

Application of Evolutionary Algorithms in the Study of Fractional Order Electromagnetic Phenomena

Isabel S. Jesus*, J. A. Tenreiro Machado*

*Department of Electrical Engineering, Institute of Engineering of Porto
Rua Dr. António Bernardino de Almeida, 4200-072 Porto, Portugal
{isj,jtm}@isep.ipp.pt

Abstract

The application of evolutionary algorithms in optimization is currently receiving growing interest from researchers with various backgrounds. A genetic algorithm (GA) is a search technique based on the natural selection process that provides a unique flexibility and robustness for process optimization. Recently, a closer look of some phenomena present in electrical systems and the motivation towards the development of comprehensive models, pointed out the requirement for a fractional calculus approach. In this work, it is applied the concept of fractional calculus to define, and to evaluate, the electrical potential of fractional order. Bearing these ideas in mind, the paper addresses the analysis and the synthesis of fractional-order multipoles, based in a GA optimization scheme.

1 Introduction

A genetic algorithm (GA) is a search process for finding approximate solutions in optimization problems. GAs are a particular class of algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, natural selection, and crossover, established by the Darwin's theory of evolution [1-2]. Some applications of GAs are in the field of robotics, strategy planning, nonlinear dynamical systems, data analysis, engineering, art, music and many others in the real world applications [3-7].

The concept of differentiation and integration to no integer order dates to 1695, when Leibniz mentioned it in a letter to L'Hopital. Since then, many scientists developed the area and notable contributions have been made, both in theory and in applications. In the last years, a new look of some phenomena present in electrical systems, such as motors, transformers and lines [8], and the demand in the development of efficient models, induced the adoption of the fractional calculus (FC) approach. Some authors [9, 10] verified that well-known expressions for the electrical potential are related through integer-order integral and derivatives, and have proposed its generalization based on the concept of fractional-order poles. Nevertheless, the mathematical generalization towards FC lacks a comprehensive method for its practical implementation.

Bearing these ideas in mind, in this article we address the synthesis of fractional-order multipoles. In section 2 we recall the classical expressions for the static electric potential and we analyze them in the perspective of its generalization through the FC. In section 3 we develop a genetic algorithm (GA) scheme for

implementing fractional-order approximations for the electrical potential. Finally, in section 4 we outline the main conclusions.

2 Classical Expressions for the Static Electric Potential

For a homogeneous, linear and isotropic media, the electric potential φ at a point P produced by a single charge (1a), a dipole (1b), a quadrupole (1c), an infinite straight filament carrying a charge λ per unit length (2a), two opposite charged filaments (2b), and a planar surface with charge density σ (3), are given by [4, 5]:

$$\varphi = \frac{q}{4\pi\epsilon_0} \frac{1}{r} + C \quad (1a)$$

$$\varphi = \frac{ql \cos \theta}{4\pi\epsilon_0} \frac{1}{r^2} + C, r \gg l \quad (1b)$$

$$\varphi = \frac{ql^2(3\cos^2 \theta - 1)}{4\pi\epsilon_0} \frac{1}{r^3} + C, r \gg l \quad (1c)$$

$$\varphi = -\frac{\lambda}{2\pi\epsilon_0} \ln r + C, C \in \Re \quad (2a)$$

$$\varphi = \frac{\lambda l \cos \theta}{2\pi\epsilon_0} \frac{1}{r} + C, r \gg l \quad (2b)$$

$$\varphi = -\frac{\sigma}{2\varepsilon_0}r + C, C \in \mathfrak{R} \quad (3)$$

where ε_0 represents the permittivity, q the electric charge, r the radial distance, θ the corresponding angle with the multipole axis and $C \in \mathfrak{R}$.

Analyzing expressions (1)-(3) we verify the relationship $\varphi \sim r^{-3}, r^{-2}, r^{-1}, \ln r, r$ that corresponds to the application of integer-order derivatives and integrals.

3 Implementation of the Fractional Potential

The integer-order differential nature of the potential expressions motivated several authors [3] to propose its generalization in a FC perspective. Therefore, a fractional multipole produces at point P a potential $\varphi \sim r^{-\alpha}$, $\alpha \in \mathfrak{R}$. Nevertheless, besides the abstract manipulation of mathematical expressions, the truth is that there is no practical method, and physic interpretation, for establishing the fractional potential [2, 3, 6, 7, 8].

Inspired by the integer-order recursive approximation of fractional-order transfer functions [9, 10], we adopt a genetic algorithm (GA) [11, 12, 13] for implementing a fractional order potential. Similarly to what occurs with integer/fractional transfer a function, the electrical integer-order potential has a *global* nature and fractional-order potentials can have only a *local* nature. By other words, it is only possible to capture a fractional potential in a restricted region of space. This observation leads to an implementation approach conceptually similar

to the one described in [6, 9, 10, 14, 15] that is, to an approximation scheme based on a recursive placement of integer-order functions.

In this line of thought, we develop a one-dimensional GA that places n charges at the positions x_i and determines the corresponding values q_i ($i = 1, 2, \dots, n$). Our goal is to compare the approximate potential φ_{app} given by:

$$\varphi_{app} = \sum_{i=1}^n \frac{q_i}{4\pi\epsilon_0|x-x_i|} \quad (4)$$

that mimics the desired reference potential $\varphi_{ref} = kx^\alpha$ in a given interval Δ_x . The position $x = 0$ corresponds to the center of the multipole.

It is important to refer that a reliable execution and analysis of a GA usually requires a large number of simulations to guarantee that stochastic effects are properly considered [16, 17, 18, 19]. Therefore, in this study the experiments consist on executing the GA several times, for generating a combination of positions and charges that leads to an electrical potential with a fractional slope similar to the desired reference potential. The values of the GA parameters are: population number $P = 40$, crossover $C(\%) = 85.0\%$, mutation $M(\%) = 1.0\%$ and an elitist strategy $ES(\%) = 10.0\%$. The chromosome has $2n$ genes: the first n genes correspond to the charges and the last n genes indicate their positions. The gene codifications adopts a Gray code with a string length of $l = 16$ bits. The optimization fitness function corresponds to the minimization of the index:

$$J = \sum_{k=1}^m (\varphi_{app} - \varphi_{ref})^2 \quad (5)$$

where m is the number of sampling points along the interval $\Delta_x = x_{min} < x < x_{max}$. We establish a maximum number of iterations $I_{max} = 1000$ and a stopping scheme when $J < 10^{-1}$ for the best individual (*i.e.*, solution) of the GA population.

For an approximation with $n = 10$ charges, figure 1 shows φ_{app} and $\varphi_{ref} = 1.0x^{-1.5}$ (with a scale factor of $\times (4\pi\epsilon_0)^{-1}$), $\Delta_x = 0.2 < x < 0.8$, obtained for two different GA solutions:

- case A: $\{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\} = \{0.6564, 0.9534, -0.3816, 0.4514, 0.5028, 0.7531, -0.8771, -0.9381, -0.5641, -0.7395\}$ [C] (with scale factor $\times (4\pi\epsilon_0)^{-1}$), located at $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} = \{-0.0780, -0.0617, 0.0397, 0.1608, 0.1654, 0.4075, 0.7148, 0.7790, 0.7900, 0.8804\}$ [m], respectively;
- case B: $\{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\} = \{0.7719, 0.8944, 0.2151, 0.2392, 0.1264, -0.9352, -0.8805, -0.6431, -0.1115, 0.4729\}$ [C] (with scale factor $\times (4\pi\epsilon_0)^{-1}$), located at $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\} = \{-0.0738, -0.0583, 0.1109, 0.2639, 0.4409, 0.5615, 0.7249, 0.8441, 0.8610, 0.8688\}$ [m], respectively.

In the case A the GA needs $I_A = 34$ iterations to satisfy the adopted fitness function stopping threshold, and in the case B the GA needs $I_B = 115$ iterations.

The results show a good fit between φ_{ref} and φ_{app} and we verify that it is possible to find more than one ‘good solution’.

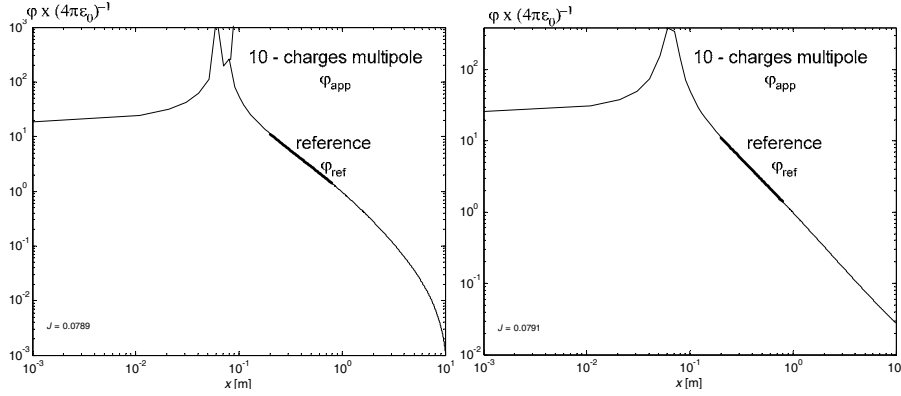


Fig. 1. Comparison of the electrical potential φ_{app} and φ_{ref} versus the position x for $\varphi_{ref} = 1.0 x^{-1.5}$ [volt], $\Delta_x = 0.2 < x < 0.8$ [m], and a $n = 10$ charges approximation, in both cases.

With the purpose of evaluating the influence on the charges and positions when we increase the length of the range of the approximations, we execute the GA for $x_{min} = -0.2$ and $x_{max} = \{0.2, 0.6, 1.0, 1.4, 1.8, 2.0\}$. For reducing the stochastic effects the GA is repeated 10 times and we analyze the average of the results. Figure 2 depicts the average of the charges versus the average of locations for $n = \{1, \dots, 10\}$.

We conclude that the GA places the first charge approximately in the same value and with the same location for all the cases. The pattern for the rest of the cases is to decrease the value of the charges and to extend their location when the total number of charges increases.

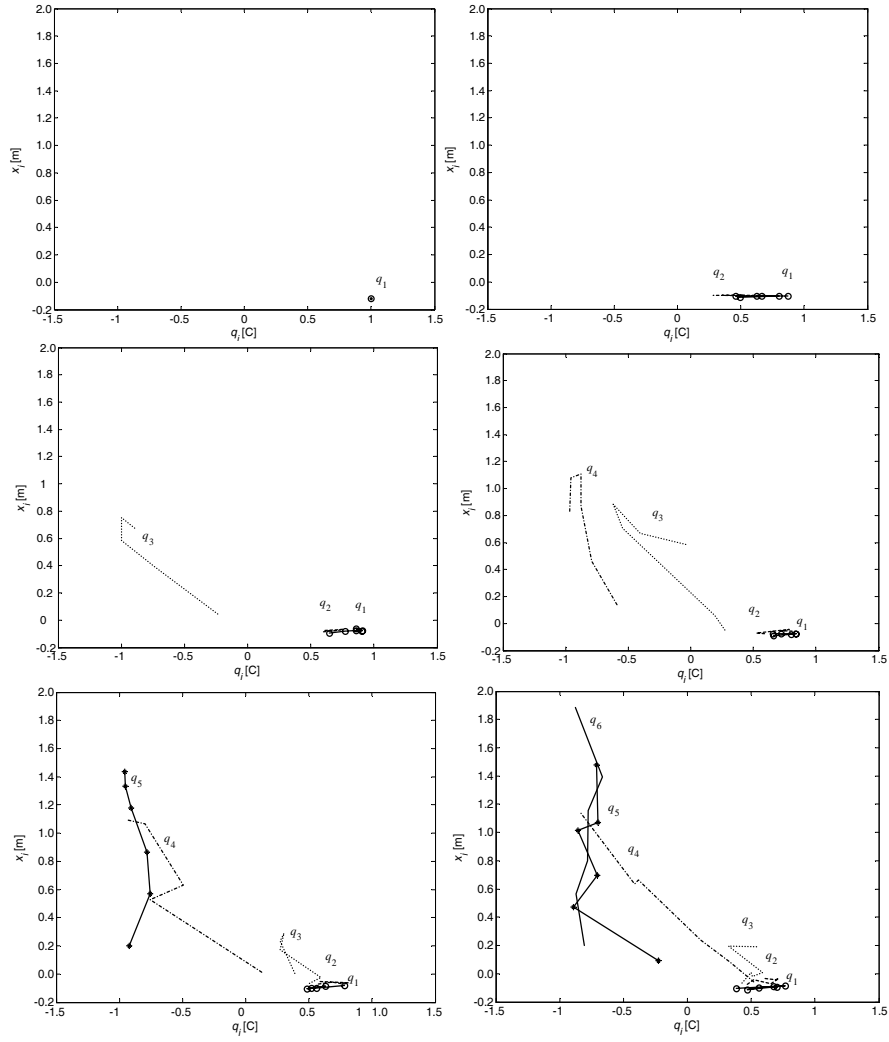


Fig. 2 (part 1). Locus of (μ_{qi}, μ_{xi}) : average of the charges μ_{qi} versus average of the positions μ_{xi} , $n = \{1, \dots, 10\}$

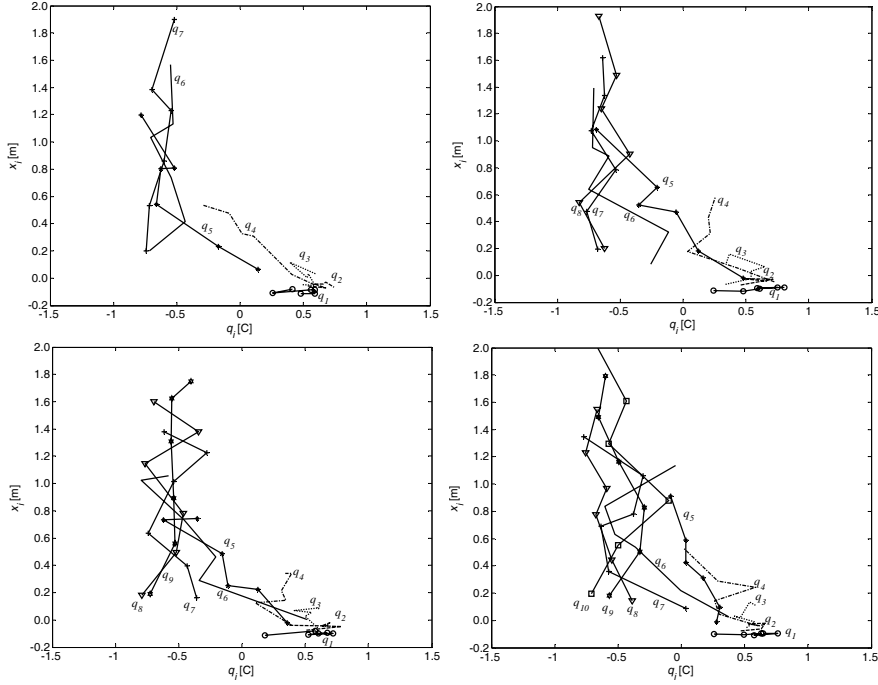


Fig. 2 (part 2). Locus of (μ_{q_i}, μ_{x_i}) : average of the charges μ_{q_i} versus average of the positions μ_{x_i} , $n = \{1, \dots, 10\}$

We study also the statistics of the charge values and positions for $n = \{1, \dots, 10\}$. Figures 3 and 4 show the averages μ_{q_i} and μ_{x_i} of the charges and their positions. Figures 5 and 6 depict the standard deviation σ_q and σ_x of the charge values and positions, respectively. Finally, figure 7 shows the minimum and maximum errors, $Min(J)$ and $Max(J)$, between φ_{ref} and φ_{app} , for the different number of charges.

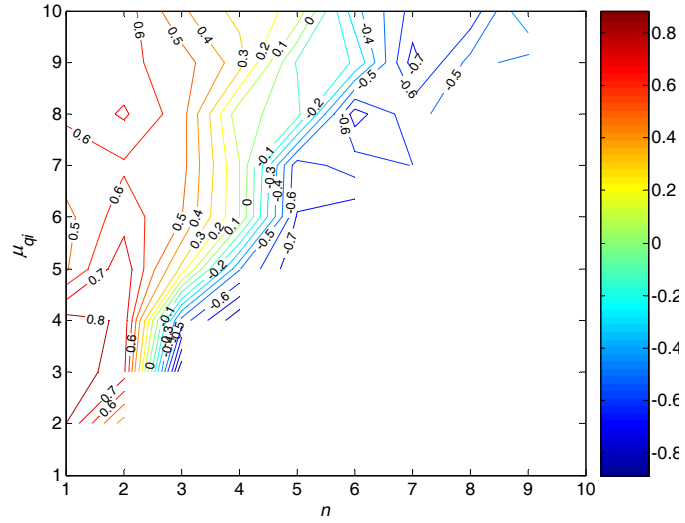


Fig. 3. Average of the charges μ_{qi} for $\phi_{ref} = 1.0 x^{-1.5}$ [volt], $\Delta_x = -0.2 < x < 1.0$ [m] and $n = \{1, \dots, 10\}$

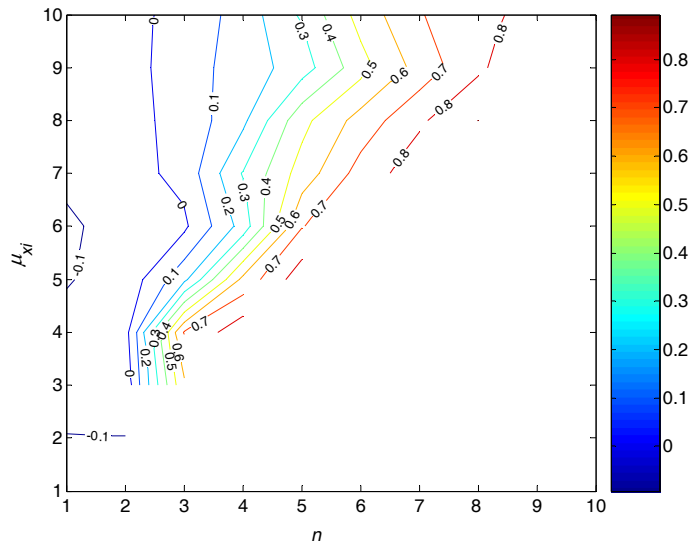


Fig. 4. Average position of the μ_{xi} for $\phi_{ref} = 1.0 x^{-1.5}$ [volt], $\Delta_x = -0.2 < x < 1.0$ [m] and $n = \{1, \dots, 10\}$

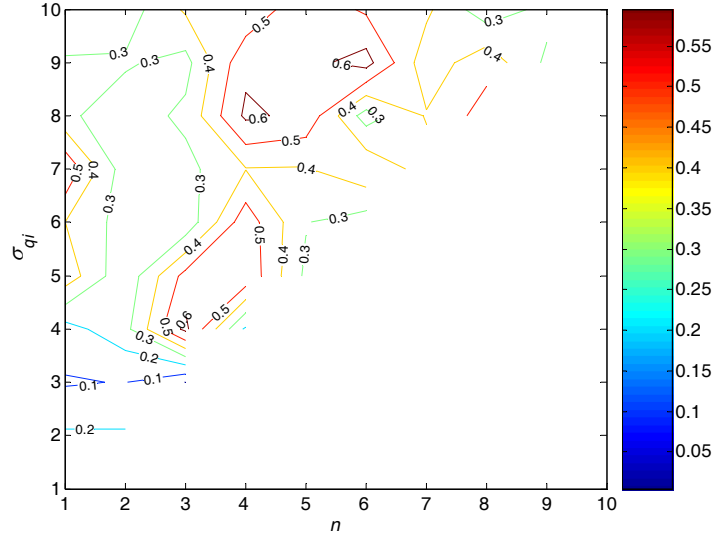


Fig. 5. Standard deviation of the charges σ_{qi} for $\phi_{ref} = 1.0 x^{-1.5}$ [volt], $\Delta_x = -0.2 < x < 1.0$ [m] and $n = \{1, \dots, 10\}$

Figure 7 reveals clearly that, as expected, the errors, both the minimum and the maximum, decrease with n . Therefore, for a given application, the larger the precision, the higher the required number of charges. We verify also that it is difficult to reveal a clear pattern in the solution space.

This lack of ‘order’ is due to the large number of possible solutions. Therefore, the GA has a high freedom, choosing solutions that are almost not correlated. However, we believe that further study imposing more strict restrictions may lead to the emergence of a comprehensive pattern.

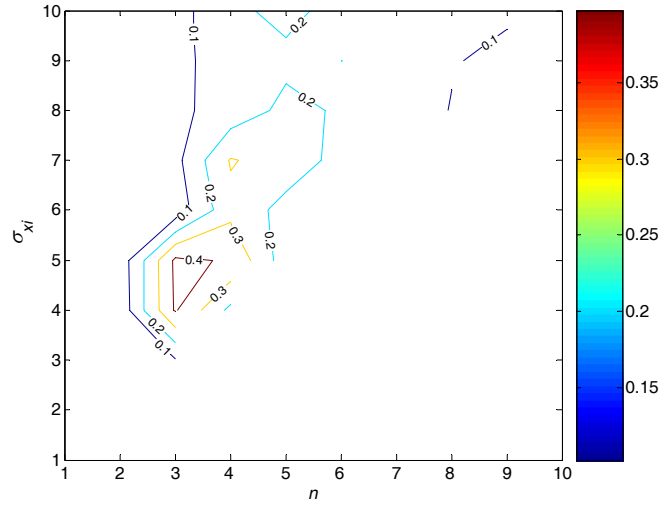


Fig. 6. Standard deviation of the positions σ_{xi} for $\phi_{ref} = 1.0 x^{-1.5}$ [volt], $\Delta_x = -0.2 < x < 1.0$ [m] and $n = \{1, \dots, 10\}$

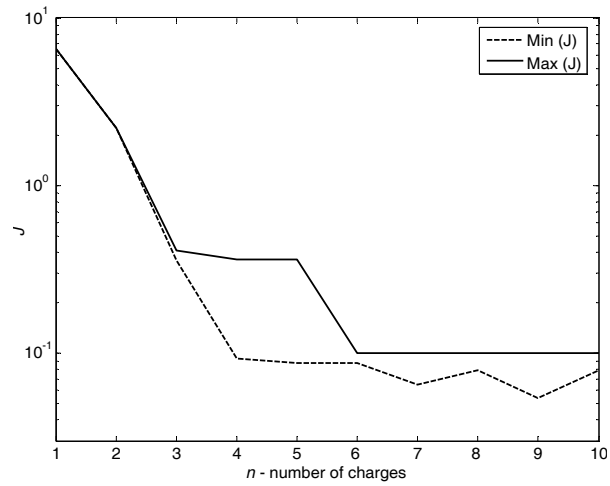


Fig. 7. Minimum and maximum errors between ϕ_{app} and $\phi_{ref} = 1.0 x^{-1.5}$ [volt], $\Delta_x = -0.2 < x < 1.0$ [m] for a sample of ten GA executions and $n = \{1, \dots, 10\}$

4 Conclusions

This paper addressed the implementation of a fractional-order electrical potential through a GA. The results reveal the necessity of a large number of charges for decreasing the approximation error. On the other hand, it was verified that increasing the number of charges leads to an higher dispersion of their location.

Bibliography

- [1] N. Engheta (1996) On fractional calculus and fractional multipoles in electromagnetism. *IEEE Transactions on Antennas and Propagation*, 44(4):554-566.
- [2] J. T. Machado, Isabel Jesus, Alexandra Galhano, Albert W. Malpica, Fernando Silva, József K. Tar (2005) Fractional order dynamics in classical electromagnetic phenomena. *Proc. of The Fifth EUROMECH Nonlinear Dynamics Conference*, Eindhoven, 1322-1326.
- [3] J. T. Machado, Isabel Jesus, Alexandra Galhano (2005) A fractional calculus perspective in electromagnetics. *Proc. of ASME - Int. Design Engineering Technical Conf. & Computers and Information in Engineering Conference - 5th Int. Conf. on Multibody Systems, Nonlinear Dynamics and Control*, U.S.A..
- [4] K. Küpfmüller (1939) *Einführung in die Theoretische Elektrotechnik*. Springer-Verlag, Berlin.
- [5] L. Bessonov (1968), *Applied Electricity for Engineers*. MIR Publishers, Moscow.
- [6] Isabel S. Jesus, J. A. Tenreiro Machado, J. Boaventura Cunha (2006) Application of genetic algorithms to the implementation of fractional electromagnetic potentials. *Proc. of The Fifth International Conference on Engineering Computational Technology*, Spain.
- [7] J. T. Machado, Isabel Jesus, Alexandra Galhano, J. Boaventura Cunha (2006) Fractional order electromagnetics. *Signal Processing (EURASIP/Elsevier)*, Special Issue on Fractional Calculus Applications in Signals and Systems, 86(10):2637-2644.
- [8] J. T. Machado, Isabel Jesus, Alexandra Galhano (2005) Electric fractional order potential. *Proc. of the ISEF 2005 – XII International Symposium on Electromagnetics Fields in Mechatronics, Electrical and Electronic Engineering*, Spain.
- [9] Keith B. Oldham and Jerome Spanier (1974) *The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order*. Academic Press.
- [10] A. Oustaloup (1991) *La Commande CRONE: Commande Robuste d'Ordre Non Entier*. Hermes.
- [11] D. E. Goldberg (1989) *Genetic Algorithms in Search Optimization and Machine Learning*. Addison-Wesley.
- [12] Z. Michalewicz (1996) *Genetic Algorithms + Data Structures = Evolution Programs*. Springer-Verlag.
- [13] Melanie Mitchell (1998) *An Introduction to Genetic Algorithms*. MIT Press.
- [14] Isabel S. Jesus and J. A. Tenreiro Machado (2008) Fractional Calculus and Applied Analysis. *FCAA - Fractional Calculus & Applied Analysis*, 3(11): 237-248.
- [15] Isabel S. Jesus, J. A. Tenreiro Machado and Ramiro S. Barbosa (2008), Implementing an electrical fractional potential through a genetic algorithm. *Proc. of NSC'08 - 2nd Conference on Nonlinear Science and Complexity*, Porto, Portugal.

- [16] Ting-Yu Chen, Yung Sheng Hsu (2006) A multiobjective optimization solver using rank-niche evolution strategy. *Advances in Engineering Software*, 37:684-699.
- [17] Shun-Fa Hwang, Rong-Song He (2006) Improving real-parameter genetic algorithm with simulated annealing for engineering problems. *Advances in Engineering Software*, 37:406-418.
- [18] C.K. Dimou, V.K. Koumouis (2003) Competitive genetic algorithms with application to reliability optimal design. *Advances in Engineering Software*, 34:773-785.
- [19] Hyun-Su Kim, Paul N. Roschke (2007) GA-fuzzy control of smart base isolated benchmark building using supervisory control technique. *Advances in Engineering Software*, 38:453-465.