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A Fractional Order Identification of a Mechanical Transmission System with Known Failures

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Abstract — *To compete in the global market the manufacturers must introduce new high quality products in short time. This ideal requires a fast evaluation of new products and the guarantee of availability of the critical machines in a production line, typically by reducing the maintenance shut-downs. Roughly, the diagnosis' process consist in comparing the machine's actual behavior with the known one under a specific (common) failure, namely monitoring any change of its dynamics. Unfortunately, the manufacturers rarely supply any dynamical model and consequently it is identified using naive transfer function structures of integer order. In this respective, we evaluate an integer second order model and a similar structure of a fractional one to identify a system under different types of failures. The fractional order model outperforms the integer one, suggesting that it represents better the machine condition when using compact equations.*

1 Introduction

For competing in the global market industries must introduce high quality products and have agile fabrication [1]. This ideal just may be approached by ensuring that the production line is fully available at any time [2]. Unfortunately this is not always possible as the machines get wear and fails, or must shut-down during a programed maintenance task, reducing its availability. In the other hand, the quality of the new merchandise must be quickly evaluated before being introduced in the market. In both cases, the maintenance/quality inspection must be done in short time in order to avoid increasing the production time. Therefore, a competitive factory requires a fast and accurately quality inspection technique that involves short training time and a simple interpretation by the maintenance personal.

The classical approach to supervise condition in rotatory machines is the signature inspection technique. It consist of a frequency analysis of the vibratory/acoustic signal emitted by the device [3, 4, 5, 6]. Typically this strategy needs trained technicians with a high knowledge of the machine operation: frequency of movement of each piece, expected harmonics in the vibratory/acoustic signal, among others [7, 8]. Another strategy mentioned in the literature uses multiresolution analysis, that is to say, a good localization of a failure signal both in time and frequency. However, that approach implies considerable additional data, that are typically complex to analyze, and consequently, requires very specialized workers [9, 10]. Finally, another strategy is based in the model identification of the machine [11, 12, 13]. In that approach, it is possible to locale failures by interpreting the different parameters of the model. Nevertheless, building accurate models is not always possible due to the non-linearities within and the inherent complexity of many systems. Furthermore, the maintenance team may not know perfectly the dynamics of each machine in the product line. Alternately, a dynamical system with only a few parameters is sometimes a valid approach to handling the real machine in a limited frequency range [14]. The more accurate the model that represent the actual state, better is the diagnosis associated with it [15].

Unfortunately a low Integer Order Model (IOM) hardly captures high order dynamics resulting of the complex interaction of several pieces within the machine. In contrast, the literature reports that the Fractional Order Models (FOM) deal with complex systems of high order dynamics using a few parameters as shown in [16, 17, 18, 19].

In this work, we test the FOM's ability to accurately represents the data acquired from a system with several known failures and compares the fitting accuracy with an IOM.

Bearing this ideas in mind, the article is organized as follows. Section 2 introduces the fundamentals of our method and the methodology used to evaluate it. Section 3 presents the experimental case of study. Section 4 discusses the major results of identification based on data taken form a real device. Finally, section 5 outlines the main conclusions.

2 Methods

Nowadays, manufacturer's recommendations guide the frequency of maintenance over a particular machine with the aim of preventing damages on the machine due to well known failures [20, 21, 22]. However, periodic maintenance is a low efficient strategy, because the action may be programed unnecessarily early, or so rarely that the failures appear before a programed maintenance action [23]. Consequently, in the literature it is proposed an alternative kind of strategy in which the machine is subjected to a maintenance action just when it is necessary. To achieves this ideal, the maintenance experts monitor a set of signals looking for anomalies corresponding to a failure signature, that is, a change in the machine behavior [24, 25]. Unfortunately, this is a task that requires a considerable experience of high qualified technicians.

It is well accepted that a change in the system dynamics explains deterministic changes in any signal taken from the machine. Hence, an intuitive alternative is to compare a dynamical model, obtained from the machine in a normal operation point, with the current behavior. Although, obtaining accurate models is not a simple task, because the machines components interact in a complex manner (memory effects, non-linear behavior, among others) and lead to in a high order model that is difficult to identify. A common solution is to suppose that the entire system obeys a simple model, namely a second order model, disregarding other effects. This scheme simplifies the identification task but penalizes the model accuracy.

It is presently known that fractional order models capture dynamical behaviors that the integer ones overlook. In fact this is, due to the generality that gives an arbitrary order by reducing or expanding the amount of memory and complexness related to each fractional coefficient. Consequently, this type of models not only represent complex behaviors with compact equations, but also reduces the identification effort.

2.1 Fractional Order Model

A FOM is defined by set of differential equations of arbitrary order. A FOM allows the Bode representation of systems with decaying/rising slopes different to $20dB$ per decade which is typical in systems with multiples interactions [26, 27]. In the Laplace space, the fractional order equations have a similar handling as the integer one. Formally, being ${}_0D_t^\alpha$ the differential α-order operator actuating between 0 and t, and F the Laplace's transform of the function f, the Laplace's transform of a fractional order derivative is defined as [28]:

$$\mathcal{L}\{{}_0D_t^\alpha\} = s^\alpha F(s) - \sum_{j=0}^{n-1} s^j \left[{}_0D^{\alpha-j-1}f(0)\right], \quad n-1 < \alpha < n, \qquad n \in \mathbb{Z} \tag{1}$$

Note that this representation follows the Laplace's transform of the integer order derivative if $\alpha \in \mathbb{Z}^+$.

By extension, a linear invariant fractional order model is defined as:

$$G(s) = \frac{\sum_{i=0}^{N} a_i s^{\alpha_i}}{\sum_{j=0}^{M} b_j s^{\beta_j}}, \quad \{a_i, b_j\} \in \Re, \quad \{\alpha_i, \beta_j\} \in \Re^+ \tag{2}$$

2.2 Identification Algorithm

In case of system identification, it is necessary to follow four sequential steps [14]:

- To obtain and pre-process the data, this typically comes from several sensors located strategically in the machine of interest. It is necessary to filter the data as it maybe contaminated with noise from the sensors, the environment and the digital acquisition.
- To suppose a model structure: it is a common practice to use physical knowledge about the system, but it is not always possible, because the manufacturer rarely shares this kind of information. In those cases it is common to suppose a system with compact representation as the second order ones and to test how well it fits the data.
- To identify the parameters of the model. It is common to use a set of data to find the best set of parameters that explain the data.
- To validate the model by comparing it to a set of data that was not used to find the model's parameters.

Accordingly, system identification can be seen as a minimization of the error between the data and the proposed model or, by other words, to, find the best parameter vector

$\vec{\boldsymbol{p}}$ that minimizes an objective function of error f_e between actual system $G_a(s)$ and the model $G_m(\vec{\boldsymbol{p}}, s)$:

$$f_e(\vec{\boldsymbol{p}}) = |G_a(s) - G_m(\vec{\boldsymbol{p}}, s)| \tag{3}$$

Many alternatives can be applied to minimize the Eq. 3 without loosing generality. Specifically, in this work we use the *simplex search* [29], that consists of an iterative algorithm that looks for a candidate point by computing the centroid of three initial test points. It is checked if the centroid is a better candidate than any of the test points. Then, it replace the worst point of the initial set. The algorithm executes again until get a convergence point or after a number of iteration specified by the user.

3 Case of Study

We propose simple mechanical transmission as experimental setup, because its ubiquity in rotatory machines. Using it we compare the fitting ability of IOM and FOM in systems with known failure. The experiment consist in a permanent magnet DC motor as a torque input to a four stage gearbox composed by four spur gears of nine teeth each. Figure 1 shows a detail of the transmission. The input signal is taken from an accelerometer located at the output of the gearbox, sensing the vibration due to a given failure. In the other hand, this anomalous vibration must change the motor current signature. Therefore, it corresponds to the output system signal and it is sensed by a resistance in series with the motor circuit. As it is impossible to acquire the failure signature directly in its source, the signal is modulated by the transmission path, it is, the physical pathway that the wave related to the failure travel across from the its source until a sensor. As an instance, the Figure 2 introduce the transmission path between a failure located at the gear 4 and the accelerometer (vibration sensor) and the resistance (motor's current sensor). In this configuration, we acquired the vibration on the output bushing and motor motor-current signals using and acquisition toolkit compatible with Labview®. In this work we analyze four possible cases of failure:

Case 1. Normal condition. The system does not present any failure and it is working in the normal conditions.

Case 2. A tooth broken in the gear 2. The second gear does not have one of the nine teeth. The vibration signal, related to the failure, travels through the gears 2, 3 and 4 to be sensed by the accelerometer. Hence, it is the test failure that less affects the accelerometer measurement. On the other hand, the signal signature makes a shorter travel passing by the gears 1 and 2 and the motor circuit affecting the current applied.

Case 3. A tooth broken in the gear 3. The pattern of the vibration signal related to a failure in the gear 3 is modulated by the gear 3 and 4 (transmission path to the accelerometer). The failure signal that affects the motor's current travels through the gears 1, 2, 3 and the motor's circuit.

Case 4. A tooth broken in the gear 4. In the absence of a tooth in the gear 4, the transmission path between the failure and the accelerometer is the is the body of the gear

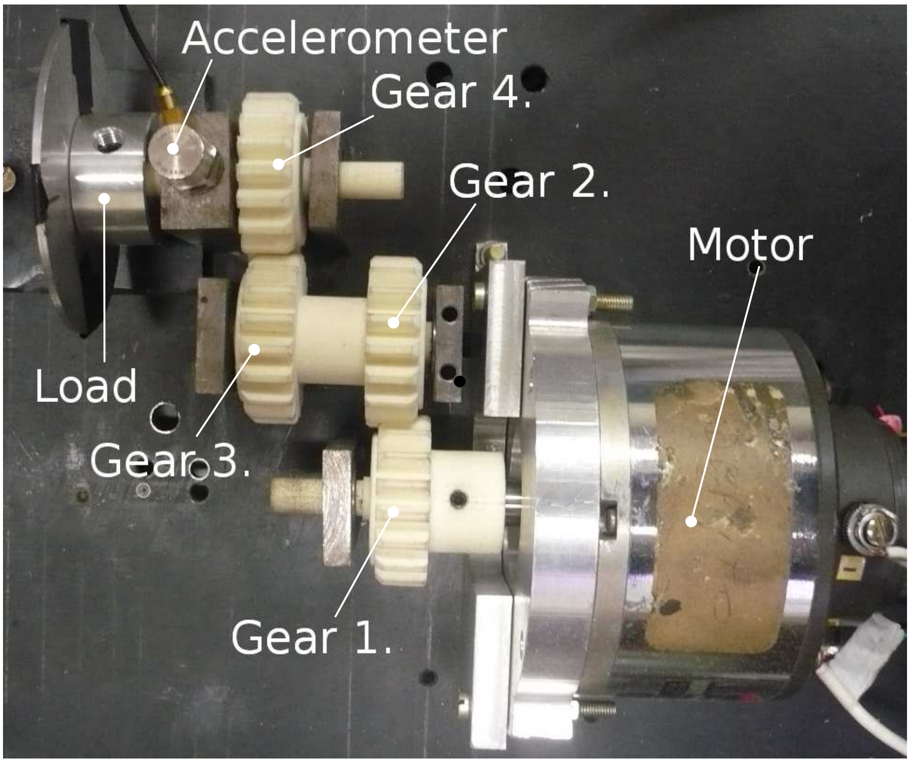

Figure 1: Principal components of the mechanical transmission. Here, the motor provides the torque necessary to rotate the load. Each of the gears may have a known failure that affects the vibration and current signatures.

4, and the gears 1,2,3 and the motor's circuit to the current sensor as shown in Figure 2.

Each of those cases is tested at several speeds, in order to provide a signal composed by a wide range of frequencies. After that, we filter each signal using a moving average (MA) filter to reduce the environmental noise. Later we compute a windowed Fourier transform over a Hanning's window of one seconds lenght, in order to reduce the noise due to the digitization process. Hence, for any of the cases, we take the motor current (I) as the model output and the voltage generated at the accelerometer as the input (V), we define the actual system as the *empirical transfer function estimate* (ETFE) [14] as:

$$G_{ETFE_i}(\omega) = \frac{\mathcal{F}\{I_i(t)\}}{\mathcal{F}\{V_i(t)\}} \quad i \in \{1, 2, 3, 4\} \tag{4}$$

where $\mathcal{F}(\cdot)$ represents the Fourier's transform over a function, i denotes the i-th case and ω the frequencies of analysis, in this cases between 100 to 1000 rad/s, because it was verified that the meshing gear angular frequency lie in this bandwidth.

We propose the identification of each $ETFE_i$ model using two types of structures: a classical second order model as shown in (5) and a fractional order model with the structure defined in (6).

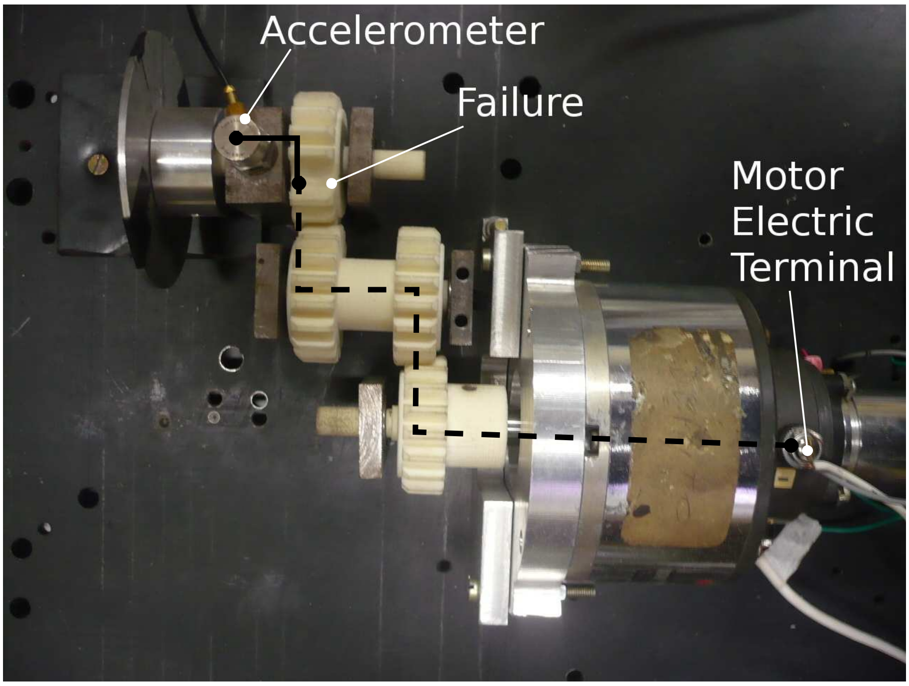

Figure 2: Transmission line between a failure at the gear 4 and each sensor. In solid black the transmission path to the accelerometer, the vibration wave travels through gear 4. In dashed line, the path of the vibration wave until be sensed as a change in the motor current signature.

$$G_i(s) = \frac{1}{as^2 + bs + c}, \qquad \{a, b, c\} \in \Re \tag{5}$$

$$G_i(s) = \frac{1}{as^\alpha + bs^\beta + c}, \qquad \{a, b, c, \alpha, \beta\} \in \Re \tag{6}$$

Those models were adjusted using the algorithm explained in the section 2.2, adopting 20 aleatory sets of one second of duration to train and 10 sets of data to test.

4 Results

In order to evaluate the efficiency of FOM and IOM to describe complex dynamics and how the they are sensitive to failures in a machine, we formulate naive models with the structures presented in (5) and (6) for each case of failure using 20 aleatory sets of data. After that the resultant models were tested with a new dataset of 10 ETFEs not previously presented in order to find the parameters. As result, Figure 3 shows the residuals of each model with the test dataset using a logarithmic scale. Note that the FOM approach consistently gets a better fitting behavior to the data than the integer one. It is due to a main reasons: The FOM actually captures very high order dynamics, in this case due to the interaction between several pieces [30, 31].

Accordingly, an IOM requires a more careful structure choice, locating a priori enough degrees of freedom (poles and zeros) increasing, therefore, the amount of knowledge in-

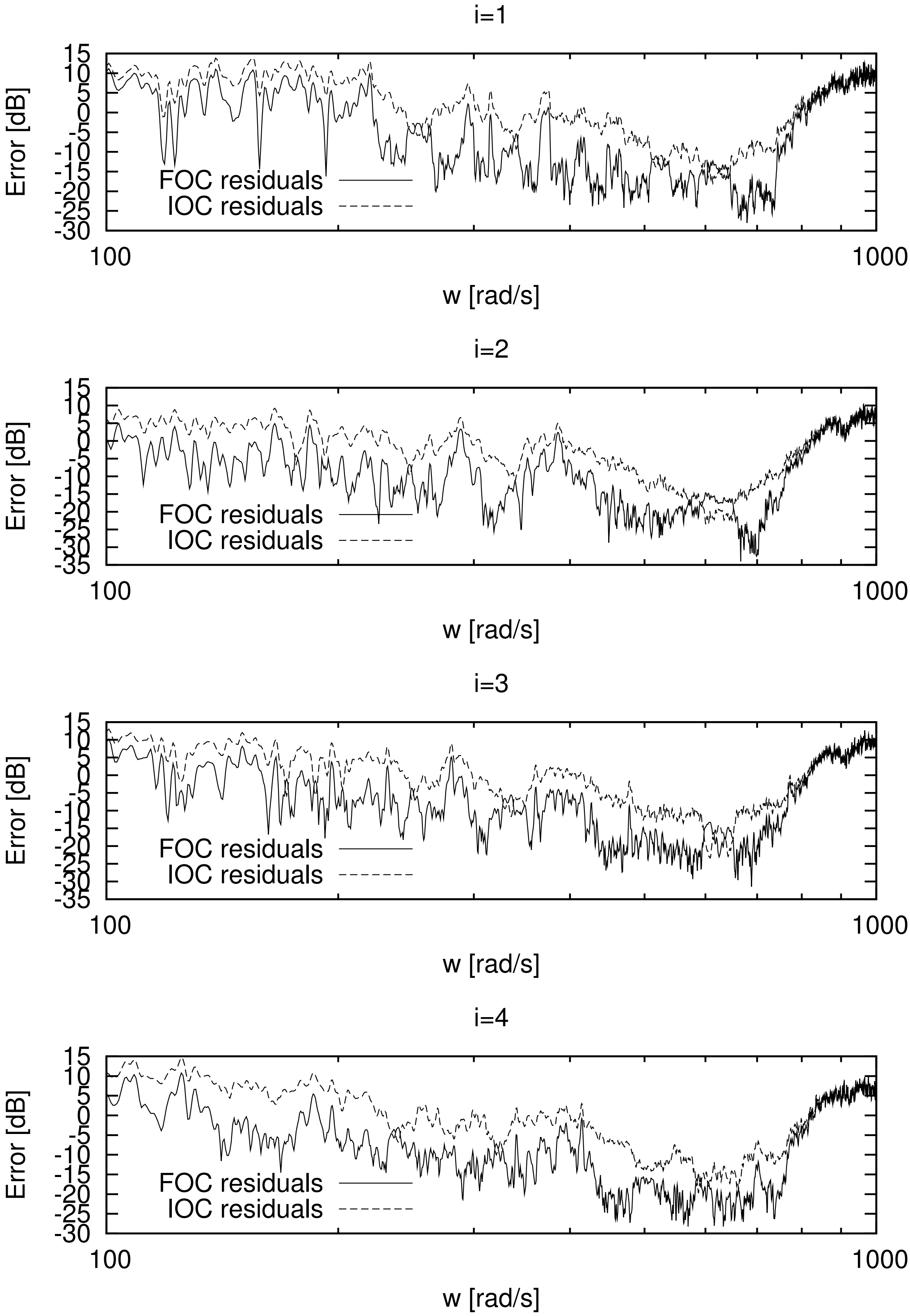

Figure 3: Fractional and integer fitting to the ETFE in each case of study. The more negative the residual, the better the approximation.

	Train error $\pm$ deviation		Test Error $\pm$ deviation	
	IOM	FOM	IOM	FOM
Case 1	3.23 ± 0.18	1.84 ± 0.15	3.47 ± 0.27	2.06 ± 0.23
Case 2	2.26 ± 0.05	1.12 ± 0.05	2.17 ± 0.03	1.05 ± 0.02
Case 3	2.43 ± 0.38	1.44 ± 0.34	2.37 ± 0.10	1.41 ± 0.09
Case 4	1.36 ± 0.06	0.74 ± 0.05	1.51 ± 0.07	0.89 ± 0.07

Table 1: Train and test mean errors $\pm$ standard deviation of the integer and the fractional order models when compared with real data. As expected the FOM fits better the whole dataset than the IOM.

volved in this process. Moreover, to model a high order degree system using integer order equations, we need a large number of parameters, penalizing the identification convergence as it will be affected by the course of dimensionality [32]: the higher the number of unknown parameters, the harder the identification becomes.

Table 1 presents the mean square error (MSE) and the standard deviation between the dataset and FOM and IOM approaches, it is computed as the difference between the actual ETFE values with the model's ones (G_{m_i}) for the i-case at the angular frequency (ω), and N sampling frequencies:

$$MSE_i = \frac{1}{N}\sum_{n=1}^{N}\left(G_{ETFE_i}(\omega_n) - G_{m_i}(\omega_n)\right)^2 \quad i \in \{1,2,3,4\} \tag{7}$$

Although the results shows a comparable variability, the fractional order is in average better suited to fit the data. In fact, analysing the results, we verify that FOM outperforms clearly the IOM approach both on the train and in the test datasets.

5 Conclusions

In this paper we compared the fitting flexibility of two models, namely, a second order integer model and a fractional one with a similar structure and we use a common optimization method to identify their parameters. The fractional order model demonstrate it ability to lead with complex data, consistently better than the integer approach. It is owing to the arbitrary order brings an additional degree of freedom that regulates by itself the complexness represented by the model.

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References

[1] Dragan Djurdjanovic, Jay Lee, and Jun Ni. Watchdog agent–an infotronics-based prognostics approach for product performance degradation assessment and prediction. *Advanced Engineering Informatics*, 17(3-4):109 – 125, 2003. Intelligent Maintenance Systems.

[2] Marzio Marseguerra, Enrico Zio, and Luca Podofillini. Condition-based maintenance optimization by means of genetic algorithms and monte carlo simulation. *Reliability Engineering and System Safety*, 77:151–166, 2002.

[3] Pratesh Jayaswal, A. K. Wadhwani, and K. B. Mulchandani. Machine fault signature analysis. *International Journal of Rotating Machinery*, 2008.

[4] S. S. Rao. *Vibrações Mecânicas*. Pearson Prentice Hall, 2008.

[5] Babak Eftekharnejad and D. Mba. Seeded fault detection on helical gears with acoustic emission. *Applied Acoustics*, 70:547–555, 2009.

[6] T. H. Loutas, J. Kalaitzoglou, G. Sotiriades, and V. Kostopoulos. A novel approach for continuous acoustic emission monitoring on rotating machinery without the use of slip ring. *Journal of Vibration and Acoustics*, 130, 2008.

[7] Ž. Hocenski and T. Keser. Failure detection and isolation in ceramic tile edges based on contour descriptor analysis. *Mediterranean conference on control and automation*, 2007.

[8] Juan Dai, C.L. Philip Chen, Xiao-Yan Xu, Ying Huang, Peng Hu, Chi-Ping Hu, and Tao Wu. Machinery vibration signals analysis and monitoring for fault diagnosis and process control. *ICIC*, pages 696–703, 2008.

[9] Naim Baydar and Andrew Ball. Case history and comparatives study of acoustic and vibration signals in detection of gear failures using wigner-ville dsitribution. *Mechanical Systems and Signal Processing*, 15(6):1091–1107, 2001.

[10] F. K. Choy, D. H. Mugler, and J. ZhouJ. Damage identification of a gear transmission using vibration signatures. *Transactions of the ASME*, 125:394–403, June 2003.

[11] A. Vania and P. Pennacchi. Experimental and theoretical application of fault identification measures of accuracy in rotating machine diagnostics. *Mechanical Systems and Signal Processing*, 18:329–352, 2004.

[12] A. Duvar, V. Eldem, and N. Saravanan. A system identification approach for failure diagnosis and detection. *Proceedings of the IEEE International Workshop on Intelligent Motion Control*, pages 61–64, 1990.

[13] Simon Rofe. Signal processing method for gearbox fault detection. Technical report, Australian Goverment - Department of Defense, Airframes and Engines Division, 1997.

[14] Lennard Ljung. *System identification, theory for de user*. Prentice Hall, University of Linköping, 1987.

[15] Kamal Medjaher, Rafael Gouriveau, and Noureddine Zerhouni. A procedure for failure prognosis in dynaic systems. *IFAC Symposium on Information Control Problems in Manufacturing*, 13:1–6, 2009.

[16] Igor Podlubny. The laplace transform method for linear differential equations of the fractional order. Technical report, Slovak Academy of Sciences Institute of Experimental Physics, 1994.

[17] R.L. Magin and M. Ovadia. Modeling the cardiac tissue electrode interface using fractional calculus. *Journal of Vibration and Control*, 14(9-10):1431–1442, 2008.

[18] J.M. Rosario, D. Dumur, and J.A. Tenreiro Machado. Analysis of fractional - order robot axis dynamics. *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and its Applications*, 2(1), July 2006.

[19] Lokenath Debnath. Recent applications of fractional calculus to science and engineering. *International Journal of Mathematics and Mathematical Sciences*, 2003(54):3413–3442, 2003.

[20] Bankim Shikari and C.M. Sadiwala. Automation in condition based maintenance using vibration analysis. Technical report, Dept. Of Mechanical Engineering. Maulana Azad National Institute Of Technology Bhopal, India, 2004.

[21] Jefferson Luiz Bosa. Sistema embarcado para a manutenção inteligente de atuadores elétricos. Master's thesis, Universidade Feredal do Rio Grande do Sul, 2009.

[22] Christin Groba, Sebastian Cech, Frank Rosenthal, and Andreas Gossling. Architecture of a predictive maintenance framework. *Computer Information Systems and Industrial Management Applications, International Conference on*, 0:59–64, 2007.

[23] R. C. M. Yam, P. W. Tse, L. Li, and P. Tu. Intelligent predictive decision support system for condition-based maintenance. *The International Journal of Advace Manufacturing Technology*, 17:383–391, 2001.

[24] Mitchell Lebold, Katherine McClintic, Robert Campbell, Carl Byington, and Kenneth Maynard. Review of vibration analysis methods for gearbox diagnostics and prognostics. *Proceedings of the 54th Meeting of the Society for Machinery Failure Prevention Technology*, pages 623–634, May 2000.

[25] Pedro Saavedra González, Carlos Figueroa, and Marcelo Iribarren. Análisis dinámico de máquinas utilizando la distribución de wigner-ville. *Mecánica computacional*, 16, September 1996.

[26] Isabel S. Jesus, J. A. Tenreiro Machado, and J. Boaventura Cunha. Fractional electrical dynamics in fruits and vegetables. *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and its Applications*, 2(1), July 2006.

[27] A. Oustaloup, J. Sabatier, and X. Moreau. From fractal robustness to the crone approach. *Proceedings of Fractional differential systems: Models, methods and applications*, 5:177–192, 1998.

[28] Keith B. Oldham and Jerome Spanier. *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, volume 1. Dover Publication, New York, 2006.

[29] J. C. Lagarias, J. A. Reeds, M. H. Wright, and P. E. Wright. Convergence properties of the nelder-mead simplex method in low dimensions. *SIAM Journal of Optimization*, 9(1):112–147.

[30] Dumitru Baleanu and Sami I.Muslih. Nonconservative systems within fractional generalized derivatives. *Proceedings of the 2nd IFAC Workshop on Fractional Differentiation and its Applications*, 2(1), July 2006.

[31] T. Poinot and J.-C. Trigeassou. A method for modelling and simulation of fractional systems. *Signal Processing*, 83:2319–2333, 2003.

[32] Richard T. Woodward, Yong-Suhk Wui, and Wade L. Griffin. Living with the curse of dimensionality: Closed-loop optimization in a large-scale fisheries simulation model. *American Journal of Agricultural Economics*, 87(1):48–60, 2005.