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Control of Robots with Nonlinear Friction and Backlash in the Joints

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Abstract

This paper investigates the control of robots with nonlinear friction and dynamic backlash in the joints. The study is based on the describing function (DF) of nonlinear systems. The controllers studied are the first order model and the second order model variable structure controllers (FOM-VSC and SOM-VSC). We start by studying nonlinear systems consisting of a mass subjected to Coulomb plus viscous friction or two masses subjected to backlash. The worst case for the limit cycle generation is found to be the closed-loop system having two masses with backlash. On the other hand, the closed-loop system of a mass with nonlinear friction is found to be amenable to control without limit cycles. By decoupling the joints of the robot we find similar results within a good accuracy range.

1 Introduction

The progress in computational systems made possible the intensive study of nonlinear systems through simulation while the development of adequate control strategies complemented the computer-based techniques [1]. In this perspective, this paper investigates the dynamics of robots with friction and backlash in the joints through the describing function (DF) method. These nonlinear dynamic phenomena have been an area of active research, however well established conclusions are still lacking. Dupont in [2] studied the effect of Coulomb friction in the existence and uniqueness of the solution of the direct dynamics. Dupont showed that the problems of existence and uniqueness occur even for a system with a single degree-of-freedom. Studies about nonlinear friction modelling can be found also in [3-9]. In these papers the position and velocity dependence of the friction phenomena was investigated. A computer simulation of the stick-slip friction was also developed by Karnopp in [10], that presented an efficient algorithm for the problem. The compensation of the nonlinear friction is found in [11-12]. In these studies the model of the nonlinear friction for the development of efficient control systems is employed. Recently, de Wit *et al.* in [13] modelled the friction through bristles with good results. Nevertheless, in order to compare results with previous studies, in this article we adopt the classical Coulomb and viscous friction model, to get a simple mathematical treatment.

The backlash phenomenon is found in many physical systems. Tao and Kokotovic in [14] considered this problem and developed an algorithm for the compensation of the (geometric) backlash based on an adaptive controller and an unknown backlash model of the system; though, they did not consider the impact effects. Also, in [15-16] there are studies of the backlash with some simplifications in the dynamical models. In [17] one can find a study about compliant actuators with backlash. Other cases of backlash compensation and control can be found in [18-19].

In this line of thought, this paper is organised as follows. In section 2 we formulate the problem treated in this paper and we introduce the DF method. In section 3 we describe two variable structure controllers (VSC's) and we calculate their DF's. In sections 4 and 5, we study the control of a mass and a 2R robot systems having nonlinear friction and backlash, respectively. Finally, in section 6, we present the main conclusions.

2 The Describing Function and the Prediction of Limit Cycles

In this section we present a summary of the DF method and its application to limit cycle prediction in nonlinear systems. The purpose of this paper is to analyse the controller performance in robots with nonlinear friction and backlash at the joints. Our strategy is to analyse the DF evolution in the Nyquist diagram of each controller and system. By this way, we can study the stability and we can predict the occurrence of limit cycles.

It is well-known that many relationships among physical variables are not linear, although they are often approximated by linear equations, mainly for mathematical simplicity. This simplification may be satisfactory as long as the resulting solutions are in agreement with experimental results. In fact, it has been demonstrated that this is the case with the approximation of nonlinear systems by a DF where limit cycles can be predicted with reasonable accuracy [1]. It must be emphasised that the DF method is not the only one tractable to limit cycle prediction, being the most important others the frequency balance and the amplitude dependent gain margin methods.

Nevertheless, in the condition of limit cycle occurrence all of the methods are equivalent to the DF method. Next, we will introduce the fundamental aspects of the DF method.
Suppose that the input to a nonlinear element is sinusoidal. The output of the nonlinear element is, in general, not sinusoidal. Assume that the output is periodic with the same period as the input, containing higher harmonics in addition to the fundamental harmonic component. In the DF analysis, we assume that only the fundamental harmonic component of the output is significant. Such assumption is often valid since the higher harmonics in the output of a nonlinear element are of smaller amplitude than the amplitude of the fundamental component. Moreover, many controlled systems are "low-pass filters" with the result that the higher harmonics are further attenuated.
The DF of a nonlinear element is defined as the complex ratio of the fundamental harmonic components of the output $Y_1\angle\Phi_1$ and the input $a\angle 0$, that is:

$$N = \frac{Y_1}{a}\angle\Phi_1 \qquad (1)$$

where the symbol N represents the DF, a is the amplitude of the input sinusoid and Y_1 and Φ_1 are the amplitude and the phase shift of the fundamental harmonic component of the output, respectively. Several DF's of simple nonlinear systems can be found in [1]. In general, the DF can be computed evaluating the expression:

$$N(a,\omega) = \frac{2}{aT}\int_{t_1}^{T+t_1} y(\omega t)e^{-j\omega t}dt \qquad (2)$$

where ω is the angular frequency of the input and output waveforms and $T = 2\pi/\omega$.
Once calculated, the DF can be used for the stability analysis of a nonlinear control system. Let us consider the standard system shown in Fig. 1 where the block N denotes the DF of the nonlinear element.

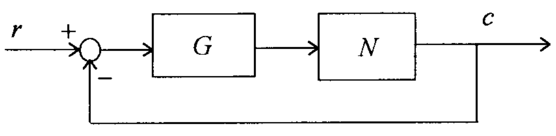

Fig. 1 Nonlinear control system.

If the higher harmonics are sufficiently attenuated, N can be treated as a real or complex variable gain and the characteristic equation becomes:

$$1 + NG(j\omega) = 0 \;\Leftrightarrow\; G(j\omega) = -1/N \qquad (3)$$

If equation (3) is satisfied, then the system will exhibit a limit cycle which may be found to be stable or unstable through graphical and mathematical analysis.

3 The DF's of Variable Structure Controllers

In this section we introduce the VSC concept and we calculate the corresponding DF.
The VSC's were introduced in automatic control systems by Utkin in [21]. These algorithms are robust and present low computational requirements. Since then, VSC's have been studied for a large variety of applications. Some examples of other studies about VSC's can be found in [20, 23].
If the controller G is nonlinear, as is the case of a VSC, we may employ the DF of the controller to the analysis of the closed-loop limit cycles. In fact, consider the VSC sketched in Fig. 2.

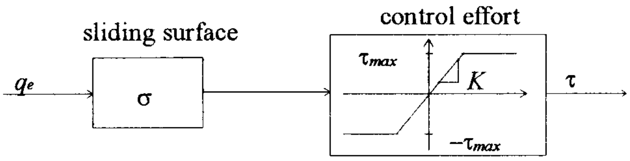

Fig. 2 The block diagram of the VSC.

A "standard" expression for the control effort block is:

$$\tau = \begin{cases} \tau_{max} & ,\sigma \ge \tau_{max}/K \\ K\sigma & ,|\sigma| < \tau_{max}/K \\ -\tau_{max} & ,\sigma \le -\tau_{max}/K \end{cases} \qquad (4)$$

The VSC becomes of first order (FOM-VSC), if the sliding surface σ obeys the expression:

$$\sigma = \dot{q}_e + cq_e \qquad (5)$$

where the parameter c is the corresponding eigenvalue, q_e is the input to the VSC and τ is the respective output. For a second order model VSC (SOM-VSC), the sliding surface σ is given by [20]:

$$\sigma = \ddot{q}_e + 2\xi\omega_n\dot{q}_e + \omega_n^2 q_e \qquad (6)$$

where ξ is the damping ratio and ω_n is the undamped natural frequency.
The DF for the FOM-VSC is given as follows:

$$N_{FOM}(a,\omega) = K(c + j\omega), \quad a \le \frac{\tau_{max}}{K\sqrt{c^2+\omega^2}} \qquad (7a)$$

$$
\begin{aligned}
N_{FOM}(a,\omega) &= \frac{2Kc\phi_1}{\pi} - \frac{K\sqrt{4\pi^2+c^2T^2}}{\pi T}\sin(2\phi_1)\cos(\phi_2) + \\
&+ \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\cos(\phi_2) \\
&+ j\left[\frac{4K\phi_1}{T} + \frac{K\sqrt{4\pi^2+c^2T^2}}{\pi T}\sin(2\phi_1)\sin(\phi_2) + \right. \\
&\left. + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\sin(\phi_2)\right], \quad a > \frac{\tau_{max}}{K\sqrt{c^2+\omega^2}}
\end{aligned}
\tag{7b}
$$

$$
\phi_1 = \arcsin\left(\frac{\tau_{max}T}{aK\sqrt{4\pi^2+c^2T^2}}\right), \quad \phi_2 = \arctan\left(\frac{2\pi}{cT}\right), q_e = a\cos(\omega t)
\tag{7c}
$$

For the SOM-VSC we get:

$$
N_{SOM}(a,\omega) = K\left[(\omega_n^2-\omega^2) + j2\omega\xi\omega_n\right], \quad a \le \frac{\tau_{max}}{K\sqrt{\left[(\omega_n^2-\omega^2)^2 + (2\omega\xi\omega_n)^2\right]}}
\tag{8a}
$$

$$
\begin{aligned}
N_{SOM}(a,\omega) &= \frac{2(T^2\omega_n^2-4\pi^2)}{\pi T^2}\operatorname{sgn}(a\xi\omega_n\alpha)\phi_1 + \\
&+ \frac{K\sqrt{\alpha}}{\pi T^2}\sin(2\phi_1)\sin(\phi_2) - \frac{4\tau_{max}}{\pi a}\sin(\phi_2)\cos(\phi_1) + \\
&+ j\left[8\operatorname{sgn}(a\alpha)\frac{K\xi\omega_n}{T}\phi_1 - \right. \\
&\left. - \frac{K\sqrt{\alpha}}{\pi T^2}\sin(2\phi_1)\cos(\phi_2) + \frac{4\tau_{max}}{\pi a}\cos(\phi_1)\cos(\phi_2)\right], \\
a &> \frac{\tau_{max}}{K\sqrt{\left[(\omega_n^2-\omega^2)^2 + (2\omega\xi\omega_n)^2\right]}}
\end{aligned}
\tag{8b}
$$

$$
\begin{aligned}
\alpha &= 16\pi^4 + 8\pi^2T^2\omega_n^2(2\xi^2-1) + T^4\omega_n^4, \\
\beta &= 16\pi^4a^2K^2 + 8\pi^2a^2K^2T^2\omega_n^2(2\xi^2-1) + T^2(a^2K^2\omega_n^2 - \tau_{max}^2) \\
\phi_1 &= \arctan\left(\frac{T^2\tau_{max}}{\sqrt{\beta}}\right), \quad \phi_2 = \arctan\left(\frac{4\pi^2 - T^2\omega_n^2}{4\pi T\xi\omega_n}\right), \\
\omega &= 2\pi / T
\end{aligned}
\tag{8c}
$$

The DF's of a FOM-VSC with $K = 10$, $\tau_{max} = 10$ and $c = 2.5\ s^{-1}$ is depicted in Fig. 3. Note that the points corresponding to $a \to 0$ depend only on T and lie in a vertical straight line.
We verify that the DF of the SOM-VSC is nearly a straight line passing through the origin, with different slopes according with ω. Furthermore, the DF of a FOM-VSC is always on the first quadrant, while the DF of a SOM-VSC may be either on the first or on the second quadrants of the Nyquist plane.

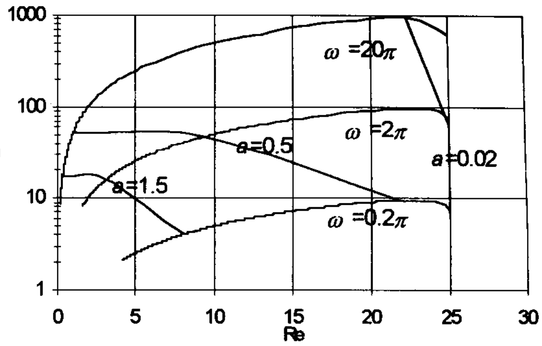

Fig. 3 The DF of a FOM-VSC ($K = 10$, $\tau_{max} = 10$, $c = 2.5\ s^{-1}$).

The "pseudo" DF's of the well-known PD and PID controllers (that are just the frequency response of the system) are not amplitude dependent. Therefore, the Nyquist diagram of the PD controller is just a vertical straight line starting (on the first quadrant) at a point (K_p, 0). The Nyquist diagram of the PID controller is also a vertical line with real part equal to K_p and passing through the fourth and first quadrants. Due to these characteristics, we shall verify in the sequel, that VSC's are more appropriate to avoid limit cycles than standard PID controllers.
The DF of a SOM-VSC for $K = 10$, $\tau_{max} = 10$, $\xi = 2.5$ and $\omega_n = 10$ rad s^{-1}, is represented in Fig. 4.

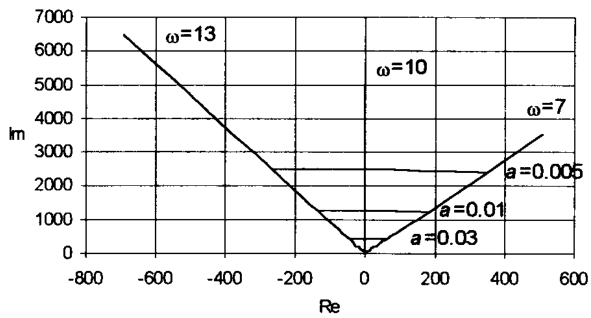

Fig. 4 The DF of a SOM-VSC ($K = 10$, $\tau_{max} = 10$, $\xi = 2.5$ and $\omega_n = 10$ rad s^{-1}).

4 Systems with Nonlinear Friction

In this section we calculate the DF of dynamical systems with nonlinear friction [22, 24].
Let us consider a system with a mass M, moving on a horizontal plane, under the effect of a Coulomb (K) plus a viscous friction (B). This type of friction is depicted in Fig. 5.
The steady-state position response $x(t)$, to a sinusoidal input force $F = a\cos(\omega t)$, becomes:

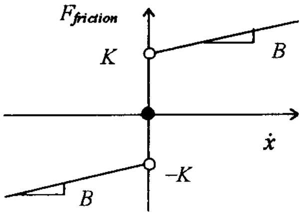

Fig. 5 Model of a Coulomb and viscous friction.

$$x(t)=\begin{cases}\alpha_1\sin(\omega t+\phi)+k_1+k_2 e^{-\frac{B}{M}t}-\frac{K}{B}t, & \dot{x}>0\\ \alpha_1\sin(\omega t+\phi)+k_3+k_4 e^{-\frac{B}{M}t}+\frac{K}{B}t, & \dot{x}<0\end{cases} \quad (9)$$

where the parameters ϕ, k_1, k_2, k_3 and k_4 cannot be calculated in closed-form. Therefore, the DF must be determined numerically. Fig. 6 shows $-1/N(a, \omega)$ for a system with $M = 1$ Kg, $K = 1$ N and $B = 0.1$ Ns/m.

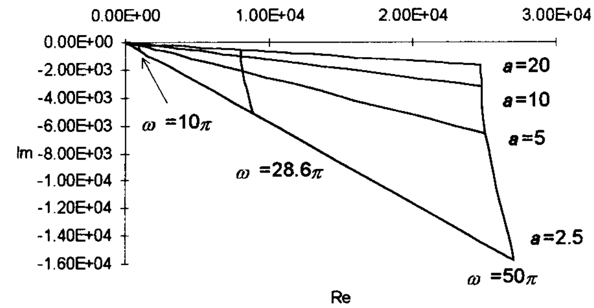

Fig. 6 The function $-1/N(a, \omega)$ for a system with a mass subjected to Coulomb and viscous friction.

We conclude that limit cycles may only arise with a PID because this controller has part of its (pseudo) DF on the fourth quadrant. If we decouple the joints of a robot (i.e. we ignore the non-diagonal terms of the inertia matrix and the other components of the robot dynamics), we get a DF for each axis that leads to results consistent with simulations. Figure 7 shows a limit cycle for the PID control of a 2R robot with nonlinear friction in the joints.
The frequency of the oscillation is in accordance with the DF prediction, while the magnitude reveals a significant error because, for this variable, the intersection of the two DF's, is nearly tangential.
Fig. 8 present the phase-plane response of the 2R robot controlled with a FOM-VSC. Clearly the VSC eliminate the limit cycle at the cost of a small steady-state error.

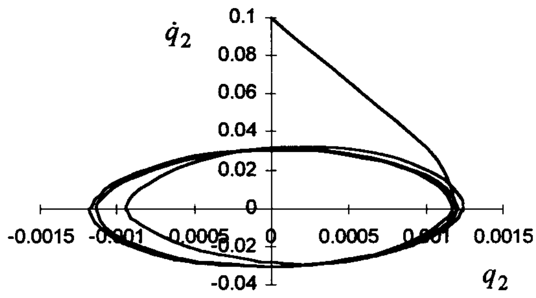

Fig. 7 Phase-plane evolution of joint 2 of a robot system with nonlinear friction and a PID controller.

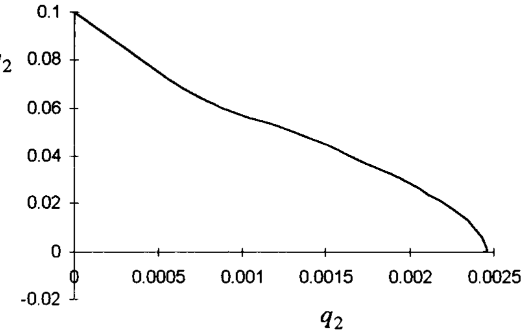

Fig. 8 Phase-plane evolution of joint 2 of the 2R robot with nonlinear friction and a FOM-VSC controller.

5 Systems with Dynamic Backlash

In this section we analyse a system with backlash and its control requirements through the DF method.
Let us consider two masses subjected to backlash (Fig. 9).

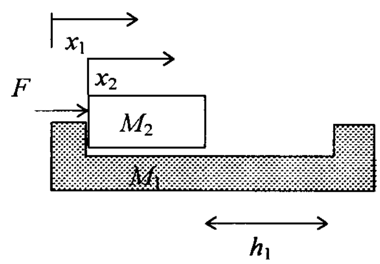

Fig. 9 System with two masses with backlash.

A collision between the masses M_1 and M_2 occurs when $x_1 = x_2$ or $x_2 = h_1+x_1$. In this case we can compute the velocities of masses M_1 and M_2 after the impact ($\dot{x}_1'$ and $\dot{x}_2'$, respectively) by applying the Newton's law:

$$\dot{x}'_{12} = -\varepsilon\,\dot{x}_{12}, \quad 0 \le \varepsilon \le 1 \tag{10}$$

where $x_{12} = x_1 - x_2$ and ε is the coefficient of restitution. On the other hand, by the principle of conservation of momentum it comes:

$$M_1\dot{x}'_1 + M_2\dot{x}'_2 = M_1\dot{x}_1 + M_2\dot{x}_2 \tag{11}$$

From equations (10) and (11) we obtain:

$$\begin{cases} \dot{x}'_1 = \dfrac{\dot{x}_1(M_1 - \varepsilon M_2) + \dot{x}_2(1+\varepsilon)M_2}{M_1 + M_2} \\ \dot{x}'_2 = \dfrac{M_1(1+\varepsilon)\dot{x}_1 + (M_2 - \varepsilon M_1)\dot{x}_2}{M_1 + M_2} \end{cases} \tag{12}$$

For this system we found $-1/N(a, \omega)$ numerically. The sinusoidal input force was applied to mass M_2 and the output position x_1 monitored. For example, Fig. 10 shows the DF for a system having $M_1 = M_2 = 0.5$ Kg, $\varepsilon = 0.8$, $h_1 = 2$ mm, under the action of an input force $F = a\cos(\omega t)$ where $a = 20$ N. The upper curve represents the standard geometric backlash, while the lower curve describes the dynamic backlash presently under study. The standard backlash corresponds to the DF of a mass $M_1 + M_2$ followed by a geometric backlash having input and output position variables [1].

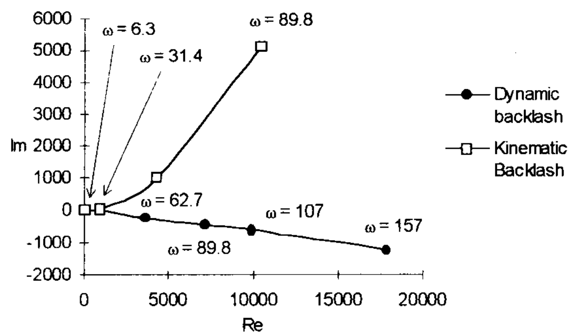

Fig. 10 Plot of $-1/N(a, \omega)$ for a plant with backlash for $a = 20$ N.

Analysing the DF of the controllers under study, we concluded, for the 2R robot, that intersection between $-1/N$ and G can occur in the first quadrant, which makes this system more prone to limit cycle generation. As an example, let us consider the 2R robot with dynamic backlash in the joints under the control of a FOM-VSC. The coefficient of restitution of the joint material is assumed to be $\varepsilon = 0.8$. The phase-plane response of the closed-loop system is depicted in Fig. 11 showing a (stable) limit cycle.

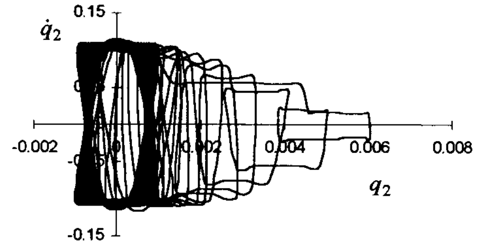

Fig. 11 Phase-plane response of joint 2 of the 2R robot having dynamic backlash, under the control of a FOM-VSC.

Employing a SOM-VSC to control the 2R robot with joint backlash, we obtain a slightly smaller position error. Nevertheless, the SOM-VSC (Fig. 12) did not remove the limit cycle.

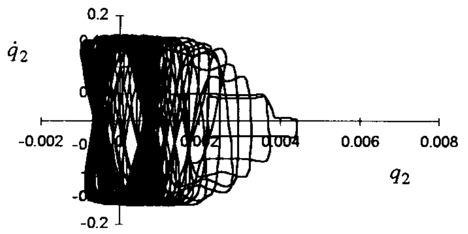

Fig. 12 Phase-plane response of joint 2 of the 2R robot having dynamic backlash, under the control of a SOM-VSC.

6 Conclusions

This paper studied, through the DF method, the dynamical properties of systems with nonlinear friction or dynamic backlash. The worst system, in terms of stability, is the one with dynamic backlash because it is more prone to limit cycles. The DF method of predicting limit cycles has shown a very good accuracy in terms of the frequency of the oscillation. We analysed the dynamic backlash instead of the standard backlash, because we have taken into consideration the impact phenomenon in accordance with the laws of physics. The DF's of a FOM-VSC and a SOM-VSC were calculated and compared with classical PD and PID algorithms. The study demonstrated that, in general, the PID controller presents the worst controllability properties, while VSC's revealed a superior performance and robustness.

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