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A new zoom algorithm and its use in frequency estimation

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Abstract: This paper presents a novel zoom transform algorithm for a more reliable frequency estimation. In fact, in many signal processing problems exact determination of the frequency of a signal is of paramount importance. Some techniques derived from the Fast Fourier Transform (FFT), just pad the signal with enough zeros in order to better sample its Discrete-Time Fourier Transform. The proposed algorithm is based on the FFT and avoids the problems observed in the standard heuristic approaches. The analytic formulation of the novel approach is presented and illustrated by means of simulations over short-time based signals. The presented examples demonstrate that the method gives rise to precise and deterministic results.

Keywords: Zoom algorithm; frequency estimation; FFT

1 Introduction

Frequency estimation usually involves the use of FFT-based estimators, either directly, as it is the case of periodogram, or indirectly, as in the MEM, MUSIC and other similar methods. In all cases, we are looking for the exact position of the spectral peaks in a given spectral estimate $S(f)$, $|f| < \frac{1}{2}$ (where f denotes frequency). Nevertheless, when using the FFT, $S(f)$ is sampled over an uniform grid and, as a consequence, it is very unlikely that we are successful in obtaining the true peak positions. A better estimation of the positions of the spectra peaks can be obtained using large zero padding, thus leading to very large FFT lengths. In [3] a warped discrete Fourier transform is used and its performance is compared with several other procedures, namely: Dichotomous-search, Tretter's linear regression, Kay's phase difference and chirp $\mathbb{Z}$-transform methods. Further readings and applications of this technique can be found in [2, 4, 5, 8–10].

We have an interpolation problem in the frequency domain, but it is very special case since we know the interpolating function, which is the Fourier transform. The real problem appears because we are using a DFT implementation. As this defines completely the Fourier transform, we can approach this problem from a quite different point of view: the zooming of a small portion of the spectrum that includes the peak position. To perform the spectral zoom, two different methods of interpolation have been proposed and usually referred as the zoom transform [1]. However, since these methods imply a return to time, modulation and filtering, they are not very useful when dealing with short-time signals. An alternative and simpler algorithm was proposed in [6] that merely explores the fact that the FFT (DFT) is a sampling of the Fourier Transform and, so, it has the whole information we need.

This paper further details this topic and is organized as follows. In section 2 we present a general formulation that allows us to choose the frequency search grid. Furthermore, it is also shown that the proposed algorithm is useful in converting a spectral representation from a linear to logarithm scale. In section 3 we present some numerical results. Finally section 4 outlines the main conclusions.

2 The Zoom Algorithm

Let $x(n)$, $n = 0, \cdots, L-1$, denote an L-length sample sequence. Every $N \geq L$ point DFT sequence represents samples of the Discrete-Time Fourier Transform (DTFT):

$$X\left(e^{j\omega}\right) = \sum_{n=0}^{L-1} x(n)\, e^{-j\omega n}. \quad (1)$$

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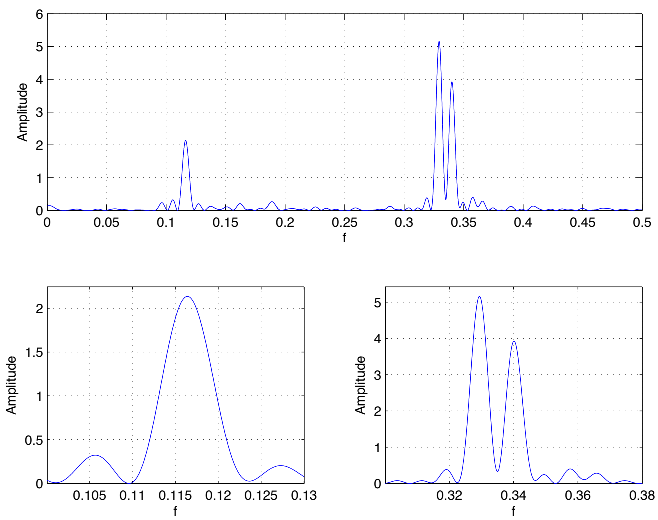

Figure 1: Zoom in two different bands of a spectrum.

This relation defines an infinite number of DFTs, each one characterized by one sampling grid. We do not need any additional information to pass from one to another.

Let $X_N(k)$ denote the DFT of $x(n)$, corresponding to sampling $X\left(e^{j\omega}\right)$ at a uniform grid:

$$X_N(k) = DFT\left[x(n)\right] = X\left(e^{j\frac{2\pi}{N}k}\right), \quad k = 0, \cdots, N-1, \; N \geq L. \tag{2}$$

Accordingly to what we said we have many DFTs: one for each grid that can be defined by N. Each of these DFTs has the same inverse given by:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_N(k)\, e^{j\frac{2\pi}{N}kn}, \quad n = 0, \cdots, N-1. \tag{3}$$

Substituting equation (3) into equation (1) results in:

$$X\left(e^{jw}\right) = \frac{1}{N}\sum_{k=0}^{N-1} X_N(k)\, G(\omega, k), \quad k = 0, \cdots, N-1, \tag{4}$$

where $G(\omega, k)$ is given by

$$G(\omega, k) = \frac{1 - e^{-j\left(\omega - \frac{2\pi}{N}k\right)L}}{1 - e^{-j\left(\omega - \frac{2\pi}{N}k\right)}}, \tag{5}$$

for $|\omega| \leq \pi$ and $0 \leq k < N$. This relation allows us to compute the Fourier coefficients corresponding to one grid from the ones of another grid. It is straightforward to show that:

$$G(\omega, k) = \frac{sinc\left[\left(\omega - \frac{2\pi}{N}k\right)\frac{L}{2}\right]}{sinc\left[\left(\omega - \frac{2\pi}{N}k\right)\frac{1}{2}\right]} e^{j\left(\omega - \frac{2\pi}{N}k\right)\frac{L-1}{2}}, \tag{6}$$

where $sinc(x) = \frac{\sin(x)}{x}$. Since $\omega = 2\pi f$, we obtain:

$$G(f, k) = \frac{sinc\left[\left(f - \frac{k}{N}\right)L\right]}{sinc\left(f - \frac{k}{N}\right)} e^{j\pi\left(f - \frac{k}{N}\right)(L-1)}. \tag{7}$$

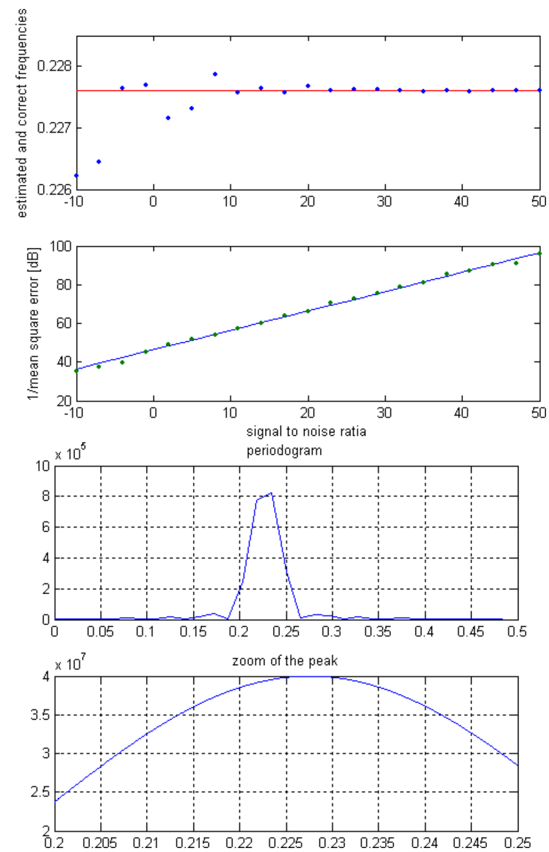

Figure 2: a) Estimated frequency as a function of the SNR by zooming showing on red the exact frequency value (top) and the mean square error in (– dB) with the Cramer-Rao bound in blue (bottom), b) Periodogram and zoom of the peak for a sinusoidal signal, $f_0 = 0.2276$ Hz.

So, equations (4) and either (6) or (7) allows us to zoom into the frequency region of interest. Of course, we are not interested in zooming the whole spectrum[1], just a given band (see Fig. 1).

3 Simulation results

3.1 Frequency Estimation

In this section, we present some simulation results obtained with a sinusoidal signal of angular frequency $\omega_0 = 1.43$ ($f_0 = 0.2276$ Hz) for different values of the signal-to-noise ratio (SNR), spanning from −10 up to +50 dB. The Cramer-Rao bound is included for reference

1 But we can do it, if we find it useful.

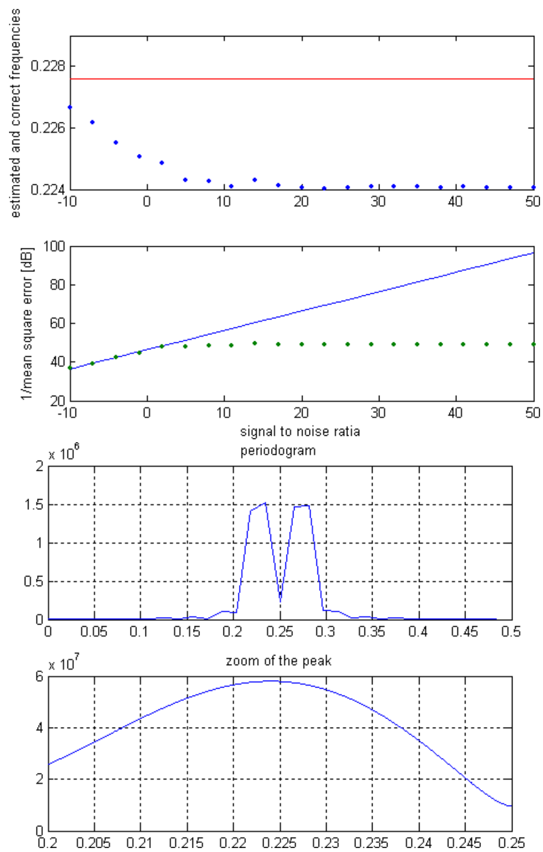

Figure 3: Frequency estimation as a function of the SNR of a sinusoidal signal (f_0 = 0.2276 Hz) corrupted by a sinusoid with identical amplitude, for $f_1 = 1.2f_0$ and noise, showing, a) from top to bottom: estimated frequency, inverse of mean square error (dB), b) from top to bottom: periodogram and zoom of the peak.

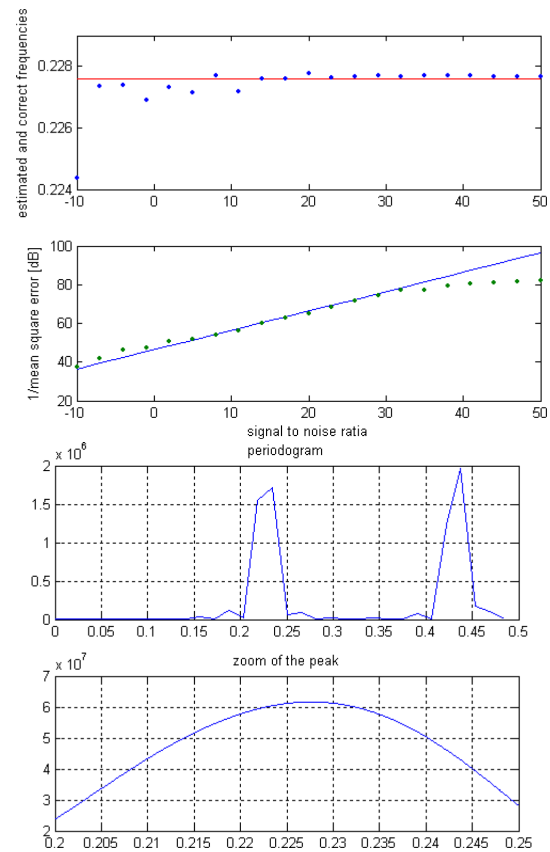

Figure 4: Frequency estimation as a function of the SNR of a sinusoidal signal (f_0 = 0.2276 Hz) corrupted by a sinusoid with identical amplitude, for $f_1 = 1.9f_0$ and noise, showing a) from top to bottom: estimated frequency, inverse of mean square error (dB), b) from top to bottom: periodogram and zoom of the peak.

as in [3] and the reciprocal value of the variance in dB is also shown. For the calculations the values $L = 24$, $N = 64$ are adopted. Furthermore, the band of frequencies to be zoomed is $f \in [0.20, 0.25]$ Hz, where 1000 points are computed in a total of 100 runs (Figure 2a)). Figure 2b) depicts the periodogram and the corresponding peak zoom for the same simulations of Figure 2a).

Other numerical calculations are performed and their results are presented in Figures 3 and 4. The values $L = 24$, $N = 64$ are considered, and the band of frequencies to be zoomed consists of $f \in [0.20, 0.25]$, where 1000 points are computed in a total of 100 runs. In both cases, a sinusoid with identical amplitude is now added to the original signal, plus noise as before. The new sinusoid has a frequency equal to $1.2f_0$ (Fig. 3) and equal to $1.9f_0$ (Fig. 4). We verify that the presence of the second sinusoid produces a bias in the estimated frequencies and, as expected, when the two frequencies are closer, the bias becomes higher.

The saturation effect we observe in Figure 3 is due to the numerical errors.

3.2 Comparision to FFT Zoom

In this section we present several comparison experiments of the proposed zoom algorithm against a “classical” FFT based zoom: a non-destructive zoom Fast Fourier transform of a time history [7]. We ran both algorithms in the same circumstances for $N = 32$, $N = 64$ and $N = 128$ as pictured in Figures 5, 6 and 7, respectively. The increased performance of the proposed algorithm both in terms of the frequency estimation and its error variance is evident. However, as the time sequence length increases, both algorithms approach their performances.

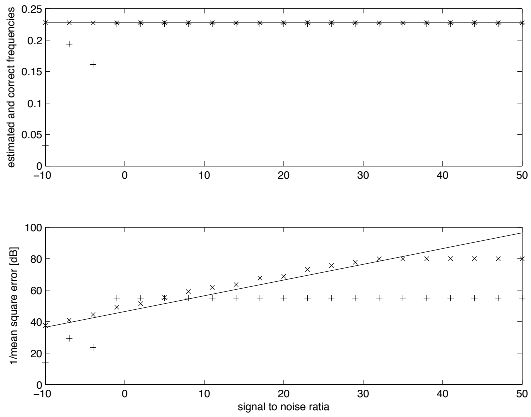

Figure 5: Frequency estimation as a function of the SNR of a sinusoidal signal ($f_0 = 0.2276$ Hz) corrupted white noise, showing from top to bottom: estimated frequency and inverse of mean square error (dB), where '+' corresponds to the FFT zoom and '×' to the proposed algorithm, for $N = 32$.

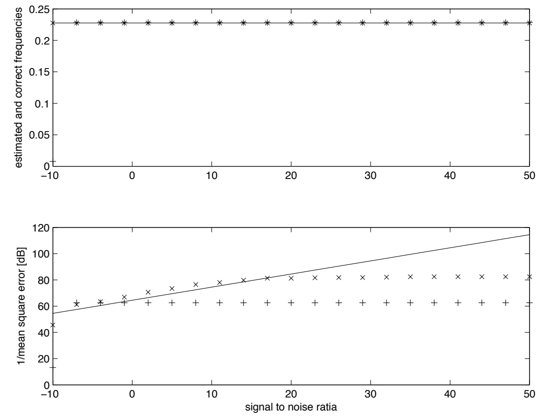

Figure 7: Frequency estimation as a function of the SNR of a sinusoidal signal ($f_0 = 0.2276$ Hz) corrupted white noise, showing from top to bottom: estimated frequency and inverse of mean square error (dB), where '+' corresponds to the FFT zoom and '×' to the proposed algorithm, for $N = 128$.

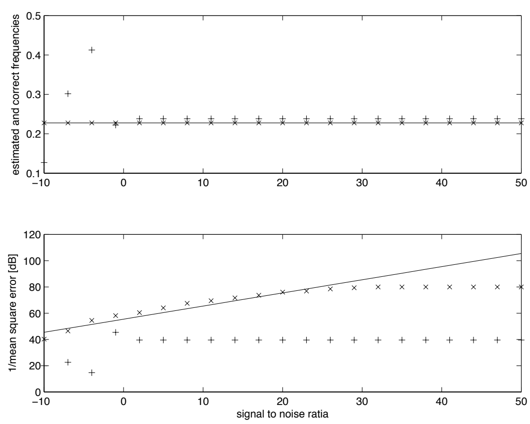

Figure 6: Frequency estimation as a function of the SNR of a sinusoidal signal ($f_0 = 0.2276$ Hz) corrupted white noise, showing from top to bottom: estimated frequency and inverse of mean square error (dB), where '+' corresponds to the FFT zoom and '×' to the proposed algorithm, for $N = 64$.

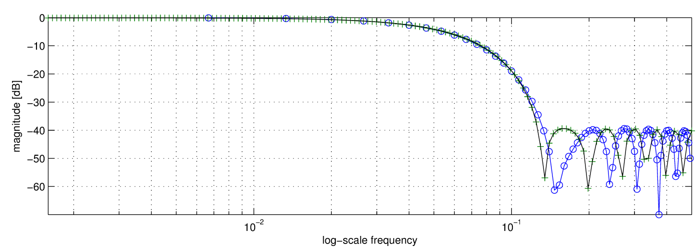

Figure 8: Example of linear ('o' and blue line) to log ('+' and green line) frequency conversion of a 10th-order low-pass FIR filter with a cut-off frequency of 0.0125 Hz.

3.3 Frequency Scale Conversion

Another application of this algorithm is the conversion of a linear frequency scale into a logarithmic one. In Figure 8 we use the algorithm for converting the transfer function of a 10th-order low-pass FIR filter, with a bandwidth of 0.0125 Hz, from a linear (blue plot) to log frequency scale (green plot). This conversion can be easily done using equation (7) as an interpolation factor for the filter frequency response to the new frequency scale.

4 Conclusions

A zoom algorithm was presented, with applications ranging from frequency estimation to frequency scale conversion. Often the frequency estimation problem involves the usage of FFT-based estimators, either directly (e.g., periodogram), or indirectly (e.g., MEM, MUSIC and similar methods). In all the cases, we are looking for the exact position of the peaks in a given spectral estimate. The underlying idea of several existing algorithms is to refine the sampling of the DFT, but with the expense of large computational requirements. Other methodologies [1] try to solve this problem by zooming a small portion of the spectrum where the peak position is located. A first draft of the algorithm was proposed in [6] and was now further developed and explored. The scheme was applied to spectral estimation with good results.

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