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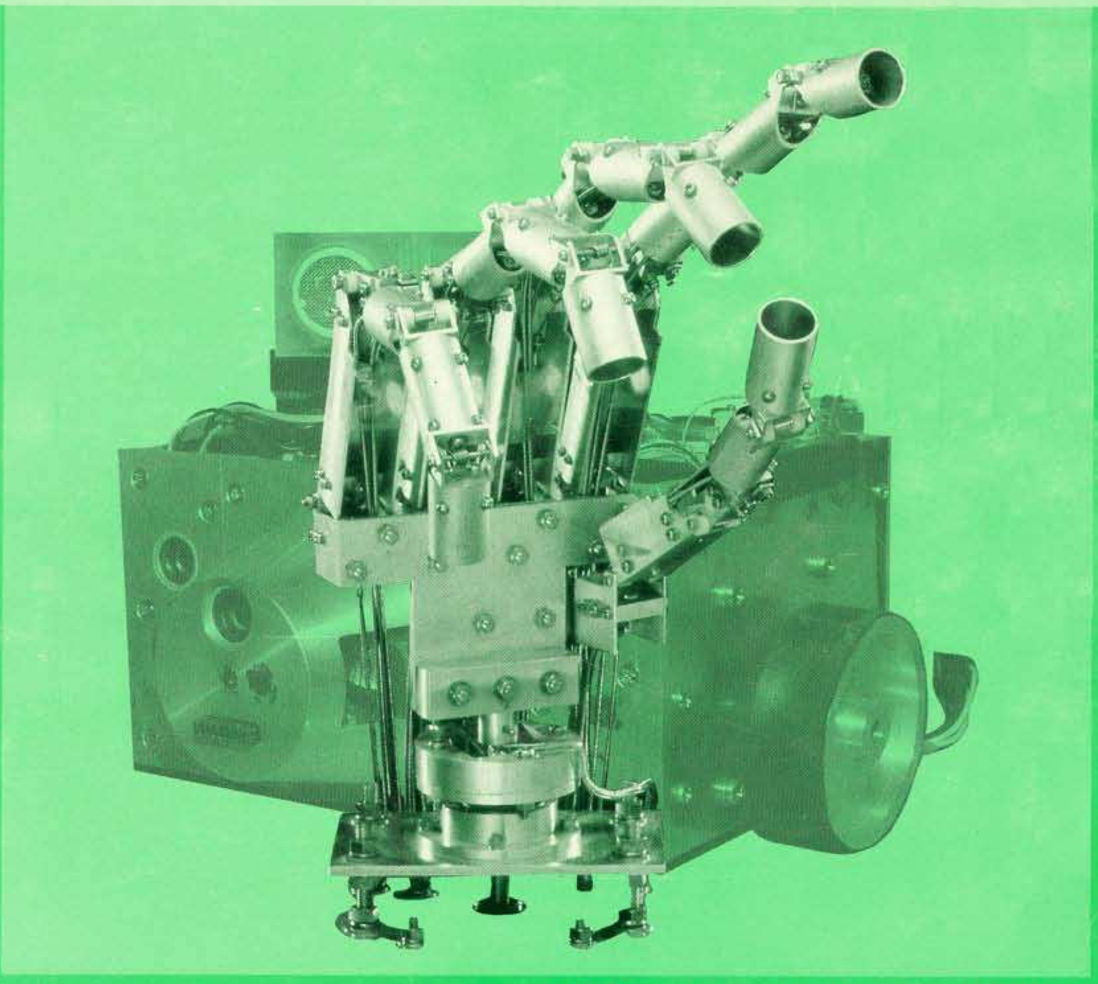

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A STATISTICAL APPROACH TO THE ANALYSIS AND DESIGN OF ROBOT MANIPULATORS

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Abstract-A new approach to the analysis and design of robot manipulators is announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, can be very complex to tackle. This motivates the need of models based on other mathematical concepts having distinct characteristics. The proposed statistical scheme is a step in that direction which gives clear guidelines towards the robot kinematic and dynamic optimization. Furthermore, the inherent use of histograms allows not only fast calculation procedures, but above all, the use of experimental data; consequently, complex dynamic modelling exercises can be avoided.

I. Introduction

In the last two decades the use of robot manipulators has spread out to a large range of industrial and scientific purposes. However, many of the recent applications pose challenging issues to the available industrial manipulators. Usually, these systems consist of a mechanical arm, made of rigid links coupled by linear or rotational joints, and a controller based on a computer system interconnected with some power drive electronics.

The relatively poor performance of present day industrial robot manipulators, when compared with the human arm, motivated the research and development of each of the aforementioned sub-systems. The present paper deals with the problem of the mechanical arm performance optimization. Some research in this area has already been done leading to the implementation of direct drive mechanical arms [1] and to the study of lightweight flexible manipulators [2]. Another different strategy consisted in the development of mathematical and computer tools to study the kinematic and dynamic properties of robot manipulator systems. Examples of such research on kinematics are the work of Tsai and Soni [3] and Yoshikawa [4]; more recently the dynamic characterization has been investigated by Asada [5], Yoshikawa [6], Peltomaa and Koivo [7], Salisbury and Abramowitz [8], Yang and Tzeng [9], Youcef-Toumi and Asada [10], Khatib [11], and Homsup and Anderson [12].

From those studies it is clear that a simple optimization criterion comprising both the kinematics and dynamics is still lacking. In this paper we address this problem, and our presentation is organized as follows: in section 2 the robot manipulator model and the associated optimization criterion are discussed, in section 3 a new model which leads to a natural optimization criterion is formulated and applied to a 2R robot manipulator, and finally in section 4 future developments of this work are discussed and some conclusions are drawn.

II. On the Robot Manipulator Model and Optimization Criterion Formulation

The modelling of rigid robot manipulator systems is well known; for the kinematic model, a set of equations relating the operational space and the joint space, can be found to be of the form:

$$p=\Phi(q) \tag{1a}$$
$$\dot{p}=\mathcal{J}(q)\dot{q} \tag{1b}$$
$$\ddot{p}=\mathcal{J}(q)\ddot{q}+N(q,\dot{q}) \tag{1c}$$

where $\mathcal{J}(q)$ is the jacobian matrix of the transformation (1a) and $N(q,\dot{q})$ is a nonlinear position and velocity dependent n-vector.

Associated with the kinematic model we have the static model, that is the model relating the operational space forces Γ with the joint actuator torques T:

$$T=\mathcal{J}(q)^{T}\Gamma \tag{2}$$

Finally, the dynamic description neglecting non-rigid link behaviour, actuator dynamics saturation, backlash, high frequency mechanical resonant modes and other dynamic phenomena, is described by a set of nonlinear differential matrix equations

$$T=J(q)\ddot{q}+C(q,\dot{q})+G(q) \tag{3}$$

This dynamic joint space model can also be reduced to a dynamic operational space model.

If an "optimal performance" manipulator is to be designed then we should bear in mind the following points:

a) Which "optimal performance" criterion is to be obeyed? In fact, there is a multitude of proposed criteria such as maximum workspace, kinematic and dynamic manipulability measures, minimization of power requirements, minimization of torque requirements, minimization of the nonlinear components of the torque requirements etc. Moreover, some criteria such as the dynamic manipulability measure, still allow several different numerical indices.

b) Kinematic criteria may be inconsistent, or difficult to relate with dynamic criteria. If an optimization algorithm is to be chosen, then there should be some form of interrelationship.

c) If possible, such criteria should provide some insight into related areas like trajectory planning or controller architectures.

d) An optimization criterion, should provide a clear and easy to use method of manipulator design; moreover, it should be well adapted to CAD environments. Methods failing to provide any structural guidelines, such as those restricted to mere hints, should be avoided.

e) Although there are a large number of determining parameters, like masses, inertias, link lengths etc., the optimization algorithm should be computationally efficient, if it is to be used in a CAD environment. Moreover, the influence of each parameter should be clearly understood.

To obey the above considerations a non-standard formulation of the optimization criterion should be sought after. As this criterion must be closely related to the robot manipulator model, then the previous statement is somewhat equivalent to the requirement of a new type of mathematical model.

At this point further analysis of the implications of consideration e) may give some hints towards a new approach. Although model (1-3) is correct, the large number of parameters involved gives rise to a cumbersome work, either in a design or in an analysis stage. On the other hand, a model that is based on a statistical description of the robot manipulator system satisfies the above mentioned requirements. If, with such a model, we lose the certainty of the deterministic model (1-3), we gain a simpler and more intuitive one. This approach in a simplified way has already been used by Mooring and Pack [13] for the kinematics and by Scheinman and Roth [14] for the dynamics, of robot manipulators. In the present study we refer to the new model as the statistical model in contrast with the standard deterministic model. Our model comprises:

-The statistical description of a set of input variables, that is variables that are free to change independently.

-The statistical description of a set of output variables, that is, variables that are functions of the the previous ones.

-A set of parameters that, like the input variables, influence the output variables, but that are to be optimized in the design stage.

One should note that the above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is shown in Fig. 1. This option enables a definition of the required kinematic performances on the operational space (i.e. the description of the desired statistical properties of p, $\dot{p}$ and $\ddot{p}$), which are more natural to the designer. The set of joint positions, velocities and accelerations (i.e. q, $\dot{q}$ and $\ddot{q}$), acts like output variables in the kinematic model, but plays the role of input variables set in the dynamic model. This methodology, provides a relationship between kinematics and dynamics in a form amenable to performance optimization criteria as defined in the sequel and obeying point b) above. The set of dynamic output variables consists of the required joint torques, and the parameter set is made of the link lengths, masses and inertias.

For the statistical description of the involved variables, one could think of a

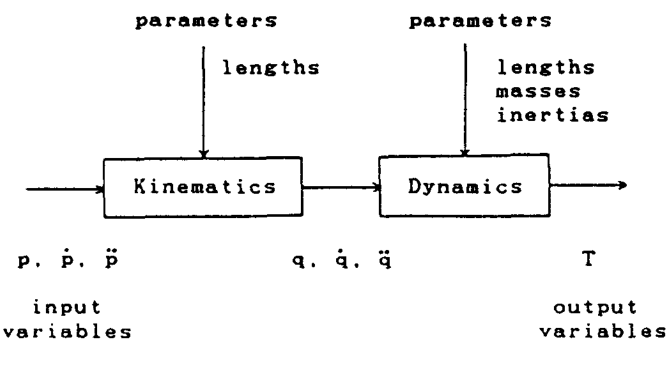

Fig. 1. Statistical model diagram for robot manipulators.

stochastic model. Nevertheless, such a model would be more complex than the deterministic one; therefore, to achieve, as desired, a simple to use model, the implicit time variable is not considered, that is variables which are related through the time derivative operator (p, $\dot{p}$, $\ddot{p}$ and q, $\dot{q}$, $\ddot{q}$), are considered independent of each other. In other words we use a statistical time independent model.

Let us now look at the performance optimization procedure. As mentioned in a) it is hard to find the best option. Here instead of predefining an optimization criterion, we look ahead of the model to gain a feel for such a criterion. A more precise statement of these issues is presented in the next section with the help of a 2R (i.e. with two rotational d.o.f.) robot manipulator example.

III. A Statistical Model for the 2R Robot Manipulator

In this section we use the 2R robot manipulator as the support for the development and implementation of the concepts presented in section 2. This strategy is justified because this manipulator configuration already includes all the kinematic and dynamic phenomena that appear in manipulators with more degrees of freedom. The kinematic and dynamic equations, corresponding to the 2R robot, are listed in Appendix 1.

Let us start by introducing our approach in the kinematic case; in the second sub-section we shall analyse the dynamic case.

Kinematics

The set of kinematic input variables consists of position, velocity and acceleration that our prototype model is required to perform in the operational space. Therefore, it is necessary to characterize them in statistical terms, namely by defining appropriate probability density functions (p.d.f.'s) for each one. As there is no a priori knowledge about the typical behaviour we start with some reasonable assumptions:

-For the position variable $p=[x,y]^T$ we consider a bidimensional uniform p.d.f.

$$f_P(p)=\begin{cases} C & \text{if } (r_1-r_2)^2 \le x^2+y^2 \le (r_1+r_2)^2 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

with $C=1/\{\pi[(r_1+r_2)^2-(r_1-r_2)^2]\}$.

In the sequel we will see how to modify the input p.d.f. in order that the kinematic performances are optimized. It is also necessary to define the p.d.f.'s for velocity and acceleration. By the same above arguments, we decided to use bidimensional Gaussian p.d.f.'s with zero mean

$$f_V(\dot{p})=1/(2\pi\sigma_V^2)\ \exp[-(\dot{x}^2+\dot{y}^2)/(2\sigma_V^2)] \quad (5)$$
$$f_A(\ddot{p})=1/(2\pi\sigma_A^2)\ \exp[-(\ddot{x}^2+\ddot{y}^2)/(2\sigma_A^2)] \quad (6)$$

Moreover, using these p.d.f.'s we impose some interesting properties, such as:

a) The random variables position, velocity and acceleration in the operational space are independent of each other.

b) The velocity and acceleration vectors are made of two independent components, that is $\dot{x}$ is independent of $\dot{y}$ and $\ddot{x}$ is independent of $\ddot{y}$.

The "excitation" of the (inverse) kinematic system (A1-A3) produces output random variables q, $\dot{q}$ and $\ddot{q}$ (i.e. the position, velocity and acceleration in the joint space), with p.d.f.'s which are related to the previous ones by:

$$f_q(q)=J_P\ f_P(\alpha) \tag{7a}$$
$$f_{q\dot{q}}(q,\dot{q})=J_V\ f_V(\beta) \tag{7b}$$
$$f_{q\dot{q}\ddot{q}}(q,\dot{q},\ddot{q})=J_A\ f_A(\gamma) \tag{7c}$$

where

$$J_P=\det\begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} \end{bmatrix} \tag{8a}$$

$$J_V=\det\begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & & \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & & \\ \frac{\partial \dot{x}}{\partial q_1} & \frac{\partial \dot{x}}{\partial q_2} & \frac{\partial \dot{x}}{\partial \dot{q}_1} & \frac{\partial \dot{x}}{\partial \dot{q}_2} \\ \frac{\partial \dot{y}}{\partial q_1} & \frac{\partial \dot{y}}{\partial q_2} & \frac{\partial \dot{y}}{\partial \dot{q}_1} & \frac{\partial \dot{y}}{\partial \dot{q}_2} \end{bmatrix} \tag{8b}$$

$$J_A=\det\begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & & & & \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & & & & \\ \frac{\partial \dot{x}}{\partial q_1} & \frac{\partial \dot{x}}{\partial q_2} & \frac{\partial \dot{x}}{\partial \dot{q}_1} & \frac{\partial \dot{x}}{\partial \dot{q}_2} & & \\ \frac{\partial \dot{y}}{\partial q_1} & \frac{\partial \dot{y}}{\partial q_2} & \frac{\partial \dot{y}}{\partial \dot{q}_1} & \frac{\partial \dot{y}}{\partial \dot{q}_2} & & \\ \frac{\partial \ddot{x}}{\partial q_1} & \frac{\partial \ddot{x}}{\partial q_2} & \frac{\partial \ddot{x}}{\partial \dot{q}_1} & \frac{\partial \ddot{x}}{\partial \dot{q}_2} & \frac{\partial \ddot{x}}{\partial \ddot{q}_1} & \frac{\partial \ddot{x}}{\partial \ddot{q}_2} \\ \frac{\partial \ddot{y}}{\partial q_1} & \frac{\partial \ddot{y}}{\partial q_2} & \frac{\partial \ddot{y}}{\partial \dot{q}_1} & \frac{\partial \ddot{y}}{\partial \dot{q}_2} & \frac{\partial \ddot{y}}{\partial \ddot{q}_1} & \frac{\partial \ddot{y}}{\partial \ddot{q}_2} \end{bmatrix} \tag{8c}$$

$$J_P=r_1r_2S_2 \tag{8d}$$
$$J_V=J_P(r_1r_2S_2) \tag{8e}$$
$$J_A=J_V(r_1r_2S_2) \tag{8f}$$

Each of the formulae (7) is made of two distinct factors:

a) Weighting factors - the jacobians J_P, J_V and J_A - which depend solely on the system kinematic properties

b) The "excitation" p.d.f.'s, which are a measure of the task performances.

These factors can be interpreted in a dynamical system theoretic framework, namely the weighting factor corresponds to the characterization of the system intrinsic properties, and the second factor corresponds to the system response to the excitation variables.

Note the similarities of these interpretations with the well known free and forced response of linear systems, although here these terms are multiplicative.

Bearing these facts in mind, several experiments were performed, having:

a) the total link length constant, L=1.8.

b) several ratios $\mu=r_1/r_2$, namely 0.4, 0.6, 0.8, 1, 1.2, 1.4 and 1.6

c) The operational space categories

1. manipulator with low requirements both in velocity and acceleration: $\sigma_V=0.1$, $\sigma_A=0.1$
2. manipulator with low velocity but high acceleration requirements: $\sigma_V=0.1$, $\sigma_A=1$
3. manipulator with high velocity but low acceleration requirements: $\sigma_V=1$, $\sigma_A=0.1$
4. manipulator with high requirements both in velocity and acceleration: $\sigma_V=1$, $\sigma_A=1$.

d) Excitation of the kinematic system with a numerical random sample of 1000 operational state points obeying the p.d.f.'s (4-6).

e) Analysis of the resulting histograms of the output variables amplitude. To simplify matters, only marginal p.d.f.'s were considered.

TABLE 1
VALUES OF THE 2R ROBOT PARAMETER SET

$r_1=1m$, $r_2=0.8m$, $J_1=5Kgm^2$, $J_2=5Kgm^2$, $m_1=0.5Kg$	
loaded case $m_2=6.25Kg$	unloaded case $m_2=0.25Kg$

After a large number of experiments using the numerical set of parameters depicted in Table 1 [15], we concluded that the shape of the resulting p.d.f.'s varied significantly from variable to variable, but all of them showed symmetry around zero. Moreover, the characterization of the resulting histograms by the mean value and standard deviation, failed in the acceleration and torque histograms, due to the lack of convergence of the corresponding defining integrals. Those facts motivated the definition of a scalar index that somehow characterizes each histogram and doesn't suffer from the aforementioned problem. We decided to use for this scalar index the upper limit of the integral which corresponds to 95% of probability (this definition coincides with two standard deviations for Gaussian type p.d.f.'s).

The resulting histograms are condensed through this index and depicted in Fig. 2. We can observe in the majority of the charts a minimum about $\mu=1$; nevertheless, this conclusion can be easily inferred from (7). In fact, for symmetrical histograms about the y axis, a larger value of the jacobian corresponds to a smaller dispersion of the random variable, which in turn means average smaller amplitude requirements posed to that variable. Therefore, we found an algorithm that leads naturally to an optimization criterion, in the sense that it is not <u>forced externally</u>, but instead it <u>arises naturally from the model</u>.

As the maximization of the (kinematic) jacobians J_P, J_V and J_A requires the same steps, we have for:

$$r_1+r_2=L,\quad r_1/r_2=\mu \tag{9}$$

then a maximum occurs when

$$\mu=1,\quad q_2=\pi/2 \tag{10}$$

which coincide with the results obtained (using different methodologies) by Tsai and Soni [3] and Yoshikawa [4]. Furthermore, our optimization criteria enables new conclusions and procedures:

-Because J_P, J_V and J_A are consecutive powers of $r_1r_2S_2$, we see that for a given deviation from the optimal values (10) we have an increasingly degradation of the cost function with the powers of $r_1r_2S_2$. In other words this means that for a given deviation, we have, by increasing order of sensitivity, position, velocity and acceleration.

-Due to (1b) and (2) a kinematic optimization is equivalent to a static optimization;

-If further optimization is desired, then the next step will be the selection of an optimal "excitation" p.d.f.. This second step of optimization will define, in a statistical sense, an optimum (kinematic) class for the manipulator trajectories. As far as velocity and acceleration are concerned we have seen that the kinematic system did not point out any special p.d.f. class. It shoud be emphasi-

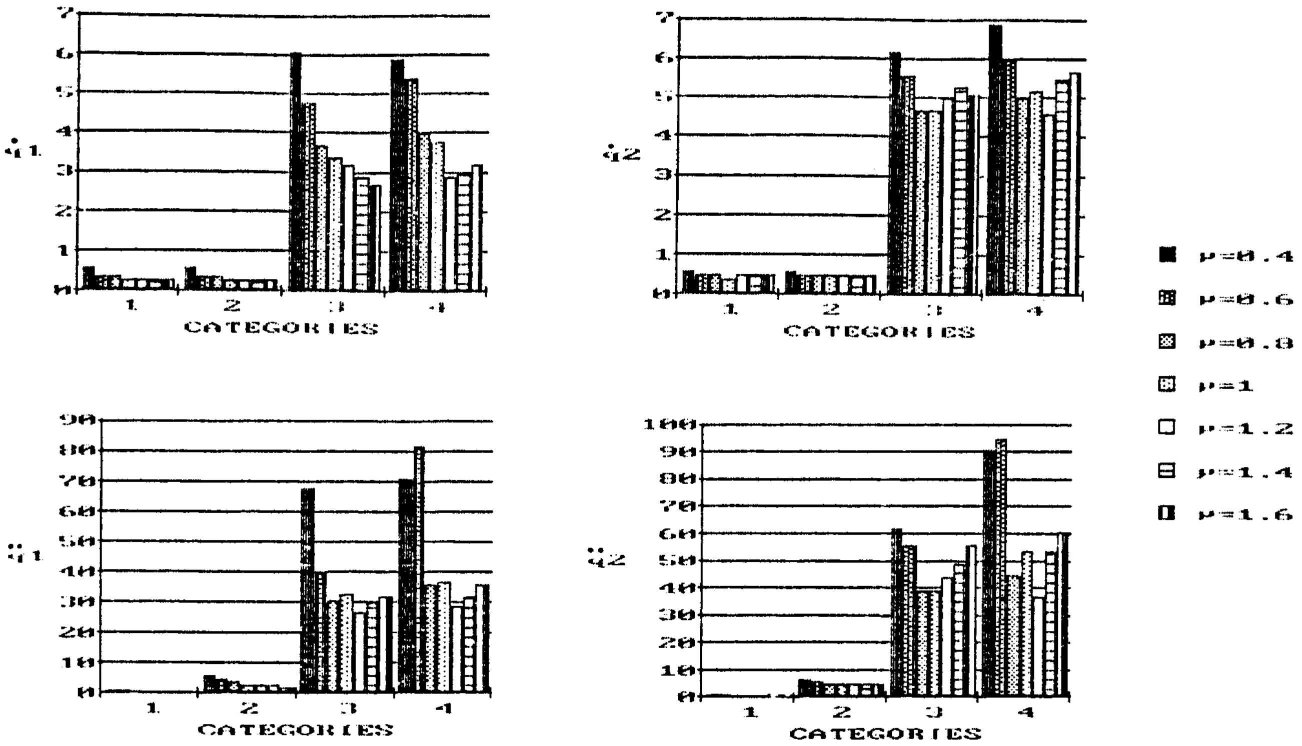

Fig. 2. Comparison charts for the 2R robot kinematic performances, having different geometric configuations μ.
Category 1: $\sigma_V=0.1$ m/s $\sigma_A=0.1$ m/s², Category 2: $\sigma_V=0.1$ m/s $\sigma_A=1$ m/s²
Category 3: $\sigma_V=1$ m/s $\sigma_A=0.1$ m/s², Category 4: $\sigma_V=1$ m/s $\sigma_A=1$ m/s²

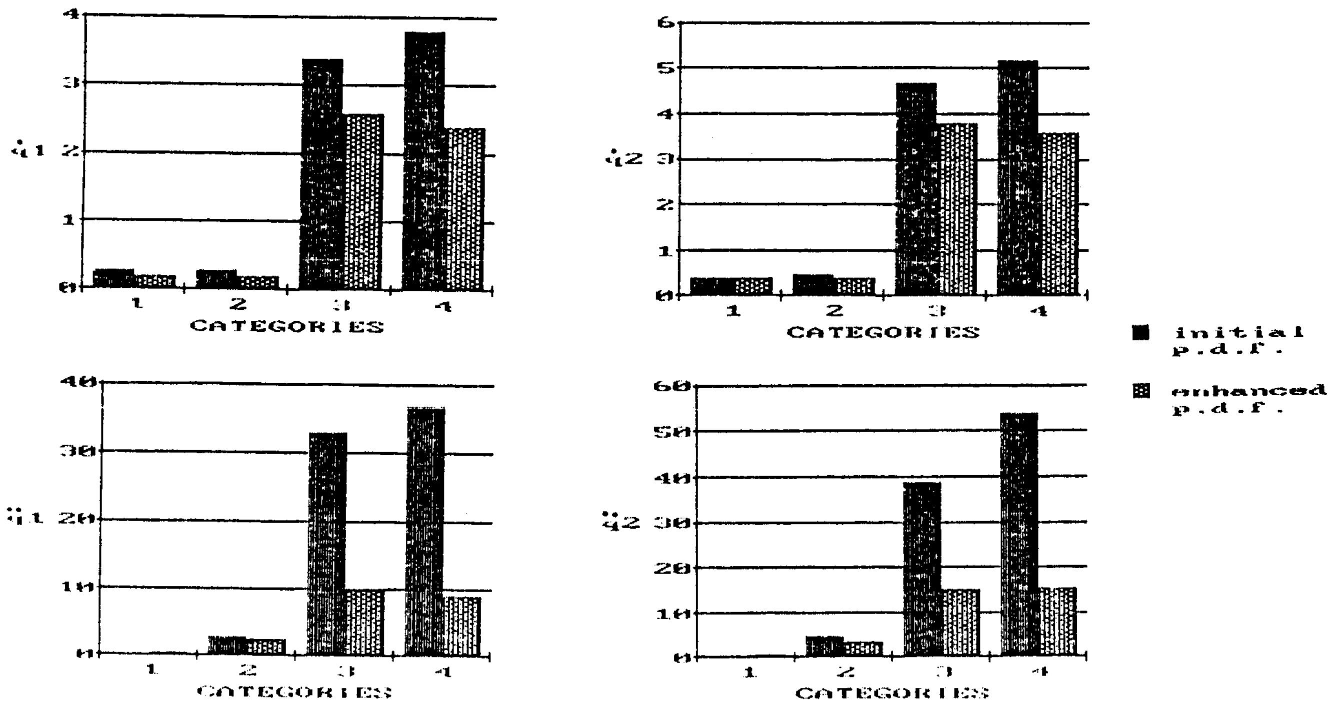

Fig. 3. Comparison charts of the kinematic performances for a 2R robot with geometric configuration μ=1.
Initial p.d.f: $f_Q(q_1,q_2)=const.*S_1$, enhanced p.d.f: $f_Q(q_1,q_2)=const.*S_2^2$
Category 1: $\sigma_V=0.1$ m/s $\sigma_A=0.1$ m/s², Category 2: $\sigma_V=0.1$ m/s $\sigma_A=1$ m/s²
Category 3: $\sigma_V=1$ m/s $\sigma_A=0.1$ m/s², Category 4: $\sigma_V=1$ m/s $\sigma_A=1$ m/s²

zed the statistical nature of these conclusions; they suggest "average" guidelines, therefore leaving room for deterministic rules like Sahar and Hollerbach results [16].

Given the complexity of the problem, we restrict ourselves to the following family of p.d.f.'s (with $K \geq 1$)

$$f_P(x,y)=constant*\{1-[(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2)]^2\}^{(K-1)/2} \quad (11)$$

which corresponds to

$$f_Q(q_1,q_2)=constant * S_2^K \quad (12)$$

For $K=1$ we have the uniform p.d.f. on the operational space $\{O,x,y\}$, and that for $K \to \infty$ we get Dirac type p.d.f. ($\delta(.)$)

$$f_P(x,y)=\delta[x^2+y^2-(r_1^2+r_2^2)] \quad (13a)$$
$$f_Q(q_1,q_2)=1/2\ [\delta(q_2+\pi/2)+\delta(q_2-\pi/2)] \quad (13b)$$

Notice that this choice agrees with the previous conclusion that an optimal choice should maximize S_2.

To test the above conjectures, the previous results (i.e. the uniform position p.d.f.) for $\mu=1$ are compared with a new case using $\mu=1$ and $K=3$ in (11) and (12). This has revealed a remarkable performance improvement as shown in Fig. 3, particularly for high velocity requirements (categories 3 and 4).

Dynamics

The previous considerations motivated the development of a new model that lead naturally to an optimization criterion. In this subsection those concepts are applied to the robot manipulator dynamics (A4).

Following the same methodology, we need to find a set of dynamic equations that play the same role of equations (7,8) in the kinematic case. As the gravitational, Coriolis/centripetal and inertial torques are additive and they depend on $\{q\}$, $\{q,\dot{q}\}$ and $\{q,\ddot{q}\}$ input variables, respectively, we decided to use them as the dynamic output variables (i.e. those to be optimized), resulting:

$$f_G(q)=J_G\ f_Q(q) \quad (14a)$$
$$f_C(q,\dot{q})=J_C\ f_{Q\dot{Q}}(q,\dot{q}) \quad (14b)$$
$$f_I(q,\dot{q},\ddot{q})=J_I\ f_{Q\dot{Q}\ddot{Q}}(q,\dot{q},\ddot{q}) \quad (14c)$$

where

$$J_G=[m_2(m_1+m_2)g^2r_1r_2S_1S_{12})^{-1} \quad (15a)$$
$$J_C=J_GJ_C^*=J_G[(2m_2r_1r_2S_2)^2q_1q_{12}]^{-1} \quad (15b)$$
$$J_I=J_CJ_I^*=J_C\{[(m_1+m_2)r_1^2+J_1](m_2r_2^2+J_2)+ +J_2(m_2r_2^2+2r_1r_2m_2C_2)-(r_1r_2m_2C_2)^2\}]^{-1} \quad (15c)$$

Unlike the kinematic situation, where the optimization was similar for all the jacobians, we have now different effects according to each dynamic term. The Coriolis/centripetal torques affect J_C through J_C^* and the inertial torques affect J_I through J_I^*; consequently, gravitational, Coriolis/ centripetal and inertial optimization procedures must now deal with J_G, J_C^* and J_I^* respectively. As far as J_C^* is concerned its maximization agrees with the kinematic results, given the presence of the factor $r_1r_2S_2$, but needs further analysis in the case of J_G and J_I^*. The study of this optimization, as well as the integration these concepts on a trajectory planning strategy are the subject of a companion paper [17].

Another important aspect is that a robot manipulator, which mimics the human arm, is much more sensitive to velocity than to acceleration requirements. In other words, a robot manipulator is by excellence a "position/acceleration machine" and not a "velocity machine", as can be observed in Fig. 4. Although obvious, this aspect is somewhat overlooked in the literature. Moreover, it points out that the usual electrical, hydraulic or pneumatic robotic actuators, which are developments of standard "velocity machines" are not well adapted to robotic applications. Alternative solutions such as muscle like "position/acceleration" actuators [18,19] will allow more efficient robot structures.

IV. Conclusions

A new approach to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, on the other hand, can be very complex and hard to tackle. This motivates the need of models based on other mathematical concepts having distinct characteristics. The proposed statistical scheme is a step in that direction which has been shown to present interesting properties and to enable new procedures.

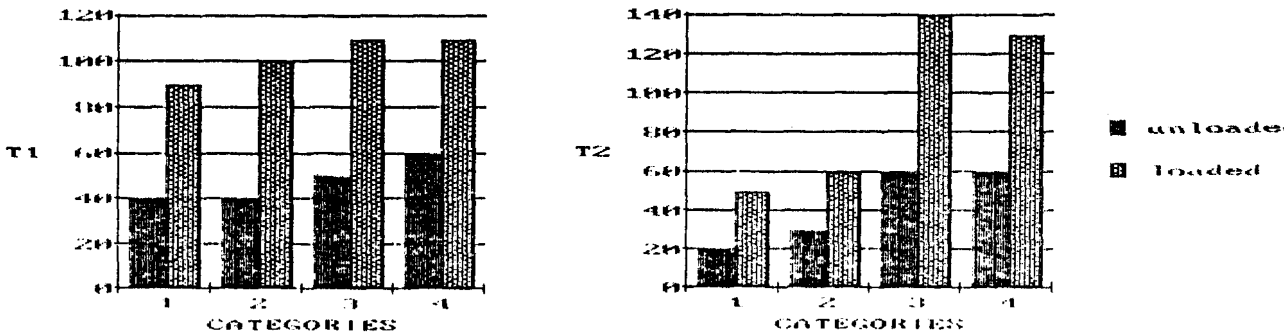

Fig. 4. Comparison charts of the actuator torques for a 2R robot with geometric configuration $\mu=1$, in the loaded and unloaded cases, and using the enhanced p.d.f.: $f_Q(q_1,q_2)=constant * S_2^3$

In this paper the new method provides a framework giving clear guidelines towards the robot mechanical optimization. As a result, the manipulator design procedure, both kinematic and dynamic, leads to simple and intuitive conclusions. Previously proposed schemes give similar results for the simple kinematic case; unfortunately, they are difficult to apply in the (more complex) dynamic stage. In the proposed algorithm both situations appear as natural and clear extensions of a common and systematic methodology. Furthermore, the inherent use of histograms allows not only fast calculation procedures, but above all, the use of experimental data; consequently, complex dynamic modelling exercises can be avoided.

Also, it should be highlighted the results pointing out some characteristics of the trajectory planning block, defining optimal rest and active regions, and ideal-actuator properties, as "position/acceleration" devices instead of "velocity" machines. This observation is of utmost importance as it gives a clear basis to new mechanical robot manipulator structures, with performances near to the muscle-actuated biological systems.

Appendix

Inverse Kinematics of the 2R robot manipulator

$$q=\alpha(p) \qquad (A1a)$$

$$\begin{cases} C_2=(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2) \\ q_1=\tan^{-1}(y/x)-\tan^{-1}[r_2S_2/(r_1+r_2C_2)] \end{cases} \qquad (A2b)$$

$$\dot{q}=\beta(\dot{p},q) \qquad (A2a)$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = d \begin{bmatrix} r_2C_{12} & r_2C_{12} \\ -r_1C_1-r_2C_{12} & -r_1S_1-r_2S_{12} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \qquad (A2b)$$

$$\ddot{q}=\gamma(\ddot{p},q,\dot{q}) \qquad (A3a)$$

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_{12} \end{bmatrix} = d \left\{ \begin{bmatrix} r_2C_{12} & r_2S_{12} \\ -r_1C_1 & -r_1S_1 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} r_1r_2C_2 & r_2 \\ -r_1^2 & -r_1r_2C_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_{12}^2 \end{bmatrix} \right\}$$

where $d=(r_1r_2S_2)^{-1}$

Inverse Dynamics of the 2R robot manipulator

$$T=J(q)\ddot{q}+C(q,\dot{q})+G(q) \qquad (A4a)$$

$$J(q)=\begin{bmatrix} (m_1+m_2)r_1^2+m_2r_2^2+2m_2r_1r_2C_2+J_1 & m_2r_2^2+m_2r_1r_2C_2 \\ m_2r_2^2+m_2r_1r_2C_2 & m_2r_2^2+J_2 \end{bmatrix} \qquad (A4b)$$

$$C(q,\dot{q})=\begin{bmatrix} -m_2r_1r_2S_2\dot{q}_2^2-2m_2r_1r_2S_2\dot{q}_1\dot{q}_2 \\ m_2r_1r_2S_2\dot{q}_1^2 \end{bmatrix} \qquad (A4c)$$

$$G(q)=\begin{bmatrix} g[m_1r_1C_1+m_2(r_1C_1+r_2C_{12})] \\ m_2gr_2C_{12} \end{bmatrix} \qquad (A4d)$$

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