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# Intelligent Maintenance of Linear Systems: A Fractional Order Identification Approach

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**Abstract** — *Compete in the global market requires high quality of products with short time of manufacture, so it needs to minimize the time that the machinery is stopped, as well as a rapid quality control of manufactured products. The lack of the integer order system identification relys on the hardness of find a good model for the actualy system. Although, the behavior of complex systems can be approximated accurately by a few parameters if we assume them as a fractional order equivalent giving a new tool for quickly health evaluation.*

## 1 Introduction

A failure is something does not allow a machine to keep working, because it is impossible to turn it back or safety. Typically are caused by the use of a material primed for the task, apply a force in a different direction that it was designed, cyclic loading, fatigue, wear, etc. When a failure occurs and the machine continues working tends to be worse and cause other problems. Maintenance is as good as the knowledge of the cause of failure is determined to thereby maintain the parts to exchange, have the tools to replace it, experts and workers, everything in place and time needed to repair the equipment before the fault occurs seriously and keeping it the shortest time in detention. For this, it is not enough to know that the machine is failing, but also the source of failure [1]. In order to solve this problem in the industry has proposed several strategies maintenance strategies in order to reduce costs of production over time.

In mechatronics, the machines are increasingly expensive and complex, containing multiple integrated components of several technologies ( as electrical, mechanical, etc.), adding components to the signals used in diagnosis. This fact complicates the task of maintenance as it currently skilled workers diagnose systems based on experience, so the more complex the system is, more difficult to isolate the problem and increase the economic cost of the expert, as an instance between 1975 and 1991 in the United States the maintenance cost increase in a 10-15 % a year [2].

In order avoid this problem, some researchers try to automate the maintenance task, therefore some works have propose the use of artificial intelligence techniques over signals usually analyzed by experts [3, 4, 5], but they are few applied because the signals are so complex and the academic level of the workers must increase and because of the high investment on equipment without total trust in computer decisions [6].

Bearing this ideas in mind, the rest of the article is organized as follows: In section 2 introduce generally the strategy of intelligent maintenance, the section 3 presents some basis of the fractional order calculus and its applications in identification of dynamical systems. In the section 5 the result of apply fractional order identification to a complex system is introduced and finally in section 6 the main conclusions are presented.

## 2 Intelligent Maintenance

In the global market, customers have suppliers of several qualities around the world. Therefore to remain competitive the factories need to produce goods of high quality and in a short time, so that satisfy the international demand of clients and customers recently acquired [7]. Consequently the production chain is more vulnerable to various disturbances, the possibility of failure and the time needed to repair it. A perfect balance only can be achieved when the factory is in operation in several shifts a day and the machines are fully functional. Therefore it requires to apply a maintenance strategy that allows to approach the ideal situation described above [8].

Each system in the chain presents problems due to deterioration of parts due to use or stochastic failures, such as dropped a tool, machine dovetailed bad, etc. In order to minimize costs of repair and stop the production, there are used some procedures to prevent, predict and correct a failure, whose together are called maintenance [9]. In literature, the strategies of maintenance are classified in [10, 9, 4]: (1) Correction, over time this strategy is more expensive, the maintenance action takes place when the system symptoms are evident. (2) Timely, when the system has minor flaws as to keep the machine in operation. When a failure happens more, takes advantage of the maintenance stop in to play all the parts worn. (3) Preventive, Based on the information delivered by the manufacturer and experience of staff of the plant are planned periodic maintenance actions, in which possibly the machine is stopped. Moreover the failure may occur before the time of maintenance for unusual wear of parts or because of a random failure. (4) Predictive: The system must be constantly monitored and the signals analyzed at the time. When the operator observes that the machine presents a possible situation of failure in the near future, will be held the maintenance action.

If the maintenance action needs to hold the device, has three possible effects: (1) the frequency of maintenance is adequate and the machine has no additional stops, only to repair random failures, (2) the frequency is low and the machine fails before the scheduled maintenance action, therefore presents an additional stop for a random or undetected failure, or (3) the frequency is so high that it increases the maintenance costs unnecessarily. In the 1990s another idea starts to be used in industry, where catch signs of equipment constantly, and when the signal is abnormal and analyzing diagnostic even without stopping the production, performing the maintenance action only when it is need [11, 12].

Making decisions maintenance based on condition (CBM) requires a tough one to predict failure and its severity. It has three goals: (1) Design a strategy for the maintenance of sophisticated equipment in complex operating environments, (2) reduce cost of storage of spare parts and finally (3) reduce catastrophic failures and eliminate unscheduled stops [6]. The typical techniques used in CBM are: (1) Statistics index, a single value indicating health asset of the machine is obtained by calculating one of the next measures: (1) the root mean square, the pick to pick average ratio or kurtosis of the signal. However this value is easy to interpret does not give information about the failure localization. (2) Frequency analyzes, the data is transformed to another space where it is less correlated. By analyzing the signal carefully, it is possible to identify and localize a singular failure, but the information is typically hard to understand as it is noisy. Therefore, the analyzer is a high trained operator. This method do not give any information about when the failure occurs. (3) System Identification, it could give the exact location of the failure and how it affect the machine behavior. Although, obtain a good model of the actual machine is difficult as an accurately model must consider interaction between several pieces, nonlinear behavior and high order dynamics. In addition if an accurately complex model is obtained, rarely is applicable, because it consumes a lot of computational resources. Therefore, this designer must find the balance between accurately and usability.

The use of a fractional order technique could fill the lack of the integer system identification strategy, because it models complex systems with few parameters.

## 3 Fractional Order Calculus

Fractional order calculus (FOC) was little used in engineering because of its complexity, the apparent sufficiency of the integer order calculus (IOC) and the lack of a physical or geometric interpretation [13, 14]. However, it models more accurately the behavior of some systems in nature related to several areas of engineering, and it is used as a promising tool in bioengineering [15, 16], viscoelasticity [17, 18], electronics [19, 20], robotics [21, 22, 23], control theory [24, 25] and signal processing [26, 27] among others.

In the latter years these concepts have attracted the attention of engineers because it models the behavior of many physical systems nonlinear compact taking into account non-local features as “infinite memory” [28, 29, 30]. Some examples are the phenomenon of heat diffusion [31], electrical impedance of fruits and vegetables [32], modeling the love triangles between human [33], the behavior of water in the pores of the dyes, where the radio damping is constant regardless of the mass of water in motion [34], etc. On the other hand, directing the behavior of a process with fractional-order controllers is an advantage, since the system response is not restricted to the addition of exponential functions, therefore there is a wide range of behaviors reached where the integer order response is just a particular case [35].

The concept of fractional order calculus has existed since the creation of the integer order calculus, this can be proved across a letter from Leibniz to L’opital in 1965 [36]. This is a generalization of the IOC in real or complex order [37]. Formally for real order can be written as:

$$D^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & \alpha > 0, \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases} \tag{1}$$

With $\alpha \in \Re$.

One possible cause for their because it is little used in engineering is that FOC has multiple definitions [30, 38], hindering their geometric interpretation, and that the IOC seemed to be sufficient to model nature. However many phenomena are better described by fractional order formulations, since it takes into account past behavior and have the ability to express with few coefficients dynamic systems considered of high order [39, 40].

A tools of interest in engineering is the Laplace transform, which is still valid to simplify operations such as convolution and can be used to solve differential equations of fractional order. FOC in the Laplace transform is defined as [41]:

$$\mathcal{L}\{_0D_t^\alpha\} = s^\alpha F(s) - \sum_{j=0}^{n-1} s^j \left[_0D^{\alpha-j-1} f(0)\right] \tag{2}$$

with $n-1 < q < n, \qquad n \in \mathbb{Z}$. Thus, the transform takes into account all the initial conditions from the first to the $n$-th derivative $-1$. Using this result is clear that any dynamic system of an arbitrary order could described by transfer functions of the form [42]:

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}} \tag{3}$$

With $\alpha, \beta \in \Re$, $\alpha_n > \alpha < n-1 > \cdots > \alpha_0$ e $\beta_m > \beta < m-1 > \cdots > \beta_0$

Many real systems can be identified from the theory of fractional systems [28, 43], whereas the transfer function is of a fractional or that the response time could not be achieved through a linear combination of exponential functions [44], in addition the order is a variable degree of freedom that lets to adjust accurately to the system describing it in a compact way [45]. Djouambi [46] used this fact to identify a fractal system, bringing data to the equation:

$$F(s) = \frac{K}{s^m + a}, \qquad m \in \Re \tag{4}$$

Adjusted the template to find the parameters $\{K, a, \alpha\}$ that minimize the mean error when compared with the actual data.

## 4 Algorithm for System Identification

We use an approach based on the Levenberg-Marquardt algorithm in order to obtain the best parameters that fit the real data [47] which is the typical strategy for nonlinear optimization, but other are allowed without loosing specificity of the method. It consist in look for the best parameter vector $\mathcal{P}$ that minimize the error $\epsilon$ between the original data $y$ and the data obtained with the model $\hat{y}$, formally, the model is calculated as a function $f$ over the obtained parameter vector:

$$\hat{y} = f(\mathcal{P}) \tag{5}$$

A vector $P$ minimize the error between real and obtained data as:

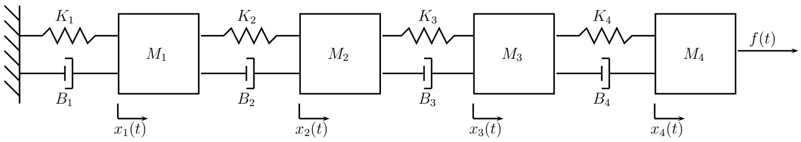



Figure 1: Case of Study. A high order system with several coupled subsystems.

$$\epsilon = \underset{\mathcal{P}}{\operatorname{argmin}}(y - \hat{y}(\mathcal{P})) \tag{6}$$

the author propose a hill descent strategy based on the first derivative, for that it is used the Taylor expansion over the Eq.5 obtaining at the evaluation point $\mathcal{A}$:

$$f(\mathcal{A}) \approx f(\mathcal{A}) + \frac{\partial f(\mathcal{A})}{\partial \mathcal{P}}(\mathcal{P} - \mathcal{A}) \tag{7}$$

Then the problem is solved by looking for the best $\mathcal{P} - \mathcal{A}$ argument iteratively.

# 5 Results

In order to evaluate the efficiency of the fractional order calculus to describe complex systems and how the fractional order approximation is sensitive to failures in a machine, we propose the model shown in the Fig. 1 as experimental setup. It was modeled in the space state presented in the equation 8. Each state is equivalent to a function of the system as shown in table 1. The parameters used in simulation as the normal point of operation of the system are presented in table 2.

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \\ \dot{X}_5 \\ \dot{X}_6 \\ \dot{X}_7 \\ \dot{X}_8 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{K_4}{M_4} & -\frac{B_4}{M_4} & \frac{K_4}{M_4} & \frac{B_4}{M_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{K_3}{M_3} & \frac{B_3}{M_3} & 0 & 0 \\ \frac{K_4}{M_3} & \frac{B_4}{M_3} & -\frac{K_4+K_3}{M_3} & -\frac{B_4+B_3}{M_3} & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{K_3+K_2}{M_2} & -\frac{B_3+B_2}{M_2} & \frac{K_2}{M_2} & \frac{B_2}{M_2} \\ 0 & 0 & \frac{K_3}{M_2} & \frac{B_3}{M_2} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{K_2}{M_1} & \frac{B_2}{M_1} & -\frac{K_2+K_1}{M_1} & -\frac{B_2+B_1}{M_1} \\ 0 & 0 & 0 & 0 & & & & \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \end{pmatrix} + \begin{pmatrix} 0 \\ -F \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

The system was exited with a sinusoidal force with unitary amplitude, varying the frequency of oscillation between 1 until 100 $Hz$. It was supposed that there is a single sensor

| State | Variable | State | Variable |
|---|---|---|---|
| $X_1$ | $x_4(t)$ | $X_5$ | $x_2(t)$ |
| $X_2$ | $\frac{dx_4(t)}{dt}$ | $X_6$ | $\frac{dx_2(t)}{dt}$ |
| $X_3$ | $x_3$ | $X_7$ | $x_1(t)$ |
| $X_4$ | $\frac{dx_3(t)}{dt}$ | $X_8$ | $\frac{dx_1(t)}{dt}$ |

Table 1: Space state definition.

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $K_1$ | 100 | $B_1$ | 200 |
| $K_2$ | 300 | $B_2$ | 150 |
| $K_3$ | 300 | $B_3$ | 320 |
| $K_4$ | 300 | $B_4$ | 50 |
| $M_1$ | 3 | $M_3$ | 5 |
| $M_2$ | 4 | $M_4$ | 4 |

Table 2: Physical parameters of the model.

of displacement monitoring at $x_2(t)$. With those information a Bode's plot of the system in normal operation was constructed as shown in Fig. 2(a) and other equivalents to add failures at the parameters $K_1$, $K_3$, $B_2$ as shown in Figs. 2(b), 2(c) and 2(d) respectively and approximated by:

$$G(s) \approx T\frac{(s^\beta + b)(s^\gamma + g)}{s^\alpha + a}, \qquad \{\alpha, \beta, \gamma\} \in \Re \tag{9}$$

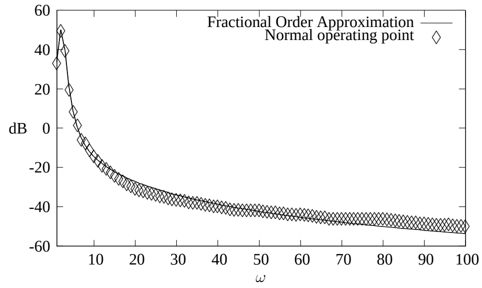
adjusted via the Levenberg-Marquardt algorithm explained above.

Change any of the physical parameters of the system results in a severe change of all parameters in the transference function (Eq. 9), i.e., the behavior of a system in the normal point of operation is very different to the behavior of the system when a failure is present. Therefore, this fact would be used to identify the presence of particular failure. In order to compare two identified parameters, $p_1$ a parameter of the system in normal operation and $p_2$ the parameter identified in the system with any failure, we use a metric of distance $d$ in percentage between $p_1$ and $p_2$, it is:

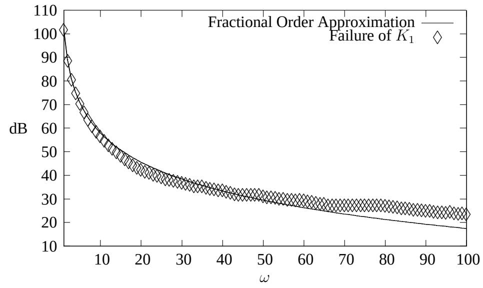
$$d = \frac{\max(p_1, p_2) - \min(p_1, p_2)}{\max(p_1, p_2)} \tag{10}$$

Applying this metric over several cases of failure the table 3 was calculated. Note that the distance $d$ is long enough to classify the system as healthy or not, and the parameters let identify the type of failure in the machine.

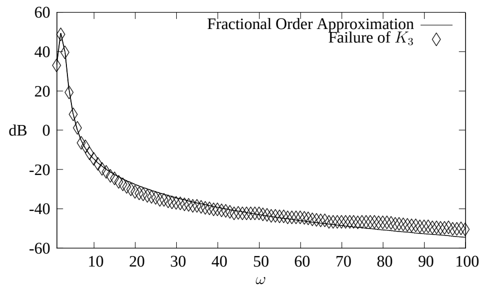
Therefore the presence of a singular particular failure could change the value of the parameters an average of 84% in a system that use less parameters than the exact integer one.



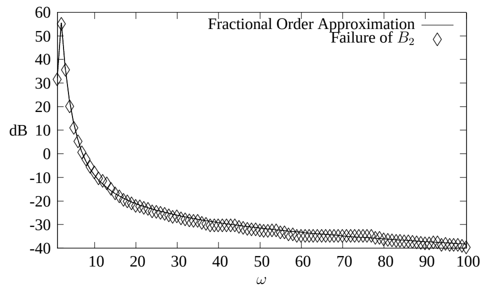

(a) Approximation of the system in "normal condition" via a fractional order system.




(b) Approximation of the system when the spring $K_1$ is broken via a fractional order system.




(c) Approximation of the system when the spring $K_3$ is broken via a fractional order system.




(d) Approximation of the system when the damper $B_2$ is broken via a fractional order system.

Figure 2: Approximation of a system with different failures via $G(s) = T\frac{(s^\beta+b)(s^\gamma+g)}{s^\alpha+a} \mid_{\{T,a,b,g,\alpha,\beta,\alpha\}\in\Re}$. Note that it is a good approximation for almost all systems in the frequency band of $[1Hz - 100Hz]$.

| Failure | $T$ | $a$ | $\alpha$ | $b$ | $\beta$ | $g$ | $\gamma$ | Mean Error |
|---|---|---|---|---|---|---|---|---|
| Not failing | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $K_1$ Broken | 90.47 | 99.96 | 84.52 | 99.36 | 42.43 | 83.42 | 90.54 | 84.39 |
| $K_2$ Broken | 6.10 | 2.07 | 10.83 | 13.75 | 3.71 | 14.79 | 90.79 | 20.29 |
| $K_3$ Broken | 75.35 | 99.79 | 72.62 | 91.05 | 53.41 | 84.41 | 85.28 | 80.27 |
| $K_4$ Broken | 1.01 | 4.37 | 0.24 | 40.98 | 56.57 | 56.44 | 42.72 | 28.9 |
| $B_2$ Broken | 97.07 | 99.99 | 59.98 | 99.99 | 33.82 | 44.69 | 99.19 | 76.39 |
| $B_3$ Broken | 19.76 | 99.84 | 78.94 | 99.78 | 93.79 | 75.22 | 86.83 | 79.17 |
| $B_4$ Broken | 94.76 | 99.74 | 42.67 | 99.86 | 99.40 | 33.71 | 98.42 | 81.22 |

Table 3: Distance in percentage between identified parameters of a system with a failure and one in the normal operation point.

## 6 Conclusions

Nowadays the availability of a method to measure the quality of a machine or a product in short time minimizing production line stops is a paramount problem in the industry. Here were introduced some aspects of condition based monitoring (CBM) and how the fractional order calculus (FOC) would identify and evaluate some failures over a known system.

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