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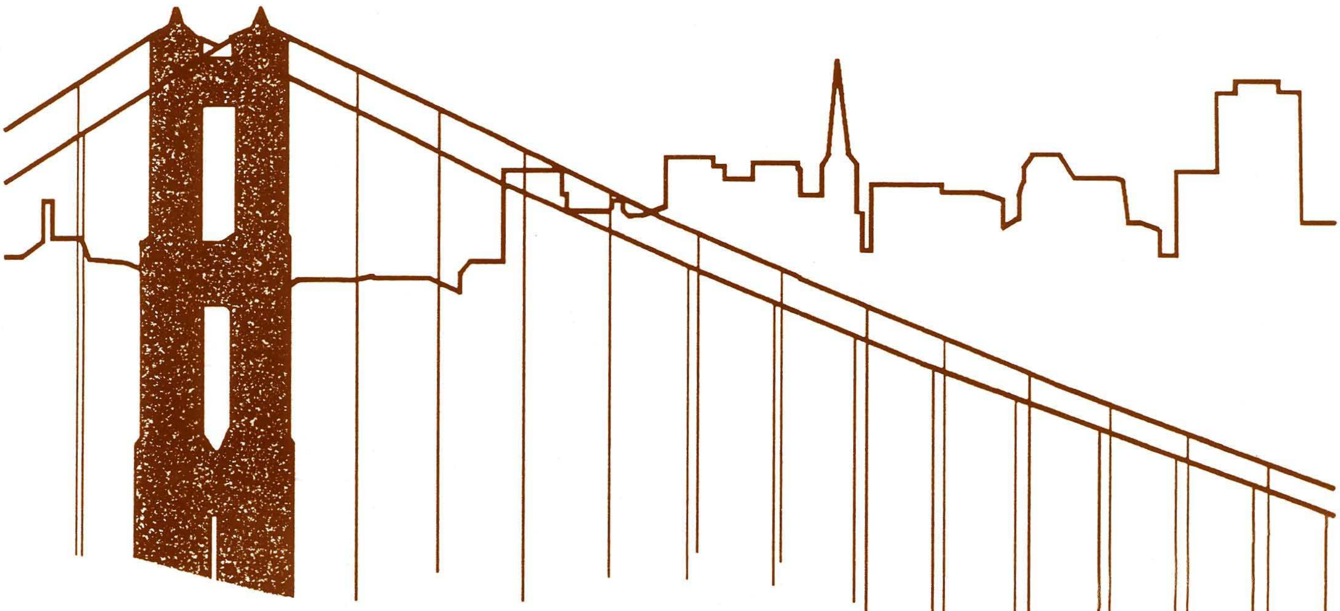

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VARIABLE STRUCTURE CONTROL OF SYSTEMS WITH NONLINEAR FRICTION AND DYNAMIC BACKLASH

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Abstract: This paper investigates the control of systems with nonlinear friction and dynamic backlash. The study is based on the describing function (DF) of nonlinear systems. The controllers studied are the first order model and the second order model variable structure controllers (FOM-VSC and SOM-VSC). The nonlinear plants consist on a mass with kinetic plus viscous friction or two masses subjected to backlash. The worst case for the limit cycle generation is found to be the closed-loop system having two masses with backlash. On the other hand, the closed-loop system of a mass with friction is found to be amenable to control without limit cycles.

Keywords: Variable structure control, Describing functions, Backlash, Friction, Nonlinear systems.

1. INTRODUCTION

The progress in computational systems made possible the intensive study of nonlinear systems through simulation and the development of nonlinear control strategies. In this perspective, this paper investigates the dynamics of systems with friction and backlash through the describing function (DF) method. These nonlinear dynamic phenomena have been a major area of research but well established conclusions are still lacking. Dupont (1992 and 1993) studied the effect of Coulomb friction in the existence and uniqueness of the solution of the direct dynamics. Studies about nonlinear friction modelling can be found also in Armstrong-Hélouvry (1991), Armstrong--Hélouvry *et al.* (1994), Gogoussis and Donath (1990), Haessig and Friedland (1991), Hu (1994), Southward and Radcliffe (1991) and de Wit *et al.* (1995). These papers investigated the position and velocity dependence of the friction phenomena. More recently, an efficient computer simulation of the stick-slip friction was developed by Karnopp (1985). The compensation of the nonlinear friction is found in the articles of Cai and Song (1994) and Walrath (1984). In these studies it is employed the model of the dynamic nonlinear friction for the development of efficient control systems.

The phenomenon of backlash is also found in several physical systems. Tao and Kokotovic (1993 and 1995) analysed this problem and developed an algorithm for the compensation of kinematic backlash based on an adaptive controller. Also, Choi and Noah (1989) and Dubowsky *et al.* (1987) studied the backlash phenomenon with simplified models while Stepanenko and Sankar (1986) investigated the case of compliant actuators with dynamic backlash. Other studies of backlash compensation and control can be found in Allan and Levy (1980), Bradenburg *et al.* (1986), Dagalakis and Myers (1985), Luh *et al.* (1983) and Machado (1995).

In this line of thought, this paper is organised as follows. In section 2 we formulate the problem treated in this paper and we introduce the DF method. In section 3 we introduce two variable structure controllers (VSC's) and we calculate their DF's. In sections 4 and 5 we study the control of a mass system having nonlinear friction and dynamic backlash, respectively. Finally, in section 6, the main conclusions are drawn.

2. DESCRIBING FUNCTIONS AND PREDICTION OF LIMIT CYCLES

In this section we present a summary of the DF method and its application to limit cycle prediction in nonlinear systems in the perspective of analysing controller performance in the presence of systems with friction and backlash.

Suppose that the input to a nonlinear element is sinusoidal. The output of the nonlinear element is, in general, not sinusoidal. Consider that the output is periodic with the same period as the input, containing higher harmonics in addition to the fundamental harmonic component. In the DF analysis, we assume that only the fundamental harmonic component of the output is significant. Such assumption is often valid since the higher harmonics in the output of a nonlinear element are usually of smaller amplitude than the amplitude of the fundamental component. Moreover, most control systems are "low-pass filters" with the result that the higher harmonics are further attenuated.

The DF of a nonlinear element is defined as the complex ratio of the fundamental harmonic components of the output $Y_1\angle\phi_1$ and the input $a\angle 0$, that is:

$$N = \frac{Y_1}{a}\angle\Phi_1 \tag{1}$$

where N represents the DF, a is the amplitude of the input sinusoid and Y_1 and Φ_1 are the amplitude and the phase shift of the fundamental harmonic component of the output, respectively. Several DF's of simple nonlinear systems can be found in Atherton (1975). In general, the DF can be computed evaluating the expression:

$$N(a,\omega) = \frac{2}{aT}\int_{t_1}^{T+t_1} y(\omega t)e^{-j\omega t}dt \tag{2}$$

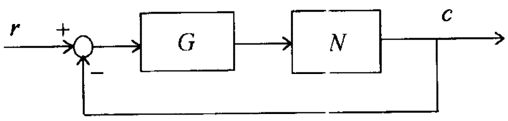

Fig. 1. Nonlinear control system.

where ω is the angular frequency of the input and output waveforms and $T = 2\pi/\omega$.

Once calculated, the DF can be used for the stability analysis of a nonlinear control system. Let us consider the standard system shown in Fig. 1 where the block N denotes the DF of the nonlinear element.

If the higher harmonics are sufficiently attenuated, N can be treated as a real or complex variable gain and the closed-loop frequency response becomes:

$$\frac{C(j\omega)}{R(j\omega)} = \frac{NG(j\omega)}{1+NG(j\omega)} \tag{3}$$

The characteristic equation is:

$$1 + NG(j\omega) = 0 \tag{4}$$

If equation (4) is satisfied, then the system will exhibit a limit cycle which may be found to be stable or unstable through mathematical and graphical analysis.

3. THE DF's OF VARIABLE STRUCTURE CONTROLLERS

The VSC's were introduced in automatic control systems by Utkin (1977). These algorithms are robust and present low computational requirements. Since then, VSC's have been studied for a large variety of applications. Some examples of other studies about VSC's are the works of Asada and Slotine (1986) and Young (1978).

If the controller G is nonlinear, as is the case of a VSC, we may employ the DF of the controller in the analysis of the

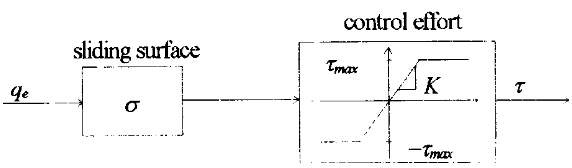

Fig. 2. The block diagram of the VSC.

closed-loop limit cycles. In fact, consider the first order model VSC (FOM-VSC) sketched in Fig. 2.

This VSC is of first order, because the sliding surface σ is given by the expression:

$$\sigma = \dot{q}_e + cq_e \tag{5}$$

where c is the corresponding eigenvalue. For a second order model VSC (SOM-VSC), the sliding surface σ is given by (Machado, 1993):

$$\sigma = \ddot{q}_e + 2\xi\omega_n\dot{q}_e + \omega_n^2 q_e \tag{6}$$

where ξ is the damping ratio and ω_n is the undamped natural frequency. A "standard" expression for the control effort block is:

$$\tau = \begin{cases} \tau_{max} & , \sigma \geq \tau_{max} / K \\ K\sigma & , |\sigma| < \tau_{max} / K \\ -\tau_{max} & , \sigma \leq -\tau_{max} / K \end{cases} \tag{7}$$

The DF for the FOM-VSC is given as follows:

$$N(a,\omega) = K(c + j\omega), \quad a \leq \frac{\tau_{max}}{K\sqrt{c^2+\omega^2}} \tag{8a}$$

$$N(a,\omega) = \frac{2Kc\phi_1}{\pi} - \frac{K\sqrt{4\pi^2+c^2T^2}}{\pi T}\sin(2\phi_1)\cos(\phi_2) + \\ + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\cos(\phi_2) \\ + j\left[\frac{4K\phi_1}{T} - \frac{K\sqrt{4\pi^2+c^2T^2}}{\pi T}\sin(2\phi_1)\sin(\phi_2) + \right. \\ \left. + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\sin(\phi_2)\right], \quad a > \frac{\tau_{max}}{K\sqrt{c^2+\omega^2}} \tag{8b}$$

$$\phi_1 = \arcsin\left(\frac{\tau_{max}T}{aK\sqrt{4\pi^2+c^2T^2}}\right),$$

$$\phi_2 = \arctan\left(\frac{2\pi}{cT}\right), q_e = a\cos(\omega t) \tag{8c}$$

For the SOM-VSC the expressions are very long and, therefore, will not be presented here. Nevertheless, for "very small a" the expression becomes:

$$N(a,\omega) = K\left[\left(\omega_n^2 - \omega^2\right) + j2\omega\xi\omega_n\right] \tag{9}$$

For example, the DF of a FOM-VSC with $K = 10$, $\tau_{max} = 10$ and $c = 2.5$ s^{-1} and several amplitudes of the sinusoidal input signal is depicted in Fig. 3. Note that the points corresponding to $a \to 0$ depend, only, on T and lie in a vertical straight line. On the other hand, the DF of a SOM-VSC for $K = 10$, $\tau_{max} = 10$, $\xi = 2.5$ and $\omega_n = 10$ rad s^{-1}, is represented in Fig. 4. We verify that the DF of the SOM-VSC is nearly a straight line passing through the origin, for $a = 0$ to $a \to \infty$. Furthermore, the DF of a FOM-VSC is always on the first quadrant, whereas the DF for the SOM-VSC may be either on the first or on the second quadrants of the Nyquist plane.

The "pseudo" DF's of the well-known PD and PID controllers are just the frequency response of the system and, therefore, are not amplitude dependent. The Nyquist diagram of the PD controller is just a vertical straight line starting (on the first quadrant) at a point (K_p, 0). The Nyquist diagram of the PID controller is also a vertical line with real part equal to K_p and passing through the fourth and first quadrants. Therefore, as we shall verify in the sequel, the VSC's are more appropriate to avoid limit cycles than the PID controllers, because their DF's do not pass through the fourth quadrant.

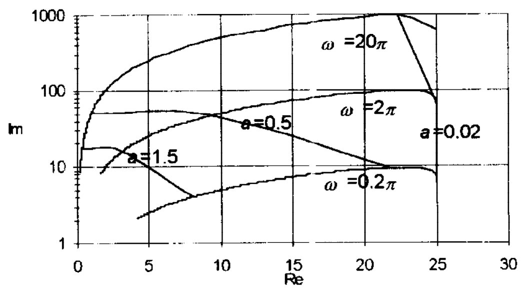

Fig. 3. The DF of a FOM-VSC ($K = 10$, $\tau_{max} = 10$, $c = 2.5$ s^{-1}).

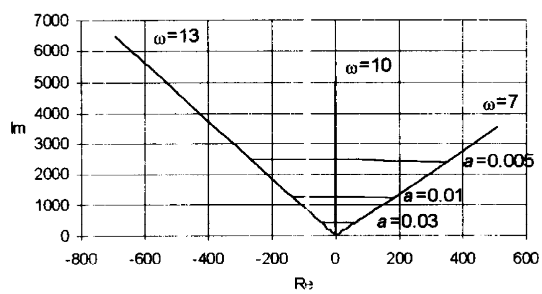

Fig. 4. The DF of a SOM-VSC ($K = 10$, $\tau_{max} = 10$, $\xi = 2.5$ and $\omega_n = 10$ rad s^{-1}).

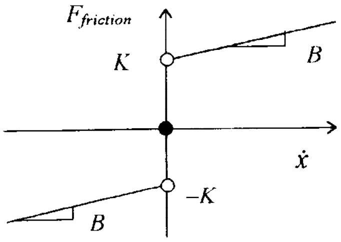

Fig. 5. Model of a Coulomb and viscous friction.

4. SYSTEMS WITH NONLINEAR FRICTION

In this section we calculate the DF of a dynamical system with nonlinear friction and we study its dynamical behaviour and controllability. Let us consider a system with a mass M, moving on a horizontal plane, under the effect of a Coulomb (K) plus a viscous friction (B). This type of friction is depicted in Fig. 5.

The steady-state response to a sinusoidal input force $F = a\cos(\omega t)$ becomes:

$$x(t) = \begin{cases} \alpha_1 \sin(\omega t + \phi) + k_1 + k_2 e^{-\frac{B}{M}t} - \frac{K}{B}t, & \dot{x} > 0 \\ \alpha_1 \sin(\omega t + \phi) + k_3 + k_4 e^{-\frac{B}{M}t} + \frac{K}{B}t, & \dot{x} < 0 \end{cases} \quad (10)$$

where the parameters ϕ, k_1, k_2, k_3 and k_4 cannot be determined in closed-form. Therefore, the DF must be determined numerically. Fig. 6 shows the function $-1/N(a, \omega)$ for a system with $M = 1$ Kg, $K = 1$ N and $B = 0.1$ Ns/m.

We conclude that limit cycles are 'avoided' by VSC's but may arise with PID controllers, because this algorithm has

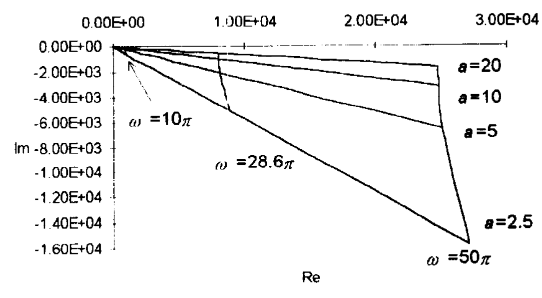

Fig. 6. The function $-1/N(a, \omega)$ for a system with a mass subjected to Coulomb and viscous friction.

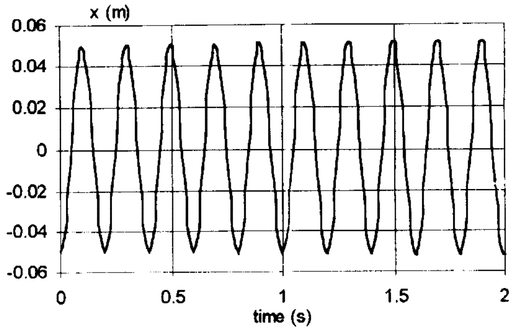

Fig. 7. Time response of a system mass with nonlinear friction and a PID controller.

part of its pseudo-DF on the fourth quadrant. Fig. 7 show a limit cycle for the PID control of a mass with nonlinear friction and parameters $K = 1$ N, $B = 0.1$ Ns/m, $M = 1$ Kg, $K_p = 987.6$, $K_i = 495.7\times10^3$, $K_d = 500$.

The frequency of the oscillation is in accordance with the one predicted by the DF method. On the other hand, the magnitude reveals a significant error. In fact, the intersection of the two DF's. is nearly tangential (i.e. unreliable), while for the frequencies the intersection is nearly perpendicular (i.e. with good accuracy).

In the next section we shall see that a system with dynamic backlash is a more complex case and, therefore, more difficult to control efficiently.

5. SYSTEMS WITH DYNAMIC BACKLASH

In this section we analyse a system with dynamic backlash and its control requirements through the DF method. In this perspective, let us consider a plant consisting on two masses subjected to dynamic backlash (Fig. 8).

A collision between the masses M_1 and M_2 occurs when $x_1 = x_2$ or $x_2 = h_1+x_1$. In this case the velocities of masses M_1 and M_2 after the impact ($\dot{x}_1'$ and $\dot{x}_2'$, respectively) obey

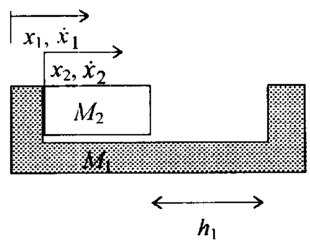

Fig. 8. System of two masses with dynamic backlash.

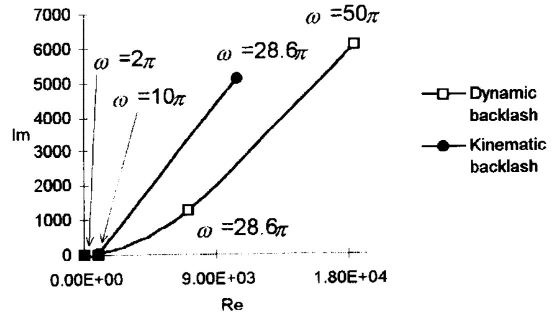

Fig. 9. Plot of $-1/N(a, \omega)$ for a system with backlash.

the Newton's law (11).

$$\dot{x}_{12}' = -\varepsilon\,\dot{x}_{12}, \quad 0 \le \varepsilon \le 1 \tag{11}$$

where $x_{12} = x_1 - x_2$ and ε is the coefficient of restitution. On the other hand, by the principle of conservation of momentum it comes:

$$M_1\dot{x}_1' + M_2\dot{x}_2' = M_1\dot{x}_1 + M_2\dot{x}_2 \tag{12}$$

From equations (12) and (13) we obtain:

$$\begin{cases} \dot{x}_1' = \dfrac{\dot{x}_1(M_1 - \varepsilon M_2) + \dot{x}_2(1+\varepsilon)M_2}{M_1 + M_2} \\ \dot{x}_2' = \dfrac{M_1(1+\varepsilon)\dot{x}_1 + (M_2 - \varepsilon M_1)\dot{x}_2}{M_1 + M_2} \end{cases} \tag{13}$$

We found $-1/N(a, \omega)$ for this plant via numeric simulation. The input sinusoidal force was applied to mass M_2 and the output position x_1 monitored. For example, for a system with $M_1 = M_2 = 0.5$ Kg, $\varepsilon = 0.5$, $h_1 = 2$ mm, and a input force $F(t) = 20\cos(\omega t)$ the DF is presented in Fig. 9. The upper curve corresponds to a kinematic backlash, while the lower curve describes the dynamic backlash. The kinematic backlash corresponds to the (ideal) DF of a mass $M_1 + M_2$ subjected to a input position sinusoid having a geometric backlash at the output (Atherton, 1975).

Analysing the DF of the controllers under study, we conclude that an intersection between $-1/N$ and G can occurs in the first quadrant, making this system prone to limit cycle generation when using PID controllers. In this case VSC's reveal a better response by avoiding large oscillations. As an example, let us consider a system with dynamic backlash and a FOM-VSC with parameters $K = 1$ Ns/m, $c = 2.5$ s^{-1} and $\tau_{max} = 10$ N. The time response of the closed-loop system is depicted in Fig. 10 that shows a significant overshoot and settling time. The oscillation remains in the steady state.

The time response of the same system with a SOM-VSC having the first eigenvalue of the sliding surface equal to the eigenvalue c adopted in the FOM-VSC and the second ten times higher is shown in Fig. 11. The result reveals a steady-state limit cycle, but the SOM-VSC presents a smaller overshoot (about 10 times smaller), a faster settling time and a reduced amplitude of oscillation.

6. CONCLUSIONS

This paper studied, through the DF method, the dynamical properties of systems with nonlinear friction or dynamic backlash. The worst system, in terms of controllability, is the one with dynamic backlash because it is more prone to limit cycles. The DF method of predicting limit cycles has shown a very good accuracy in terms of the frequency of the oscillation. We analysed the dynamic backlash instead of the kinematic backlash, because we have taken into

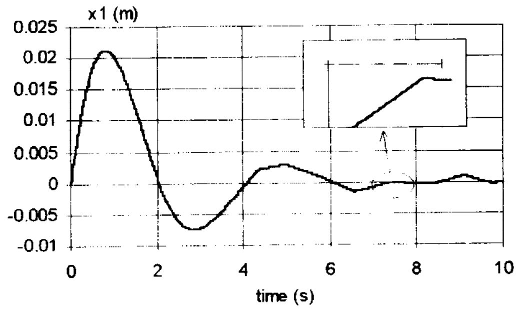

Fig. 10. Time response of a system with dynamic backlash under the control of a FOM-VSC.

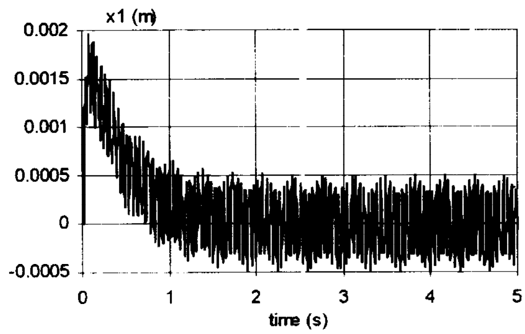

Fig. 11. Time response of a system with dynamic backlash and a SOM-VSC.

consideration the phenomenon of impact. The DF's of a FOM-VSC and a SOM-VSC were calculated and compared with classical PD and PID algorithms. The study demonstrated that, in general, the PID controller presents the worst controllability properties, such as, for example, in the simpler case of a system with nonlinear friction.

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