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SURVEY PAPER

RANKING THE SCIENTIFIC OUTPUT OF RESEARCHERS IN FRACTIONAL CALCULUS

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Abstract

This paper analyses the citation profiles (CP) of 130 researchers in fractional calculus. In a first phase, the Canberra distance is used to measure the similarities between the researchers' CP, and the multidimensional scaling technique (MDS) is adopted for processing and visualizing the information. In a second phase, the gamma probability distribution is used to fit the normalized CP and the gamma parameters are used to characterize the researchers. The MDS results and the gamma distribution parameters are represented graphically in 2- and 3-dimensional locus depicting the relative positions of the researchers.

MSC 2010: Primary 01-XX, 26A33, 01A60, 01A61, 01A67; Secondary 34A08, 35R11, 60G22

Key Words and Phrases: science metrics, fractional calculus, Canberra distance, multidimensional scaling, visualization

1. Introduction

Quantifying the scientific output of researchers, namely their productivity (volume) and/or impact (recognition), is important in many circles, such as universities, journals, funding agencies, promotion committees and employers [38, 6].

The h-index was proposed in 2005 by Hirsch [22] to measure the scientific output of individual researchers [7]. A researcher has index $h \in \mathbb{N}$ if h is the largest number such that his h most cited publications have at least h citations each. For computing h we start by sorting the number of

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citations per publication in decreasing order, obtaining the citation profile (CP) given by the function $\phi(k)$, where $k \in \mathbb{N}$ represents the rank. Afterwards, for the array $H = \min(\phi(k), k)$, we obtain $h = \max(H)$. This means that h is close to the intersection between the curve $\phi(k)$ and a 45 degree straight line. The h-index has a time memory, since it captures the accumulation of citations [38].

Within a short period of years the h-index became popular and widely used as a criterion for establishing rankings [6]. Its domain of application surpassed its original purpose [6, 11, 27] and was adopted for assessing the scientific impact of journals [9, 31], measuring collective scientific output of scientists [45, 12, 1], and quantifying the volume of work in certain topics [4].

The h-index has the advantage of incorporating in a single-number criterion both the quantity and visibility of publications [13, 45]. Moreover, it is equally robust to modest and highly cited works [11, 17, 40]. Nevertheless, this index also has some weaknesses, as any other one-parameter measure, since it withdraws the multidimensional nature of the CP [45]. Additional shortcomings are its inability to differentiate between active and inactive researchers [47, 43], its sensitivity to long scientific careers [15] and to discipline-dependent CP [8, 23], or its difficulty to reflect the role of co-authorship [46, 2]. The limitations of h measure led to the proposal of complementary, or alternative, indices to assess the scientific output [22, 44, 14, 25]. These alternative measures can be grouped into four categories: *citation*, *year weighted*, *author weighted*, and *year and author weighted* indices [10]. The *citation* indices include the Hirsch's index and some variations, like the g and h^2 indices [14, 30], that give more weight to highly cited publications, and the e-index, that tries to differentiate between researchers with similar h, but different CP [48]. The *year weighted* indices take into account the age of the publications as, for example, the Jin's AR-index [26]. The *author weighted* indices include the effects of co-authorship, as occurs with the $hI, norm$-index [20]. The *year and author weighted* indices seek to include both effects, that is, the co-authorship and the year of the publications. Two examples of this group are the Jin's and the Harzing's AR-indices normalized per authors [26, 20].

Fractional Calculus (FC) generalizes the classical integral and differential operations to non-integer orders [28, 18, 3]. FC dates back to year 1695, starting with the exchange of letters between l'Hôpital and Leibniz about the meaning, and apparent paradox, of a n-order time derivative of a function, $f(t)$, $\frac{d^n f(t)}{dt^n}$, for $n = \frac{1}{2}$. During the last decades FC was recognized as playing an important role in modeling and control of many important physical phenomena. Nowadays the FC community includes a considerable number of researchers in different scientific fields, namely in mathematics, physics, engineering, biology, finance and geophysics [24, 34, 39, 36, 35, 37].

In this paper the CP of 130 researchers with work in FC are analyzed. Firstly, the Canberra distance is used to measure the similarities between CP. The multidimensional scaling (MDS) is adopted for processing and visualizing the information. Secondly, the gamma probability distribution is fitted in the CP data and the resulting parameters are adopted to characterize the researchers productivity. The MDS results and the gamma distribution parameters are depicted in 3- and 2-dimensional locus, respectively, providing identical interpretations for the relative locations of the points, that represent researchers.

In this line of thought, the paper is organized as follows. Sections 2 and 3 present the dataset and the mathematical and computational tools, respectively. Section 4 processes the data and discusses the results. Finally, Section 5 draws the main conclusions.

2. **The dataset**

Bibliometric data are available in several databases, such as Clarivate Analytics' Web of Science (http://www.clarivate.com/), Elsevier's Scopus (http://www.scopus.com), and Google Scholar (https://scholar.google.com). Scopus is now a well-established and accepted resource, used in many prestigious international rankings of universities, as is the case of the Times Higher Education ranking [21]. Elsevier claims that Scopus is now the largest curated abstract and citation database of peer-reviewed literature, featuring smart tools to track, analyze and visualize scientific research. The database is updated daily, and the titles covered are evaluated regularly [16].

The data used in this paper is publicly available, and corresponds to the CP of 130 FC researchers retrieved on August, 26^{th} 2018. Researchers with identical names and researchers that use different short names in their publications pose difficulties in the searching process. For minimizing errors caused by counting incorrectly the number of publications and/or citations, we adopt a combination of several searching fields, namely the author name, address, and affiliation. In the numerical analysis shown in the follow-up we identify the researchers by a 2-letter code (Table 2 in Appendix). It should be noted that the 130 researchers in the list were chosen to obtain a good geographic and gender representativeness. However, the sample includes a small subset of researchers with work in FC and, therefore, we apologize to all who were left out of this study.

The CP allow the determination of different indices to quantify the scientific output. The formulation of measures is an active topic and we can find many different proposals. In general, those indices are correlated to some extent. Figure 1 shows the Pearson correlation value, $r_{pq} = r_{qp}$, and the plot among all pairs $\{p, q\}$, $p, q = 1, \cdots, 14$, in the sample of Table 1, using the CP of 130 FC researchers. The histograms of the 14 variables appear along the matrix diagonal. We verify that most indices are highly correlated, with the exception of h^n that reveals different behavior from the others. Nevertheless, the question remains of knowing what index is the best for ranking CP.

Index type/number	Name	Abbreviation	Reference
citation			
1	Hirsch's h-index	h-index	[22]
2	Egghe's g-index	g-index	[14]
3	Jin's A-index	A-index	[25]
4	Kosmulski's h^2-index	h^2-index	[30]
5	Zhang's e-index	e-index	[48]
6	Sidiropoulos' normalized h-index	h^n-index	[43]
year weighted			
7	Sidiropoulos' contemporary h-index	h^c-index	[43]
8	Jin's AR-index	AR^J-index	[26]
9	Harzing's AR-index	AR^H-index	[20]
author weighted			
10	Batista's individual h-index	hI-index	[5]
11	Harzing's individual h-index	$hI, norm$-index	[19]
12	Schreiber's multi-authored h-index	hm-index	[42]
year and author weighted			
13	Jin's AR-index normalized per authors	AR_n^J-index	[26]
14	Harzing's AR-index normalized per authors	AR_n^H-index	[20]

Table 1. Sample of bibliometric indices.

3. Mathematical and computational tools

This section introduces the mathematical tools for processing the data, namely the Canberra distance, the MDS technique and the gamma distribution.

3.1. The Canberra distance. The Canberra distance was proposed, and latter modified, by Lance and Williams [32, 33]. Given 2 points in a K-dimensional space, $X = (x_1, \cdots, x_K)$ and $Y = (y_1, \cdots, y_K)$, the Canberra distance between X and Y is given by:

$$d_C(X, Y) = \sum_{k=1}^{K} \frac{|x_k - y_k|}{|x_k| + |y_k|}. \tag{3.1}$$

Equation (3.1) is a metric widely used for quantifying data scattered around an origin. The Canberra distance has several interesting properties,

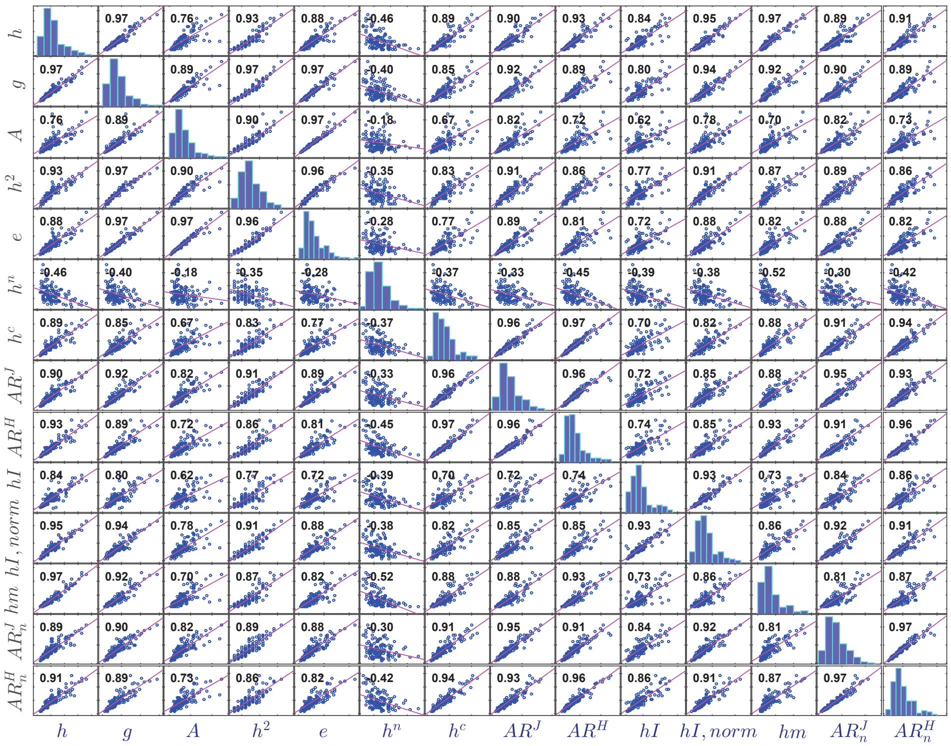

FIGURE 1. Pearson correlation between the 14 indices in Table 1 considering the CP of 130 FC researchers.

namely it is unitary when the arguments are symmetric, biased for measures around the origin, and highly sensitive for values close to zero.

3.2. **Multidimensional scaling.** MDS is a computational technique for clustering and visualizing data [41]. In a first phase, given s items and a measure of dissimilarity, a $s \times s$ symmetric matrix, $\mathbf{\Delta} = [\delta_{ij}]$, $(i,j) = 1, \cdots, s$, of item to item dissimilarities is calculated. The matrix $\mathbf{\Delta}$ represents the input information for starting the MDS numerical scheme. The MDS rational is to assign points for representing items in a multidimensional space and to try to reproduce the measured dissimilarities, δ_{ij}. In a second phase, MDS evaluates different configurations for maximizing some fitness function, arriving at a set of point coordinates (and, therefore, to a symmetric matrix of distances $\mathbf{D} = [d_{ij}]$ that represent the reproduced dissimilarities) that best approximates δ_{ij}. A common fitness function is the raw stress:

$$\mathcal{S} = [d_{ij} - f(\delta_{ij})]^2 , \tag{3.2}$$

where $f(\cdot)$ indicates some type of transformation.

The MDS interpretation is based on the patterns of points that can be visualized in the generated map. Similar (dissimilar) objects are represented by points that are close to (far from) each other. So, the information retrieval is not based on the point coordinates, or the geometrical form of the clusters, and we can rotate or translate the map because the distances remain identical. The MDS axes have neither special meaning nor units.

The quality of the MDS mat can be assessed by means of the stress and Shepard plots. The stress plot represents $\mathcal{S}$ versus the number of dimensions m of the MDS map. The plot $\mathcal{S}(m)$ is a monotonic decreasing chart and choosing the value of m is a compromise between achieving low values of $\mathcal{S}$ or m. Often are adopted the values $m = 2$ or $m = 3$ since they allow a direct visualization. The Shepard diagram, for a particular value m, compares d_{ij} and δ_{ij}. A narrow scatter around the 45 degree line represents a good fit between d_{ij} and δ_{ij}.

3.3. **Gamma distribution.** The gamma probability distribution is widely used to model continuous, positive and skewed random variables. Some well-known distributions, namely the exponential, the Erlang, and the chi-squared are special cases of gamma. Its probability density function, $f(x)$, can be expressed using different parametrizations, but we often use [29]:

$$f(x, \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \; x, \alpha, \beta > 0, \tag{3.3}$$

where α and β denote the shape and rate parameters, and $\Gamma(\cdot)$ represents the gamma function.

4. Data analysis and results

In this section we adopt a procedure that compares CP. Therefore, the Canberra distance is used for quantifying dissimilarities between researchers' profiles, and the MDS is adopted for clustering and visualization.

4.1. **Comparing and visualizing scientific output by means of MDS.** Given the CP (ϕ_i, ϕ_j) corresponding to researchers i and j, respectively, we first calculate a 130×130 symmetric matrix $\mathbf{\Delta} = [\delta_{ij}]$, $(i, j) = 1, ..., 130$, where δ_{ij} denotes $d_C(\phi_i, \phi_j)$, as given in (3.1), and K represents the length of the larger CP of (i, j). The smaller profile is filled with trailing zeroes for obtaining equal lengths for the ϕ_i and ϕ_j vectors. The resulting matrix $\mathbf{\Delta}$ is the input of the MDS algorithm. It should be noted that other distance measures were tested for constructing the matrix $\mathbf{\Delta}$, but several numerical simulations revealed that d_C leads to reliable and easily interpretable results.

Figure 2 depicts the 3D locus (i.e., $m = 3$) generated by the MDS, while Figures 3 and 4 represent the Shepard and stress plots, respectively. The Shepard diagram shows a good distribution of points around the 45 degree line, which means a good fit of the distances to the dissimilarities. The stress plot reveals that a 2D representation is insufficient, that 3D is a good choice, while higher dimensions lead to limited improvements. However, even adopting $m = 3$, the question remains of visualizing easily the information, since for 3D representations it is difficult to perceive assertively the real location of the objects in space.

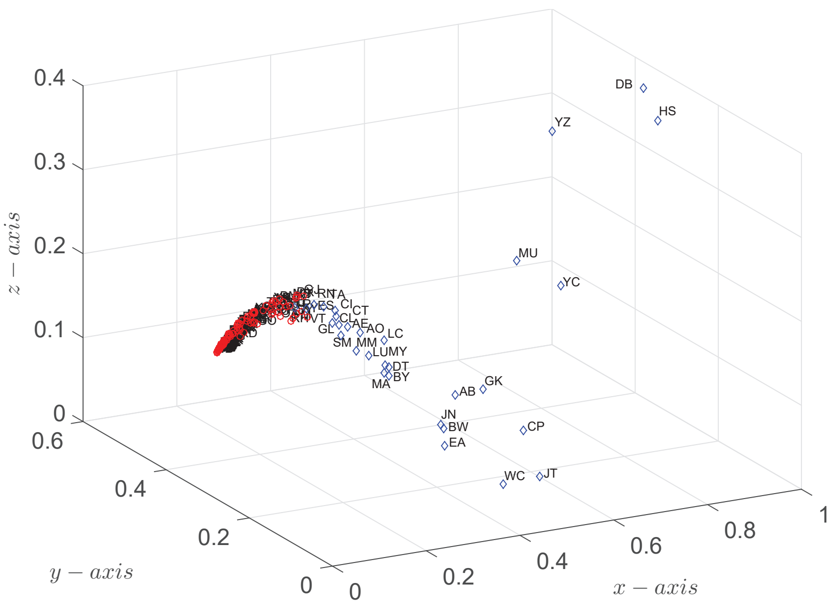

FIGURE 2. The 3-dimensional map generated by the MDS with d_C for $s = 130$ FC researchers. The blue diamonds represent 31 top cited researchers.

Despite all the constraints mentioned, we verify that points representing top cited researchers, that is, those on the mid-right hand side of Figure 2, stand out from the rest, but any other possible patterns (in case they exist) are hidden by the large number of points. This visual effect occurs because top cited researchers saturate the graph and difficult establishing relationships among the others. Magnifying the cloud of points mitigates the problem, but does not solve the problem significantly. If we remove the researchers that saturate the plot and calculate the corresponding MDS we have a new (different) locus based on the pruned data. Figure 5 depicts the 3D MDS locus with d_C, for $s = 99$, that is, after removing 31 top researchers

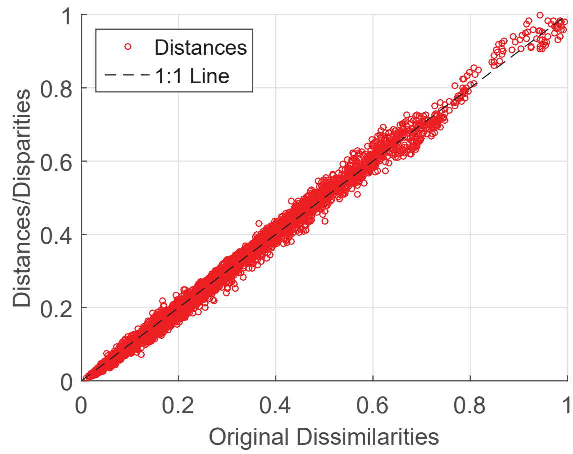

FIGURE 3. The Shepard diagram generated by the MDS with d_C for $s = 130$ FC researchers.

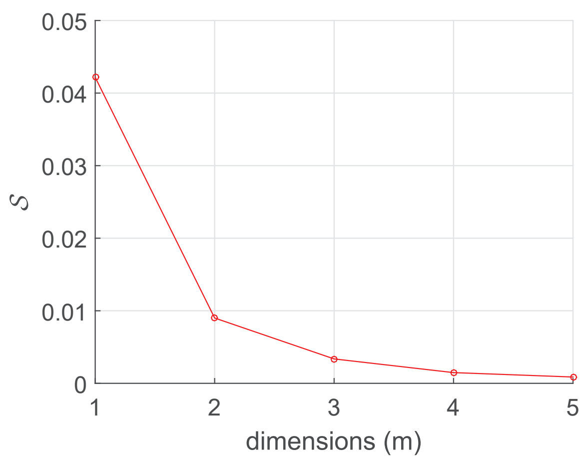

FIGURE 4. The stress map generated by the MDS with d_C for $s = 130$ FC researchers.

from the original data. The Sheppard and stress plots are very close to the previous ones and they are omitted here. We verify that the relative location of mid cited researchers is now clear, but, again, lower cited ones are still overlapping. So, this procedure can be adapted, either withdrawing top, or bottom, cited researchers, for obtained easily interpretable results.

4.2. **Modeling and visualizing scientific output by means of the gamma distribution.** For overcoming the difficulties in visualizing the

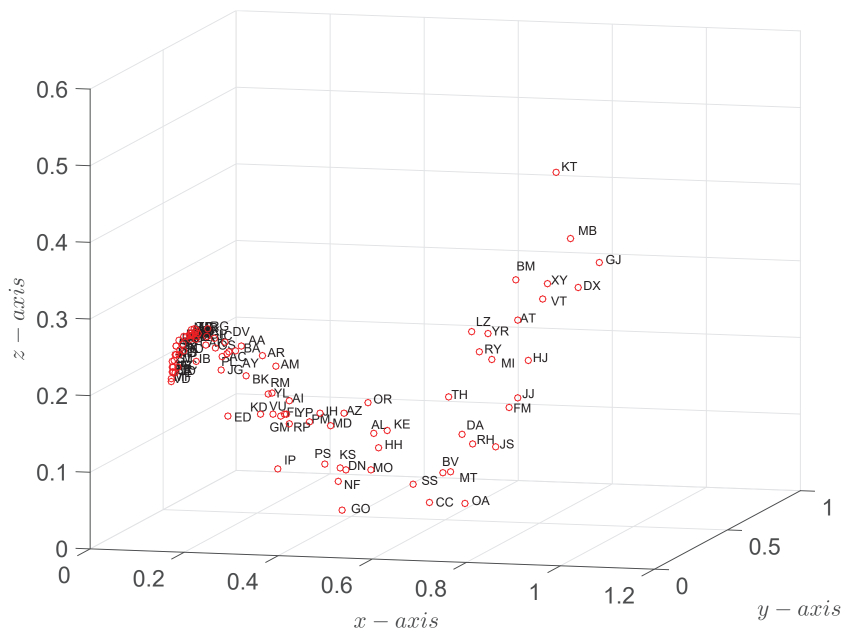

FIGURE 5. The 3-dimensional map generated by the MDS with d_C, for $s = 99$, that is, after removing 31 top researchers from the original data.

MDS results, we model the researchers' CP, $\phi(k)$, by a suitable 2-parameter function and we compare their scientific output based on the parameters locus.

We start by normalizing each $\phi(k)$ by the corresponding total number of citations, obtaining

$$\phi_n(k) = \frac{1}{\sum_k \phi(k)} \phi(k). \tag{4.4}$$

We have $\sum_k \phi_n(k) = 1$ and thus we can interpret ϕ_n as a density probability function that we approximate by means of a gamma distribution (3.3).

For example, Figure 6 depicts the original and fitted data for one researcher, where $(\alpha, \beta) = (0.8720, 0.0496)$, demonstrating the suitability of $f(x, \alpha, \beta)$ for approximating ϕ_n.

Figures 7 and 8 depict the locii of (α, β) for the two cases in Subsection 4.1, that is, when considering $s = 130$ and $s = 99$ FC researchers. The relative positions of the points in these plots are comparable with those produced by the MDS and depicted in Figures 2 and 5, respectively. Therefore, we verify that both for the MDS and the gamma parameters representations, clear patterns emerge based on the scientific CP. Nonetheless, the locii (α, β) of the gamma parameters has the advantage of being simpler to visualize.

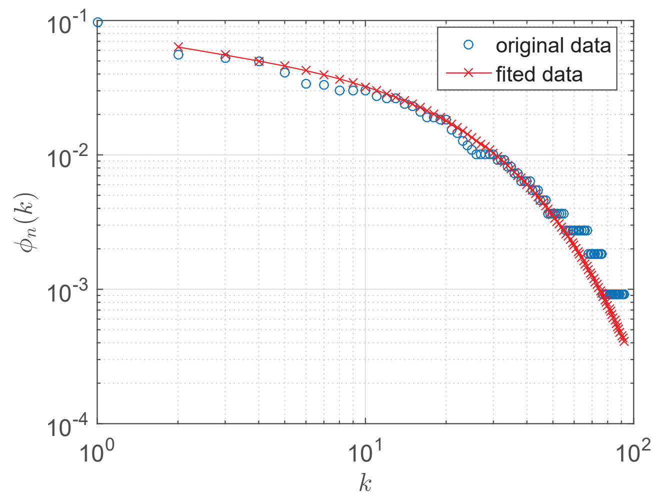

FIGURE 6. Normalized CP, C_n and the gamma density probability function approximation, $f(x, \alpha, \beta)$, for one researcher.

5. Conclusions

In this paper the CP of 130 FC researchers was compare. First, the Canberra distance was used to quantify the similarities between researchers' CP, and the MDS technique adopted for processing and visualizing the information. Second, the gamma distribution was used to approximate the CP viewed as a histogram, and the locus of parameters adopted to depict the relative positioning of the researchers. Both the MDS and the gamma distribution lead to identical results and conclusions, but the locus of the gamma parameters has the advantage of leading to a more direct visualization.

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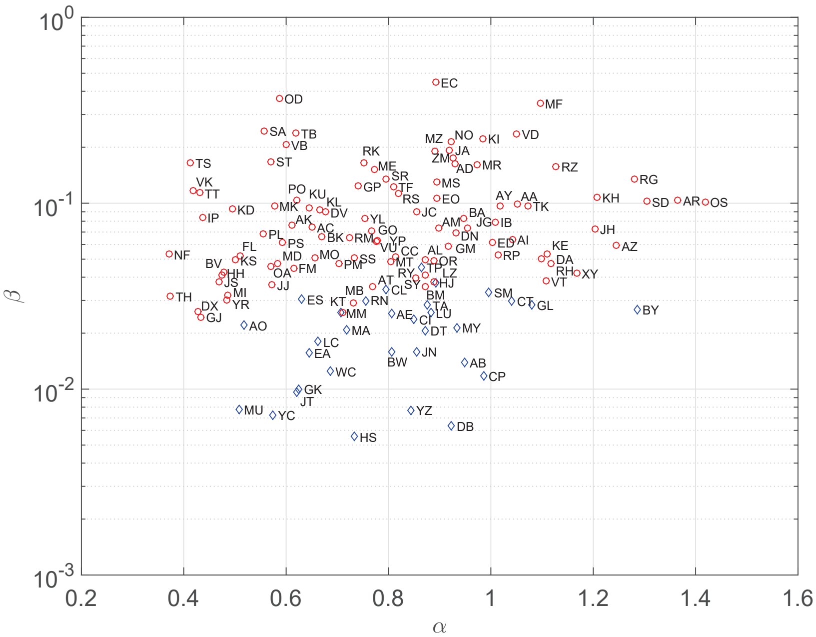

FIGURE 7. Locus of the gamma distribution parameters, (α, β), for $s = 130$ FC researchers. The blue diamonds represent 31 top cited researchers.

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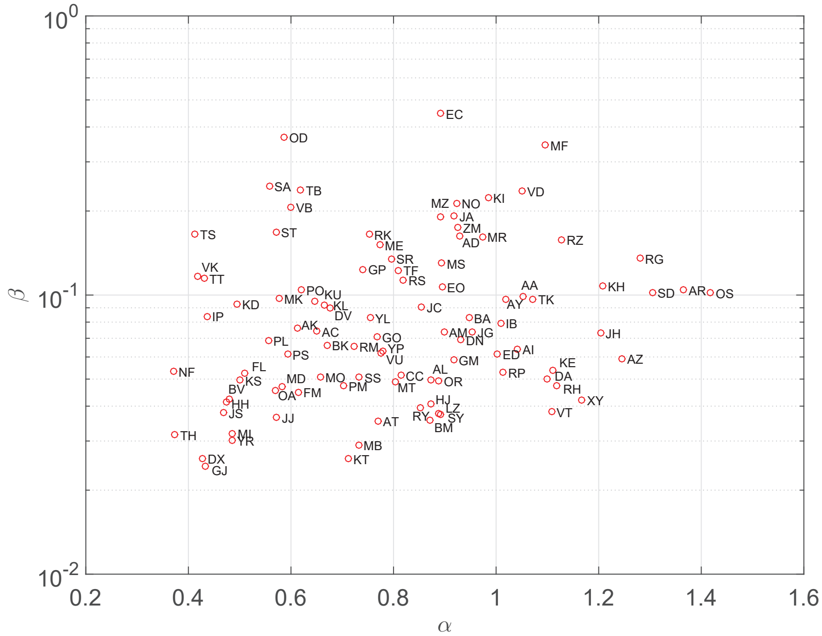

FIGURE 8. Locus of the gamma distribution parameters, (α, β), for $s = 99$, that is, after removing 31 top researchers from the original data.

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Appendix – Table 2

Code	Name	h-index	Code	Name	h-index
AA	Alireza Alfi	20	LC	Liping Chen	27
AB	Alexander Blumen	55	LU	FaWang Liu	43
AC	Abdelfatah Charef	14	LZ	Carlos Lizama	19
AD	Amar Debbouche	13	MA	Richard L. Magin	43
AE	A.S. Elwakil	30	MB	Maamar Bettayeb	21
AI	Alexander Iomin	17	MD	Mark Malamud	17
AK	Anatoly Kochubei	13	ME	Mark Edelman	13
AL	António M. Lopes	20	MF	Masataka Fukunaga	6
AM	Agnieszka B. Malinowska	19	MI	A. M. Mathai	19
AO	Alain Oustaloup	31	MK	Malgorzata Klimek	12
AR	Omar Abu Arqub	21	MM	Mark M. Meerschaert	46
AT	Abdon Atangana	21	MO	Manuel D. Ortigueira	20
AY	M.A. Abdelkawy	15	MR	Margarita Rivero	15
AZ	Agacik Zafer	17	MS	Mohammed Al-Smadi	13
BA	Mauro Bologna	14	MT	Mohammad Saleh Tavazoei	26
BK	Karabi Biswas	15	MU	Michael Unser	72
BM	B. Maundy	19	MY	Masahiro Yamamoto	33
BV	Blas M. Vinagre	24	MZ	M.A. Zaky	14
BW	Bruce J. West	49	NF	Neville J. Ford	23
BY	Ali Bhrawy	43	NO	Necati Ozdemir	8
CC	Carlos F. M. Coimbra	28	OA	Om Prakash Agrawal	32
CI	Clara Ionescu	21	OD	Ozlem Defterli	7
CL	Changpin Li	41	OR	Enzo Orsingher	19
CP	Alberto Carpinteri	49	OS	Adel Ouannas	13
CT	Carlo Cattani	27	PL	Patrick Lanusse	15
DA	E. H. Doha	28	PM	Pierre Melchior	15
DB	Dumitru Baleanu	50	PO	Piotr Ostalczyk	8
DN	Diego del-Castillo-Negrete	23	PS	Ivo Petras	23
DT	Delfim Torres	32	RG	Roberto Garrappa	16
DV	Duarte Valério	15	RH	Rudolf Hilfer	34
DX	Dingyu Xue	21	RK	Reyad El-Khazali	11
EA	Elias C. Aifantis	52	RM	Rachid Malti	16
EC	Eduardo Cuesta	8	RN	Raoul Nigmatullin	25
ED	Eva Dulf	10	RP	Richard B. Paris	13
EO	Edmundo Capelas de Oliveira	12	RS	Renat Sibatov	9
ES	Enrico Scalas	27	RY	Santanu Saha Ray	19
FL	Carl F. Lorenzo	17	RZ	Rico Zacher	11
FM	Francesco Mainardi	36	SA	Sadia Arshad	6
GJ	Guy Jumarie	21	SD	Shantanu Das	16
GK	George Em Karniadakis	66	SM	Shaher Momani	47
GL	Grzegorz Litak	26	SR	Sergei Rogosin	9
GM	Guido Maione	16	SS	Stefan G. Samko	22
GO	Rudolf Gorenflo	28	ST	Pablo Raúl Stinga	8
GP	Gianni Pagnini	14	SY	Santos Bravo Yuste	28
HH	Hans J. Haubold	18	TA	Teodor M. Atanackovic	25
HJ	Hossein Jafari	28	TB	Catalina Tobón	6
HS	Hari M. Srivastava	51	TF	Todd Freeborn	12
IB	Inés Tejado Balsera	10	TH	Tom T. Hartley	19
IP	Igor Podlubny	22	TK	Jan Terpak	8
JA	Jay L. Adams	6	TP	Viet-Thanh Pham	31
JC	Jacky Cresson	15	TS	Tomas Skovranek	8
JG	J.F. Gómez-Aguilar	11	TT	Thiab Taha	12
JH	Jordan Hristov	21	VB	Vicente Feliu Batlle	6
JJ	Juan J. Trujillo	26	VD	Vladimir Despotovic	5
JN	Juan Nieto	56	VK	Virginia Kiryakova	14
JS	Jocelyn Sabatier	25	VT	Vasily E. Tarasov	31
JT	J. Tenreiro Machado	42	VU	Vladimir Uchaikin	17
KD	Kai Diethelm	22	WC	Wen Chen	39
KE	Mokhtar Kirane	20	XY	Xiao-Jun Yang	31
KH	Katica R. Hedrih	9	YC	YangQuan Chen	61
KI	Mateusz Kwaśnicki	9	YL	Yury Luchko	21
KL	Konstantinos Lazopoulos	12	YP	Yuriy Povstenko	17
KS	R.K. Saxena	21	YR	Yuriy A. Rossikhin	20
KT	József K. Tar	16	YZ	Yong Zhou	51
KU	Vladimir V. Kulish	11	ZM	Zeinab S. Mansour	7

Table 2 (see previous page). List of 130 researchers with work in FC. The h-index was retrieved on August, 26^{th} 2018 on the Elsevier's Scopus website https://www.scopus.com/.

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