group of experiments are analyzed the characteristics of the *STF* matrix for roads with several lanes considering identical traffic conditions (*i.e.*, $\phi_k(t; 0) \in [0.12, 1]$ vehicles s^{-1} , $k = 1, 2$, $l = 1000$, $\Delta v = 20 \text{ km h}^{-1}$). Figure 8a) depicts the amplitude Bode diagram of $T_{1,1}(s; 1000, 0)$ and $T_{1,2}(s; 1000, 0)$ for $v_{av} = 50 \text{ km h}^{-1}$ (*i.e.*, $v_k(t; 0) \in [30, 70] \text{ km h}^{-1}$).

Figure 8b) presents the amplitude Bode diagram of $T_{1,1}(s; 1000, 0)$ and $T_{1,2}(s; 1000, 0)$ for $v_{av} = 90 \text{ km h}^{-1}$ (*i.e.*, $v_k(t; 0) \in [70, 110] \text{ km h}^{-1}$).

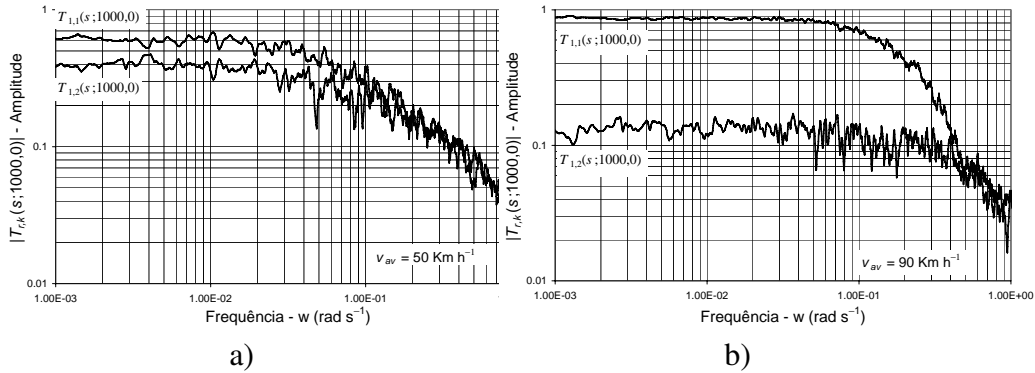


Figure 8: Bode diagram of $T_{r,k}(s; 1000, 0)$ for a) $v_{av} = 50 \text{ km h}^{-1}$ and b) $v_{av} = 90 \text{ km h}^{-1}$, $n_l = 2$, $l = 1000$ m, $\phi_k(t; 0) \in [0.12, 1]$ vehicles s^{-1} , $\Delta v = 20 \text{ km h}^{-1}$, $k = 1, 2$

We verify that $T_{1,1}(s; 1000, 0) \approx T_{2,2}(s; 1000, 0)$ and $T_{1,2}(s; 1000, 0) \approx T_{2,1}(s; 1000, 0)$. This property occurs because SITS uses a lane change logic where, after the overtaking,

the vehicle tries to return to the previous lane. Therefore, lanes 1 and 2 have the same characteristics leading to identical *STF*.

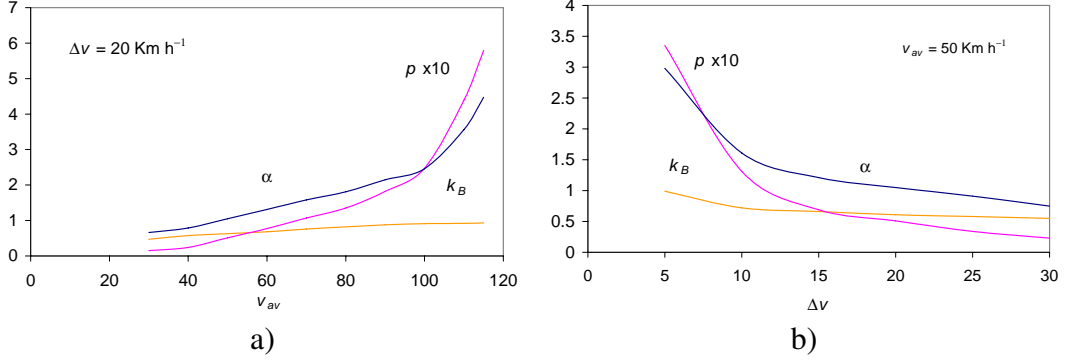


Figure 9: Parameters (k_B , p , α) versus a) v_{av} and b) Δv , for $T_{1,1}(s; 1000, 0)$ with $n_l = 2$, $l = 1000 \text{ m}$ and $\phi_1(t; 0) \in [0.12, 1]$ vehicles s^{-1}

Comparing Figure 8a) and these results, we conclude that the transfer matrix elements vary significantly with v_{av} . Moreover, the *STF* parameter dependence is similar to the one-lane case represented previously. Figure 9a) and Figure 9b) show the variation of parameters (k_B , p , α) for $T_{1,1}(s; 1000, 0)$ versus v_{av} (with $\Delta v = 20 \text{ km h}^{-1}$) and Δv (with $v_{av} = 50 \text{ km h}^{-1}$), respectively, for $n_l = 2$.

We conclude that:

- i. The time delay τ is independent of the number of lanes n_l , because in these experiments the lanes have all the same input flow $\phi_k(t; 0) \in [0.12, 1]$ vehicles s^{-1} .
- ii. For a fixed set of parameters we have for each *STF* gain \times bandwidth \approx constant.
- iii. For each row of the transfer matrix, the sum of the *STF* gains is the unit.
- iv. The gains and the poles of the diagonal elements of the *STF* matrix are similar. The gain of the non-diagonal elements, that represent dynamic coupling between the lanes, are lower (due to iii), but the corresponding poles are higher (due to ii).
- v. The fractional order α increases with v_{av} . Nevertheless, the higher the number of lanes the lower the low-pass filter effect, that is, the smaller the value of α .

5 Traffic Control

Based on the previous dynamic description, we study a new traffic control concept, as shown in Figure 10. In this perspective, it is adopted a Variable Speed Limit Indicator (VSLI) to control the vehicle speed. The reference speed v_{ref} is displayed by the VSLI, at a given position x_{VSLI} , while a Feedforward Sensor (FS) is placed at distance x_{FS} ahead of the VSLI. The reference speed is given by:

$$v_{ref} = \rho v_{max} + (1 - \rho)v_{average}, \quad 0 < \rho < 1 \quad (9)$$

where: v_{max} is the maximum speed allowed in the lane and $v_{average}$ is the average traffic speed of the FS readings.

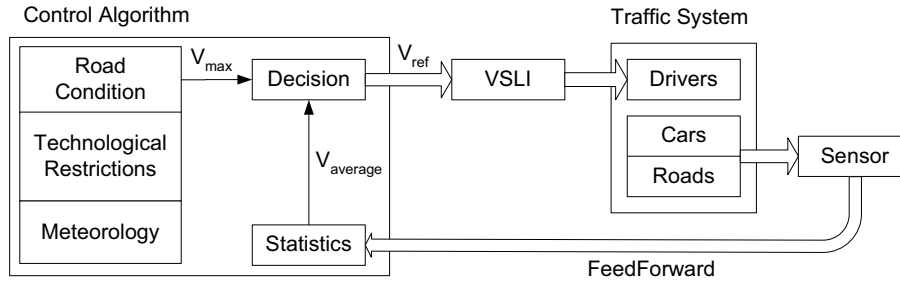


Figure 10: Overall traffic control concept

Therefore, in the perspective of control theory v_{ref} consists of the ‘reference signal’, $v_{average}$ is the ‘feedforward’ and equation (9) is the ‘controller’. In the experiments we have $x_{VSLI} = 205$ m, $x_{FS} = 5$ m, $l = 1000$ m, $n_l = 1$, $v_{max} = 100$ km h⁻¹, $\rho = 0.5$ and the speed limit displayed by the VSLI is computed with a sampling interval $T_s = 100$ sec.

The dynamics of the closed-loop system (*i.e.*, with controller (9)) differs somehow from the previous open-loop case as can be verified in Figure 11. An analytical expression for the *STF*, fitting closely the resulting data, requires a large number of poles and zeros. In order to simplify the comparison of the open and close loop dynamics, in the sequel it is adopted expression (8), since we can still get a reasonable curve fitting.

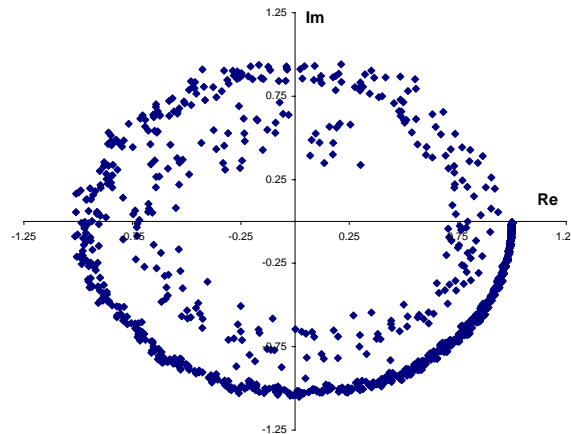


Figure 11: Polar diagram of the *STF* $T_{1,1}(s; 1000, 0)$ for a closed-loop case with $\phi_1(t; 0) \in [0.12, 1]$ vehicles s⁻¹, $v_1(t; 0) \in [30, 70]$ km h⁻¹ ($v_{av} = 50$ km h⁻¹, $\Delta v = 20$ km h⁻¹, $l = 1000$ m, $n_l = 1$)

Figure 12 depicts the variation of the *STF* parameters (τ , p , α) *versus* v_{av} , for the open and closed loop cases, with $\Delta v = 20$ km h⁻¹ and $\phi_1(t; 0) \in [0.12, 1]$ vehicles s⁻¹ it can be observed that reducing v_{av} , for the closed-loop case, yields:

- i.* The time delay τ remains almost constant which is justified by the VSLI control effect.
- ii.* The pole p increases corresponding to a larger bandwidth.
- iii.* The variation of α seems related with the elimination of noise associated with uncontrolled (*i.e.*, with large Δv) traffic. Nevertheless, a clear understanding of the

complete phenomena is still under research.

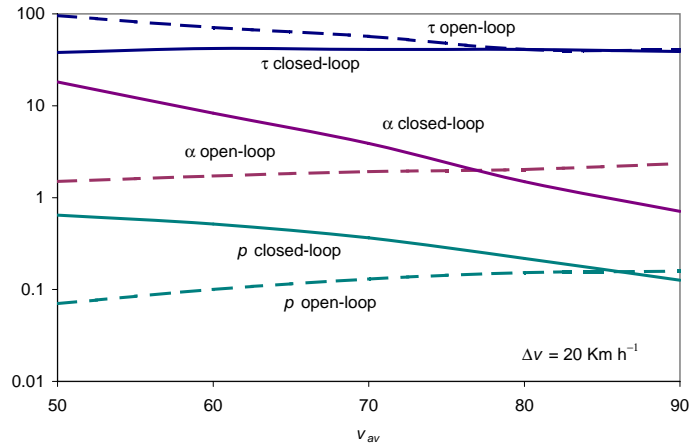


Figure 12: Parameters τ , p and α versus v_{av} for the open-loop and the closed-loop cases, with $\Delta v = 20 \text{ km h}^{-1}$, $n_l = 1$, $l = 1000 \text{ m}$ and $\phi_1(t; 0) \in [0.12, 1] \text{ vehicles s}^{-1}$

6 Conclusions

In this paper was described a software tool based on a microscopic simulation approach, to reproduce real traffic conditions in an urban or non-urban network. At this stage of development the SITS considers different types of driver behaviour model, namely car following, free flow, and lane changing logic. Several experiments were carried out in order to analyse the dynamics of the traffic systems. In this perspective it was adopted a formalism based on the tools of systems theory. Moreover, the new dynamic description integrated the concepts of fractional calculus lead to a more natural treatment of the continuum of the TF parameters intrinsic in this system. Motivated by the dynamical analysis a new control algorithm was also developed and its performance was analysed. The results pointed out that it is possible to study traffic systems, including the knowledge gathered with automatic control algorithms.

References

- [1] L. Figueiredo, I. Jesus, J. Machado, J. Ferreira, and J. Santos. Towards the development of intelligent transportation systems. In *Proceedings of the Fourth IEEE Intelligent Transportation Systems Conference*, pages 1207–1212, 2001.
- [2] J. Sussman. *Introduction to Transportation Systems*. Artech House, 2000.
- [3] S. Ghosh and T. Lee. *Intelligent Transportation Systems - New Principles and Architectures*. CRC Press, 2000.
- [4] Bob McQueen and Judy McQueen. *Intelligent Transportation Systems Architectures*. Artech House, 1999.
- [5] E. Lieberman and Ajay K. Rathi. *Traffic flow theory*. Oak Ridge National Laboratory, 1997.
- [6] Matti Pursula. Simulation of traffic systems - an overview. *Journal of Geographic Information and Decision Analysis*, pages 1–8, 1999.
- [7] D. Gerlough and M. Huber. *Traffic flow theory - A monograph*. TRB Report 165, 1975.

- [8] J.F. Gabbard. *Car-Following Models*. Concise Encyclopedia of Traffic and Transportation Systems, Pergamon Press, 1991.
- [9] L. Figueiredo, J. Machado, and J. Ferreira. Dynamical analysis of freeway traffic. *IEEE Transactions on Intelligent Transportation Systems*, pages 259–266, 2004.
- [10] D. Manstetten, W. Krautter, and T. Schwab. Traffic simulation supporting urban control system development. In *Proceedings of the 5th World Conference on Intelligent Transportation Systems*, 1998.
- [11] J. Barcelo and J. Casas. Dynamic network simulation with aimsun. In *Proceedings of International Symposium on Transport Simulation*, 2002.
- [12] L. Figueiredo, J. Machado, and J. Ferreira. On the dynamics analysis of freeway traffic. In *Proceedings of the sixth IEEE Intelligent Transportation Systems Conference*, pages 358–363, 2003.
- [13] J. A. Tenreiro Machado and Alexandra M. S. F. Galhano. A statistical perspective to the fourier analysis of mechanical manipulators. *Journal Systems Analysis-Modelling-Simulation*, pages 373–384, 1998.
- [14] J. A. Tenreiro Machado. A probabilistic interpretation of the fractional-order differentiation. *FCAA - Journal of Fractional Calculus Applied Analysis*, pages 73–80, 2003.

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