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FRACTIONAL ELECTRICAL DYNAMICS IN FRUITS AND VEGETABLES

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Abstract: Fractional calculus (FC) was originally developed in a pure mathematical viewpoint. However, nowadays FC is applied in many emerging fields of physics and engineering. This paper studies the fractional electrical impedance of vegetables and fruits having FC modelling in mind. In this line of thought, are developed several experiments for measuring the impedance of botanical elements, based in the Bode and polar diagrams. An electrical circuit that models these systems is presented and conclusions are drawn. *Copyright © 2006 IFAC*

Keywords: Fractional Calculus, Electrical, Impedance, Fruits, Vegetables.

1. INTRODUCTION

Fractional calculus (FC) is a generalization of the integration and differentiation to a non-integer order. The fundamental operator is ${}_aD_t^{\alpha}$, where the order α is a real or, even, complex number, and the subscripts a and t are the two limits related to the operation, (Oldham, *et al*., 1974; Samko, *et al*., 1993; Oustaloup, 1995; Miller, 2002).

Recent studies have brought FC into attention revealing that many physical phenomena can be modeled by fractional differential equations. The importance of fractional order mathematical models is that it can be used to make a more accurate description and to give a deeper insight into the physical processes underlying long range memory behaviours.

Fractional-order systems have witnessed an increasing interest lately (Machado, *et al*., 2004; Jesus, *et al*., 2006). Capacitors are one of the crucial elements in integrated circuits and are used extensively in many of them, such as sample and holds, radio-frequency oscillators and mixers (Samavati, *et al*., 1998).

Jonscher (Jonscher, 1993) demonstrated that the ideal capacitor cannot exist in nature, because an impedance of the form $1/[(j\omega)C)]$ would violate causality (Bohannan, 2002). The dielectric material exhibits a realistic fractional behaviour for impedances $1/[(j\omega)^{\alpha}C]$, with $\alpha \in \Re^{+}$.

Bearing these ideas in mind, this paper analyzes the fractional-order dynamics in botanical electrical impedances and is organized as follows. Section 2 presents the fundamental electrical concepts. Section 3 describes the research work in the field of fractional order impedances. Section 4 formulates the fractional model for the impedance. Finally, section 5 draws the main conclusions.

2. ON THE ELECTRICAL IMPEDANCE

In an electrical circuit the voltage $u(t)$ and the current $i(t)$ can be expressed as a function of time t:

$$u(t) = U_0 \cos(\omega t) \quad (1)$$

$$i(t) = I_0 \cos(\omega t + \phi) \quad (2)$$

where U_0 and I_0 are the amplitudes of the signals, ω is the frequency and ϕ is the current phase shift. The voltage and current can be expressed in complex form as:

$$u(t) = \mathrm{Re}\left\{U_0 e^{j(\omega t)}\right\} \quad (3)$$

$$i(t) = \mathrm{Re}\left\{I_0 e^{j(\omega t + \phi)}\right\} \quad (4)$$

Consequently, the electrical impedance $Z(j\omega)$ is:

$$Z(j\omega) = \frac{U(j\omega)}{I(j\omega)} = Z_0 e^{j\phi} \quad (5)$$

A brief reference about the constant phase elements (CPE) and Warburg impedance is presented here due to their application in the work. In fact, to model an electrochemical phenomenon it is often used a CPE because the surface is not homogeneous (Barsoukov, *et al.*, 2005). So, with a CPE:

$$Z(j\omega) = \frac{1}{(j\omega)^{\alpha} C} \quad (6)$$

C is the ideal capacitance and α is a parameter that can change between 0 and 1, being an ideal capacitor for $\alpha = 1$.

On the other hand, in electrochemical systems with diffusion, the impedance is modelled by the so-called Warburg element (Ho, *et al.*, 1980, Barsoukov, *et al.*, 2005). The Warburg element arises from one-dimensional diffusion of an ionic species to the electrode. If the impedance is under an infinite diffusion layer, the Warburg impedance is:

$$Z(j\omega) = \frac{R}{(j\omega)^{0.5} C} \quad (7)$$

where R is the diffusion resistance. If the diffusion process has finite length, the Warburg element becomes:

$$Z(j\omega) = R\frac{\tanh(j\omega\tau)^{0.5}}{(j\omega\tau)^{0.5}} \quad (8)$$

with $\tau = \delta^2 / D$, where R is the diffusion resistance, τ is the diffusion time constant, δ is the diffusion layer thickness and D is the diffusion coefficient.

3. STUDY OF FRACTIONAL ORDER ELECTRICAL IMPEDANCES

The structure of fruits and vegetables have cells that are sensitive to heat, pressure and other stimuli. These systems constitute electrical circuits exhibiting a complex behaviour. Bearing these facts in mind, in our work we study the electrical impedance for several botanical elements, under the point of view of fractional order systems.

We apply sinusoidal excitation signals $v(t)$, to the botanical system, for several distinct frequencies ω (Fig. 1) and the impedance $Z(j\omega)$ is measured based on the resulting voltage $u(t)$ and current $i(t)$. Moreover, we measure the environmental temperature, the weight, the length and width of all botanical elements. This criterion helps us to understand how these factors influence $Z(j\omega)$.

In this study we develop several different experiments for evaluating the variation of the impedance $Z(j\omega)$ with the amplitude of the input signal V_0, for different electrode lengths of penetration inside the element Δ, the environmental temperature T, the weights W and the dimension D.

The value of R is changed for each experiment, in order to adapt the values of the voltage and current to the scale of the measurement device.

We start by analyzing the impedance for an amplitude of input signal of $V_0 = 10$ volt, a constant adaptation resistance R_a = 15 kΩ, applied to one *Solanum Tuberosum* (potato), with an weight $W = 1.24\ 10^{-1}$ kg, environmental temperature $T = 26.5$ degree Celsius, dimension $D = 7.97\ 10^{-2} \times 5.99\ 10^{-2}$ m, and the electrode length penetration $\Delta = 2.1\ 10^{-2}$ m.

Figure 2 presents the Bode diagrams for $Z(j\omega)$. The results reveal that the system has a fractional order impedance. In fact, approximating the experimental results in the amplitude Bode diagram through a power function namely by $|Z(j\omega)| = a\omega^{-b}$, we obtain $(a, b) = (4.91\ 10^3, 0.0598)$, at the low frequencies, and $(a, b) = (7.94\ 10^5, 0.5565)$, at the high frequencies.

It is interesting to compare the polar diagram and the admittance loci for *RLC*, series or parallel, circuits. We verify that our systems have similarities with the *RC* parallel circuit and, therefore, we conclude that this vegetable has proprieties similar to a kind of capacitor.

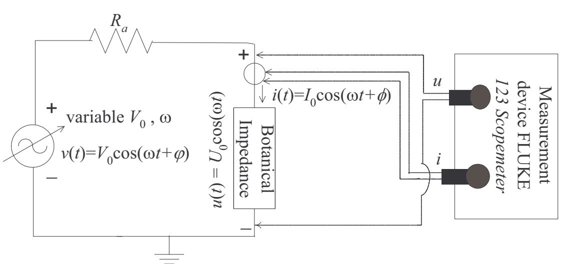

Fig 1. Electrical circuit for the measurement of the botanical impedance $Z(j\omega)$

In order to analyze the system linearity we evaluate $Z(j\omega)$ for different amplitudes of input systems, namely, $V_0 = \{5, 15, 20\}$ volt, maintaining constant the adaptation resistance R_a = 15 kΩ. The impedance $Z(j\omega)$ has a fractional order and this characteristic

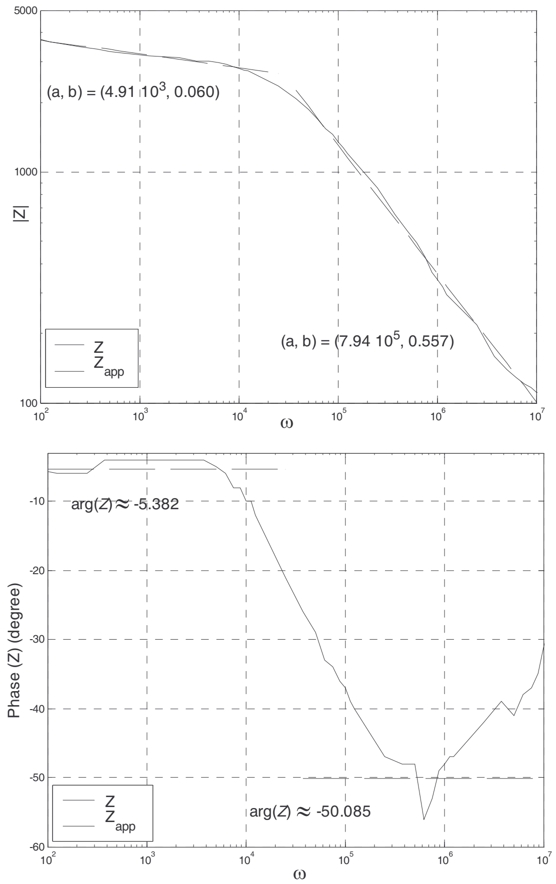

Fig 2. Bode diagrams of the impedance $Z(j\omega)$ for the potato.

does not change significantly with the variation of input signal amplitude (Table I). Therefore, we can conclude that this system has a linear characteristic.

In a second experiment, we vary the length Δ of the electrode penetration inside the potato, and we evaluate its influence upon the value of the impedance. Therefore, we adjust the electrode to $\Delta = 1.42\ 10^{-2}$ m, with $V_0 = 10$ volt and adaptation resistance $R_a = 5$ kΩ, leading to $|Z(j\omega)|$ approximations $(a, b) = (5.48\ 10^3, 0.0450)$, at the low frequencies, and $(a, b) = (1.00\ 10^6, 0.5651)$, at the high frequencies. With these results, we conclude that the length of wire inside the potato does not change significantly the values of the fractional orders. Also the linearity was again confirmed.

The last experiment with the potato is related with the variation of environmental temperature. In this case, we use the first potato and the same conditions of first experience, but with an temperature $T = 25.7$ degree Celsius. The amplitude impedance $|Z(j\omega)|$ has the values: $(a, b) = (8.91\ 10^3, 0.0555)$, at the low frequencies, and $(a, b) = (7.10\ 10^5, 0.5010)$, at the high frequencies. Once more we verify the small variation of the fractional order.

Another issue that may influence the results is the weight. Therefore, we apply an input signal with amplitude $V_0 = 10$ volt, adaptation resistance $R_a = 15$ kΩ, with environmental temperature $T = 26.5$ degree Celsius, and electrode penetration

Table I . Comparison of the values of $|Z(j\omega)| \approx a\omega^{-b}$ for different amplitudes of the input signal

Amplitude (volt)	**low ω**		**high ω**	
	a	***b***	***a***	***b***
5	$4.79\ 10^3$	0.062	$6.52\ 10^5$	0.542
10	$4.91\ 10^3$	0.060	$7.94\ 10^5$	0.557
15	$4.54\ 10^3$	0.054	$5.66\ 10^5$	0.530
20	$4.65\ 10^3$	0.055	$5.86\ 10^5$	0.530

Table II . Characteristics of the vegetables and fruits

Vegetable or Fruit / Specie	**Weight (kg)**	**Length (m)**	**Width (m)**
Carrot / *Daucus Carota L.*	$8.85\ 10^{-2}$	$1.55\ 10^{-1}$	$3.39\ 10^{-2}$
Garlic / Allium sativum *L.*	$2.99\ 10^{-3}$	$1.38\ 10^{-2}$	$6.00\ 10^{-3}$
Onion / *Allium cepa L.*	$8.33\ 10^{-2}$	$5.86\ 10^{-2}$	$5.77\ 10^{-2}$
Potato / *Solanum tuberosum*	$1.24\ 10^{-1}$	$7.97\ 10^{-2}$	$5.99\ 10^{-2}$
Pimento / *Capsi-cum annuum*	$1.30\ 10^{-1}$	$1.23\ 10^{-1}$	$8.20\ 10^{-2}$
Tomato/*Lycoper-sicom esculentum*	$1.46\ 10^{-1}$	$5.57\ 10^{-2}$	$6.88\ 10^{-2}$
Turnip / *Brassica napobrassica*	$7.90\ 10^{-2}$	$7.26\ 10^{-2}$	$5.43\ 10^{-2}$
Apple / *Malus domestica*	$1.39\ 10^{-1}$	$6.36\ 10^{-2}$	$7.15\ 10^{-2}$
Banana / *Musa ingens*	$1.11\ 10^{-1}$	$1.49\ 10^{-1}$	$3.42\ 10^{-2}$
Kiwi / *Actinidia deliciosa*	$8.95\ 10^{-2}$	$6.52\ 10^{-2}$	$5.50\ 10^{-2}$
Lemon / *Citrus × limon*	$1.66\ 10^{-1}$	$9.19\ 10^{-2}$	$6.58\ 10^{-2}$
Orange / *Citrus sinensis*	$1.53\ 10^{-1}$	$6.69\ 10^{-2}$	$6.98\ 10^{-2}$
Pear / *Pyrus communis*	$9.72\ 10^{-2}$	$6.51\ 10^{-2}$	$5.63\ 10^{-2}$

$\Delta = 2.1\ 10^{-2}$ m to another potato with dimension $D = 7.16\ 10^{-2} \times 3.99\ 10^{-2}$ m, weight $W = 5.89\ 10^{-2}$ kg. The asymptotic results for $|Z(j\omega)|$ are $(a, b) = (7.17\ 10^3, 0.0546)$, at the low frequencies and $(a, b) = (2.00\ 10^6, 0.5990)$, at the high frequencies.

Again, this experience does not reveal significant variations in the fractional order while the linearity is also confirmed.

In conclusion, the impedance does not change significantly with the factors analyzed. In this line of thought, we organize similar experiments with other vegetables and fruits.

The results correspond to experiments adopting an amplitude of input signal $V_0 = 10$ volt and an electrode penetration $\Delta = 2.1\ 10^{-2}$ m. Similar experiments are developed for several fruits. Table II presents the characteristics of the vegetables and fruits respectively. Figures 3 and 4 depict Re $\{Z(j\omega)\}$ and Im $\{Z(j\omega)\}$ for the vegetables and fruits under study, and the corresponding approximation values,

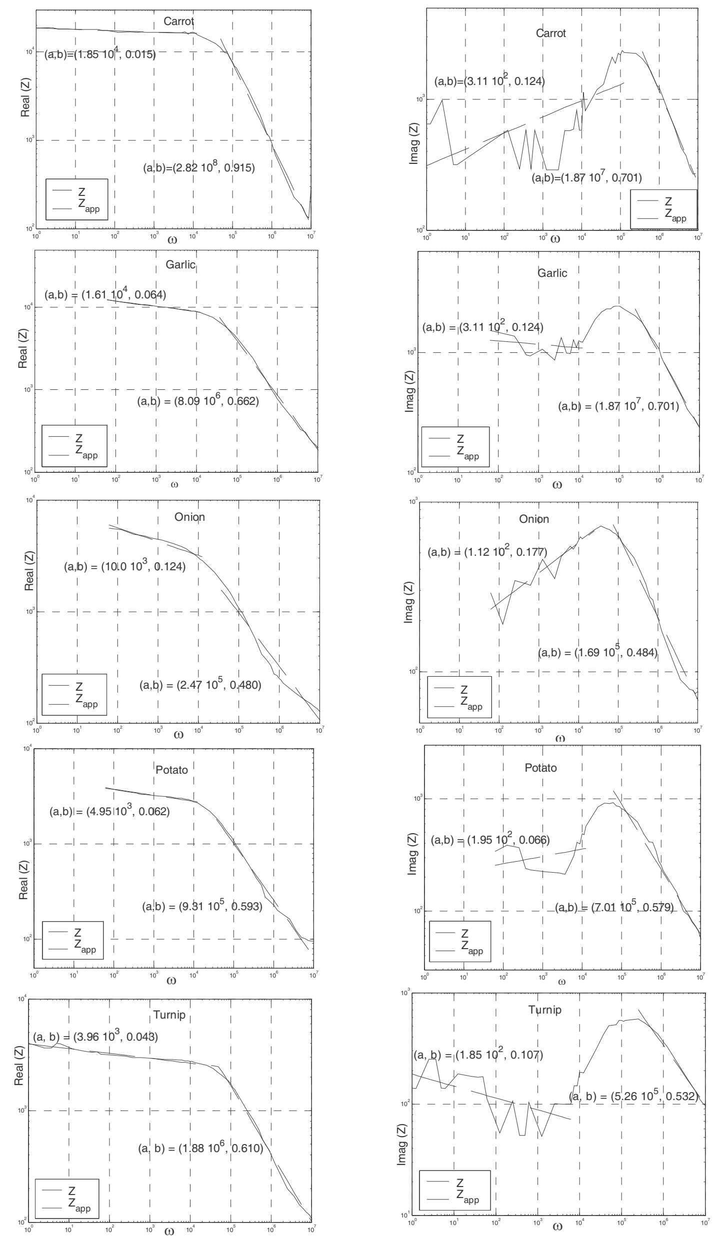

Fig 3. Diagrams of real Re [$Z(j\omega)$] and imaginary Im [$Z(j\omega)$] parts of the electrical impedance for several vegetables: carrot (with R_a = 4.7 kΩ), garlic (with R_a = 15.0 kΩ), onion (with R_a = 2.7 kΩ), potato (with R_a = 15.0 kΩ) and turnip (with R_a = 2.2 kΩ).

respectively. In these experiences, the adaptation resistance R_a is changed for each case.

The results reveal that $Z(j\omega)$ has distinct characteristics according with the frequency range. For low frequencies, the impedance is approximately constant, but for high frequencies, it is clearly of fractional order.

4. THE IMPEDANCE MODEL

In the previous section we verified that it is difficult to find a model for $Z(j\omega)$ within the whole frequencies range. In this section, we apply the circuit of Fig. 5, often adopted in the area of electrochemistry, where R_0 and R_1 are resistances and *CPE* is given in (6).

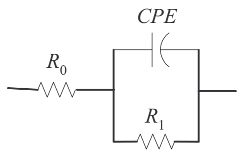

Fig. 5. The Randles circuit

The numerical values of R_0, R_1, C and α for the different impedances are shown in Table III. The results reveal a very good fit for several vegetables and fruits. Figure 6 presents the amplitude and the phase Bode diagrams, for the garlic, potato, tomato, kiwi and pear. Fig. 7 depicts the corresponding polar diagrams. It is clear that adopting circuits with more components, and other configuration, we can have better approximations. Therefore, in future development we will study new circuits for modelling the impedance of other materials.

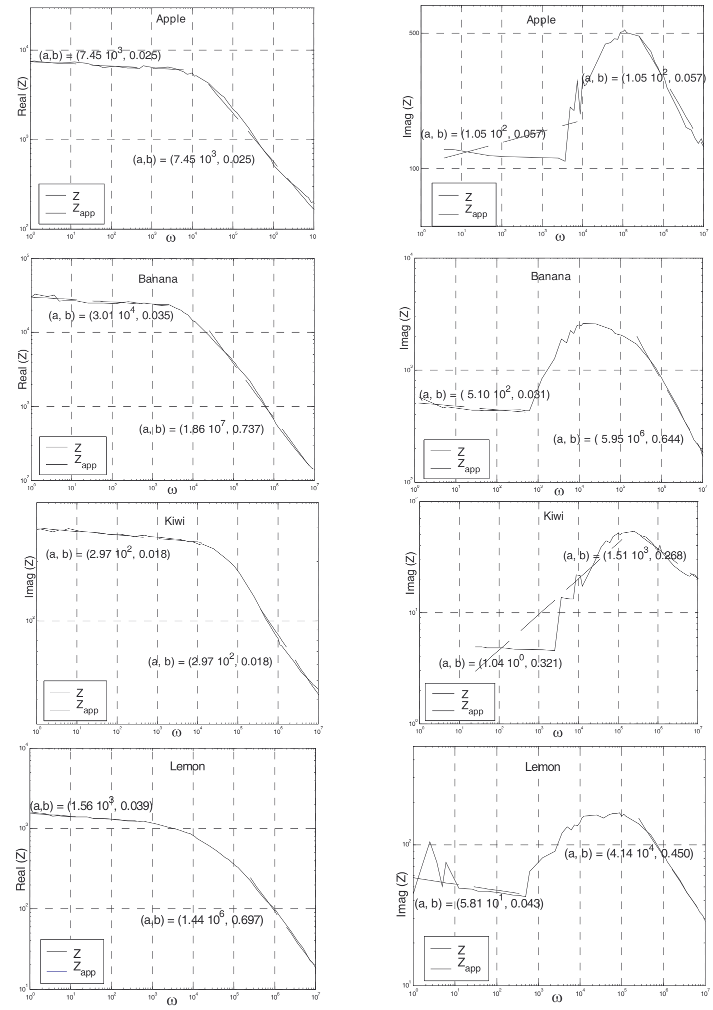

Fig 4. Diagrams of real Re [$Z(j\omega)$] and imaginary Im [$Z(j\omega)$] parts of the electrical impedance for several fruits: apple (with R_a = 1.0 kΩ), banana (with R_a = 5.5 kΩ), kiwi (with R_a = 750 Ω) and lemon (with R_a = 750 Ω).

Recent research focus on the implementation of fractional order capacitances, often called fractances. Patents and commercial products are presently available, opening promising areas of application in electronics and control (Bohannan, 2002).

This article follows an alternative strategy, studying natural living systems instead of technological artificial elements. Consequently, it points out interesting new directions towards the design of devices capable of measuring how mature is the fruit and vegetable, or to give an estimative of its life span for storage purposes.

Table III . Values of the elements of the Randles circuit for the garlic, potato, tomato, kiwi and pear

Vegetable / fruits	R_0 [Ω]	R_1 [Ω]	C [F]	α
Garlic	1	$9.7\ 10^3$	$1.81\ 10^{-7}$	0.609
Potato	57	$3.15\ 10^3$	$2.40\ 10^{-7}$	0.677
Tomato	35.04	240.30	$5.00\ 10^{-6}$	0.565
Kiwi	28.04	242.00	$7.67\ 10^{-6}$	0.531
Pear	44.04	409.00	$1.14\ 10^{-6}$	0.619

5. CONCLUSION

The idea of fractional calculus is not new. Fractional derivatives are almost as old as integer-order definition. The area of fractional calculus has primarily been the domain of mathematicians, and only had the theoretical foundation. Nowadays, this concept is employed in physics, engineering, biology, economy and other scientific fields. In our work, we apply the concepts of FC and the theory of electrical impedance to botanical elements. The fractional order behaviors of these types of systems are studied and the relation with the electrical impedance is formulated. The results reveal that all elements have different characteristics for low and high frequencies; however, the impedance remains linear when the system conditions are modified. The equivalent circuit model was presented according to the experiments made with vegetables and fruits. The close fit of the modelled and the experimental results, indicate that this model can be used to optimize the development of electrode designs. Moreover, the impedances have fractional order characteristics for high frequencies and reveal similarities with electrical fractional capacitors, called fratances.

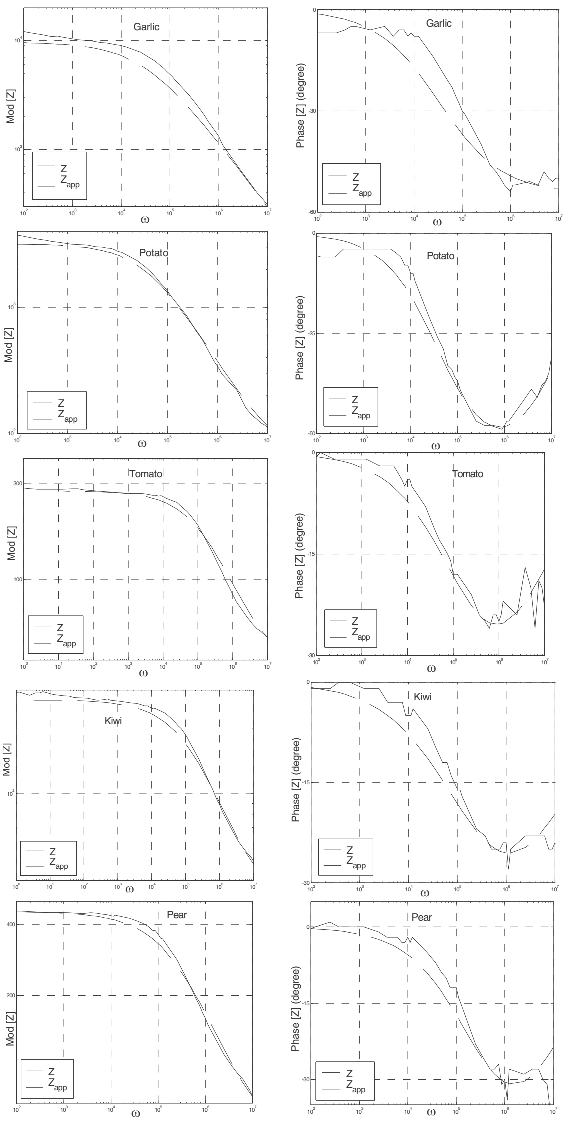

Fig 6. Amplitude and phase Bode diagrams of $Z(j\omega)$ for several vegetables and fruits: garlic, potato tomato, kiwi and pear.

We conclude that fractional calculus is an important tool to describe physical phenomena, adopting different concepts of classical methodologies.

6. ACKNOWLEDGEMENTS

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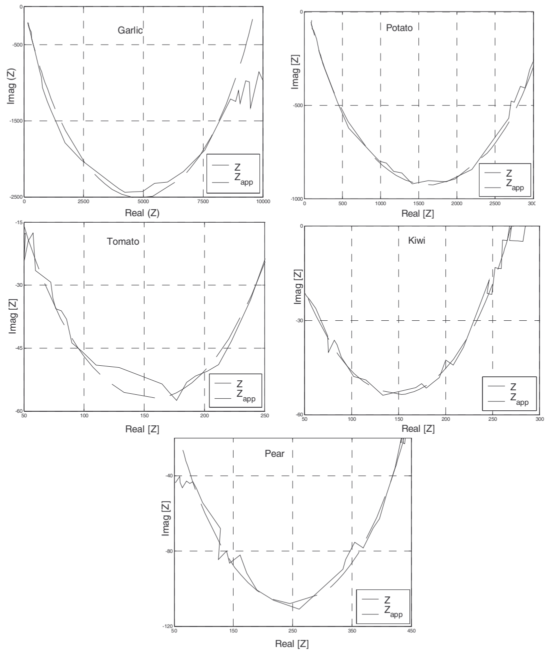

Fig 7. Polar diagrams of the impedance $Z(j\omega)$ for several vegetables and fruits: garlic, potato tomato, kiwi and pear.

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