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LIMIT CYCLE PREDICTION OF ROBOT SYSTEMS WITH NONLINEAR PHENOMENA IN THE JOINTS

Abílio Azenha and J. A. Tenreiro Machado
Department of Electrical and Computer Engineering
Faculty of Engineering, University of Porto
Rua dos Bragas, 4099 Porto Codex, Portugal
Phone (351) 2-317105 Fax (351) 2-319280 E-Mail: azenha@tom.fe.up.pt, jtm@fe.up.pt

ABSTRACT

This paper analyses the control of industrial robots with dynamic phenomena at the joints such as friction, backlash and flexibility. The article adopts the describing function (DF) method for the study of stability and the prediction of limit cycles. For the robot, the DF's are evaluated by decoupling the link inertias and considering only the diagonal constant inertial effects. The controllers studied are the first order model and the second order model variable structure controllers. The results show that the best case is the robot system with nonlinear friction, while the worst is the manipulator system with backlash.

1 INTRODUCTION

The progress in computational systems made possible the intensive study of nonlinear systems through simulation while the development of adequate control strategies complemented the computer-based techniques. In this perspective, this paper investigates the dynamics of robots with friction, backlash and flexibility in the joints through the describing function (DF) method. These nonlinear dynamic phenomena (present in all industrial robots) have been an area of active research, however well established conclusions are still lacking. For example, in Armstrong-Hélouvry *et al.* (1994) one can find an extensive survey about the friction present in systems where there is contact between two surfaces. Studies about nonlinear friction modelling can be found also in Armstrong (1991), Gogoussis and Donath (1990), Haessig and Friedland (1991), Held and Maron (1988), de Wit *et al.* (1995), Dupont (1993) and Hu (1994). In what concerns the presence of backlash, Tao and Kokotovic (1993 and 1995) developed an algorithm for its compensation based on an adaptive controller and an unknown backlash model. Also, Choi and Noah (1989), Dubowsky *et al.* (1987) and Stepanenko and Sankar (1986) studied the phenomenon of backlash with some simplifications in the dynamical models. The flexibility at the robot joints has also been the subject of intensive research as reported by Nicosia and Tomei (1995), Brogliato *et al.* (1995), De Luca (1995) and Readman (1994).

The objective of using the DF's (Atherton 1975) is to study the dynamic behaviour of the robot under the condition of a stable oscillation. For reasons of simplicity, the DF is evaluated by decoupling the dynamic equations of the robot (i.e. considering only the diagonal constant elements of the inertia matrix). The results of this approximation show that the oscillations predicted with the DF agree well in their frequency and, to a smaller extent, in their magnitude. In this study are used variable structure controllers (VSC's) controllers because they are robust and require a low computing power. We calculate, also, the DF of some common VSC's.

In this perspective, the paper is organised as follows. Section two presents the (simplified) DF's for the 2R robot with nonlinear phenomena at the joints. Section three is devoted to the evaluation of the DF's of some common VSC's. Section four shows several experiments for a 2R robot system and section five outlines the main conclusions.

2 THE DF METHOD APPLIED TO ROBOTS WITH NONLINEARITIES AT THE JOINTS

The dynamic equation of an ideal rigid-link-rigid-joint robot with n links is:

$$\boldsymbol{\tau} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) \qquad (1)$$

Here $\boldsymbol{\tau}$ is the $n \times 1$ vector of input torques, $\mathbf{q}$ is the $n \times 1$ vector of joint coordinates, $\mathbf{H}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{c}(\mathbf{q},\dot{\mathbf{q}})$ is a $n \times 1$ vector of centrifugal/Coriolis terms and $\mathbf{g}(\mathbf{q})$ is the $n \times 1$ vector of gravitational effects. In this study we shall adopt as prototype manipulator the 2R robot depicted in Fig. 1, with dynamics given by:

$$\mathbf{H}(\mathbf{q}) = \begin{bmatrix} (m_1 + m_2)r_1^{\,2} + m_2 r_2^{\,2} + 2m_2 r_1 r_2 C_2 + J_{1m} + J_{1g} & m_2 r_2^{\,2} + m_2 r_1 r_2 C_2 \\ m_2 r_2^{\,2} + m_2 r_1 r_2 C_2 & m_2 r_2^{\,2} + J_{2m} + J_{2g} \end{bmatrix} \qquad (2a)$$

$$\mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix} -m_2 r_1 r_2 S_2 \dot{q}_2^{\,2} - 2m_2 r_1 r_2 S_2 \dot{q}_1 \dot{q}_2 \\ m_2 r_1 r_2 S_2 \dot{q}_1^{\,2} \end{bmatrix} \qquad (2b)$$

$$\mathbf{g(q)} = \begin{bmatrix} g(m_1 r_1 C_1 + m_2 r_1 C_1 + m_2 r_2 C_{12}) \\ g m_2 r_2 C_{12} \end{bmatrix} \qquad (2c)$$

where $C_i = \cos(q_i)$, $C_{ij} = \cos(q_i + q_j)$, $S_i = \sin(q_i)$.

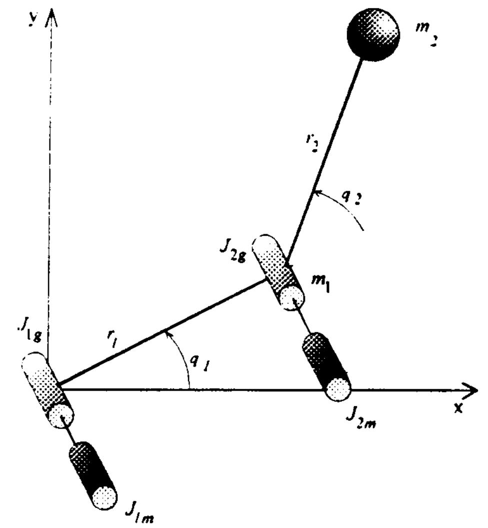


Fig. 1. The ideal 2R robot.

The approximations used to compute the DF for each joint of the 2R robot are:

- the centrifugal/Coriolis phenomena and the gravitational effects are neglected;
- the non-diagonal elements and the variable terms of inertia matrix are not considered.

Consequently, the joints are considered 'decoupled', and it is possible to calculate the individual DF's for each articulation separately.

In what concerns the dynamic phenomena at the joints, three different cases are considered: friction, backlash nd flexibility. The nonlinear friction is the sum of Coulomb plus viscous friction. The backlash is modelled through the impact between gears, considered as instantaneous and obeying the principle of conservation of momentum and the Newton's law (Azenha and Machado 1996a). The flexibility in the joints is modelled by means of a spring between the motor inertia and the robot inertia.

For kinematic/static phenomena, the corresponding DF (N) is frequency-independent and, therefore, can be calculated analytically. Nevertheless, for dynamic systems it is not always possible to find a closed-form analytical expression for the (frequency and amplitude dependent) DF. In fact, this is the case of the nonlinear friction and dynamic backlash studied in this paper and, due to this reason, their DF's are calculated numerically.

Fig. 2 shows the model of the friction employed in the robot joints.

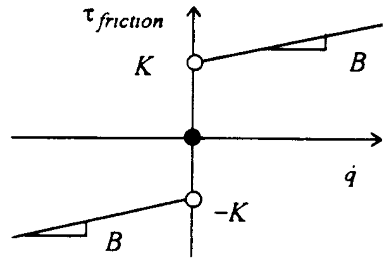


Fig. 2. Model of the joint friction.

The parameters for the 2R robot used in this study are summarised in tables I to IV. The DF for the robot with joint friction must be determined numerically. The same applies for the dynamic backlash at the joints accounting with the impact phenomena (Azenha and Machado 1996b). Indeed, for the joint backlash (i.e. gear with clearance h_i at joint q_i), we have impact phenomena between the inertias which obey the principle of conservation of momentum and the Newton's impact law resulting the equations:

$$\dot{q}_i' = \frac{\dot{q}_i(J_{ii} - \varepsilon J_{im}) + \dot{q}_{im} J_{im}(1+\varepsilon)}{J_{ii} + J_{im}} \qquad (4a)$$

$$\dot{q}_{im}' = \frac{\dot{q}_i J_{ii}(1+\varepsilon) + \dot{q}_{im}(J_{im} - \varepsilon J_{ii})}{J_{ii} + J_{im}} \qquad (4b)$$

where $0 < \varepsilon < 1$ is the Newton constant that defines the type of impact ($\varepsilon = 0$ inelastic impact, $\varepsilon = 1$ elastic impact) and $\dot{q}_i'$ and $\dot{q}_{im}'$ are the velocities of the inertias of the joint and motor after the collision, respectively. The parameter J_{ii} stands for the joint/motor inertias of joint i.

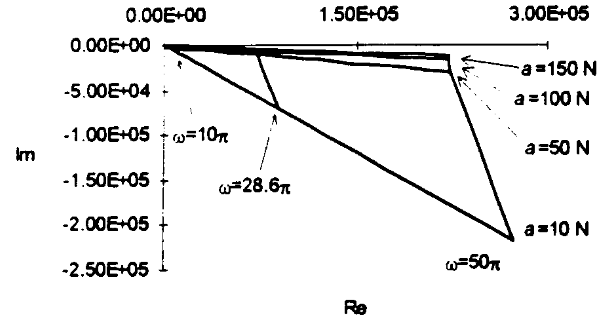
On the other hand, for the case of compliant joints, the dynamic model corresponds to (1) augmented by the equations:

$$\boldsymbol{\tau} = \mathbf{J}_m \ddot{\mathbf{q}}_m + \mathbf{B}_m \dot{\mathbf{q}}_m + \mathbf{K}_m(\mathbf{q}_m - \mathbf{q}) \qquad (3a)$$

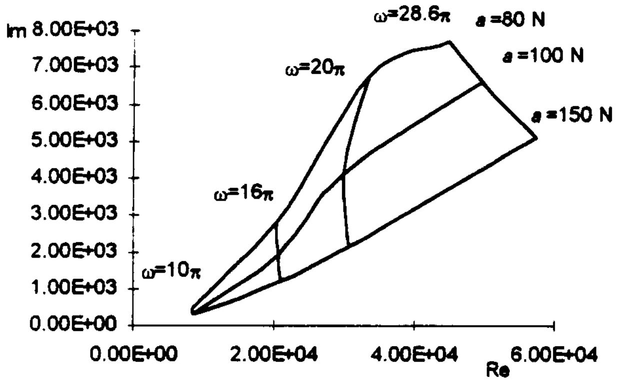
$$\mathbf{K}_m(\mathbf{q}_m - \mathbf{q}) = \mathbf{H(q)}\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g(q)} \qquad (3b)$$

where $\mathbf{J}_m$, $\mathbf{B}_m$ and $\mathbf{K}_m$ are the $n \times n$ diagonal matrices of the motor and transmission inertias, damping and stiffness, respectively. For this case the (pseudo) DF can be calculated analytically and is only a function of the frequency and the robot parameters (i.e. is not amplitude-dependent).

Figure 3 presents the chart of $-1/N$ for each of the three phenomena present at joint 2 of the 2R robot. For joint 1 the charts are similar.



a)



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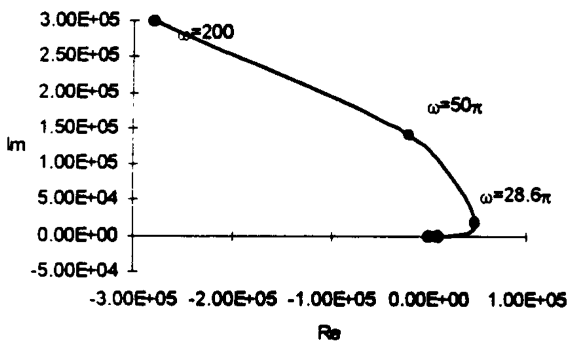


c)

Fig. 3. The $-1/N$ function of joint two with: a) nonlinear friction b) dynamic backlash c) flexibility.

i	m_i (Kg)	r_i (m)	J_{im} (Kgm^2)	J_{ig} (Kgm^2)
1	0.5	1.0	1.0	4.0
2	6.25	0.8	1.0	4.0

Table I The parameters of the 2R robot.

Joint i	K_i (Nm)	B_i (Nms/rad)
1	5	0.5
2	5	0.5

Table II Parameters of the nonlinear friction.

Joint i	h_i (rad)	ε_i
1	0.018	0.8
2	0.018	0.8

Table III Parameters of the dynamic backlash.

Joint i	K_m (Nm/rad)	B_m (Nms/rad)
1	2×10^4	10^2
2	2×10^4	10^2

Table IV Parameters of the flexibility at the joints.

In the next section it will be presented the DF's of the VSC's employed in the simulations with the 2R robot.

3 THE DESCRIBING FUNCTIONS OF VARIABLE STRUCTURE CONTROLLERS

The block diagram of the VSC's adopted in the sequel is depicted in Fig. 4 where K is the gain and τ_{max} is the maximum amplitude of the control effort.

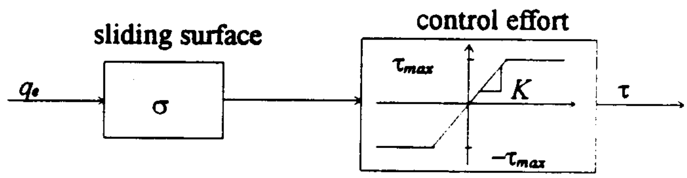


Fig. 4. Block diagram of the VSC's.

The expressions of the sliding surface (σ) for the first order model variable structure controller (FOM-VSC) (Machado 1995; Gao *et al.* 1995; Utkin 1977; Young 1978) and for the second order model variable structure controller (SOM-VSC) (Machado 1995) are:

$$\sigma = \dot{q}_e + c q_e \quad (5a)$$

$$\sigma = \ddot{q}_e + 2\xi\omega_n \dot{q}_e + \omega_n^2 q_e \quad (5b)$$

where c is the eigenvalue, ω_n is the natural frequency and ξ is the damping ratio of the sliding surface.

In both cases we can calculate the corresponding frequency (ω) and amplitude (a) dependent DF's. The DF of the FOM-VSC and the SOM-VSC are given by equations (6) and (7) respectively:

$$N_{FOM}(a,\omega) = K(c + j\omega), \quad a \le \frac{\tau_{max}}{K\sqrt{c^2+\omega^2}} \qquad (6a)$$

$$N_{FOM}(a,\omega) = \frac{2Kc\phi_1}{\pi} - \frac{K\sqrt{4\pi^2+c^2T^2}}{\pi T}\sin(2\phi_1)\cos(\phi_2) + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\cos(\phi_2) + j\left[\frac{4K\phi_1}{T} + \frac{K\sqrt{4\pi^2+c^2T^2}}{\pi T}\sin(2\phi_1)\sin(\phi_2) + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\sin(\phi_2)\right], \quad a > \frac{\tau_{max}}{K\sqrt{c^2+\omega^2}} \qquad (6b)$$

$$\phi_1 = \arcsin\left(\frac{\tau_{max}T}{aK\sqrt{4\pi^2+c^2T^2}}\right), \quad \phi_2 = \arctan\left(\frac{2\pi}{cT}\right), q_e = a\cos(\omega t) \qquad (6c)$$

$$N_{SOM}(a,\omega) = K\left[(\omega_n^2 - \omega^2) + j2\omega\xi\omega_n\right], \quad a \le \frac{\tau_{max}}{K\sqrt{\left[(\omega_n^2-\omega^2)^2 + (2\omega\xi\omega_n)^2\right]}} \qquad (7a)$$

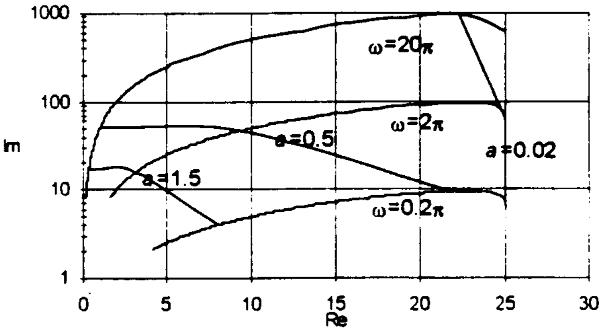
$$N_{SOM}(a,\omega) = \frac{2(T^2\omega_n^2 - 4\pi^2)}{\pi T^2}\operatorname{sgn}(a\xi\omega_n\alpha)\phi_1 + \frac{K\sqrt{\alpha}}{\pi T^2}\sin(2\phi_1)\sin(\phi_2) - \frac{4\tau_{max}}{\pi a}\sin(\phi_2)\cos(\phi_1) + j\left[8\operatorname{sgn}(a\alpha)\frac{K\xi\omega_n}{T}\phi_1 - \frac{K\sqrt{\alpha}}{\pi T^2}\sin(2\phi_1)\cos(\phi_2) + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\cos(\phi_2)\right],$$

$$a > \frac{\tau_{max}}{K\sqrt{\left[(\omega_n^2-\omega^2)^2 + (2\omega\xi\omega_n)^2\right]}} \qquad (7b)$$

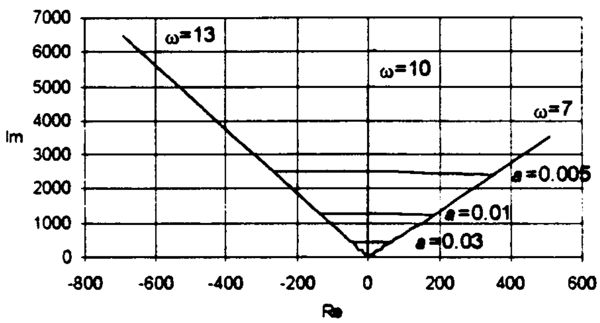
$$\alpha = 16\pi^4 + 8\pi^2T^2\omega_n^2(2\xi^2-1) + T^4\omega_n^4,$$
$$\beta = 16\pi^4a^2K^2 + 8\pi^2a^2K^2T^2\omega_n^2(2\xi^2-1) + T^2(a^2K^2\omega_n^2 - \tau_{max}^2)$$
$$\phi_1 = \arctan\left(\frac{T^2\tau_{max}}{\sqrt{\beta}}\right), \quad \phi_2 = \arctan\left(\frac{4\pi^2 - T^2\omega_n^2}{4\pi T\xi\omega_n}\right),$$
$$\omega = 2\pi / T \qquad (7c)$$

Based on these equations Fig. 5 presents a typical chart of the DF's for a FOM-VSC and a SOM-VSC.

As these charts lie in the first and second quadrants, we conclude that the DF's of the VSC's do not intersect the $-1/N$ function for a robot with friction at the joints. On the other hand, the DF's intersect the $-1/N$ function for the robot with flexibility or backlash at the joints revealing, consequently, more difficult stability cases.



a)



b)

Fig. 5. The DF of:
a) FOM-VSC (K = 10, τ_{max} = 10 and c = 2.5 s^{-1});
b) SOM-VSC (K = 10, τ_{max} = 10, ξ = 2.5 and ω_n = 10 rad s^{-1}).

Based on this analysis, in the next section we present the response of a 2R robot controlled with FOM-VSC's and SOM-VSC's, for the three dynamic cases at the joints considered in this paper.

4 ROBOTS WITH FRICTION, BACKLASH AND FLEXIBILITY AT THE JOINTS

For comparing the VSC's (5) and classical PID's, several simulations are developed to find the limit cycles for the case of a robot with friction in the joints. In fact, the $-1/N$ function of the 2R robot intersects always the (pseudo) DF of the PID controller in the fourth quadrant, resulting limit cycle oscillation. On the other hand, this is not the case of the VSC's which, therefore, are superior (Azenha and Machado 1996a).

For example, Fig. 6 shows the phase-plane (PP) trajectories for joint two of the 2R robot with nonlinear friction, dynamic backlash and flexibility, under the control of a FOM-VSC. The results reveal that, in the case of the nonlinear friction, there are no oscillations. In fact, the DF of the FOM-VSC and the −1/N function of the plant do not intersect.

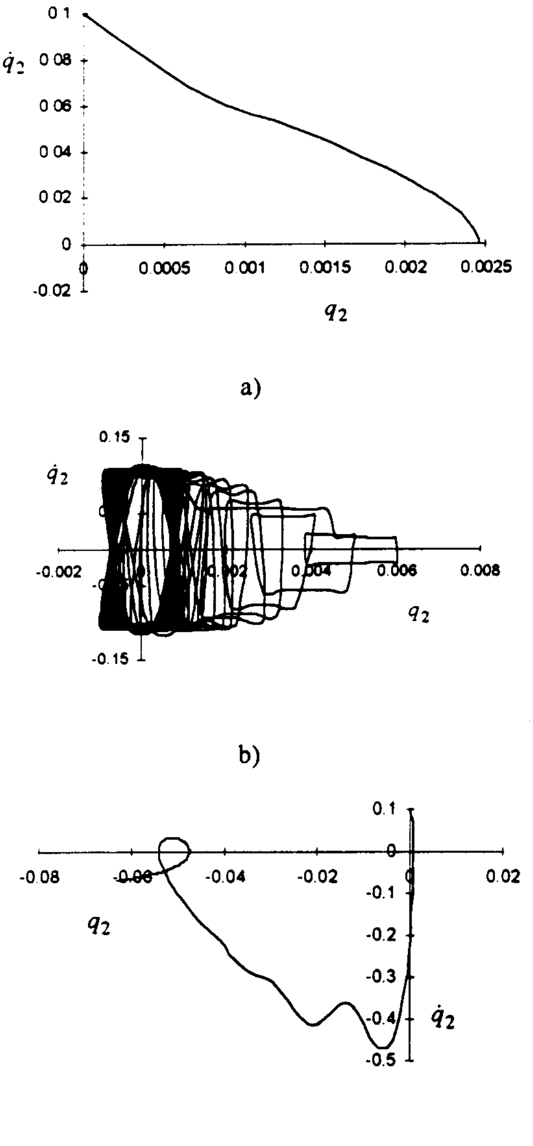


Fig. 6. PP trajectories of joint two for a 2R robot with FOM-VSC: a) nonlinear friction; b) backlash; c) flexibility.

For the case of joints with dynamic backlash it is found a limit cycle, in accordance with the DF analysis, when the material of the gear is very rigid ($\varepsilon \to 1$). This phenomenon disappears when $\varepsilon \to 0$.

The robot with flexibility in the joints represents a difficult task in terms of stability and, in fact, an unstable behaviour is observed when adopting a FOM-VSC.

The SOM-VSC leads to slightly superior results. For example, Fig. 7 shows the PP trajectory for the 2R robot with flexible joints and a SOM-VSC. Note that, in this case, the SOM-VSC has a dominant pole similar to that adopted on the FOM-VSC. We conclude that the SOM-VSC gives a slightly larger stability margin than the FOM-VSC. This extra stability margin is not required in the control of an ideal robot. However, in the case of a robot with flexible joints, the SOM-VSC is essential in order to have a stable and smooth response (Machado 1993).

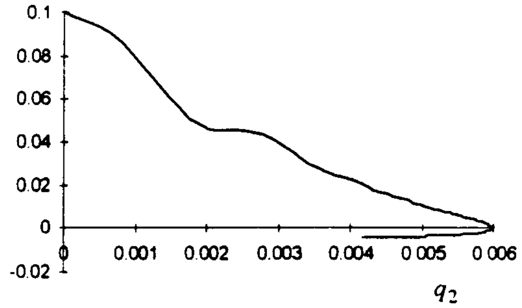


Fig. 7. PP trajectories of joint two for a 2R robot with SOM-VSC and flexibility.

5 CONCLUSIONS

This paper is concerned with the study, through the DF method, of the dynamical properties of robotic systems with nonlinear friction, dynamic backlash and flexibility at the joints. The DF method of predicting limit cycles shows a very good accuracy in terms of the frequency of the oscillation. The DF's of a FOM-VSC and a SOM-VSC are calculated and its performance compared. It is analysed a 'dynamic' backlash, because it is taken into consideration the impact phenomenon in accordance with the laws of classical physics. The most difficult system, in terms of stability, is the one with backlash because it is more prone to limit cycles.

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BIOGRAPHY

Abílio Azenha was born in Maia, Portugal, on August 27, 1968. He graduated and received the MSc degree in Electrical and Computer Engineering from the Faculty of Engineering, University of Porto, Portugal, in 1992 and 1994, respectively. Now he is a graduate student at the Department of Electrical and Computer Engineering, Faculty of Engineering, University of Porto, Portugal. He is concluding the PhD work on "Dynamical Analysis and Forces Control on Robotic Manipulators". His main research interests are robotics, control, nonlinear dynamical systems and simulation.