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THE STATISTICAL STUDY OF ROBOT MANIPULATORS

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Abstract

A new approach to the analysis and design of robot manipulators is presented. The novel feature resides on a non-standard formulation to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, in general, require laborious computations. This motivates the need of alternative models based on other mathematical concepts. The proposed statistical method gives clear guidelines towards the robot kinematic and dynamic optimization. Furthermore, the results point out structural characteristics of the trajectory planning algorithm as well as ideal-actuator properties.

Introduction

Mechanical manipulators are developed according to engineering and scientific principles which are based on fundamental concepts such as those arising from mathematics and physics. Based on these formulations, the first step on the study of a physical phenomena is the development of an adequate model. Manipulators are a typical example where we have for fundamental concepts the differential and matrix calculus and the classical newtonian physics, while the model corresponds to the standard kinematic and dynamic descriptions. Nevertheless, other classes of phenomena such as quantum physics and thermodynamics are studied using different concepts. Quantum physics requires the use of statistical methods while thermodynamics can be studied both through classical and statistical methods. These facts suggest that, for a given problem, we may develop different models, each with its own merits and drawbacks. Therefore, the classical approach may be not the unique modelling perspective. This paper presents a framework where these problems are addressed. We develop a new modelling formalism based on statistical concepts. These concepts are then illustrated on a simple mechanical joint-actuated manipulator. This example reveals not only the capabilities of the new method but also the limitations of usual robot structures. In order to develop the new method we organize this paper as follows.

In section two we formulate the fundamental modelling concepts. Section three illustrates the application of the new method to the kinematics and dynamics of mechanical joint-actuated robots. Finally, in section four conclusions are drawn.

On the Statistical Modelling of Mechanical Manipulators

The classical modelling of mechanical manipulators is well known. For a n degrees of freedom (d.o.f.) robot the kinematics is described by a set of equations relating the joint space and the operational space, of the form

$$q=\alpha(p) \tag{1a}$$
$$\dot{q}=\theta(p,\dot{p}) \tag{1b}$$
$$\ddot{q}=\Phi(p,\dot{p},\ddot{p}) \tag{1c}$$

where $\{p,\dot{p},\ddot{p}\}$ and $(\{q,\dot{q},\ddot{q}\})$ are the n-dimensional vectors of position, velocity and acceleration in the operational (joint) space. Associated with the kinematic model we have the statics model, that relates the operational space forces Γ with the joint actuator torques T:

$$T=J(q)^T\,\Gamma \tag{2}$$

where $J(q)$ is the jacobian matrix corresponding to the differential relationship $\dot{p}=J(q)\dot{q}$. The dynamics is described by a nonlinear matrix differential equation:

$$T=G(q)+C(q,\dot{q})+I(q)\ddot{q} \tag{3}$$

where $T_G=G(q)$, $T_C=C(q,\dot{q})$ and $T_I=I(q)\ddot{q}$ are the n-dimensional vectors of the gravitational, Coriolis/centripetal and inertial torques.

Based on these equations considerable research has been done on issues such as manipulator structure optimization [1-4] and path planning algorithms [5,6]. However, a more sound consideration of the whole theme reveals that these methods are far from achieving a comprehensive formulation. This observation motivates the re-evaluation of the concepts in use. In fact, expressions (1)-(3) show that the plethora of variables and parameters involved, gives rise to a cumbersome work both in the analysis and design stages. The gigantic number of possible combinations of values indicates that, in order to overcome practical problems, alternative concepts are required. Statistics is a mathematical strategy well adapted to this type of problem. If with this method, we lose the "certainty" of the deterministic model, we gain a simpler and more intuitive viewpoint. This approach has already been used by other researchers [7,8] in some restricted classes of problems. In the sequel we refer to the new approach, as the statistical model [9-10] to stress the contrast with the standard method.

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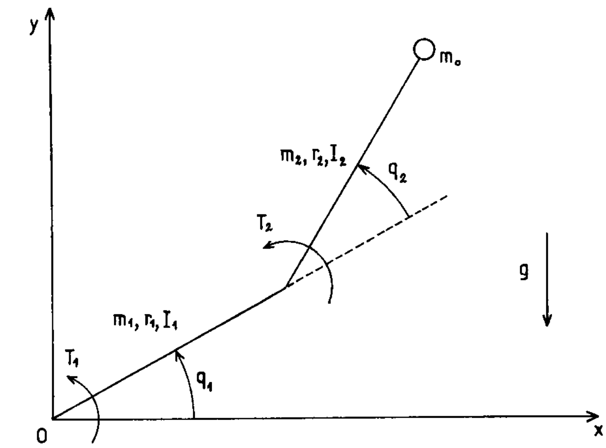

Fig. 1 The 2R joint-actuated robot manipulator

Our modelling procedure comprises:

- The statistical description of a set of input variables, that is variables that are free to change independently.
- The statistical description of a set of output variables, that is, variables that are functions of the the previous ones.
- A set of parameters which are to be optimized in the design stage.

The above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is established as follows:

- $\{p,\dot{p},\ddot{p}\}$ act as input variables of the kinematic system. This option enables a definition of the required kinematic performances on the operational space which are more natural to the designer.
- $\{q,\dot{q},\ddot{q}\}$ act as output variables of the kinematic system, but play the role of input variables set in the dynamic model. In this way we arrive at a relationship between kinematics and dynamics in a form amenable to performance optimization criteria as defined in the sequel.
- The set of dynamic output variables consists of the required joint torques $\{T\}$.
- The parameter set consists of link lengths, masses and inertias.

In other words, we are stating that in the kinematics (dynamics), p, $\dot{p}$ and $\ddot{p}$ (q, $\dot{q}$ and $\ddot{q}$) are considered as independent random variables, its probability density functions (p.d.f.'s) being similar to the histograms of a long run sampling, while q, $\dot{q}$, $\ddot{q}$ (T) are the corresponding random dependent variables. The statistical description of the involved variables, does not consider the (implicit) time variable. In this way, variables that are related through the time derivative operator - $\{p,\dot{p},\ddot{p}\}$ and $\{q,\dot{q},\ddot{q}\}$ - are considered independent of each other.

Let us now adopt the 2R joint-actuated robot manipulator (Fig. 1) as the support for the development and implementation of the new modelling concepts. In the next sub-section we begin by introducing our approach in the kinematic case. In the second sub-section we shall analyse the dynamic case and in the third sub-section we investigate the properties of the overall (i.e. kinematics + dynamics) system.

TABLE 1 Parameters of the 2R joint-actuated robot

R_1=0.05 m, R_2=0.0389 m, r_1=0.3 m, r_2=0.3 m
m_1=2.16 kg, m_2=1.68 kg, m_0=0 kg
$I_i=m_i(r_i^2/12+R_i^2/4)$, i=1,2

A Statistical Model for the Kinematics of the 2R Joint-Actuated Manipulator

The set of kinematic input variables consists of position, velocity and acceleration that our prototype manipulator is required to perform in the operational space. Therefore, it is necessary to characterize them in statistical terms, namely by defining appropriate p.d.f.'s for each variable. As there is no a priori knowledge about the typical behaviour we start with some reasonable assumptions namely, for the position variable $p=[x,y]^T$ we consider a bidimensional uniform p.d.f.

$$f_P(p)=\begin{cases} C & \text{if } (r_1-r_2)^2\leq x^2+y^2\leq(r_1+r_2)^2 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

with $C=\{\pi[(r_1+r_2)^2-(r_1-r_2)^2]\}^{-1}$.

In the sequel we will see how to modify the input p.d.f. in order that the kinematic performances are optimized. It is also necessary to define the p.d.f.'s for velocity and acceleration. By the same above arguments, we decided to use bidimensional Gaussian p.d.f.'s with zero mean:

$$f_{\dot{P}}(\dot{p})=\mathrm{EXP}[-(\dot{x}^2+\dot{y}^2)/(2\sigma_{\dot{P}}^2)]/(2\pi\sigma_{\dot{P}}^2) \quad (5)$$
$$f_{\ddot{P}}(\ddot{p})=\mathrm{EXP}[-(\ddot{x}^2+\ddot{y}^2)/(2\sigma_{\ddot{P}}^2)]/(2\pi\sigma_{\ddot{P}}^2) \quad (6)$$

Moreover, with these p.d.f.'s we impose that:

- The random variables p, $\dot{p}$, $\ddot{p}$ are independent of each other.
- $\dot{p}$ and $\ddot{p}$ are made of two independent components, that is $\dot{x}$ ($\ddot{x}$) is independent of $\dot{y}$ ($\ddot{y}$).

The "excitation" of the (inverse) kinematic system produces output random variables q, $\dot{q}$ and $\ddot{q}$, with p.d.f.'s which are related to the previous ones by:

$$f_Q(q)=J_P f_P(p) \quad (7a)$$
$$f_{Q\dot{Q}}(q,\dot{q})=J_V f_{P\dot{P}}(p,\dot{p}) \quad (7b)$$
$$f_{Q\dot{Q}\ddot{Q}}(q,\dot{q},\ddot{q})=J_A f_{P\dot{P}\ddot{P}}(p,\dot{p},\ddot{p}) \quad (7c)$$

where the jacobians J_P, J_V, J_A are

$$J_P=\partial(p)/\partial(q)=r_1r_2S_2 \quad (8a)$$
$$J_V=\partial(p,\dot{p})/\partial(q,\dot{q})=J_P(r_1r_2S_2) \quad (8b)$$
$$J_A=\partial(p,\dot{p},\ddot{p})/\partial(q,\dot{q},\ddot{q})=J_V(r_1r_2S_2) \quad (8c)$$

Each of the expressions (7) is made of two factors:

- Weighting factors - J_P, J_V and J_A - which depend solely on the system kinematic properties
- The "excitation" p.d.f.'s - $f_P(p)$, $f_{P\dot{P}}(p,\dot{p})$ and $f_{P\dot{P}\ddot{P}}(p,\dot{p},\ddot{p})$ - which are a measure of the task requirements.

These factors can be interpreted in a system theoretic framework. The jacobians characterize the system intrinsic properties, while the excitation p.d.f.'s correspond to the system response to the input variables. Bearing these facts in mind, several experiments were performed, having:

- The total link length constant, L=0.6.
- Seven robot configurations with ratios $\mu=r_1/r_2$ equal to 0.4, 0.6, 0.8, 1, 1.2, 1.4 and 1.6.
- Operational space requirements corresponding to

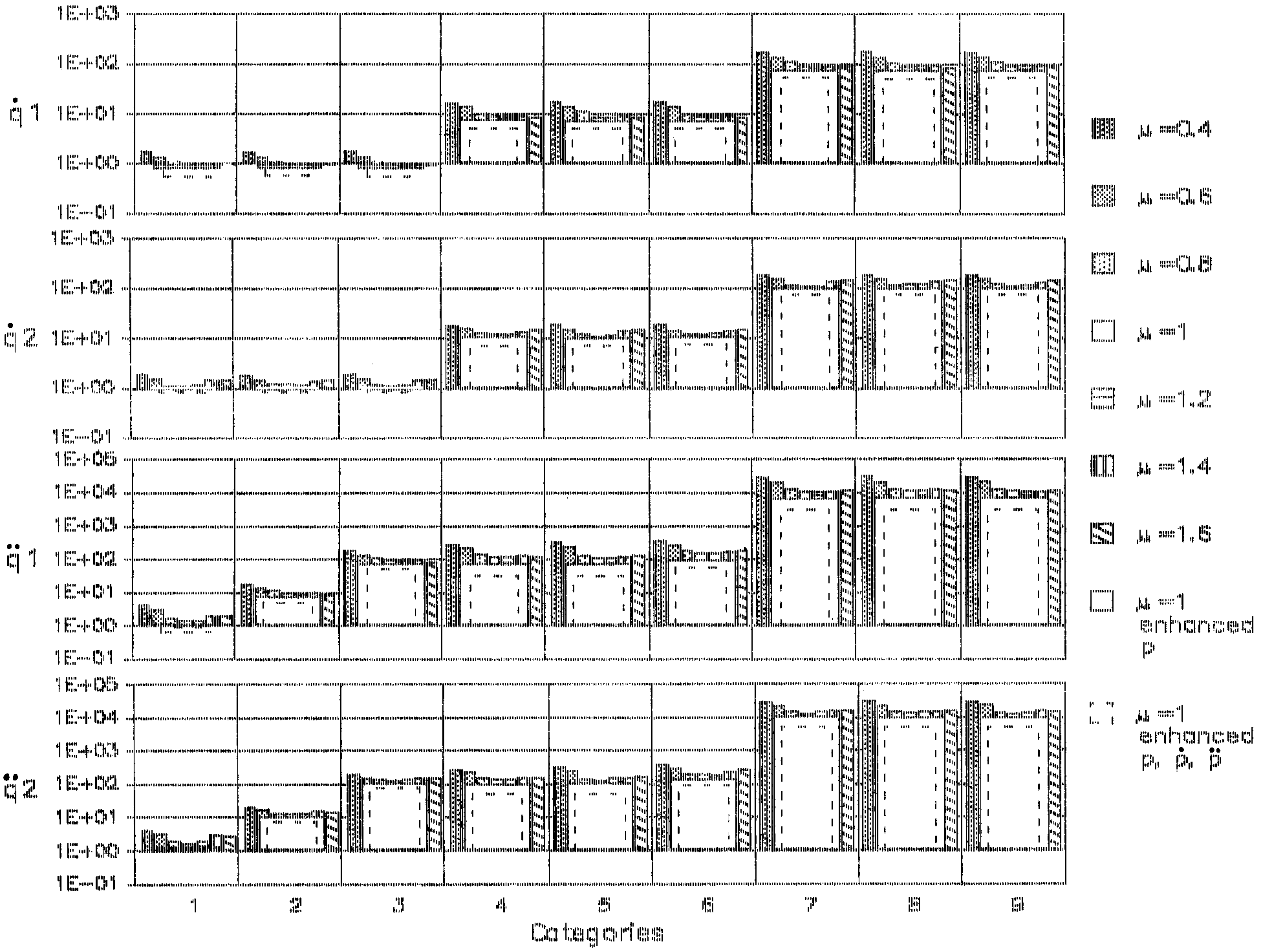

Fig. 2 **Comparison** chart for the 2R joint-actuated robot kinematic performances. The narrow columns correspond to seven geometric configurations "excited" with p.d.f.'s (4), (5) and (6). The wider columns correspond to the optimum geometric $\mu=1$ "excited" with the enhanced p.d.f. (12) and (5)-(6) for the solid borders and enhanced p.d.f.'s (12), (14) and (15) for the dotted borders.

nine categories of velocity and acceleration:

1. $\sigma_{\dot p}=0.1$ $\sigma_{\ddot p}=0.1$	4. $\sigma_{\dot p}=1$ $\sigma_{\ddot p}=0.1$	7. $\sigma_{\dot p}=10$ $\sigma_{\ddot p}=0.1$
2. $\sigma_{\dot p}=0.1$ $\sigma_{\ddot p}=1$	5. $\sigma_{\dot p}=1$ $\sigma_{\ddot p}=1$	8. $\sigma_{\dot p}=10$ $\sigma_{\ddot p}=1$
3. $\sigma_{\dot p}=0.1$ $\sigma_{\ddot p}=10$	6. $\sigma_{\dot p}=1$ $\sigma_{\ddot p}=10$	9. $\sigma_{\dot p}=10$ $\sigma_{\ddot p}=10$

• Excitation of the kinematic system with a numerical random sample of 4000 operational space variables obeying the p.d.f.'s (4)-(6).

• Analysis of the resulting histograms of the output variables amplitude. In order to simplify matters, only marginal p.d.f.'s were considered.

After a large number of experiments using the numerical set of parameters depicted in Table 1, we concluded that the shape of the resulting p.d.f.'s varied significantly from variable to variable, but all of them showed symmetry around zero. For this reason, and in order to characterize the resulting histograms by a scalar index, we decided to adopt for this index the difference between the 97.5% and 2.5% percentiles that is, the 95%-inter-percentile range. The resulting histograms are condensed through this index and depicted in Fig. 2. We can observe in the majority of the charts a minimum about $\mu=1$ yet, this conclusion can be easily inferred from (7). In fact, for symmetrical histograms about zero on the x-axis, with a peak on that point, a larger value of the jacobian corresponds to a smaller dispersion of the random variable. This, in turn, means average smaller amplitude requirements posed to that variable. Therefore, we have found an optimization criterion which is based on the new statistical modelling concepts.

As the maximization of J_P, J_V and J_A requires the same steps, we have for:

$$r_1+r_2=L, \quad r_1/r_2=\mu \qquad (9)$$

that a maximum occurs when

$$\mu=1, \quad q_2=\pi/2 \qquad (10)$$

which coincide with the results obtained (using the classical approach) in previous studies [1,3]. Furthermore, our optimization criteria enables additional conclusions:

• Because J_P, J_V and J_A are consecutive powers of $r_1r_2S_2$, we see that for a given deviation from the optimal values (10) we have an increasingly degradation of the our optimization criterion with the powers of $r_1r_2S_2$. In other words this means that for a given deviation, we have, by increasing order of sensitivity, q, $\dot{q}$ and $\ddot{q}$.

• Due to (2) a kinematic optimization is equivalent to a static optimization.

• If further optimization is desired, then the next step will be the selection of an optimum "excitation" p.d.f.. This second step of optimization will define, in a statistical sense, an optimum kinematic class for the manipulator trajectories. Obviously, we can find a multitude of different p.d.f's obeying (10); nevertheless, for the subsequent study a particular choice is of minor importance. Consequently, we decided to adopt the following family of position p.d.f.'s in the operational space (with $K\geq 1$, C'-constant):

$$f_P(p)=C'\{1-[(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2)]^2\}^{(K-1)/2} \quad (11)$$

which, in the joint space, corresponds to:

$$f_Q(q_1,q_2)=\text{constant}*S_2^K \quad (12)$$

As extreme cases, we have that for K=1 it becomes the uniform p.d.f. (4), while for $K\rightarrow\infty$ we get Dirac type p.d.f. ($\delta(.)$) optimum in the sense of (10):

$$f_P(x,y)=\delta[x^2+y^2-(r_1^2+r_2^2)] \quad (13a)$$
$$f_Q(q_1,q_2)=1/2[\delta(q_2+\pi/2)+\delta(q_2-\pi/2)] \quad (13b)$$

As far as velocity and acceleration are concerned we can see that the kinematic study does not point out any special class of p.d.f.'s. Nevertheless, these variables are affected negatively by the position deviation from the optimum configuration $q_2=\pi/2$. Therefore, we decided to study the system behaviour both for performance requirements described by p.d.f.'s (12), (5) and (6) and for the alternative situation corresponding to p.d.f. (12) associated with the "enhanced" q_2-dependent velocity and acceleration p.d.f.'s:

$$f_{\dot{P}}(\dot{p},q_2)=EXP\{-(\dot{x}^2+\dot{y}^2)/[2\sigma_{\dot{P}}^2(q_2)]\}/[2\pi\sigma_{\dot{P}}^2(q_2)] \quad (14a)$$
$$\sigma_{\dot{P}}(q_2)=\begin{cases}2\sigma_{\dot{P}}|q_2|/\pi & \text{if } 0<|q_2|\leq\pi/2\\ 2\sigma_{\dot{P}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\leq\pi\end{cases} \quad (14b)$$
$$f_{\ddot{P}}(\ddot{p},q_2)=EXP[-(\ddot{x}^2+\ddot{y}^2)/[2\sigma_{\ddot{P}}^2(q_2)]\}/[2\pi\sigma_{\ddot{P}}^2(q_2)] \quad (15a)$$
$$\sigma_{\ddot{P}}(q_2)=\begin{cases}2\sigma_{\ddot{P}}|q_2|/\pi & \text{if } 0<|q_2|\leq\pi/2\\ 2\sigma_{\ddot{P}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\leq\pi\end{cases} \quad (15b)$$

To test numerically the above conjectures, the previous results for $\mu=1$ are compared with a new case using $\mu=1$ and K=3 in (11)-(12). This has revealed a remarkable performance improvement as shown in Fig. 2, particularly for velocity-dependent requirements.

A Statistical Model for the Dynamics of the 2R Joint-Actuated Manipulator

The statistical description of the dynamics requires steps similar to those adopted in the kinematics, namely:

• Characterisation of the input variables (q, $\dot{q}$ and $\ddot{q}$) through appropriate p.d.f.'s.

• "Stimulation" of the system behaviour through numerical experiments.

• Analysis of the histograms of the output variables (T).

However, a preliminary observation shows that the dynamic study is much more complex than the kinematic one. Due to this reason, and in order to gain a deeper insight for the subsequent study we decided to consider, in a first stage, as dynamic output variables, the components of the joint torques, that is the gravitational, Coriolis/centripetal and inertial torques. Based on this preliminary analysis then, in a second stage, we consider the total joint torques. In the first stage we have:

$$f_G(q)=J_Gf_Q(q) \quad (16a)$$
$$f_C(q,\dot{q})=J_Cf_{Q\dot{Q}}(q,\dot{q}) \quad (16b)$$
$$f_I(q,\dot{q},\ddot{q})=J_If_{Q\dot{Q}\ddot{Q}}(q,\dot{q},\ddot{q}) \quad (16c)$$

where

$$J_G=\partial(q)/\partial(T_G)=[(m_1/2+m_2+m_0)(m_2/2+m_0)g^2r_1r_2S_1S_{12}]^{-1} \quad (17a)$$
$$J_C=\partial(q,\dot{q})/\partial(T_G,T_C)=J_G\{[2(m_2/2+m_0)r_1r_2S_2]^2\dot{q}_1\dot{q}_{12}\}^{-1} \quad (17b)$$
$$J_I=\partial(q,\dot{q},\ddot{q})/\partial(T_G,T_C,T_I)=J_C\{[(m_1/4+m_2+m_0)r_1^2+I_1+(m_2/2+m_0)r_1r_2C_2][(m_2/4+m_0)r_2^2+I_2]- -[(m_2/4+m_0)r_2^2+I_2+(m_2/2+m_0)r_1r_2C_2](m_2/2+m_0)r_1r_2C_2\}^{-1} \quad (17c)$$

Unlike the kinematic situation, where the optimization was similar for all the jacobians, now their effects differ according to each dynamic term. Analysing the jacobians (17) we conclude that:

• The maximizing of J_G stipulates that q_1 and q_{12} should have p.d.f.'s with maxima at 0 or π. The observation of histograms resulting from "excitation" p.d.f.'s obeying these conditions showed an interesting result. As expected the (symmetrical) histograms resembled Dirac pulses; however, those peaks were located at non-zero values. In fact, the plots showed sharp symmetrical peaks located at the maxima (positive and negative) values attained by the gravitational torques. This means that, for this case, the optimization procedure must adopt an inverse strategy, that is to say we must minimize J_G.

• The maximizing of J_C implies that q_2 must have a p.d.f. with a maximum on 0 or π. Numerical experiments showed that in this case the resulting histograms of the Coriolis/centripetal terms tended, as desired, towards a Dirac on zero.

• The analytical expression of J_I is more complex. Nevertheless, its analysis revealed a maximizing condition similar to the previous one (i.e. q_2 should have a p.d.f. with maxima at 0 or π).

Therefore, we may say that J_G defines a "rest region" while J_C and J_I define an "active region" of operation.

Now, we can proceed to the second stage, that is, the study of the (total) dynamics. The direct application of our optimizing method to the dynamics would require the mathematical and numerical treatment of 3n-dimensional p.d.f.'s. In order to avoid this intricate analysis, we decided to integrate the (partial) conclusions pointed out in the first stage (i.e. the guidelines resulting from the separate study of T_G, T_C and T_I) on the formulation of our present investigation. Furthermore, this approach will show that the new statistical concepts give considerable freedom on the research

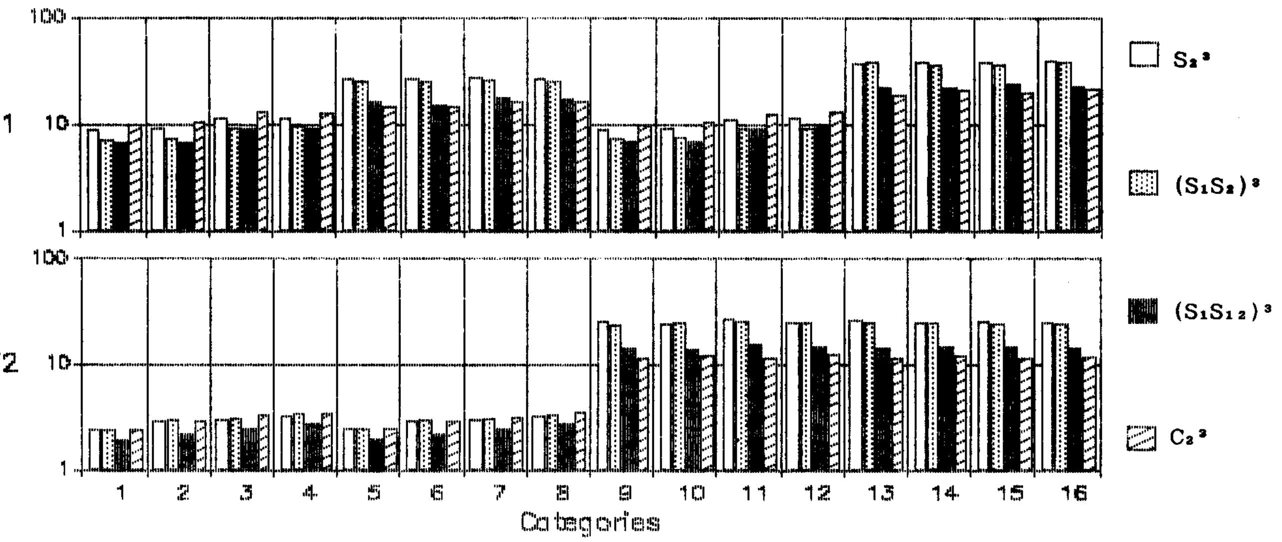

Fig. 3 Comparison charts for the 2R joint-actuated robot dynamic performances with μ=1 when subjected to "excitation" p.d.f.'s: 1st column: $f_q(q_1,q_2)=const.*S_2^3$ 2nd column: $f_q(q_1,q_2)=const.*(S_1S_2)^3$ 3rd column: $f_q(q_1,q_2)=const.*(S_1S_{12})^3$ 4th column: $f_q(q_1,q_2)=const.*C_2^3$

strategy. In this sense we decided to "excite" the dynamics with four different position p.d.f.'s (having K=3):

$$f_q(q_1,q_2)=constant*S_2^K \quad (18)$$
$$f_q(q_1,q_2)=constant*(S_1S_2)^K \quad (19)$$
$$f_q(q_1,q_2)=constant*(S_1S_{12})^K \quad (20)$$
$$f_q(q_1,q_2)=constant*C_2^K \quad (21)$$

which are suggested by the optimization of the kinematics, a compromise between kinematics and gravitational torques, the gravitational torques, and the Coriolis/centripetal and inertial torques, respectively. Due to the non-existence of optimization guidelines on $\dot q$ and $\ddot q$, we decided to consider two gaussian "excitation" p.d.f.'s (i=1,2):

$$f_{\dot q_i}(\dot q_i)=EXP[-\dot q_i^2/(2\sigma_{\dot q_i}^2)]/(2\pi\sigma_{\dot q_i}^2) \quad (22)$$
$$f_{\ddot q_i}(\ddot q_i)=EXP[-\ddot q_i^2/(2\sigma_{\ddot q_i}^2)]/(2\pi\sigma_{\ddot q_i}^2) \quad (23)$$

and the following sixteen different categories:

1. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=0.1
2. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=10
3. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=0.1
4. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=10
5. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=0.1
6. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=10
7. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=0.1
8. $\sigma_{\dot q_1}$=0.1, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=10
9. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=0.1
10. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=10
11. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=0.1
12. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=0.1, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=10
13. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=0.1
14. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=0.1, $\sigma_{\ddot q_2}$=10
15. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=0.1
16. $\sigma_{\dot q_1}$=10, $\sigma_{\dot q_2}$=10, $\sigma_{\ddot q_1}$=10, $\sigma_{\ddot q_2}$=10

Figure 3 depicts the results of T_1 and T_2 when the 95% index is applied to the corresponding histograms. These charts revealed several properties:

- T_1 (T_2) depends strongly on $\dot q_2$ ($\dot q_1$).
- The joint torques (T) have low sensitivity to acceleration ($\ddot q$) requirements.
- The suggestions pointed out by the first stage are compatible with the new results. In fact, for "rest" (or "non-active") requirements, p.d.f. (20) is the more appropriate, while for the "active" (or "non-rest") situation p.d.f. (21) is the optimal.

The Statistical Description of the Total System

Up to now we discussed the kinematics and dynamics separately however, in the real manipulator these systems can not be divided. In other words, the study of a robot manipulator, must integrate both systems. Therefore, the statistical description of the total system (i.e. both the kinematics and dynamics) will have cross-coupling effects and its influence must be evaluated. To test these effects, the total system was numerically "excited" through random samples according to p.d.f.'s (18)-(21). These position p.d.f.'s combined with the two alternative velocity and acceleration p.d.f.'s (5)-(6) or (14)-(15) (more precisely we are using their equivalent p.d.f.'s defined on the operational space), reveal that (Fig. 4):

- For low velocity/acceleration requirements (category 1), the 95% index gives almost similar results for all p.d.f.'s, because the gravitational torques predominate.
- Velocity requirements ($\dot p$) have a much stronger influence than acceleration requirements ($\ddot p$).
- Kinematic effects prevail over the dynamic ones and, therefore, the best results come from the "kinematic-dependent" p.d.f.'s (18) and (19).

In conclusion, the statistical analysis reveals that the kinematics and dynamics have different effects upon the robot system. As shown, mechanical manipulators are much more sensitive to velocity

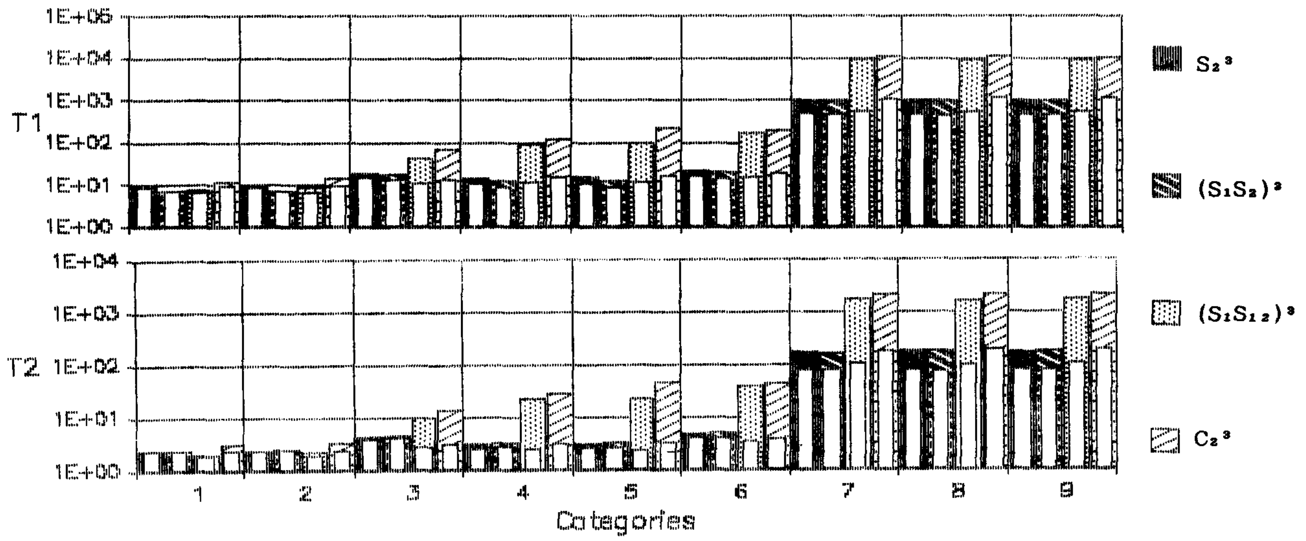

Fig. 4 Comparison chart for the 2R joint-actuated robot overall (kinematic + dynamic) performances with $\mu=1$ when subjected to "excitation" p.d.f.'s:
1st column: $f_Q(q_1,q_2)=const.*S_2^3$ 2nd column: $f_Q(q_1,q_2)=const.*(S_1S_2)^3$
3rd column: $f_Q(q_1,q_2)=const.*(S_1S_{12})^3$ 4th column: $f_Q(q_1,q_2)=const.*C_2^3$
The back columns correspond to p.d.f.'s (5)-(6) and the front white columns correspond to the enhanced p.d.f.'s (14)-(15).

requirements than to acceleration requirements and, for fast movements, kinematics is more significant than dynamics. These facts indicate that we are dealing with "position and acceleration machines" rather than "velocity machines". Although obvious, this aspect has been somewhat overlooked. Moreover, it points out that the usual robot actuators, which are developments of standard "velocity machines" are not well adapted to robotic applications. Alternative solutions, such as muscle like actuators will allow more efficient robot structures.

Conclusions

A new method to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, on the other hand, can be very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The proposed statistical method is a step in that direction which has been shown to enable a new formalism. In fact, the new method provides a framework giving clear guidelines towards the optimization both of the path planning algorithm and the robot structure. Furthermore, the results point out structural characteristics that define robotic manipulators as "position and acceleration" machines. Therefore, joint-actuated manipulators are non-optimal machines and alternative structures, such as those common in biological systems, must have adaptive mechanisms between operational exigencies and actuator requirements.

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