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The Power Law Applied to the Windowed Fourier Transform

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Abstract: The windowed Fourier transform is one of the most widely used time-frequency representations. In order to use this technique several parameters must be defined according to the signal analyzed. In this paper are studied the effects of the type, the shape, the length and the overlap of the windows. A power law/fractional window that we call fractional is defined and a new method for tuning the window parameters is presented. The study is based on the information theory and is applied to signals captured during the movement of a robotic manipulator. The experimental results demonstrate the applicability and the effectiveness of the proposed approach.

Keywords: Windowed Fourier transform, short time Fourier transform, robotics, signal processing, time-frequency analysis, mutual information, power law.

1. INTRODUCTION

When the signals parameters evolve during the time they are called non-stationary. Very often real-world processes are non-stationary containing a time-varying frequency content. In many applications we are interested in the frequency content of a signal at a given period of time. In the case of a non-stationary signal, the classical Fourier transform (FT) is not suitable for its analysis. In fact, information localized in time, such as spikes, impacts, seismic events, and high frequency bursts, are not easily detected by the FT. Therefore, a time frequency analysis is used in many fields for studying signals with a time-varying spectral content.

There are several approaches to achieve the time frequency analysis of non stationary signals. Among others, the most popular are the Wigner distribution, the Gabor transform, the windowed Fourier transform (WFT) and the wavelet transform (Allen and Mills, 2004). Textbooks that address the time-frequency representations can be referenced in (Cohen, 1995; Flandrin, 1999; Mallat, 1999). The comparison between the different approaches, for achieving the time frequency analysis, was developed by several authors (Jones and Parks, 1989; Jones and Parks, 1992; Cohen, 1989) and it was verified that the choice of the best representation depends on the application (Jones and Parks, 1989).

The WFT, also known as short time (or term) Fourier transform (STFT) or time-varying Fourier transform (TVFT), is one of the most widely used time-frequency representations. In fact, this technique is adopted in many fields of engineering, such as in audio (speech and musical) signal processing, vibration signal processing (Scheffer and Girdhar, 2004) seismic signal processing, electromagnetic radiation (Ozdemir and Ling, 1997) and robotics (Lima, *et al.*, 2006). The WFT is an extension of the classical FT, where the transform is evaluated repeatedly for a running windowed version of the time domain signal. Each FT gives a frequency domain 'slice' associated with the time instant at the window center.

There are several studies for implementing WFT recursive algorithms (Chen, *et al.*, 1993; Chen and Griswold, 1994; Tomazic and Znidar, 1996; Czerwinski and Jones, 1997). One important aspect of the WFT is the window length that is related with the time–frequency resolution. The frequency-resolution of the WFT is proportional to the effective bandwidth window. Consequently, for the WFT we have a trade-off between the time and the frequency resolutions: on one hand, a good time resolution requires a short window, while, on the other hand, a good frequency resolution requires a long window. Several authors addressed this issue (Jones and Parks, 1989; Jones and Parks, 1992; Zielinski, 2001). In order to adjust the desired resolution, the window length can be adjusted adaptively (Jones and Baraniuk, 1992; Jones and Boashash, 1997; Djurovic and Stankovic, 2003; Stankovic, 2001) based on an instantaneous quality measurement of the time frequency content.

Another aspect of the WFT is the type of window adopted (Harris, 1978; Nuttall, 1981). Several authors studied the effect of the WFT window (Allen and Mills, 2004; Oppenheim, *et al.,* 1989; Ha and Pearce, 1989) and verified that the best choice depends on the type of signal (Czerwinski and Jones, 1997).

In summary, there are distinct parameters that must be defined to use the WFT. In this line of thought the need of indices for tuning adequately the WFT motivated the work presented here. In fact the authors developed several experiments and indices that were tested for tuning the WFT. The indices included statistical, entropy and information theory approaches. In this field several authors investigated the connections between the information theory (entropies and mutual information) and the time-frequency representations (Aviyente, 2005; Aviyente and Williams, 2005; Baraniuk, *et al.*, 2001; Loughlin and Cohen, 2004). A method based on the information theory is presented in this work, revealing to be a promising strategy.

To show the behavior of the information theory approach, the WFT is applied to a set of signals captured in a robotic manipulator, which is briefly described in the following section. In the section 3 are presented the fundamental concepts. Section 4 presents the results based on experimental signals and, finally, the section 5 outlines the main conclusions.

2. APPARATUS AND EXPERIMENTAL SIGNALS

In order to analyze signals that occur in a robotic manipulator an experimental platform was developed. The platform has two main parts: the hardware and the software components (Lima, *et al.,* 2005). The hardware architecture is shown in figure 1. Essentially it is made up of a mechanical manipulator, a computer and an interface electronic system. The interface box is inserted between the arm and the robot controller, in order to acquire the internal robot signals; nevertheless, the interface captures also external signals, such as those arising from accelerometers and force/torque sensors. The modules are made up of electronic cards specifically designed for this work. The function of the modules is to adapt the signals and to isolate galvanically the robot's electronic equipment from the rest of the hardware required by the experiments.

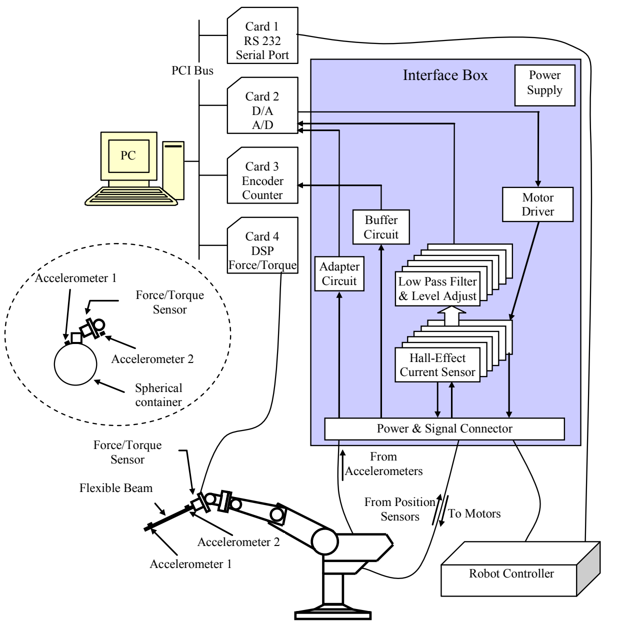


Fig. 1. Block diagram of the hardware architecture.

The software package runs in a Pentium 4, 3.0 GHz PC and, from the user's point of view, consists of two applications: (*i*) the acquisition application is a real time program responsible for acquiring and recording the robot signals; (*ii*) the analysis package runs off-line and handles the recorded data. This program allows several signal processing algorithms such as, FT, WFT, correlation, time synchronization, etc.

To test the phenomenon of mechanical impacts, in the experimental setup it is used a flexible link that consists of a long, thin, round, flexible steel rod clamped to the end-effector of the manipulator. The robot motion is programmed in a way such that the clamped rod collides with a surface and several signals are recorded with a sampling frequency of $f_s = 500$ Hz. The signals come from different sensors, such as accelerometers, wrist force and torque sensors, position encoders and joint actuator current sensors. Additionally, in another experiment, it is adopted a spherical container carrying a liquid that oscillates during the acceleration/desacceleration transients. To test the behavior of the variables in different situations, the container (figure 1) can remain empty or can be filled with a liquid or a solid. The robot motion is programmed in a way that the container moves from an initial to a final position following a linear trajectory.

Figures 4–5 depict a typical time evolution of some variables and the corresponding spectrum. Figure 4 a) shows the forces at the end-effector of the manipulator captured during a total period of $t_T = 8$ s for the impact analysis. These signals present clearly a strong variation at the instant of the impact, that occurs approximately at $t = 4$ s. The Fourier spectrum of f_z^{imp} (force z component for the case of impact) is shown in figure 4 b).

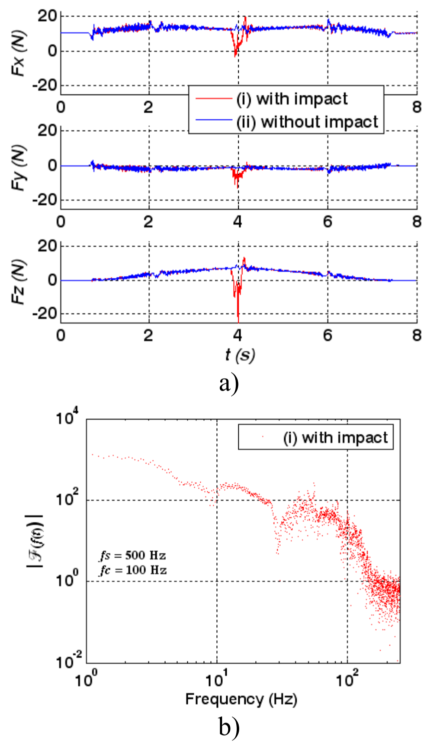
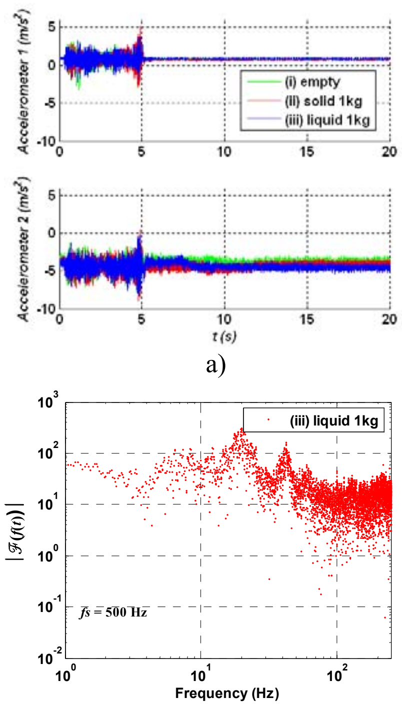


Fig. 4. a) Forces at the gripper sensor; b) f_z^{imp} spectrum.

Figure 5 a) shows the accelerations a_1^{liq} (accelerometer 1 at the clamped end of the container) and a_2^{liq} (accelerometer 2 at the terminal link of the robot) when the robot carries the liquid container. The signals are captured during a total period of $t_T = 20$ s. The a_1^{liq} signal spectrum is shown in figure 5 b).

Figures 4 b) and 5 b) show the spectrum of signals that contains information which is localized in time, due to the rod impact and the liquid vibration, respectively. Occasionally the signal spectra are scattered. In order to deal with these issues a multiwindow algorithm is used in the next sections.



b)

Fig. 5. a) a_1^{liq} and a_2^{liq} signals; b) a_1^{liq} signal spectrum.

3. MAIN CONCEPTS

3.1 The windowed Fourier transform

One way of obtaining the time-dependent frequency content of a signal is to take the FT of a function over an interval around a time instant τ. The WFT transform accomplishes this by using a general window function. The concept of this mathematical tool is straightforward. We multiply the signal to be analyzed $x(t)$ by a moving window $g(t-\tau)$ and, then, we compute the Fourier transform of the windowed signal $x(t)\, g(t-\tau)$. Each FT gives a frequency domain 'slice' associated with the time value at the window centre. Actually, windowing the signal improves local spectral estimates (Allen and Mills, 2004). The WFT for a window function centered at time τ, is represented analytically by:

$$F(\omega,\tau) = \int_{-\infty}^{+\infty} x(t) g(t-\tau) e^{-j\omega t} dt \qquad (1)$$

where $\omega = 2\pi f$ is the frequency.

Each window has a width t_w and the distance between two consecutive windows can be defined in a way so that they become overlapped during a percentage of time β in relation to t_w. Therefore, the frequencies of the analyzing signal $f < 1/t_w$ are rejected by the WFT. Diminishing t_w reduces the frequency resolution and increases the time resolution. Augmenting t_w has the opposite effect. Consequently, the choice of the WFT window entails a well-known duration-bandwidth trade-off.

The rectangular window can introduce an unwanted side effect in the frequency domain. As a result of having abrupt truncations at the ends caused by the window, the spectrum of the FT will include unwanted "side lobes". This gives rise to an oscillatory behavior in the time domain called the Gibbs phenomenon (Oppenheim, *et al.*, 1989). In order to reduce this unwanted effect, usually is used a weighting window function that attenuates the signals at their discontinuities. For this reason there are several popular windows normally adopted in the WFT as, for example, the Hanning, Hamming, Gaussian and Blackman (Oppenheim, *et al.*, 1989). Harris (Harris, 1978) and Nuttall (Nuttall, 1981) present several windows with its spectrum characteristics.

If the windows do not overlap, then it is clear that some data are lost. Additionally, if the windows overlap in a short period of time a significant part of the time signal is ignored due to the fact that most windows exhibit small values near the boundaries. To avoid this loss of data, overlap analysis must be performed.

In resume, in order to apply the WFT there are several parameters that must be defined, namely the window type, the window's width t_w and the overlapped time β. Some windows have also a parameter α that affects its shape. In this study are adopted two types of windows: the Gaussian and the power law/fractional window (PLW).

The Gaussian window has the following expression:

$$g(t) = e^{-\frac{1}{2}\left(\alpha \frac{t}{t_w/2}\right)^2}, \quad t \in \left[-\frac{1}{2}t_w, \frac{1}{2}t_w\right] \qquad (2)$$

where $\alpha, t_w \in \Re^+$ are parameters.

Expression (3) represents a window that we call power law/fractional due to the fact that the parameter $\alpha \in \Re$ can present any real value in the interval $0 < \alpha < \alpha_{max}$. The window is centered at time τ and the parameters (α, t_w) affect its shape and width.

$$g(t) = 1 - \left|\frac{t-\tau}{t_w}\right|^{\alpha}, \quad t \in \left[-\frac{1}{2}t_w, \frac{1}{2}t_w\right] \qquad (3)$$

This window is interesting due to the fact that the variation of α modifies significantly its shape. If $\alpha = 1$ it yields the well known Bartlett (or triangular) window.

Many authors studied the windows applied to the WFT in the perspective of their own characteristics. As referred previously, the choice of the window for a particular signal depends of the signal itself. Therefore, the automatic tuning of the window parameters is also dependent from the signal. Bearing these facts in mind, this article considers the window together with the signal.

3.2 Mutual information

The WFT denoted by $F(\omega,\tau)$ can be interpreted as a bidimensional probability density function with two variables ω and τ as long as we normalize it according with the expression:

$$F_1(\omega,\tau)=\frac{\left|\int_{t\min}^{t\max} x(t)g(t-\tau)e^{-j\omega t}dt\right|}{\int_\tau\int_\omega\left|\int_{t\min}^{t\max} x(t)g(t-\tau)e^{-j\omega t}dt\right|d\omega\, d\tau} \tag{4a}$$

The marginal probability distributions of the variables ω and τ are $F_2(\omega)$ and $F_3(\tau)$, respectively, according with the expressions:

$$F_2(\omega)=\int_\tau|F(\omega,\tau)|\,d\tau \tag{4b}$$

$$F_3(\tau)=\int_\omega|F(\omega,\tau)|\,d\omega \tag{4c}$$

The mutual information (Shannon, 1948; Cover and Thomas, 2006) or transinformation (Spataru, 1970), is the index that measures the dependence of two variables in the viewpoint of the information theory. The mutual information for the two values of variables ω and τ is:

$$I(\omega,\tau)=\log_2\frac{F_1(\omega,\tau)}{F_2(\omega)F_3(\tau)} \tag{5}$$

The average mutual information $I_{av}\in\Re$ between the two variables is given by:

$$I_{av}(\omega,\tau)=\int_\tau\int_\omega F_1(\omega,\tau)\log_2\frac{F_1(\omega,\tau)}{F_2(\omega)F_3(\tau)}\,d\omega\, d\tau \tag{6}$$

One application of I_{av} is to obtain the time lag required to construct the pseudo phase space. The I_{av} connects two sets of measurements with each other and establishes a criterion for their mutual dependence based on the idea of information connection. Additionally, I_{av} recognizes the non-linear properties of the variables (Trendafilova and Brussel, 2001). By other words, the mutual information presents good results both for linear and nonlinear relationships between the variables. In this line of thought, the mutual information will be tested for tuning the WFT.

4. RESULTS

To evaluate the behavior of the PLW applied to the WFT using the average mutual information $I_{av}(\omega,\tau)$, a set of signals captured in a robotic manipulator is used. In the sequel we depict only the most relevant features.

Figure 6 depicts the mutual information $I_{av}(\omega,\tau)$ for the f_x^{imp} signal (force *x* component at the gripper of the robot for the rod impact). The sensor is located at the gripper and the signal is acquired during $t_T = 8$ s. As referred previously the Gaussian (2) and the PLW (3) windows include the parameter α that affects the shape. Moreover, the overlapped time β can also vary. Therefore, the α and β parameters must be tuned. The values of α and β, for both windows, vary in the ranges $0.5<\alpha<6$ and $5<\beta<90$ %, respectively. Figures 6 a–c) show $I_{av}(\omega,\tau)$ for the Gaussian window width t_w = {0.25, 1, 3}s, respectively. Figures 6 d–f) show $I_{av}(\omega,\tau)$ for the PLW width t_w = {0.25, 1, 3}s, respectively. The dark zones represent higher values of $I_{av}(\omega,\tau)$. Comparing the corresponding figures of both windows for the same t_w, the obtained tuning for β is more accuracy for the fractional window. The corresponding value of α for both windows is different, because the shape of each window is affected differently by this parameter. The best tuning for the α and β parameters is obtained for $t_w = 3$s. The index $I_{av}(\omega,\tau)$ presents a peak at $(\beta, \alpha) = (19, 3.5)$ for the case of Gaussian window (Fig. 6c). Additionally, there are a set of higher values at $\beta = 19$% approximately. These set of values begin near $\alpha = 2.5$, which is the value usually adopted as default for the Gaussian window. In the case of the PLW the peak occurs for $(\beta, \alpha) = (19, 1)$, approximately (Fig. 6f). The results obtained for the PLW are more assertive comparing with those obtained for the Gaussian window.

5. CONCLUSIONS

The WFT is one of the most widely used time-frequency representations that is adopted in many fields of engineering. In order to use this technique several parameters must be defined according to the signal analyzed.

This work evaluates the behavior of the PLW applied to the WFT, when compared with the Gaussian window. A metric based on the mutual information was used to implement the evaluation. The results obtained for the PLW are more assertive comparing with those obtained for the Gaussian window. Additionally, the window settings obtained with the proposed index revealed to constitute a good compromise between the time and the frequency resolutions for the signals under analysis. The results based on experimental signals are promising and demonstrate the applicability and the effectiveness of the new approach.

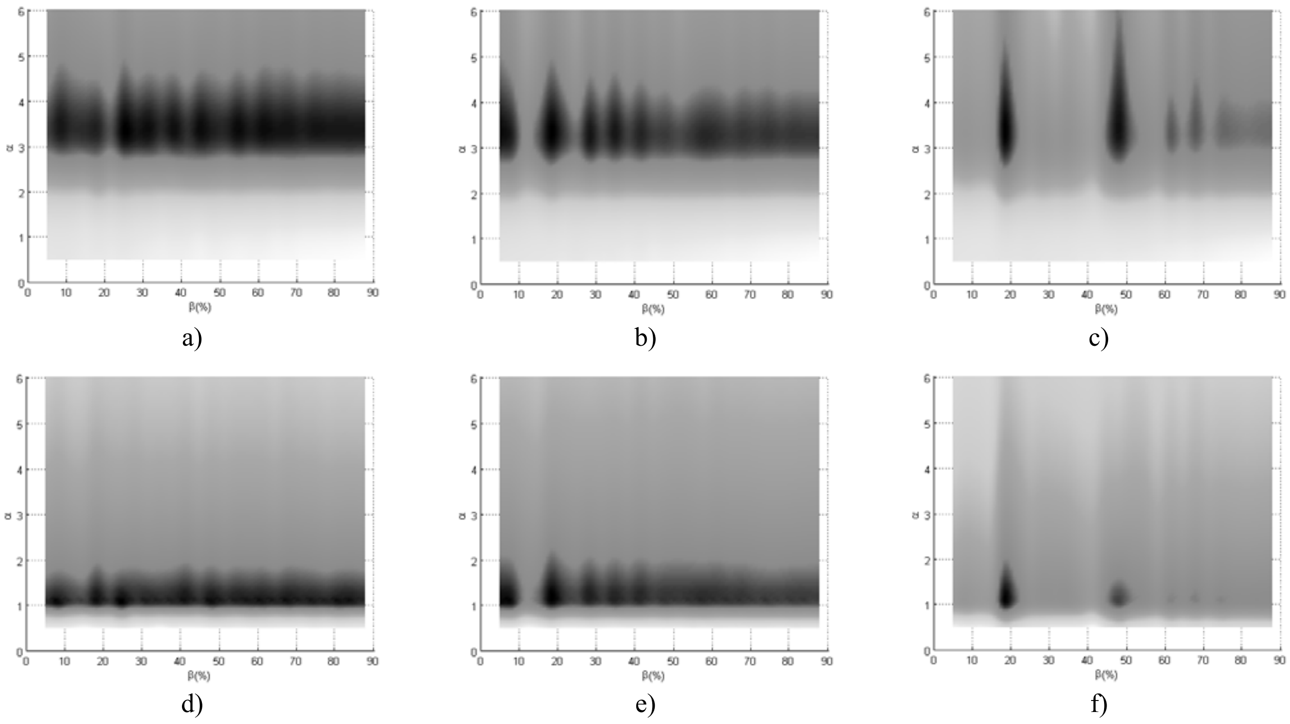


Fig. 6. The index $I_{av}(\omega,\tau)$ vs (α, β) of the f_x^{imp} signal for the a–c) Gaussian window t_w = {0.25, 1, 3}s; d–f) fractional window t_w = {0.25, 1, 3}s.

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