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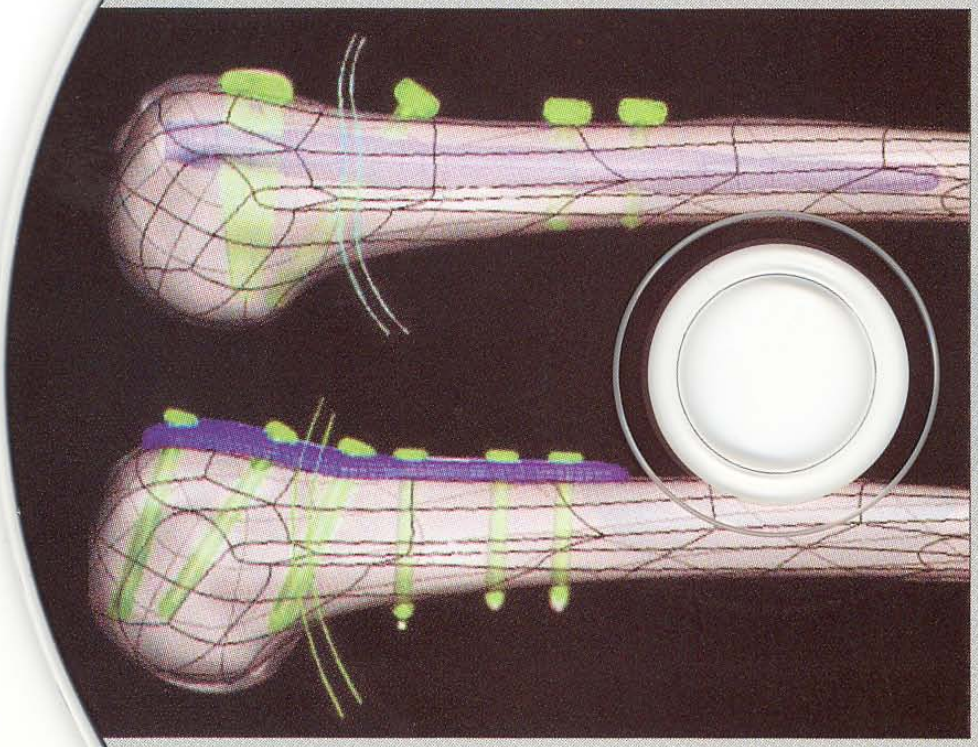

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Application of Genetic Algorithms to the Implementation of Fractional Electromagnetic Potentials

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Abstract

A genetic algorithm (GA) is a search technique based on the natural selection process. The GAs provide further flexibility and robustness that are unique for signal process. Recently, a closer look of some phenomena present in electrical systems, such as motors, transformers and lines, and the motivation towards the development of comprehensive models, seem to point out the requirement for a fractional calculus approach. Bearing these ideas in mind, we address the analysis and the synthesis of fractional-order multipoles, based in a GA optimization scheme.

Keywords: electrical potential, fractional order systems, genetic algorithm.

1 Introduction

A genetic algorithm (GA) is a search process for finding approximate solutions in optimization problems. The GAs are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, natural selection, and crossover, established by the Darwin's theory of evolution [1 – 2]. Some applications of GAs are in the field of robotics, strategy planning, nonlinear dynamical systems, data analysis, art, evolving pictures, music and many others in the real world applications.

The concept of differentiation and integration to no integer order dates of 1695, where Leibniz mentions it in a letter to L'Hopital. Since then, many scientists developed the area and notable contributions have been made, both in theory and in applications. In the last years, a new look of some phenomena present in electrical systems, such as motors, transformers and lines [3], and the demand in the development of efficient models, induced the resource of the fractional calculus (FC) approach. Several authors [4 – 5] have verified that well-known expressions for the electrical potential are related through integer-order integral and derivatives,

and have proposed its generalization based on the concept of fractional-order poles. Nevertheless, the mathematical generalization towards FC lacks a comprehensive method for its practical implementation.

Bearing these ideas in mind, we address the analysis and the synthesis of fractional-order multipoles. In section 2 we refereed the fundamental concepts of GAs. In section 3 we start by recalling the method for approximating fractional-order transfer functions based on integer order expressions. Following that, in section 4 we review classical expressions for the static electric potential and we analyze them in the perspective of FC. Based on this re-evaluation in section 5 we develop a GA for implementing fractional-order electrical potential approximations. Finally, in section 6 several conclusions are drawn.

2 Concepts of Genetic Algorithms

A genetic algorithm (GA) is a search technique used in computer science to find approximate solutions in optimization and search problems. GAs are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, natural selection, and crossover [1 – 2], [6].

The evolutionary computing was introduced in the 60s by I. Rechenberg, and the GAs were invented by John Holland that published an book in 1975 about this subject.

These algorithms begin with a set of solutions, represented by chromosomes, called population (P). Initially, a population is generated randomly. Solutions from one population are taken (parents) and used to form a new P. This is motivated by the hope that the new P will be better than the old one. Individuals are then selected to form new solutions according with their fitness; therefore, the more suitable they are the more chances they have to reproduce. This is repeated until some condition is satisfied. For example, the proximity with a best solution. Figure 1 presents the block diagram representative of the methodology used in the GAs.

In these algorithms the crossover (C) and the mutation (M) operators are the most important parts. The C is a recombination operator that combines subparts of two parent chromosomes to produce offspring that contain some parts of both parents genetic material. The simplest way to do it is to choose some randomly C point, copy everything before this point from the first parent, and then to copy everything after the C point from the other parent. There are other ways to make C, namely we can choose more C points. The most used way of encoding is a binary string; however, there are many other ways of encoding, such as to encode directly through real numbers.

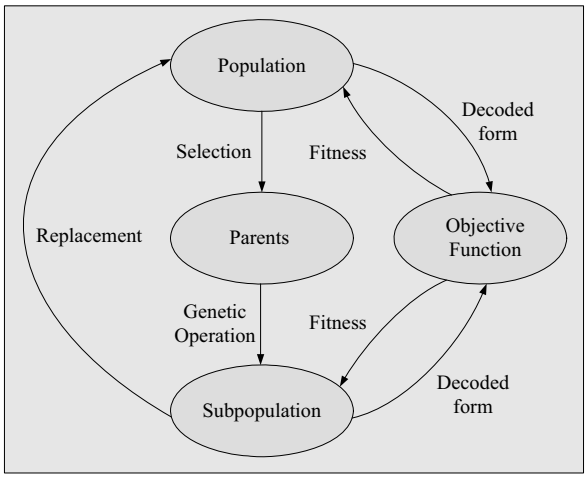


Figure 1. Block diagram of a GA

The selection of a better encoding technique depends on the problem we have to solve. The M operation randomly changes the offspring resulted from C. This procedure intended to prevent falling of all solutions in the P into a local optimum. In case of binary encoding, we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. Figure 2 and Figure 3 depict one example of C and M, respectively, both in binary encode.

Chromosome 1	11011 | 11000011110
Chromosome 2	**11011** | **00100110110**
Offspring 1	11011 | **00100110110**
Offspring 2	**11011** | 11000011110

Figure 2. Example of crossover (the symbol | represent the crossover point)

Offspring 1	11**0**111000011110
Offspring 2	110**1**1001**0**0110110
Mutated offspring 1	111**1**101000010110
Mutated offspring 2	110**0**1001**1**0110100

Figure 3. Example of Mutation

Other important concept in GA is the Elitism. The Elitism strategy (ES) was introduced by Kenneth De Jong in 1975, and is an addition to many selection methods that forces the GA to retain some number of the best individuals at each generation (G). With this tool, such individuals can be lost if they are not selected to reproduce or if they are destroyed by C or M. Many researchers have found that ES improves significantly the GA's performance [6].

The advantage of GAs is in their parallelism. GA is travelling in a search space using more individuals than other methods. However, GAs also have disadvantages namely, in the computational time, because in many times this algorithms are slower than other methodologies.

3 Approximating Fractional – Order Transfer Functions

Fractional calculus is a natural extension of the classical mathematics. In fact, since the foundation of the differential calculus the generalization of the concept of derivative and integral to a non-integer order has been the subject of distinct approaches. Due to this reason, there are several definitions [7] which are proved to be equivalent.

The Laplace definition for a derivative of order $\alpha \in C$ is a direct generalization of the classical integer-order scheme with the multiplication of the signal transform by the *s* operator yielding (for zero initial conditions):

$$L\left\{D_{0+}^{\alpha}\varphi\right\} = s^{\alpha} L\{\varphi\},\ Re(\alpha) \geq 0 \quad (1)$$

This means that frequency-based analysis methods have a straightforward adaptation to FC. The practical implementation of (1) requires an infinite number of poles and zeros obeying a recursive relationship [8]. Nevertheless, in a real approximation the finite number of poles and zeros yields a ripple in the frequency response and a limited bandwidth.

In order to analyze the frequency-based approach to (1) let us consider the recursive circuit represented on Fig. 4a) such that:

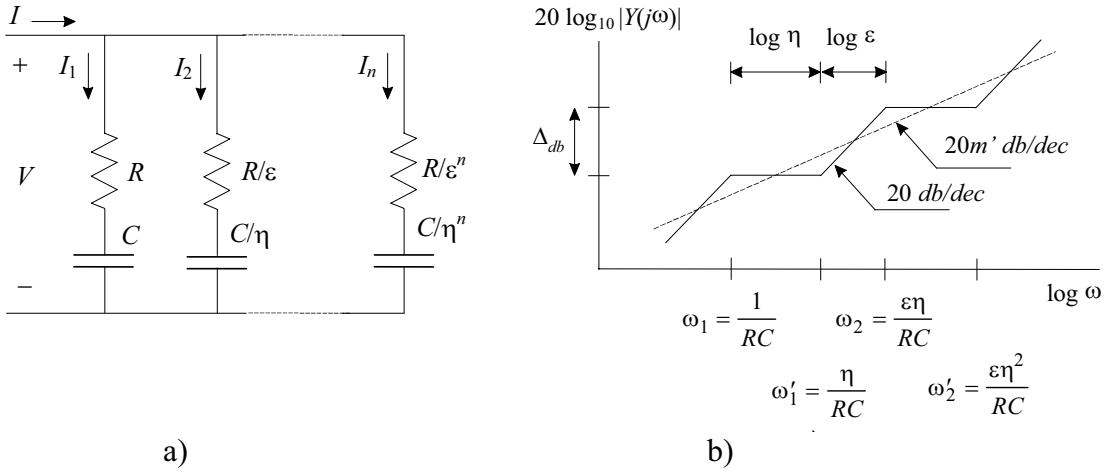
$$I = \sum_{i=1}^{n} I_i\ ,\ R_{i+1} = \frac{R_i}{\varepsilon}\ ,\ C_{i+1} = \frac{C_i}{\eta} \quad (2)$$

where η and ε are scale factors, I is the current due to an applied voltage V and R_i and C_i are the resistance and capacitance elements of the i^{th} branch of the circuit.

The admittance $Y(j\omega)$ is given by:

$$Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = \sum_{i=0}^{n} \frac{j\omega C\varepsilon^{i}}{j\omega CR + (\eta\varepsilon)^{i}} \quad (3)$$

Figure 4b) shows the asymptotic Bode diagrams of amplitude of $Y(j\omega)$.



a) b)

Figure 4. a) Electrical circuit with a recursive association of resistance and capacitance elements, b) Bode diagram of amplitude of $Y(j\omega)$.

The pole and zero frequencies (ω_i and ω'_i) obey the recursive relationships:

$$\frac{\omega'_{i+1}}{\omega'_i} = \frac{\omega_{i+1}}{\omega_i} = \varepsilon\eta\,,\ \frac{\omega_i}{\omega'_i} = \varepsilon\,,\ \frac{\omega'_{i+1}}{\omega_i} = \eta \quad (4)$$

From the Bode diagram of amplitude, or of phase, the average slope m' can be calculated as:

$$m' = \frac{log\,\varepsilon}{log\,\varepsilon + log\,\eta} \quad (5)$$

Consequently, the circuit of Fig. 1a) represents an approach to D^{α}, $0 < \alpha < 1$, with $m' = \alpha$, based on a recursive pole/zero placement in the frequency domain. In fact, this method constitutes the so-called CRONE: Commande Robuste d'Ordre Non Entier, for implementations approximations of fractional order derivatives and integrals [8].

4 Classical Expressions for the Static Electric Potential

It is well known that, for a homogeneous, linear and isotropic media, the electric potential φ at a point P by a single charge, a dipole and a quadrupole are [9 – 10]:

$$\varphi = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} + C\,,\ \varphi = \frac{ql\cos\theta}{4\pi\varepsilon_0}\frac{1}{r^2} + C, r >> l\,,\ \varphi = \frac{ql^2\left(3\cos^2\theta - 1\right)}{4\pi\varepsilon_0}\frac{1}{r^3} + C, r >> l \quad (6)$$

where ε_0 represents the permittivity, q the electric charge, r the radial distance and θ the corresponding angle with the axis.

The electric potential φ at a point P (Figure 5) for one very long straight filament carrying a charge λ per unit length, or for two opposite charged filaments are, respectively:

$$\varphi = -\frac{\lambda}{2\pi\varepsilon_0} ln\, r + C, C \in \Re,\ \varphi = \frac{\lambda l \cos\theta}{2\pi\varepsilon_0} \frac{1}{r} + C, r >> l \quad (7)$$

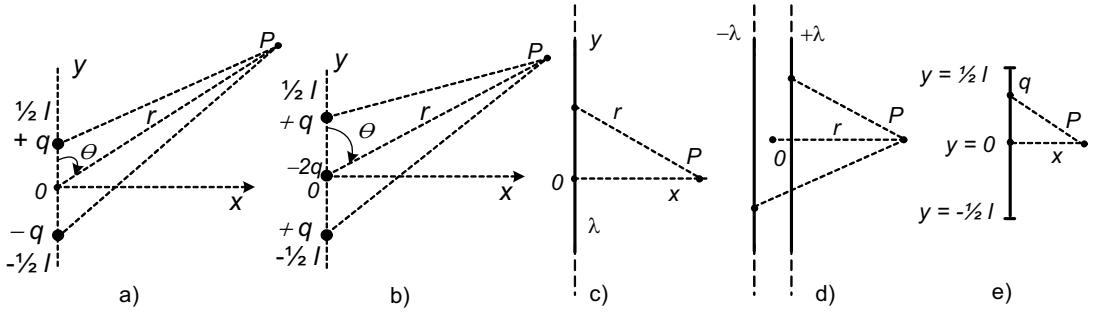


Figure 5. Electric potential of a) dipole b) quadrupole c) infinite line charge d) two opposite charged infinite filaments e) straight filament with finite length l and charge q

On the other hand, the potential resulting from a planar surface with charge density σ is given by:

$$\varphi = -\frac{\sigma}{2\varepsilon_0} r + C, C \in \Re \quad (8)$$

Analyzing expressions (6)-(8) we verify the relationship $\varphi \sim r^{-3}, r^{-2}, r^{-1}, ln\, r, r$ that corresponds to the application of integer-order derivatives and integrals.

5 Implementation of the Fractional Potential

The integer-order differential nature of the potential expressions motivated several authors [5] to propose its generalization in a FC perspective. Therefore, a fractional multipole produces at point P a potential $\varphi \sim r^{-\alpha}, \alpha \in \Re$. Nevertheless, besides the abstract manipulation of mathematical expressions, the truth is that there is no practical method, and physic interpretation, for establishing the fractional potential [4, 5].

Inspired by the integer-order recursive approximation of fractional-order transfer functions presented previously, in this section we develop a GA for implementing a fractional order potential.

We start by re-evaluating the potential produced at point $P \equiv (x,y)$ by a straight filament with finite length l and charge q (Figure 5 e):

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q}}{1} ln\left\{\frac{y+\frac{1}{2}l+\sqrt{x^2+\left(y+\frac{1}{2}l\right)^2}}{y-\frac{1}{2}l+\sqrt{x^2+\left(y-\frac{1}{2}l\right)^2}}\right\}+C,\ C\in\Re \quad (9)$$

It is well-known that for $x \to \infty$ we have $\varphi \to \frac{q}{4\pi\varepsilon_0}\frac{1}{x}+C$ and, with $y = 0$, for $x \to 0$ we have $\varphi \to \frac{1}{2\pi\varepsilon_0}\frac{q}{l} ln\left(\frac{1}{x}\right)+C$. Obviously these limit cases correspond to (6) and (7) respectively, that is, to a single charge and to an infinite filament.

Figure 6 depicts the potential (9) versus x (with $l = 1\ m$ and $y = 0$) and, for comparison, the limit cases (6a) and (7a) (for $C = 0$).

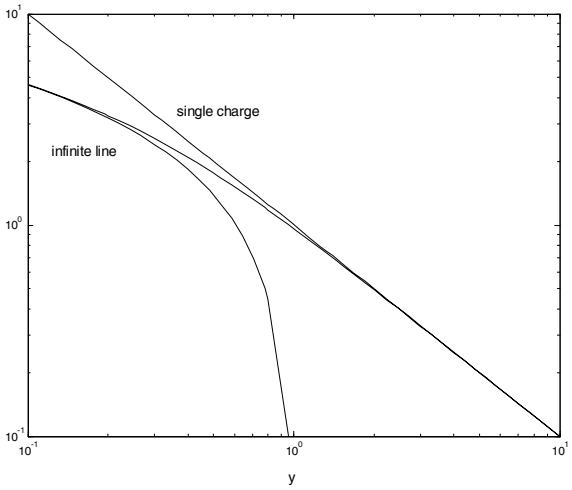


Figure 6. Comparison of the electric potential *φ versus x* for a filament with charge $q = 1$, length $l = 1\ m$, at y = 0, a single charge and an infinite line

In this chart we observe that expression (9) changes smoothly between the two limit cases. Therefore, we can have an intermediate fractional-order relationship as long as we restrict to a limited working range. For example, for $0.1 < x < 0.4$ and $0.3 < x < 1.0$ we get the approximations $\varphi \approx 1.292x^{-0.569}$ and $\varphi \approx 0.9825x^{-0.821}$, with squared coefficient of determination $R^2 = 0.993$ and $R^2 = 0.998$, respectively.

This means that standard integer-order potential relationships have a *global* nature while fractional-order potentials have a *local* nature possible to capture only in a restricted region. This conclusion leads to an implementation approach

conceptually similar to the one described in section 2 that is, to an approximation scheme based on a recursive placement of integer-order functions. Nevertheless, in the present case we do not have the analytical formalism of Bode diagrams and, therefore, we decided to adopt a numerical approach.

In this line of thought, we developed a one-dimensional GA that places recursively n charges q_i ($i = 0, \ldots, (n-1)/2$, n – odd; $i = 1, \ldots, n/2$, n – even) at the symmetrical positions $\pm x_i$ (with exception of $x_0 = 0$ that corresponds to the centre of the n-array of charges where there is a single charge q_0).

Our goal is to compare the approximate potential φ_{app}, resulting from a number of charges and the corresponding locations, with the desired reference potential $\varphi_{ref} = kx^{\alpha}$:

$$\varphi_{app} = \begin{cases} \dfrac{q_0}{|x|} + \displaystyle\sum_{i=1}^{\frac{n-1}{2}} \frac{q_i}{4\pi\varepsilon_0}\left(\frac{1}{|x-x_i|} + \frac{1}{|x+x_i|}\right) & n \text{ odd} \\ \displaystyle\sum_{i=1}^{\frac{n}{2}} \frac{q_i}{4\pi\varepsilon_0}\left(\frac{1}{|x-x_i|} + \frac{1}{|x+x_i|}\right) & n \text{ even} \end{cases}, \quad (10)$$

It is important to referee that a reliable execution and analysis of a GA usually requires a large number of simulations to provide that stochastic effects have been properly considered. Therefore, in this study the experiments consist on executing the GA several times, in order to generate a combination of positions and charges that lead to an electrical potential with fractional slope similar to the desire reference potential. In the first case of study, the values of GA parameters are: population number P = 40, crossover C = 85.0%, mutation M = 1.0%, elitist strategy ES = 10.0% and a number of generations G = 100. The optimization fitness function corresponds to the minimization of the square error J:

$$J = \sum_{k=1}^{m}\left(ln\left|\frac{\varphi_{app}}{\varphi_{ref}}\right|\right)^2, \quad \min_i(J), \quad i = 0, 1, \ldots, n-1 \quad (11)$$

where m is the number of sampling points along the x-axis.

In the present case, we consider a log-log perspective, similar to the one used in Bode diagrams, but its modification for a lin-lin case is straightforward.

For example, Figure 7 shows a pre-defined number of 5–charge approximation and $\varphi_{ref} = 1.0\, x^{-1.5}$, $0.2 < x < 0.8$, leading to case A: $q_{0A} = -0.489$ [volt],

$q_{1A} = 0.920$ [volt] and $q_{2A} = -0.077$ [volt] (with scale factor $\times(4\pi\varepsilon_0)^{-1}$), at $x_{0A} = 0.0$ [m], $x_{1A} = \pm 0.147$ [m] and $x_{2A} = \pm 0.185$ [m], respectively, and case B: $q_{0B} = +0.280$ [volt], $q_{1B} = +0.161$ [volt] and $q_{2B} = +0.361$ [volt] (with scale factor $\times(4\pi\varepsilon_0)^{-1}$), at $x_{0B} = 0.0$ [m], $x_{1B} = \pm 0.103$ [m] and $x_{2B} = \pm 0.159$ [m], respectively. In the case A the GA needs 32 interactions to satisfy the fitness function and in the case B the GA needs 36 iterations.

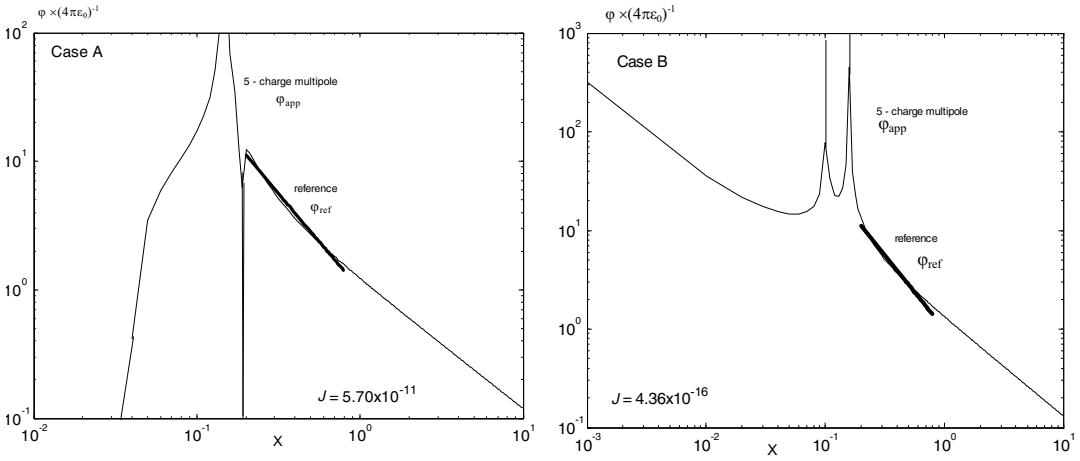


Figure 7. Comparison of the electric potential φ_{app} and φ_{ref} *versus* x for $\varphi_{ref} = 1.0\, x^{-1.5}$ [volt], $0.2 < x < 0.8$ [m], and a 5-charge approximation.

The results show a good fit between the two functions and, moreover, that it is possible to find more than one 'good' solution. Nevertheless, for a given application, a superior precision may be required and, in that case, a larger number of charges must be used. In this line of thought, we study the precision of this method for different number of charges, namely from $n = 1$ up to $n = 10$ charges, and we compare the necessary number of iterations when the number of charges increases.

Figure 8 depicts the minimum, maximum and average of min (J) *versus* n, to achieve a valid solution. This chart confirms that we have a better precision the larger the number of charges. We verify that the position of the charges varies significantly with the number of charges used in the algorithm. Therefore, the charge versus location pattern is not clear and its comparison with a fractal recursive layout is still under investigation.

Figure 9 depicts the minimum, average and maximum of number of iterations I *versus* n. This chart reveals that we need a larger number of iterations when the number of charges increases, in order to accelerate the convergence, and reduce the stochastic nature of the algorithm.

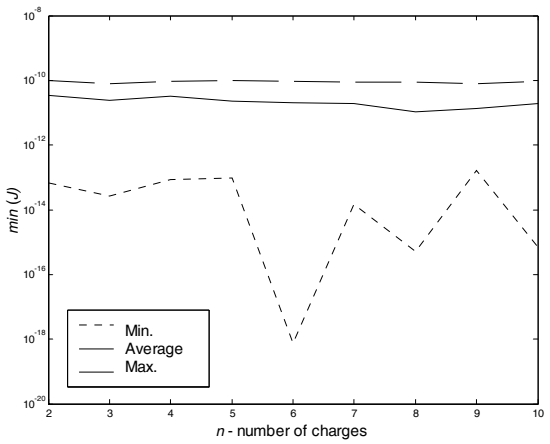


Figure 8. Approximation error min (*J*) *vs.* number charges *n* $\varphi_{ref} = 1.0\ x^{-1.5}$ [volt], $0.2 < x < 0.8$ [m]

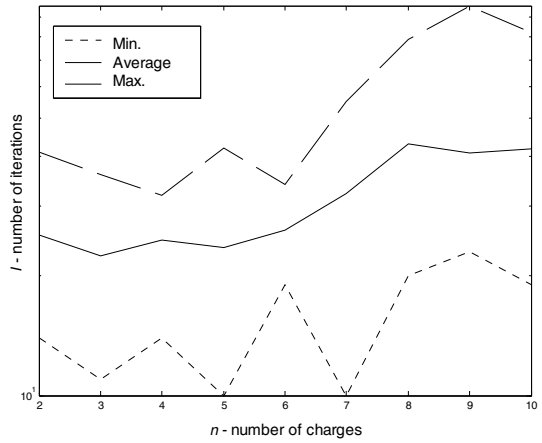


Figure 9. Number of iterations (*I*) *vs.* number charges *n* for $\varphi_{ref} = 1.0\ x^{-1.5}$ [volt], $0.2 < x < 0.8$ [m].

In order to evaluate the GA computational time *T* (in seconds) for different number of charges, we apply identical conditions at all cases, namely a population number $P = 70$, crossover $C = 85.0\%$, mutation $M = 1.0\%$, elitist strategy $ES = 10.0\%$, a number of generations $G = 100$, and the fitness function *J* given by (11). Figure 10 illustrates the corresponding *T versus n*. This chart confirm the previous conclusion, namely that the necessity of a larger number of iterations when the number of charges increases, and consequently a larger value of *T*.

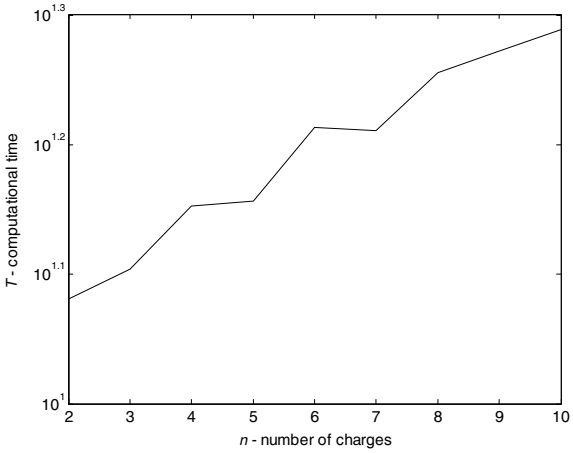


Figure 10. Computational Time (T) *vs.* number charges n for $\varphi_{ref} = 1.0\ x^{-1.5}$ [volt], $0.2 < x < 0.8$ [m].

When we compare the results with previous work, where we apply a sequential grid like numerical algorithm (NA) to solve this problem [5], we verify a smaller approximations error for all n when we apply GA, but a larger T for $n \leq 5$. For $n \geq 6$ the GA is unconditionally better than the NA. The GA reveals, in this case of study, a good commitment between T, the error and the results obtained for all charges.

6 Conclusions

This paper addressed the problem of implemented a fractional-order electric potential. It was adopted a GA scheme inspired on the Bode recursive approach. While in the Bode diagrams both numerical and analytical approaches are possible, in the present case only a numerical evaluation was implemented and the analytical counterpart remains to be investigated. In this paper, some considerations are done based in previous work developed by the authors in this field. We concluded that the GA reveals a good balance between the accuracy of the results and the computation time. The GA approach constitutes a step towards the development of a systematic design technique and, consequently, several of its aspects must be further evaluated. Research on the approximation feasibility and its convergence, error variation with the range and the number of charges, improvement when adopting an extended library of primitives rather than, merely, point charges and its extension to the three-dimensional space is presently under development.

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