OPTIMAL FUZZY PID CONTROL TUNED WITH GENETIC ALGORITHMS

Carlos Miguel Almeida Santos

Master in Computer and Electrical Engineering
Area of Specialization in Automation and Systems
Department of Electrical Engineering
Institute of Engineering of Porto
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This report fulfil, partially, the needs that are in the Unit Course of Thesis, of the 2º year, of Master in Computer and Electrical Engineering

Candidate: Carlos Miguel Almeida Santos, Nº 1090033, 1090033@isep.ipp.pt
Scientific orientation: Professor Doctor Ramiro de Sousa Barbosa, rsb@isep.ipp.pt

Master in Computer and Electrical Engineering
Area of specialization in Automation and Systems
Department of Electrical Engineering
Institute of Engineering of Porto

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I dedicate this work to my family and special to my wife.

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Abstract

Fuzzy logic controllers (FLC) are intelligent systems, based on heuristic knowledge, that have been largely applied in numerous areas of everyday life. They can be used to describe a linear or nonlinear system and are suitable when a real system is not known or too difficult to find their model. FLC provide a formal methodology for representing, manipulating and implementing a human heuristic knowledge on how to control a system. These controllers can be seen as artificial decision makers that operate in a closed-loop system, in real time.

The main aim of this work was to develop a single optimal fuzzy controller, easily adaptable to a wide range of systems – simple to complex, linear to nonlinear – and able to control all these systems. Due to their efficiency in searching and finding optimal solution for high complexity problems, GAs were used to perform the FLC tuning by finding the best parameters to obtain the best responses.

The work was performed using the MATLAB/SIMULINK software. This is a very useful tool that provides an easy way to test and analyse the FLC, the PID and the GAs in the same environment. Therefore, it was proposed a Fuzzy PID controller (FL-PID) type namely, the Fuzzy PD+I. For that, the controller was compared with the classical PID controller tuned with, the heuristic Ziegler-Nichols tuning method, the optimal Zhuang-Atherton tuning method and the GA method itself. The IAE, ISE, ITAE and ITSE criteria, used as the GA fitness functions, were applied to compare the controllers performance used in this work.

Overall, and for most systems, the FL-PID results tuned with GAs were very satisfactory. Moreover, in some cases the results were substantially better than for the other PID controllers. The best system responses were obtained with the IAE and ITAE criteria used to tune the FL-PID and PID controllers.

Keywords

Fuzzy, intelligent systems, heuristic, control, optimal, GA, PID.

Resumo

Os Controladores Lógicos Difusos (CLD) são sistemas inteligentes, baseados em conhecimentos heurísticos, que têm vindo a ser amplamente aplicados em inúmeras áreas do quotidiano. Podem ser usados para descrever sistemas lineares ou não-lineares, tornando-se apropriados quando se desconhece o modelo do sistema real ou no caso de o sistema ser difícil de modelar. Os CLD apresentam uma metodologia formal para representar, manipular e implementar um conhecimento heurístico de como controlar um sistema. Estes controladores funcionam como gestores artificiais de decisão que operam em sistemas de malha-fechada, e em tempo real.

O principal objetivo deste trabalho consistiu no desenvolvimento de um único controlador difuso ótimo, facilmente adaptável a uma vasta gama de sistemas – simples a complexos, lineares a não-lineares – e com capacidade para controlar sistemas distintos. Devido à sua eficácia na procura e descoberta de soluções ótimas, para sistemas de elevada complexidade, os Algoritmos Genéticos (AG) foram usados para a sintonia do CLD através da procura dos melhores parâmetros por forma a encontrar as melhores respostas.

O trabalho foi realizado usando o *software* MATLAB/SIMULINK. Esta é uma ferramenta útil que permite facilmente testar e analisar, no mesmo ambiente, o CLD, o PID e os AG. Por esta razão, foi proposto um controlador difuso do tipo PID, concretamente o Controlador Lógico Difuso PD+I (CLD-PID). Este controlador foi comparado com o controlador PID clássico sintonizado com o método heurístico de Ziegler-Nichols, o método ótimo de sintonização de Zhuang-Atherton e o próprio método de AG. Os critérios IAE, ISE, ITAE e ITSE, usados como as funções de avaliação dos AG, foram utilizados para comparar os desempenhos dos controladores usados neste trabalho.

De um modo geral, e para a maioria dos sistemas, os resultados do CLD-PID sintonizados com os AG foram bastantes satisfatórios. Para além disso, em alguns casos, os resultados foram consideravelmente melhores do que para os restantes controladores PID. As melhores respostas de sistemas foram obtidas com os critérios IAE e ITAE, que foram usados para sintonizar os controladores CLD-PID e PID.

Palavras-Chave

Lógica Difusa, sistemas inteligentes, heurística, controlo, ótimo, AG, PID.

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Acronyms

BOA – Bisector of Area

CA – Centre Average

CC – Cohen and Coon

CHR – Chien, Hrones and Reswick

COA – Centre of Area

COG – Centre of Gravity

CPU – Central Processing Unit

FIS – Fuzzy Inference System

FLC – Fuzzy Logic Controller

FL-PID – Fuzzy Logic-Proportional Integral Derivative

FOPDT – First Order Plus Dead Time

GA – Genetic Algorithm

GOT – Global Optimization Tool

GUI – Graphical User Interface

IAE – Integral of Absolute Error

ISE – Integral of the Square of the Error

ITAE – Integral Time Multiplied by the Absolute Error

ITSE – Integral of Time Multiplied by the Squared Error

IST^2E – Integral Squared Time-squared weighted Error

LM – Leftmost Maximum

MOM – Mean of Maxima

NB – Negative Big

NM – Negative Medium

NN – Neural Networks

NS – Negative Small

OX – Ordered Crossover

PB – Positive Big

PC – Personal Computer

PID – Proportional Integrative Derivative

PM – Positive Medium

PMX – Partially Mapped Crossover

PPX – Precedent Preservative Crossover

PS – Positive Small

RAM – Random Access Memory

RM – Rightmost Maximum

Z – Zero

ZA – Zhuang-Atherton

ZN – Ziegler-Nichols

1. INTRODUCTION

Control is present in several areas of knowledge such as science and engineering. It is used to control processes like for example the control of a car steering wheel to ensure that a car is in the right path. In industry numerous conditions such as pressure, temperature, speed, robot arms or motor positioning need to be constantly controlled and monitored in order to maintain all processes in the desired state [1][2].

Overall, there are two types of control – closed-loop control and open-loop control. In closed-loop control the output is measured and compared with a reference. In this case the control is applied to the input in order to reduce the difference between the output and the reference to zero, i.e., ideally the error is zero. Concerning the open-loop control the output is not measured and in the case of disturbances the system does not react as expected requiring a proper calibration to make sure that the system reacts according to planned. This type of control is used when the relation between inputs and outputs is well known and there are no disturbances [1].

Classical Proportional Integrative Derivative (PID) is the most used control technique, applied 95% of the times, since it is easier to implement, has a good response in transient and steady-state and presents very satisfactory results in several industrial applications. This control has three variables that must be tuned: the Proportional (P), the Integrative (I) and the Derivative (D) actions. The proportional action actuates proportionally to the error; the Integrative action has a stronger influence in the error reduction at the steady-state; while, the Derivative action has a strong influence in stability. It is important to mention that these three variables are dependent from each other [3][4].

With the need for more efficient control systems other control techniques have been developed such as Fuzzy Logic Control (FLC) or Neural Networks (NN). In the last decades it has been observed a growing interest on fuzzy controllers due to their important value for controlling complex and nonlinear industrial processes. Since the introduction of the Fuzzy Set Theory many advances have been made such as the combination of PID with fuzzy systems. In fact over the years these FLC have proven their efficiency. Opposite to classical control, fuzzy logic systems are based on heuristic knowledge rather than exact mathematics. This type of control is used to describe a linear or nonlinear system. By using this methodology, fuzzy controllers become easier to design since they use verbal language such as hot, very hot or cold rather than exact values. Therefore, it is possible to design a fuzzy controller for high performance control, even without knowing the model. Moreover, an interesting characteristic of these systems is their ability to handle numeric and linguistic information in the same framework. Fuzzy systems can evaluate a linguistic information and transpose it to exact numbers for later treatment in a computer [5][6][7][8].

Several techniques have been applied to find a balance and equilibrium in fuzzy systems. One of the most recent techniques is based on the combination of Fuzzy PID control systems with Genetic Algorithms (GAs) leading to hybrid systems.

Genetic Algorithms (GAs) are based on the Natural Selection Principle theory proposed by a biologist named Charles Darwin (1859). This theory holds the fundamental idea that the most capable and environmentally adapted individuals survive and reproduce while the less able tend to die and disappear. GAs are used to find and search for minimums or maximums in a specific proposed problem. In fact, after a few interactions (generations) the best solutions are find [9]. Thus, the evolutionary techniques used in GAs can be combined with Fuzzy PID Systems to help find the optimal parameters of the controller. The main advantage is that the designer does not need to know the process since GAs will adapt and tune the controller based on the system response; in that way this hybrid controller can be used for any given system.

The main goal of this study is to develop a system that will be able to control several systems using the same controller. For that, it will be used a hybrid system containing a PID Fuzzy Logic Controller combined with GAs. GAs were chosen due to their simplicity

and efficacy [9]. The concept is to develop an auto-adaptable fuzzy PID controller by applying the FLC, the PID and the GAs to find the optimum parameters.

1.1. CONTEXTUALIZATION

In industry there are many nonlinear processes that are difficult to model in order to achieve the proposed objectives using a PID controller. To successfully control a process it is possible to use an independently tuned FLC which is expensive and time consuming. It is important to use a control that has the ability to adapt to all processes without the need of developing a mathematical model and thus save time of the developer. An optimal fuzzy PID controller, can be a good solution, and can be achieved with the proposed project.

1.2. OBJECTIVES

The main objective of this project is to design an optimal fuzzy PID controller using GA, and MATLAB as a developing tool. The controller will be tested in six different systems with distinct responses in order to verify its efficiency and compared with a classical PID controller.

1.3. SCHEDULING

In Table 1 it is possible to observe the scheduling concerning the tasks performed during the development of this project (semester).

Table 1 Scheduling of the developed project.

Stage	Duration (weeks)	March	April	May	June	July	August	September	October	November
Control Theory Study	4									
FL Study	4									
GA Study	4									
MATLAB Study	4									
FLC development	6									
GA development	5									
Tests and results	15									
Report writing	20									
Thesis presentation	1									

1.4. OUTLINE OF THE THESIS

This thesis is outlined as follows. Chapter 1 presents some basic concepts of control systems, the scope and proposal of the work, and the outline of the thesis. Chapter 2 introduces the fundamentals of fuzzy logic control systems. Moreover, the MATLAB Fuzzy Toolbox operation is described and explained. In Chapter 3 Genetic Algorithms theory is presented as well as their fundamental concepts. In this chapter the genetic algorithm toolbox from the MATLAB Global Optimization Toolbox is briefly introduced. The obtained results and their discussion are described in Chapter 4. The conclusions are summarized in Chapter 5 as well future perspectives of work.

2. FUZZY CONTROL SYSTEMS

Fuzzy logic was first proposed by Lotfi A. Zadeh in 1965 [8]. This type of controllers is based on the idea that Humans do not think in terms of exact numbers but rather in concepts. One of the first experiences performed with fuzzy controllers revealed that the control of a vapour machine with a simple fuzzy controller was very efficient [8].

Fuzzy logic is a tool that uses ambiguous information such as hot, high or fast normally in a language that is easily understood by humans, and then converted into a numerical value that can be manipulated by a microprocessor or computer [8][10].

The implementation of a fuzzy control is appropriate when the modelling of a real world system is too difficult and when the process is very well known in terms of human experience. The development of a relatively accurate model of a dynamic system is very complex to be used in controller development, mostly in the case of conventional design control procedures that require restrictive plant assumptions such as linearity. Therefore, conventional controllers are frequently developed using simple process models behaviour that satisfies the required assumptions. Nevertheless, it is known that heuristics are present in conventional control design processes as long as the actual implementation of the

control system is considered. Conventional control engineering that uses appropriate heuristics to the design has been relatively successful. Fuzzy control provides a formal methodology to represent, manipulate and implement a heuristic of human knowledge on how to control a system [10].

2.1. FUZZY SETS

Fuzzy sets are classes with a continuous grade of membership. A class contains objects with a certain grade of membership between the interval $[0, 1]$. Contrary to conventional sets, where a number belong or not to that set, in fuzzy sets that number can partially belong to it, this means that a number belongs to the fuzzy set according to the membership of the variable. In fuzzy logic the variables are names, instead of numbers. As an example, in the case of temperature control, temperature is the linguistic variable which can take linguistic values like hot or cold. This kind of approach has more meaning in terms of human thought because humans tend to think in names rather than numbers. Another concept is the membership function that represents the way in which linguistic values belong to a fuzzy set for that linguistic value. This function must be defined in order to better fits the physical meaning of the problem in question [5][8][11].

The *universe of discourse* consist in all possible elements of concern in a particular context. Each of these elements are called a *member*, or an *element* of the universe of discourse [10][11].

For any classical set, the *characteristic function* of a set A is defined by

$$\mu_A(x) = \begin{cases} 1 \; if \; x \in A \\ 0 \; if \; x \notin A \end{cases} \tag{1}$$

The characteristic function $\mu_A(x)$ of the classical set A defined above is an indicator of members and non-members of the classical (crisp) set A, there is no ambiguity or grade of membership. An example of a classical set is shown in Figure 1.

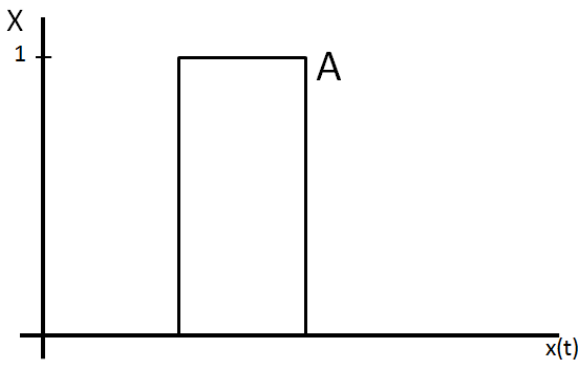


Figure 1 Characteristic function of the classical (crisp) set A.

With fuzzy sets the characteristic function must be generalized since in fuzzy sets there might be an element that can partially belong to that set or not partially belong to it. In this case larger values denote a higher degree of membership. We have to notice that the maximum degree of membership is 1, and in this case the value completely belongs to that fuzzy set, on the other hand, the minimum value is 0, and this is the case where the value does not belong to that fuzzy set. So, to generalize the characteristic function, the membership function needs to be defined in association with each set in the universe of discourse. A fuzzy set (2) is defined as an ordered pair $(\boldsymbol{X}, \mu)$, where $\boldsymbol{X}$ is the universe of discourse and μ is the a set membership function mapping $\boldsymbol{X}$ onto the interval $[0, 1]$.

$$A = \{x, \mu_A(x) : x \in \boldsymbol{X}\} \tag{2}$$

In fuzzy logic every set can have different membership functions, the function that better describes that set. Therefore, that function should describe the better way possible the characteristic represented by the set.

An example of a membership function of a fuzzy set is represented in Figure 2.

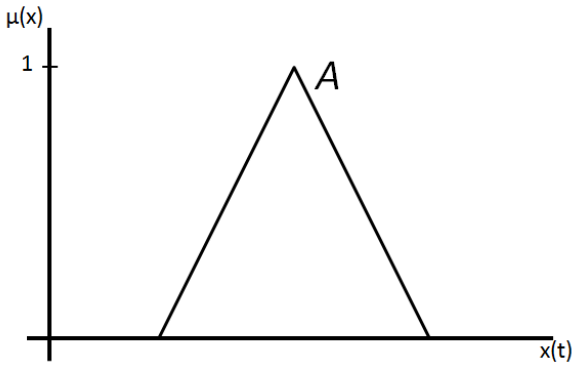


Figure 2 Membership function of the fuzzy set A.

Like in classical sets, operations can be performed between sets, the standard operations between fuzzy sets are, the *union*, *intersection* and *complement* operations.

Union

$$\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\} \tag{3}$$

$$\mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x) \tag{4}$$

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) \tag{5}$$

The *union* standard operation of two fuzzy sets can be done using the *maximum* operator (3) or the *algebraic sum* (5), although, other union operations are possible. An example of the *union* operation between two fuzzy sets using the *maximum* operator is shown in Figure 3.

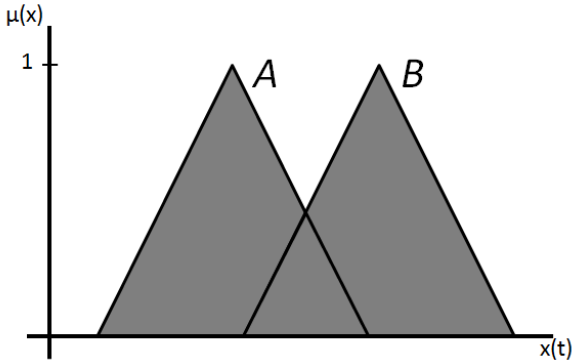


Figure 3 Union of two fuzzy subsets using *maximum* operator.

Intersection

$$\mu_{A\cap B}(x) = min\{\mu_A(x), \mu_B(x)\} \tag{6}$$

$$\mu_{A\cap B}(x) = \mu_A(x)\mu_B(x) \tag{7}$$

The *intersection* operation of two fuzzy sets can be done using the *minimum* operator (6) or the *algebraic product* (7), although, other *intersection* operations can be performed. An example of the *intersection* operation between two fuzzy sets using the *minimum* operator and the *algebraic product* are shown in Figure 4.

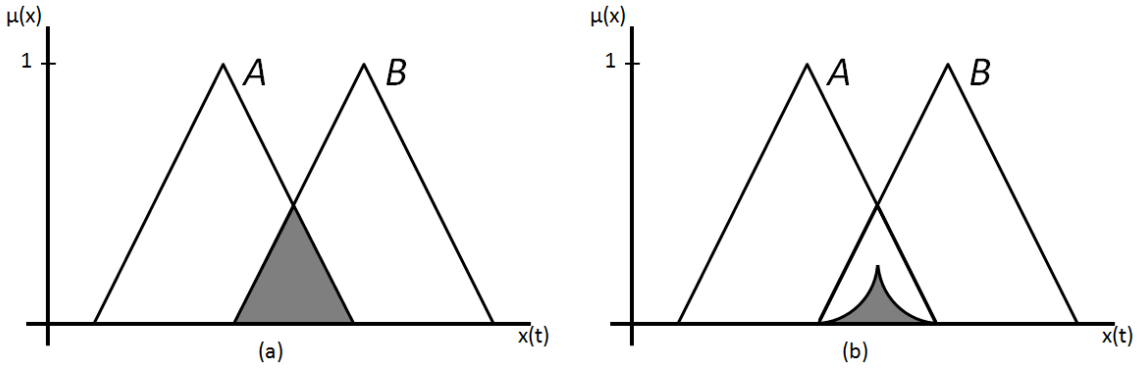


Figure 4 Interception of two fuzzy sets using the *minimum operator* (a) and *algebraic product* (b).

Complement

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{8}$$

The *complement* standard operation (8) is the same as the inverse of fuzzy set A. An example of the *complement* operation of a fuzzy set is shown in Figure 5.

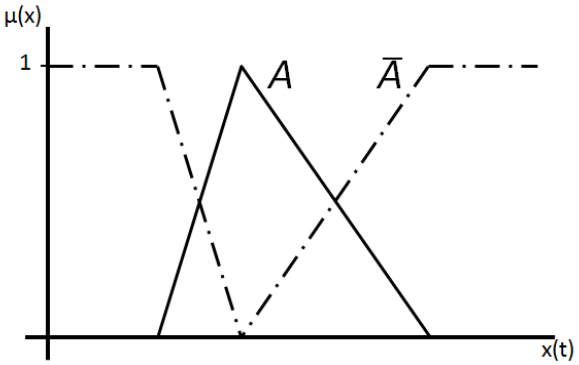


Figure 5 Complement of a fuzzy set.

There are more operations rather than the standard fuzzy operations. The aggregation operations consist in combining several fuzzy sets into only one fuzzy set. These operations of aggregation use the associative property that gives them the possibility to extend their definitions to more than three arguments. Other common operations of aggregation such as *averaging operations* and *ordered weighted averaging operations* are also found in some literature. The use of these operations of aggregation is useful to fulfil the space between the minimum operator, the *intersection*; and the maximum operator, the *union* [12].

In fuzzy sets, the Morgan's principals for classical sets are also valid. The properties of the classical sets are also valid, except for the excluded middle axioms since fuzzy sets and their complements can overlap. All the properties applied to classical sets are also used for fuzzy sets. Moreover, due to this and the fact that a classical set are a subset of the interval $[0, 1]$, classical sets are a particular case of fuzzy sets [4].

The membership function describes the information in a fuzzy set, therefore the *core* of a membership function for some fuzzy set is defined as that region of the universe that is characterized by complete and full membership in that set. The core comprises those elements x of the universe such that $\mu_A(x) = 1$. The *support* of a membership function for some fuzzy set is defined as the region of the universe that is characterized by nonzero membership in that set. The support comprises those elements x of the universe such that $\mu_A(x) > 0$. The *boundaries* of a membership function for some fuzzy set are defined as the region of the universe containing elements that have a nonzero membership but not complete membership. The boundaries comprise those elements x of the universe such that $0 < \mu_A(x) < 1$. These elements of the universe are those with some *degree* of fuzziness, or only partial membership in the fuzzy set. Figure 6 represents the universe regions that comprises the core, support and boundaries of a typical fuzzy set [12].

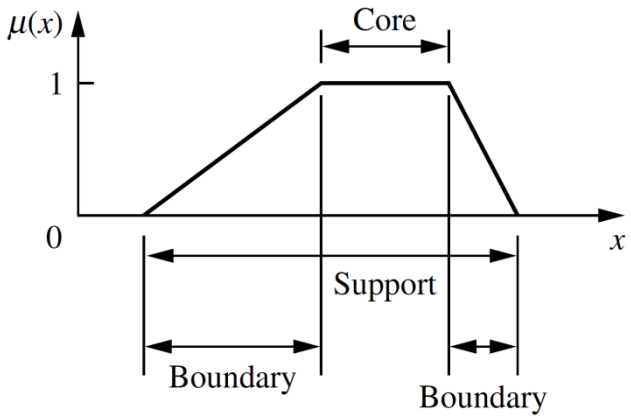


Figure 6 Core, support and boundary of a fuzzy set.

The most common forms of membership functions are the *normal* and *convex* ones. However, many operations on fuzzy sets, or operations on membership functions, result in fuzzy sets that are *subnormal* and *non-convex* [12].

A fuzzy set is said to be *normal* only if the membership function has at least one element of value one, i.e., the maximum value in a normalized universe. If the membership

function has one and one only element with the unity value, this element is typically referred to as the *prototype* of the set [12].

A fuzzy set is said to be *convex* if the membership function elements are strictly monotonically increasing, or strictly monotonically decreasing, or monotonically increasing then strictly monotonically decreasing. In a formal way, for any given element v, w, and z in a fuzzy set, with the relation $v < w < z$ that respects the statement (9) than it can be said that the fuzzy set is convex [12].

$$\mu_A(w) \geq \min\left[\mu_A(v), \mu_A(z)\right] \tag{9}$$

In Figure 7 is represented an example of normal and subnormal fuzzy sets, and in Figure 8 is represented an example of convex and non-convex fuzzy sets.

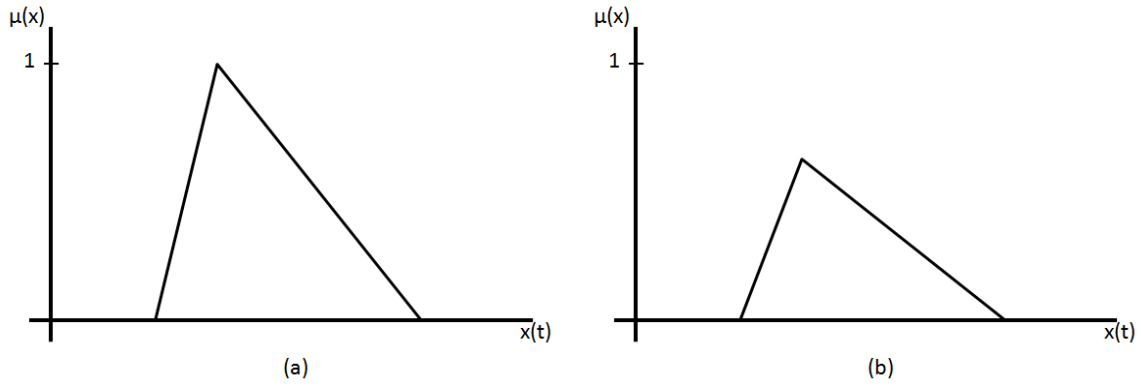


Figure 7 Normal (a) and Subnormal (b) fuzzy sets.

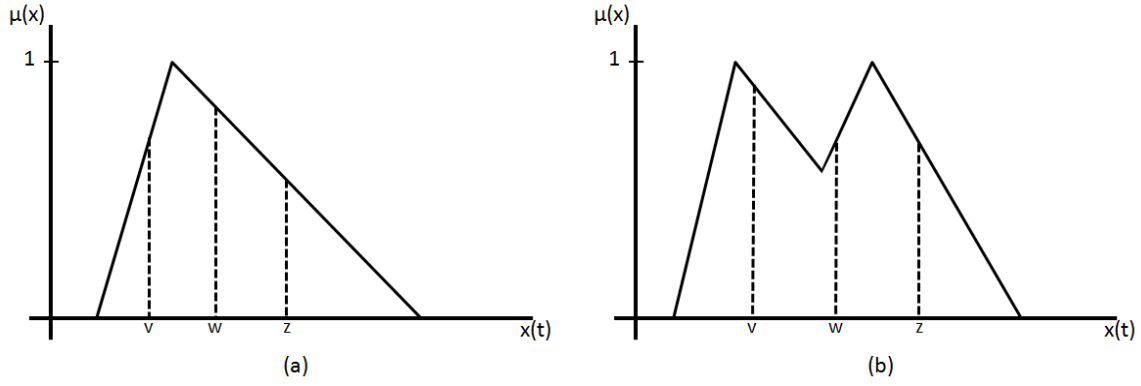


Figure 8 Convex (a) and non-convex (b) fuzzy sets.

Membership functions can be symmetrical or asymmetrical and are typically defined based on one-dimensional universe although can be described on multidimensional universes. In two dimensions curves become surfaces and for three or more dimensions these surfaces

become hyper-surfaces. These hyper-surfaces, or curves, are simple mappings of parameters' combinations in *n*-dimensional space to a membership value on interval $[0, 1]$. Once again, this membership value expresses the membership degree that the specific combination of parameters, in the *n*-dimensional space, has in a particular fuzzy set defined on the *n*-dimensional universe of discourse. The hyper-surfaces for a *n*-dimensional universe are analogous to joint probability density functions; but, of course, the mapping for the membership function is to membership, in a particular set and not to relative frequencies, as it is for probability density functions [12].

In fuzzy logic the selection of the membership functions is based on a subjective choice. Therefore, the correct and best way to choose the membership function depends on the designer knowledge and experience on the system in question. Figure 9 shows some examples of the most commonly used memberships [11][13].

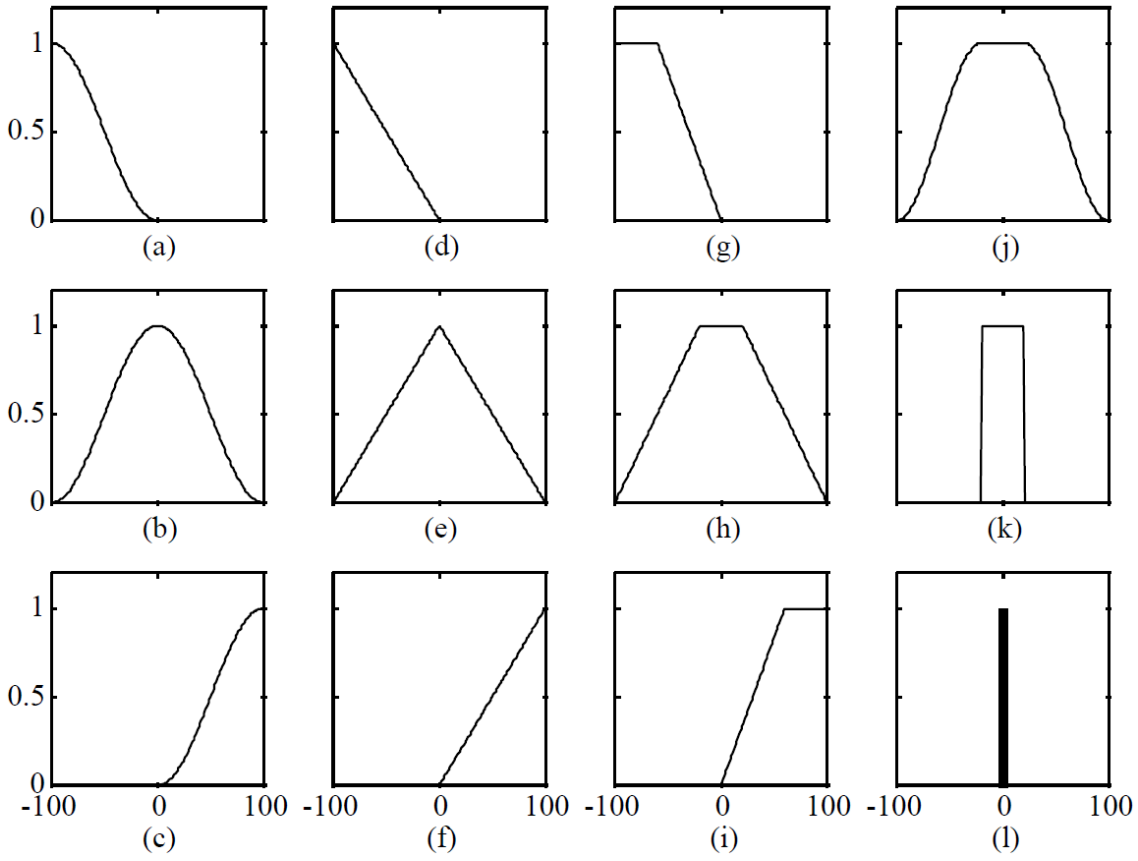


Figure 9 Commonly used membership functions [13].

2.2. FUZZY LOGIC CONTROL

To design a fuzzy controller it is necessary to collect information on how the artificial decision maker should act in a closed-loop system. This information can come from a human decision maker who performs the control task or the control designer begin understanding the process dynamics and writing a set of rules on how to control the system [10].

The fuzzy control block diagram represented in Figure 10 shows a fuzzy controller embedded in a closed-loop control system. The outputs of the system are denoted by $y(t)$, their inputs by $u(t)$, and the fuzzy controller reference input as $r(t)$.

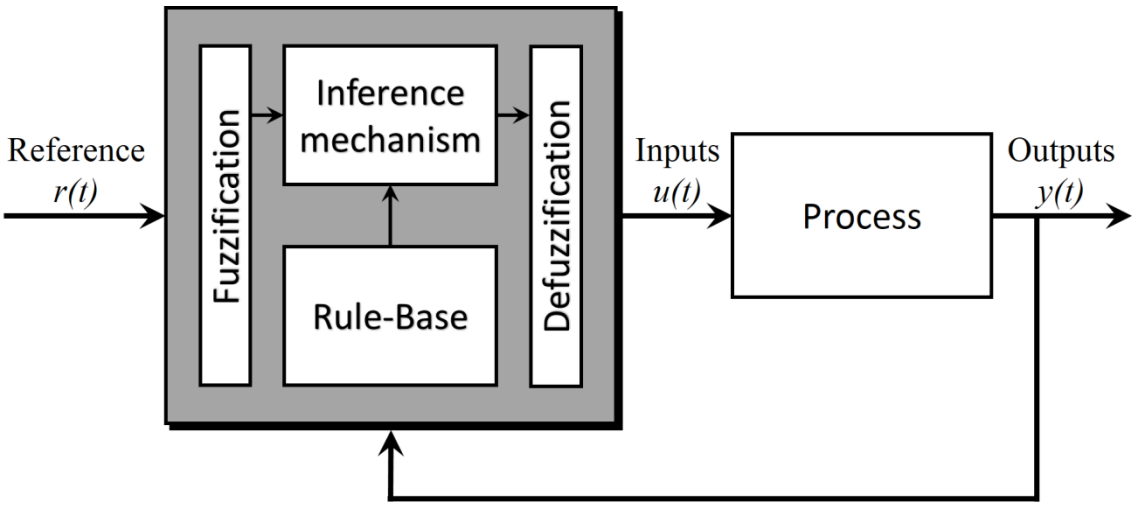


Figure 10 Fuzzy control system [10].

The fuzzy controller has four main components:

1. The *fuzzification* interface simply modifies the inputs so that they can be interpreted and compared to the rules in the rule-base.

2. The *inference mechanism* evaluates which control rules are relevant at the current time and then decides what should be the input to the system.

3. The *rule-base* holds the knowledge, in the form of a set of rules, in the best way to control the system.

4. The *defuzzification* interface converts the conclusions reached by the inference mechanism into the inputs to the system.

2.2.1. FUZZIFICATION INTERFACE

The *fuzzification* interface is the transformation process in which the crisp inputs values are turned into fuzzy linguistic values. Such transformation is realized by introduction of the membership functions, which define both a range of value and a degree of membership. For linguistic variables it is important not only which membership function a variable belongs to, but also a relative degree to which it is a member. A variable can have a weighted membership in several membership functions at the same time because of the memberships overlapping. A certain overlap between membership functions is desirable to prevent the controller to be in poorly defined states, consequently returning output values not well defined, leading to poor control [5][13].

If the process or measurement of the values has noise, it could be necessary to create fuzzy sets for the measured values. In this case, the measured values could contain some errors because of the noise, and can be converted into fuzzy sets that reflect their degree of undependability, this may be useful but is not essential and can be ignored due to its simplicity. Of course, in many cases the measurement noise can be low or not taken into account, and in these cases, the fuzzification stage consists in create singleton membership functions at the measured values, which simplifies the fuzzification mechanism [5][10][14].

2.2.2. RULE-BASE

Humans tend to make decisions based on rules in the form of IF…THEN. In classical logic, there are 4 most frequently rules based on IF…THEN; the *modus ponendo ponens* from the Latin that means "mode that affirms by affirming" or *modus ponens*, the *modus tollendo tollens* from the Latin, that means "mode that denies by denying", or *modus tollens*, the *modus ponendo tollens* from the Latin, that means "mode that denies by affirming" and the *modus tollendo ponens* from the Latin, that means "mode that affirms by denying". The IF…THEN *modus ponendo ponens* form is the most frequently used rules in fuzzy control [5][11].

The fuzzy rules are used in fuzzy control to define the relation between the fuzzified inputs of the system and their outputs. This collection of rules is called the rule-base and it comes in the form of **IF** *premise* **THEN** *consequent*. The premise or antecedent is associated with the fuzzy controller inputs, on the other hand, the consequents or actions are associated

with fuzzy controller outputs. Each of this premise can be a conjunction of more than one term. For instance, **IF** *input a* is HIGH AND *input b* is LOW **THEN** *output c* is HIGH. The number of premises and consequents are limited by the number of inputs and outputs and their linguistic values [10]. Table 2 shows an example of a rule base in a form of a table with two input variables, error $e(t)$ and error variation $\frac{d}{dt}e(t)$, and one output $u(t)$. In this example there are seven linguistic variables for each input, namely the Negative Big (NB), the Negative Medium (NM), the Negative Small (NS), the Zero (Z), the Positive Small (PS), the Positive Medium (PM) and the Positive Big (PB), which gives a total of forty nine rules, the number of maximum rules with only two inputs, thus $7 * 7 = 49$.

Table 2 Fuzzy rule-base for output *u(t)*.

$u(t)$		$e(t)$						
		NB	**NM**	**NS**	**Z**	**PS**	**PM**	**PB**
$\frac{d}{dt}e(t)$	**NB**	NM	NB	NB	NB	NB	NB	NM
	NM	NS	NM	NM	NM	NM	NM	NS
	NS	Z	NS	NS	NS	NS	NS	Z
	Z	Z	Z	Z	Z	Z	Z	Z
	PS	Z	PS	PS	PS	PS	PS	Z
	PM	PS	PM	PM	PM	PM	PM	PS
	PB	PM	PB	PB	PB	PB	PB	PM

The maximum number of rules depends on the number of inputs, the number of outputs and the number of linguistic variables for each input and output.

2.2.3. INFERENCE MECHANISM

The *inference mechanism* interface that is build in combination with the rule-base interface, maps input linguistic variables onto output linguistic variables based on the rule-base. Since input linguistic variables are weighted, the output linguistic variables can be obtained weighted as well. Traditional fuzzy logic approach comprises Mamdani type and Sugeno type inference methods. The Mamdani type method is more intuitive and assumes the output variables as a fuzzy set and is the most commonly used inference mechanism. Fuzzy rules in it contain a *precedent* part and a *consequence* part. The Sugeno type method expects the output variables to be singletons or dealing with consequents that are equations. So it is better suited for mathematical analysis, nonlinear system modelling and interpolation [14][15].

In the inference interface, the rules are statements that can be seen as restrictive statements applied to the fuzzy controller output. Most of rule-base systems are represented by more than one rule to better describe the system dynamics or nonlinearity. The most used techniques to decompose these rules are the *multiple conjunctive antecedents* and the *multiple disjunctive antecedents*.

The multiple conjunctive antecedents technique is used when the rule is on the form of:

$$IF\ x\ is\ A^1\ AND\ A^2\ \dots AND\ A^L\ THEN\ y\ is\ B^n \tag{10}$$

Assuming a fuzzy set A^n as

$$A^n = A^1 \cap A^2 \cap \dots A^L \tag{11}$$

Expressed by the means of the membership function:

$$\mu_{A^n}(x) = \min\left[\mu_{A^1}(x), \mu_{A^2}(x), \dots, \mu_{A^L}(x)\right] \tag{12}$$

Based on the definition of the standard fuzzy intersection operation the compound rule may be rewritten as

$$IF\ A^n\ THEN\ B^n \tag{13}$$

The multiple disjunctive antecedents technique is used when the rule is on the form of:

$$IF\ x\ is\ A^1\ OR\ x\ is\ A^2\ \dots OR\ x\ is\ A^L\ THEN\ y\ is\ B^n \tag{14}$$

Assuming a fuzzy set A^n as

$$A^n = A^1 \cup A^2 \cup \dots A^L \tag{15}$$

Expressed by the means of the membership function:

$$\mu_{A^n}(x) = \max\left[\mu_{A^1}(x), \mu_{A^2}(x), \dots, \mu_{A^L}(x)\right] \tag{16}$$

Based on the definition of the standard fuzzy union operation the compound rule may be rewritten as

$$IF\ x\ is\ A^n\ THEN\ y\ is\ B^n \tag{17}$$

This mechanism essentially has two stages, the first stage consist in determine the degree of firing of each rule in the rule-base interface, comparing all rule's *premises* to the controller inputs to determine which rules are on in the current situation. This process of

matching determines the certainty that each rule applies, and typically the rules with more certain are taken into account [5][10][12].

In the second stage, the inference mechanism will seek to combine the recommendations of all the rules to come up with a single conclusion. It is formed membership values for each rule premise that represent the certainty that each rule premise holds for the given inputs. Such certainty could represent the degree of confidence in each rule's applicability and would normally be a number in the interval $[0, 1]$. If the premise combination of membership functions and rules is greater than zero $\mu_{A^n}(x) > 0$ the rule is on and has a degree of confidence correspondent to that combination result. For different inputs values there will be different values of the premise certainty. Bigger values mean higher confidence while lower values mean lower confidence. The result combination of all rules will lead to a single consequent membership function [5][11].

2.2.4. DEFUZZIFICATION INTERFACE

The *defuzzification* interface mechanism is another main block of the fuzzy controller and contrary to fuzzification, performs the opposite. In fact, it transforms the output fuzzy result of the logical operations between the membership functions defined in the universe of discourse into crisp values or precise numbers to be applied into the output of the fuzzy controller, or in the input of the process to be controlled. After deffuzification the output must be scaled up to meet the physical units (e.g. volt or current), the process often contains an output gain, that can be tuned, and sometimes an integrator. The fuzzy controller can be seen as an artificial decision maker that operates in a closed-loop system in real time. It gathers a process output data $y(t)$, compares it to the reference input $r(t)$ and decides on the process input $u(t)$ to ensure that the performance objectives are found [5][10][11].

Among the defuzzification methods available, the most used are the Centre of Gravity (COG) or Centre of Area (COA). Other methods like the Bisector of Area (BOA), Centre Average (CA), Mean of Maxima (MOM), Leftmost Maximum (LM) or Rightmost Maximum (RM) can also be used [5][10][13][16].

- The COG or the COA is the most popular method, but the computational complexity is relatively high. The crisp value in the continuous universe is obtained by

$$u = \frac{\int x.\mu(x)dx}{\int \mu(x)dx} \tag{18}$$

where integrals are taken over the entire range of the output. In the case of a discrete universe the crisp value is obtained by

$$u = \frac{\sum_i \mu(x_i)x_i}{\sum_i \mu(x_i)} \tag{19}$$

This method can be seen as the weighted average of the elements in the set. In Figure 11 is shown an example of the COG method.

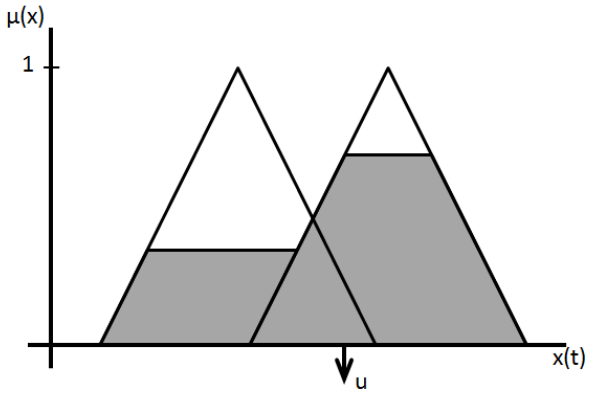


Figure 11 Centre of gravity method.

- The BOA method picks the abscissa of the vertical line that divides the area under the curve in two equal halves, and the crisp value is calculated using the expression

$$u = \left\{ x \middle| \int_{Min}^{x} \mu(x)dx = \int_{x}^{Max} \mu(x)dx \right\} \tag{20}$$

The computational complexity is relatively high, and it can be ambiguous. Because of that in the continuous case it is not defined.

- The CA method is based on the least bounded area defined as $\mu_{TR}^{(k)}(x)$. The calculation is fast due that these areas are trapezoidal. The calculation of the crisp value consists in finding the centre of area of the centre of area of each bounded subset and the overlapped area counts twice for the calculations. The expression to find the crisp value is

$$u = \frac{\sum_k \int x.\mu_{TR}^{(k)}(x)dx}{\sum_k \int \mu_{TR}^{(k)}(x)dx} \tag{21}$$

In Figure 12 is represented an example of CA method using two subsets. Note that the darker area counts twice for the calculations.

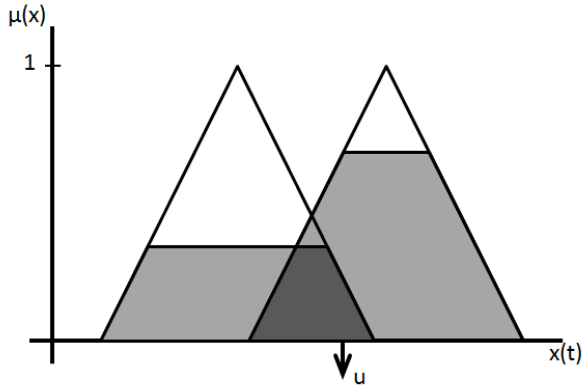


Figure 12 Centre average method.

- The MOM is computationally more efficient than other methods. However, it does not take into account the shape of the fuzzy subset. As in each subset the maximum is an interval, only the central point of that interval is taken in account, thus that point is the maximum of that subset. The crisp value is obtained using the expression

$$u = \frac{\sum_k c^{(k)}.\mu\left(c^{(k)}\right)}{\sum_k \mu(c^{(k)})} \tag{22}$$

Where $c^{(k)}$ is the central point of the maximum of the subset (k). In Figure 13 is represented an example of the MOM method.

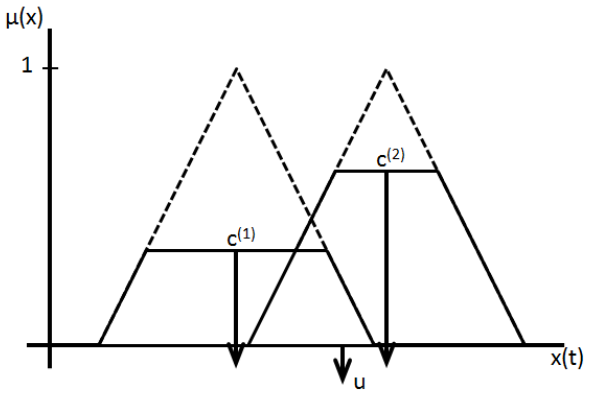


Figure 13 Mean of maxima method.

- The LM and RM are other possible methods and the computational complexity is also relatively simple. These methods choose between the left or right centre maximum of the left or right subset.

In terms of computational complexity, the crisp value should be found efficiently, to perform in real time systems. The method that take less time to compute from the methods described above is the MOM method, on the other hand, the one that takes longer is the most used, the COG [5][10][13][16].

2.2.5. FUZZY MECHANISM EXAMPLE

Figure 14 shows an example of fuzzification, Mamdani´s min-max inference mechanism, that uses the minimum operator for implication of rules and uses the maximum operator to combine the resulting membership functions of the implication operation, and the defuzzification mechanism that uses the COG to evaluate the crisp control output value.

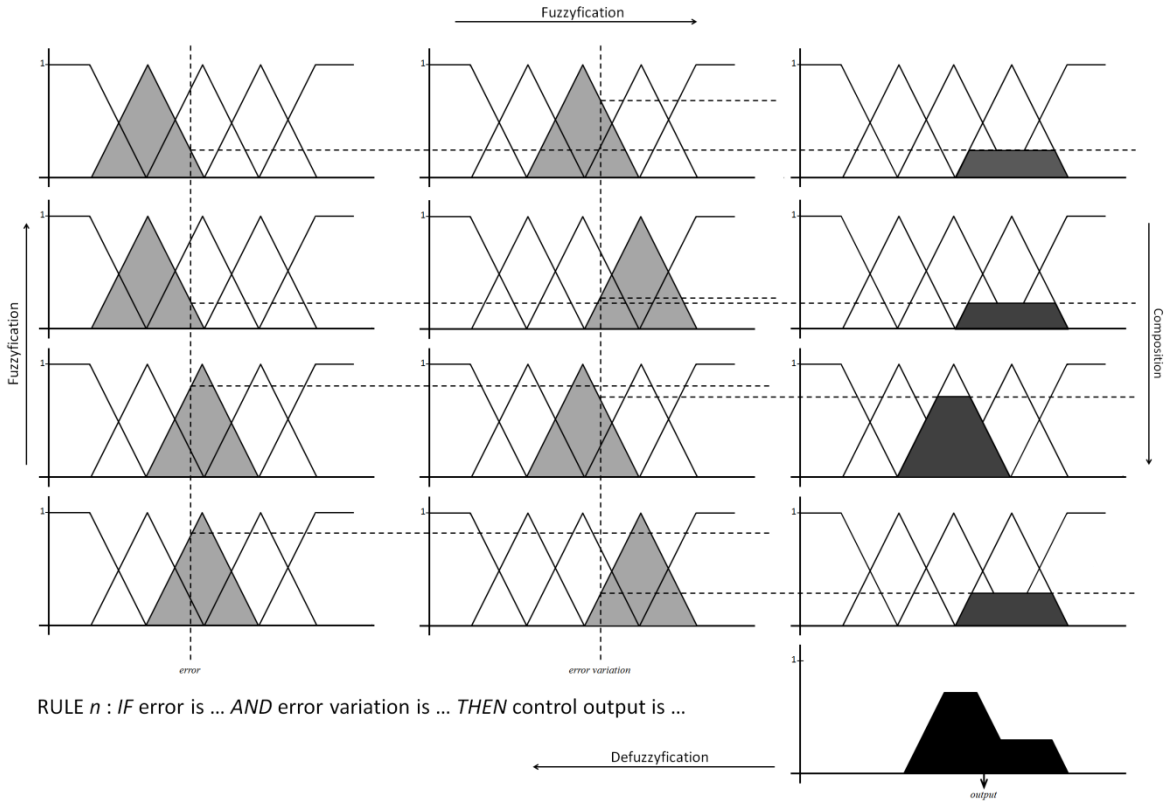


Figure 14 Example of fuzzification, Mamdani's inference and defuzzification.

In the above example the currently measured inputs values activates four rules, this means that four rules are ON and those are the rules that must be evaluated. This happens because of sets overlap, otherwise only two rules would be ON. Using Mamdani's inference min-

max mechanism, the implication of rules are made with the minimum operator because of the antecedents, therefore, the result set, shown on the right, is the minimum of the intersection of two subsets for each rule. These consequent sets are then aggregated with the union operation using the sets maximum operator that leads to the black fuzzy set in the bottom. The COG operation is used to find the crisp value that would be the output control value. This control value can now be used directly or scaled to control the system in question. After this, the process repeats, by measuring again the inputs values until find the new output control value.

2.3. FUZZY PID CONTROLLER

Proportional Integral Derivative (PID) type controllers are used to control many industrial systems. The PID has three degrees of freedom, called the PID gains.

P (proportional control)

$$u(t) = K_P e(t) \tag{23}$$

I (integral control)

$$u(t) = \frac{K_p}{T_i} \int_0^t e(\tau) d\tau \tag{24}$$

D (derivative control)

$$u(t) = K_P T_D \frac{de(t)}{dt} \tag{25}$$

With this, we can say that a PID control is a combination of all these three controls mentioned above, i.e., the sum of all terms

$$u(t) = K_P e(t) + \frac{K_P}{T_i} \int_0^t e(\tau) d\tau + K_P T_D \frac{de(t)}{dt} \tag{26}$$

Simplifying the above equation, putting the proportional gain in evidence, we get

$$u(t) = K_P \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_D \frac{de(t)}{dt} \right] \tag{27}$$

Where K_P, T_i and T_D are the proportional gain, the integral time constant and the derivative time constant, respectively. These terms are parameters that must be tuned in order to

achieve the objectives of the control. Adjusting these three gains is possible to control the overshoot, the steady-state error, the rise time, the settling time and the stability of a system response. Of course this is done with some limitations, depending on the system dynamics and user specifications. This can be achieved using several techniques like Ziegler-Nichols (ZN) rules, Zhuang-Atherton (ZA) formulas, Chien, Hrones and Reswick (CHR) method, Cohen and Coon (CC) method, GAs, fuzzy logic and others. A typical block diagram of a PID control is shown in Figure 15.

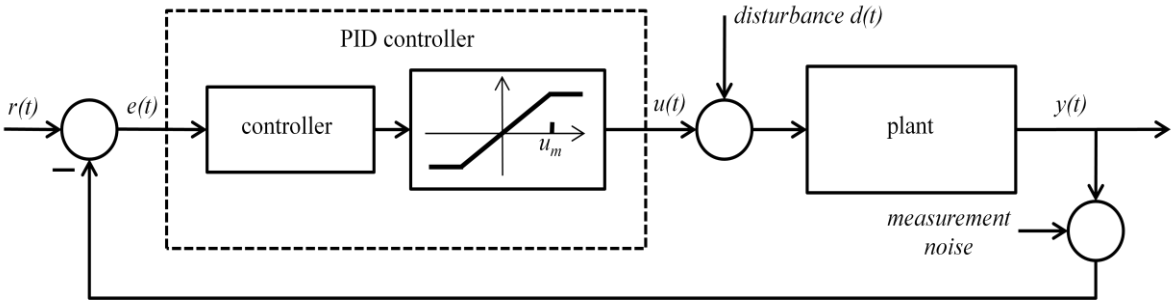


Figure 15 Typical PID control diagram.

In a fuzzy PID controller, there is also the P, the I and the D gains, but as the controller has itself a fuzzy logic on it, means that instead of three degrees of freedom, the controller has more degrees of freedom, hence, it has many parameters passive of adjusting such as the number of rules and it's combination, the membership function of each linguistic variable, the number of linguistic variables, the inference mechanism operations and the defuzzification mechanism operations. All these parameters can be tuned, what makes this type of control much superior in terms of tuning compared with a simple PID control. A typical fuzzy PID controller can be seen in Figure 16.

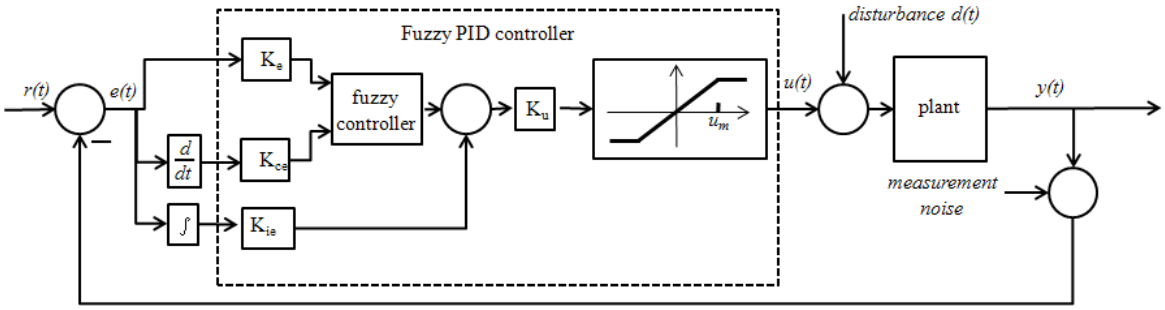


Figure 16 Typical Fuzzy PID control diagram.

The above fuzzy PID controller has two inputs, the *error* scaled with the gain K_e and the *error variation* scaled with the gain K_{ce}. These gains can be seen as the proportional and

the derivative gains, respectively. The fuzzy output is then integrated and scaled with the gain K_{ie}, which can be seen as the integral gain. The fuzzy PID controller is the controller that would be used in this work and tuned using the GAs.

2.3.1. DESIGN OF FUZZY PID CONTROLLERS

The design of a fuzzy PID controller takes more time and is more complex than design a simple PID controller, however, the advantages of a fuzzy PID sometimes justify the decision. Fuzzy controllers are more robust than PID controllers because they can cover a much wider range of operating conditions than PID, and is immune to noise and disturbances [17].

To start designing a fuzzy PID controller the designer must have a good knowledge of system that needs to be controlled. This knowledge is used to create the rules of the rule-base. The behaviour of input and output variables are important when the designer choose the number and the membership functions shape for every variable. A good option is start with symmetrical triangular membership functions, with an overlap of 50% with the two neighbours fuzzy sets, on the left and right side of the universe should be shouldered ramps. Of course these leftmost and rightmost ramp sets are not overlapped with two sets, because of its position, extreme positions. Gaps between sets should be avoid, because in these cases no rules are fired, leading to not well defined states. The sets must be sufficiently wide to allow some noise in the measurement. The number of sets must be in sufficient number to fill all the universe of discourse and describe the variable in question as better as possible [13].

2.4. MATLAB FUZZY LOGIC TOOLBOX

MATLAB is a mathematical computation tool that is used to develop and simulate many scientific and engineering problems. This tool has many toolboxes that can be used to help user to develop or simulate specific areas, one of these toolboxes is the Fuzzy Logic Toolbox. This toolbox in conjunction with SIMULINK software will be the main tool used to design the fuzzy PID controller for this work and to simulate its behaviour.

Fuzzy Logic Toolbox is a collection of functions built on the MATLAB technical computing environment. It provides tools to create and edit fuzzy inference systems within the framework of MATLAB. This toolbox relies heavily on Graphical User Interface

(GUI) tools to help users to accomplish the work, although it can be done entirely from the MATLAB command line [18].

The toolbox includes eleven built-in membership functions. These 11 functions are, in turn, built from several basic functions such as:

- piece-wise linear functions
- the Gaussian distribution function
- the sigmoid curve
- quadratic and cubic polynomial curves

The functions are [18]:

Triangular

The *triangular* membership function is one of simplest ones and the name in the toolbox is `trimf`. This function is nothing more than a collection of three points forming a triangle (Figure 17).

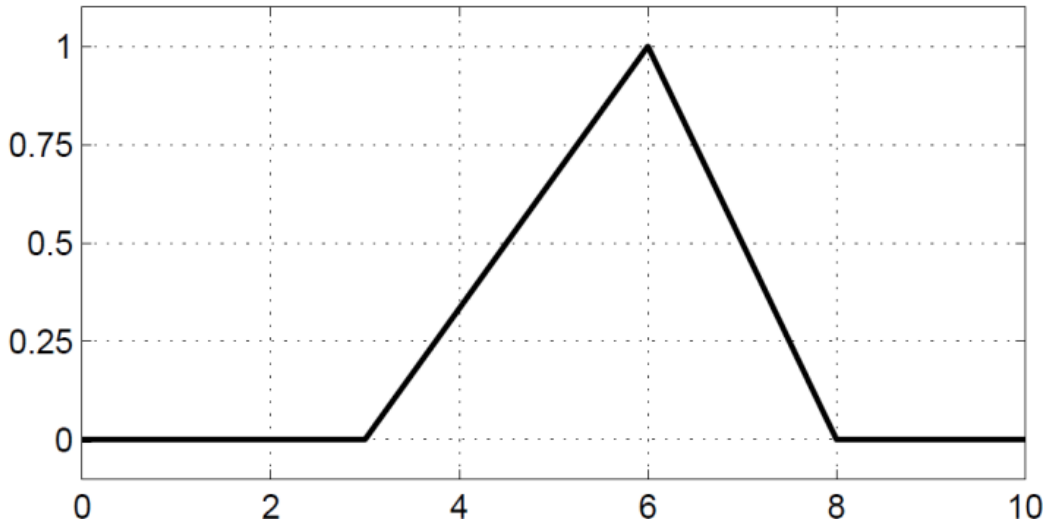


Figure 17 Triangular membership function example (`trimf`).

Trapezoidal

The *trapezoidal* membership function, the `trapmf`, has a flat top and really is just a truncated triangle curve (Figure 18).

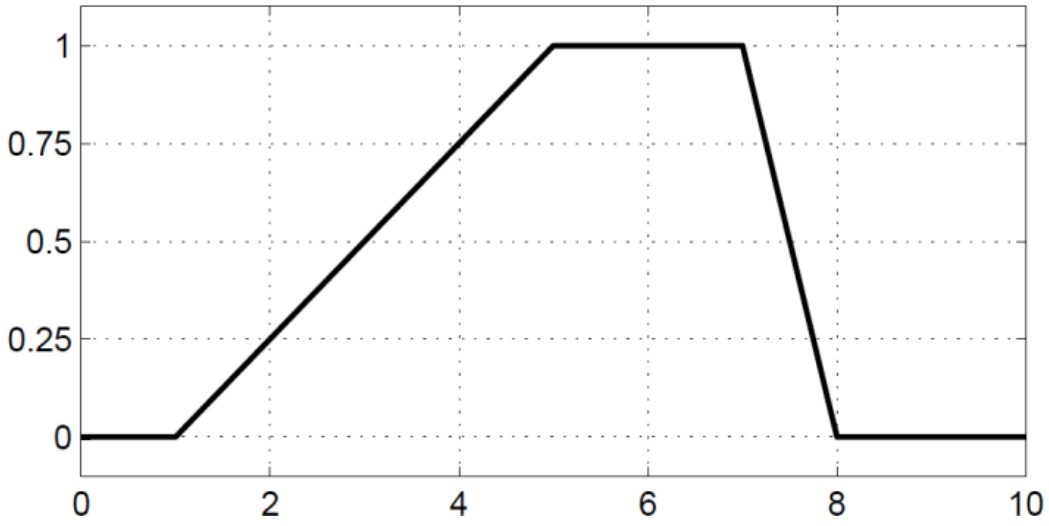


Figure 18 Trapezoidal membership function example (`trapmf`).

Simple Gaussian

The simple Gaussian membership function, the `gaussmf` (Figure 19).

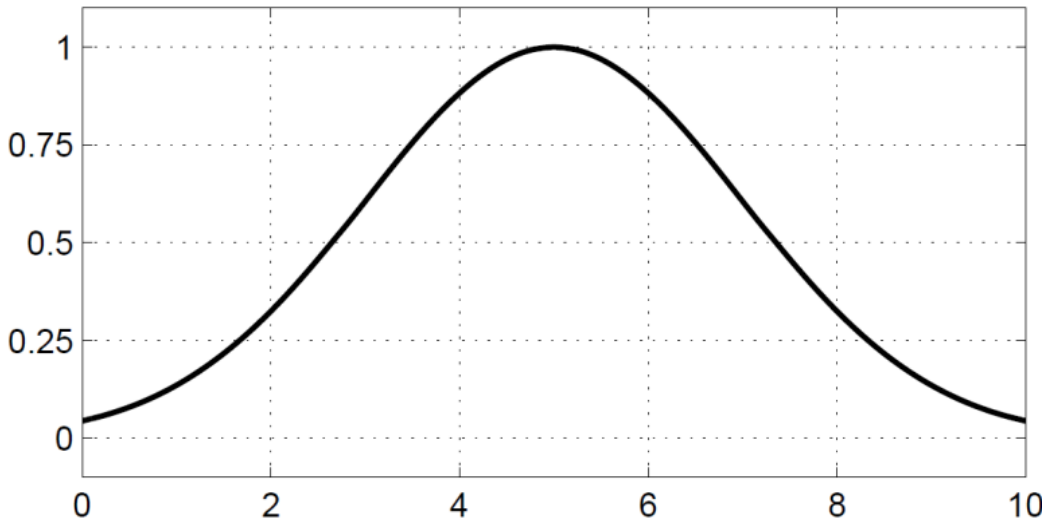


Figure 19 Simple Gaussian membership function example (`gaussmf`).

Two-sided Gaussian

The two-sided composite of two different Gaussian curves, the `gauss2mf` (Figure 20).

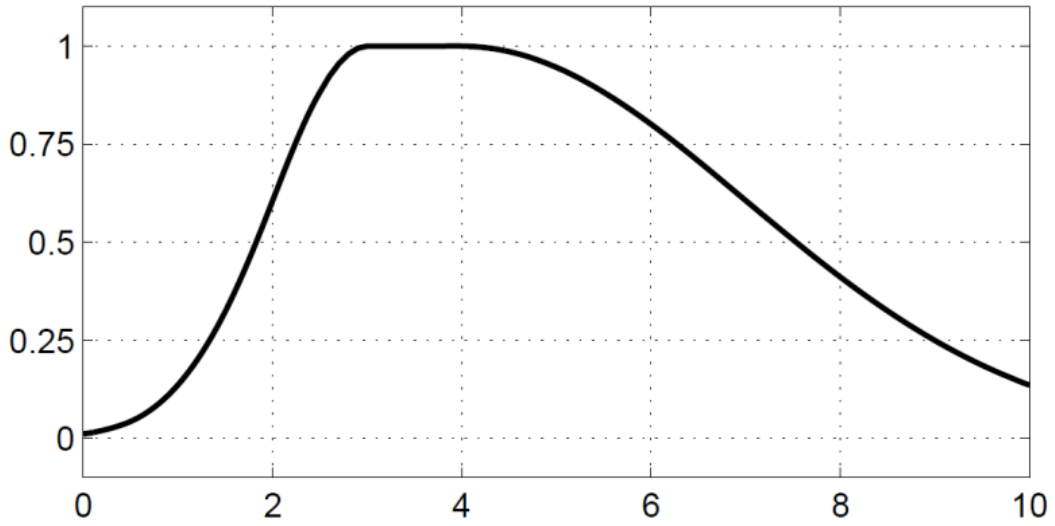


Figure 20 Two-sided Gaussian membership function example (`gauss2mf`).

Generalized Bell

The *generalized bell* membership function is specified by three parameters and has the function name `gbellmf`. The bell membership function has one more parameter than the Gaussian membership function, so it can approach a non-fuzzy set if the free parameter is tuned (Figure 21).

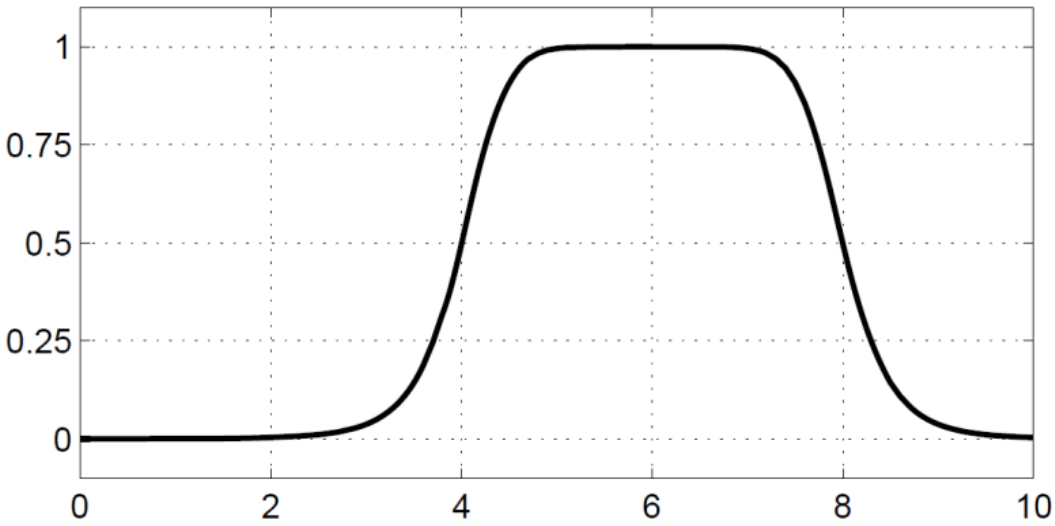


Figure 21 Generalized Bell membership function example (`gbellmf`).

Basic Sigmoidal

Sigmoidal membership function, which is either open left or right, and the name is `sigmf` (Figure 22).

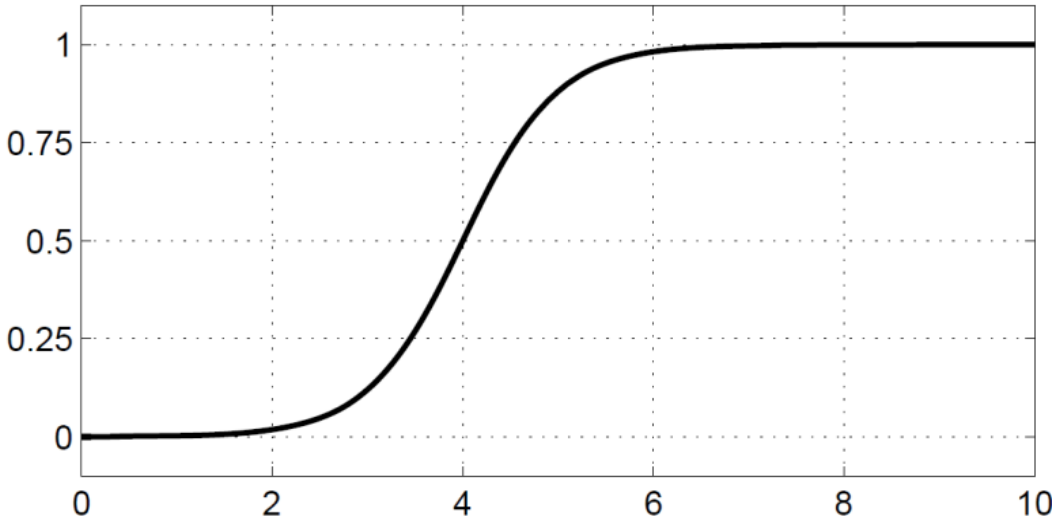


Figure 22 Basic Sigmoidal membership function example (`sigmf`).

Sigmoidal Difference

The *sigmoidal difference* membership function is the difference of two membership functions, and the name is `dsigmf` (Figure 23).

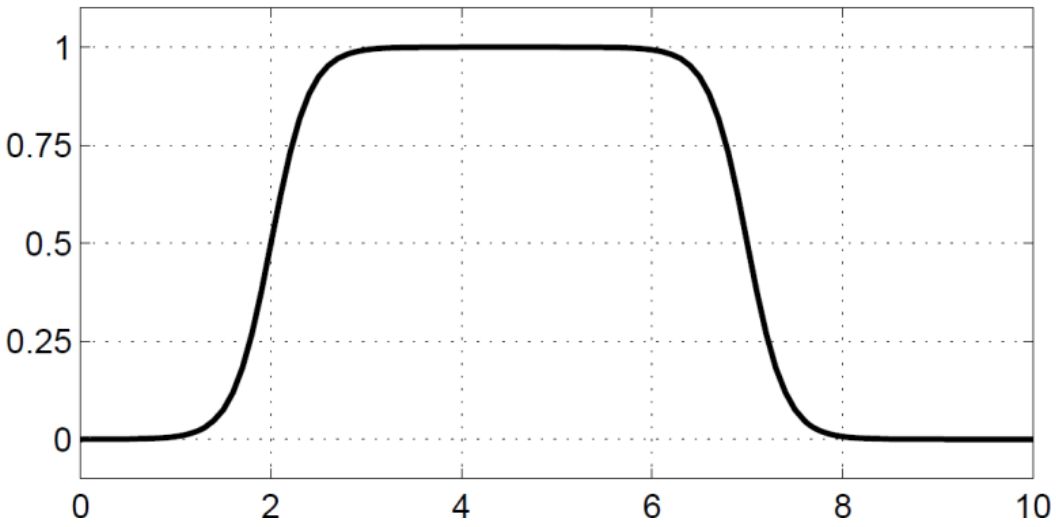


Figure 23 Sigmoidal difference membership function example (`dsigmf`).

Sigmoidal Product

The *sigmoidal product* membership function is the product of two membership functions, and the name is `psigmf` (Figure 24).

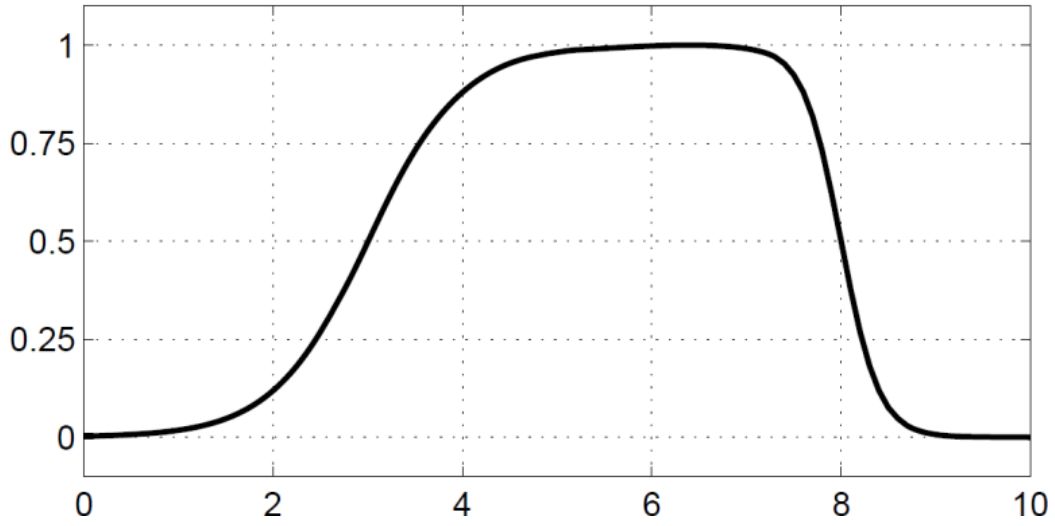


Figure 24 Sigmoidal product membership function example (`psigmf`).

Polynomial Z

Polynomial Z membership function is based on the Z function curve, the asymmetrical polynomial curve open to the left, and the name is `zmf` (Figure 25).

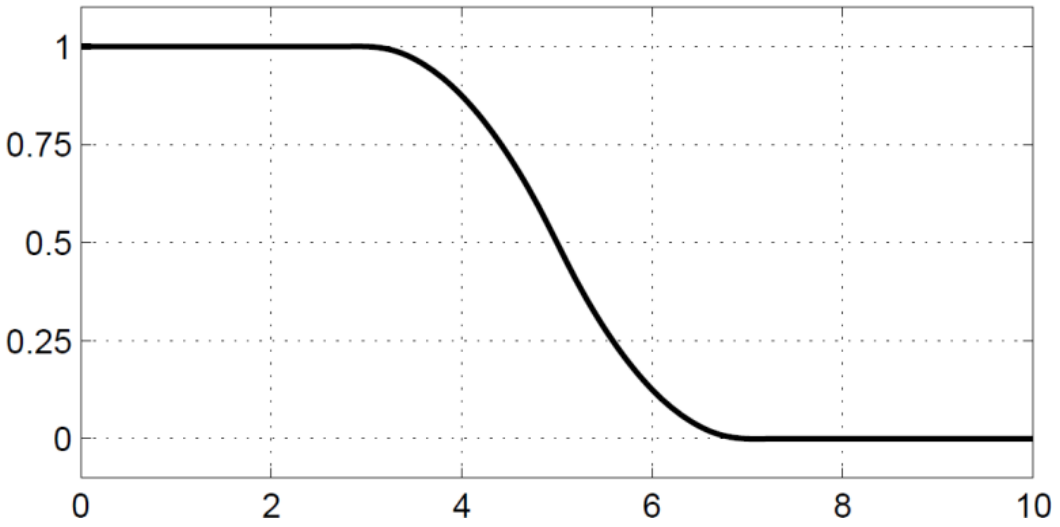


Figure 25 Polynomial Z membership function example (`zmf`).

Polynomial S

Polynomial S membership function is based on the S function curve, is the mirror-image function that opens to the right and the name is `smf` (Figure 26).

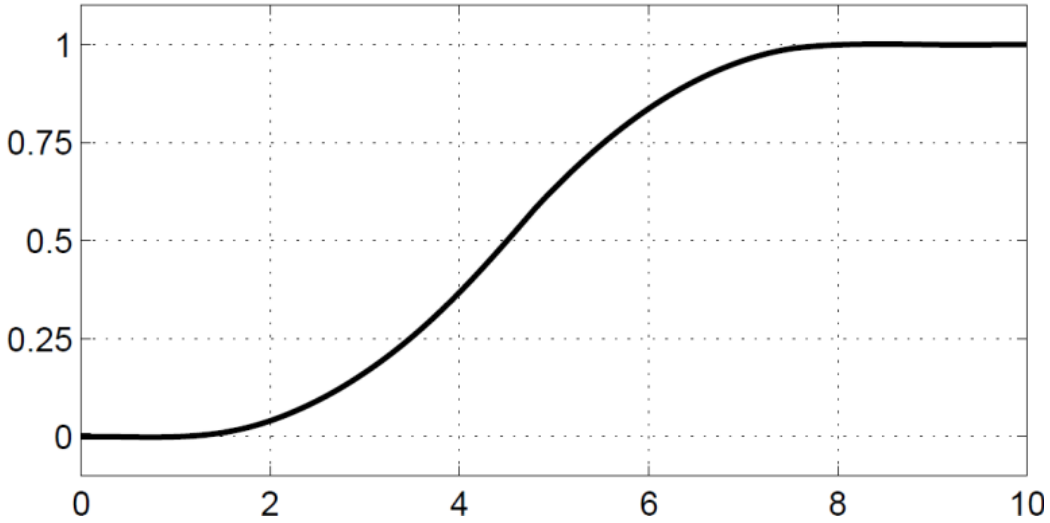


Figure 26 Polynomial S membership function example (`smf`).

Polynomial Pi

Polynomial Pi membership function is based on the Pi function curve, is zero on both extremes with a rise in the middle and the name is `pimf` (Figure 27).

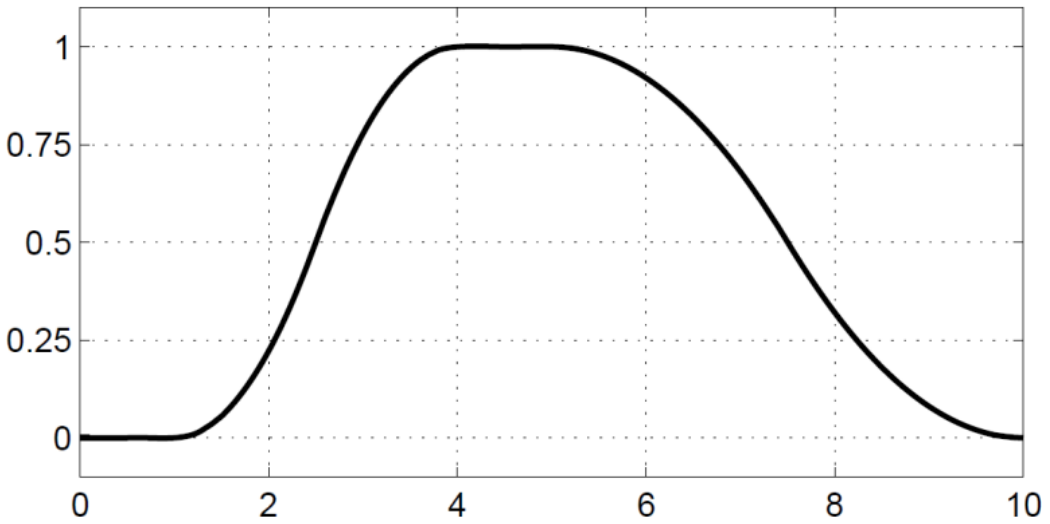


Figure 27 Polynomial Pi membership function example (`pimf`).

The Fuzzy Inference System (FIS) editor of the toolbox can be opened typing the command `fuzzy` in the MATLAB command line. With this the editor opens with default values. The graphical interface of the FIS editor looks like the diagram shown in Figure 28.

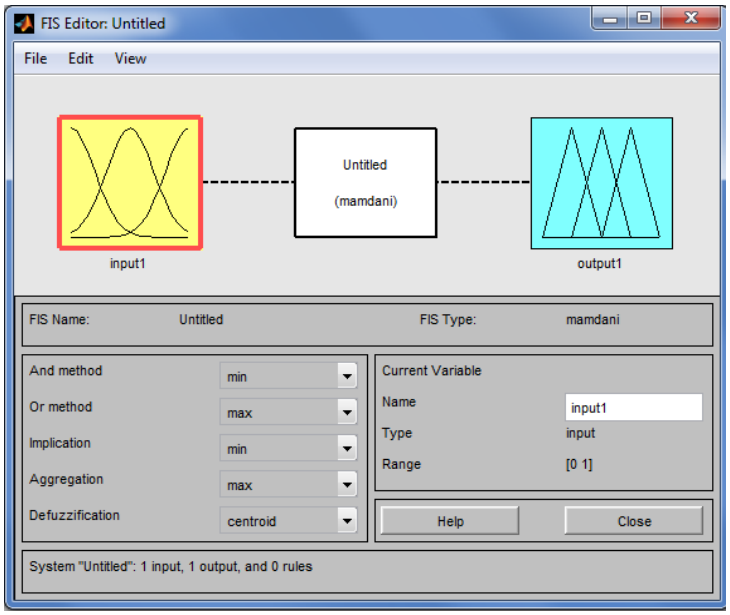


Figure 28 MATLAB Fuzzy Inference System editor graphical interface.

The default inference mechanism is the Mamdani's inference mechanism, which will be the one used in this work. The FIS editor allows to change the logic operations of rules, implication, aggregation and defuzzification method. It also permits adding inputs and outputs, likewise the membership functions and its range.

The Membership Function Editor is the tool used to display and edit the membership functions associated to every input and output for the entire inference mechanism. Figure 29 shows the graphical interface of the Membership Function Editor. This interface allow users to add and edit membership functions for every input and output, as its limit ranges.

In this interface is possible to choose between the membership functions available from the toolbox, or user defined functions, and edit its parameters, add membership functions for every variable and edit the range of the universe of discourse for that all variables. These configurations must be done for all variables independently.

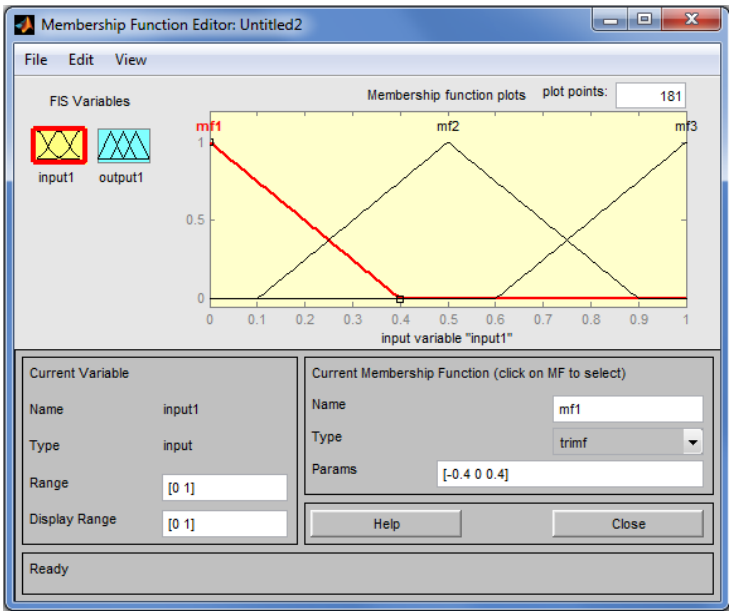


Figure 29 MATLAB Membership Function Editor graphical interface.

Another tool that is also available is the Rule Editor that is represented in Figure 30. This graphical tool allow users to add rules to the rule-base. In this interface the user can create the rule-base based on the inputs and outputs and their relations. These relations are logic operations, the antecedents, that imply a certain consequents to the outputs. The logical operations are the AND, the OR and the NOT. Is also possible to define a weight for every rule.

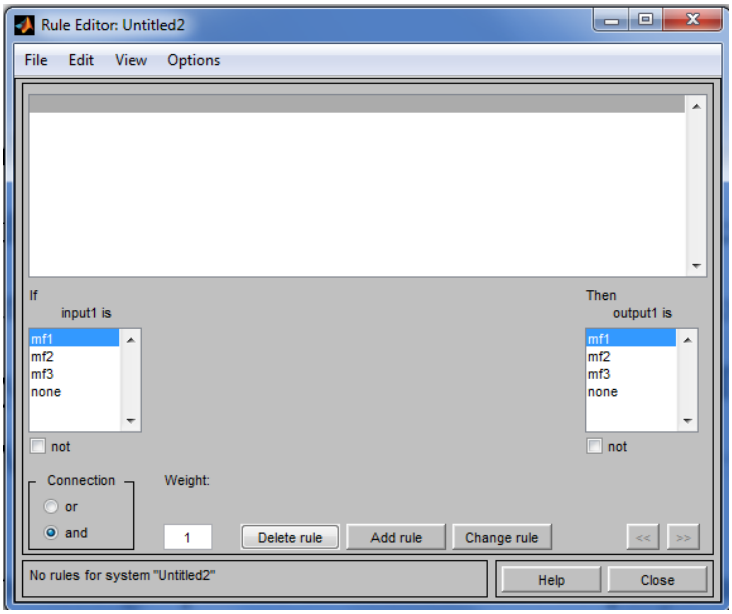


Figure 30 MATLAB Rule Editor graphical interface.

The Rule Viewer is another graphical interface that displays a roadmap of the whole fuzzy inference process. This interface represent the antecedents and consequents of the rules according to the implications and aggregation operands used. Here is possible to see how a change in the inputs affects the outputs according to the defuzzification function selected in the FIS interface. This Rule Viewer interface is shown in Figure 31.

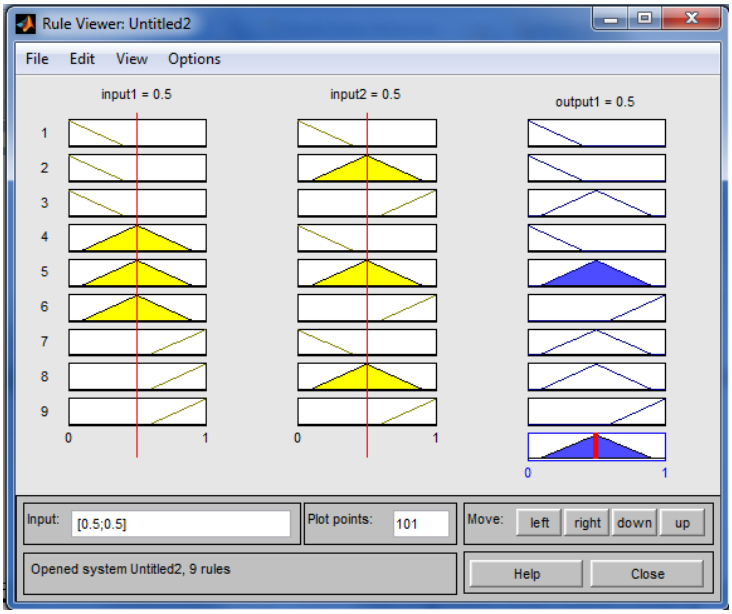


Figure 31 MATLAB Rule Viewer graphical interface.

The Surface Viewer is used to show the entire span of the output set based on the entire span of the input set. The surface can be either 2D or 3D based on number of inputs and outputs. A 2D surface needs two variables and a 3D surface need 3 variables, two inputs and one output, however, if the system has more than two inputs and more than one output, it is not possible to show a surface for all because the surface is a 3D space, only three variable are possible, thus, several surfaces are created and each one appears by selecting two inputs and one output only. The surfaces can be more or less smooth, depending on the number of plot points, that can be edited in the interface. The interface is shown in Figure 32.

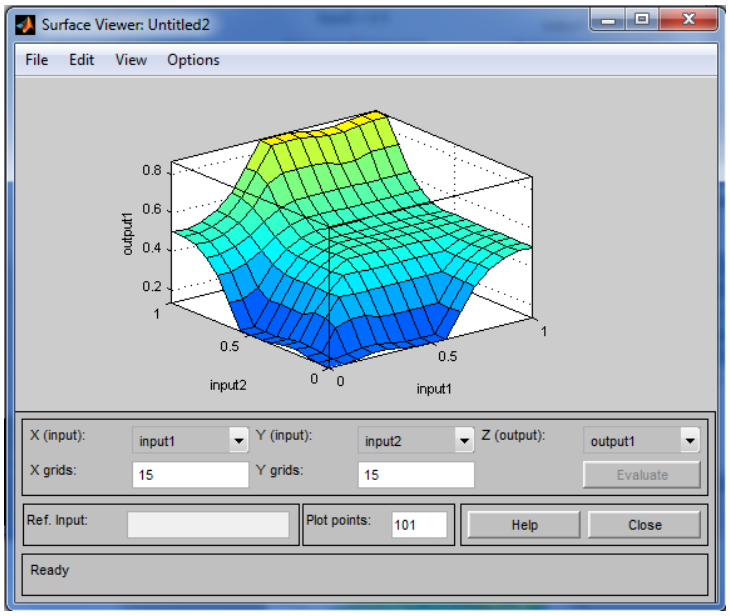


Figure 32 MATLAB Surface Viewer graphical interface.

With all this in mind, is now possible to export the project to the MATLAB workspace or to a file, where all this fuzzy logic parameters will be saved to later be used as a fuzzy logic block. This block can also be imported to the SIMULINK software and be used in simulation projects to test the behaviour of the controller. In this particular case the simulation diagram will be similar to the one shown in Figure 16, that later will be combined with the GAs.

3. GENETIC ALGORITHMS

Genetic Algorithms (GAs) are techniques used for searching minimums or maximums and for finding the solutions that satisfy the objectives of the problem. When solving a problem with GAs the best solutions are found after a few interactions or generations, where crossover and mutations happen until the best individuals, that satisfy the requisites of the problem in question, are found. These objectives are fulfilled in an evolutionary way performing operations in the codified structure, the chromosome, resembling what happens in nature.

GAs where developed by John Holland and his colleagues and students in the University of Michigan during the 60's and 70's [9]. In contrast to other evolutionary strategies and programming, the main goal of Holland, was not to develop algorithms to solve scientific problems, but to study the phenomenon of adaptation as it happens in nature, by developing ways to import these natural mechanisms to computational systems. In his book *Adaptation in Natural and Artificial Systems* (1975) he showed GA as an interpretation and adaptation of what happens in nature [9][19].

Charles Darwin's Theory of Natural Selection is the base to better understand the evolution of GAs. According to this theory, in one population of individuals, only survive and reproduce the most able individuals; the less able ones die eventually after some

generations. Charles Darwin observed with different evolutionary characteristics in birds of the same species, in different Galapago's islands. As an example, he observed birds from the same species with different nozzles shape, as an attempt of a better adaptation to the habitat they were in. This adaption is the result of several generations that were high lightening the expression of the advantage of that characteristic to easily find food. Darwin's theory was very contested for not totally answering how this individuals diversity or their characteristics are passed over future generations.

In the XX century, with the discovery of genetic material, genetics started to answer these questions. All living organisms are composed of cells; the cells are then composed of genes that have on its structure a combination of basic units named DNA (deoxyribonucleic Acid). These small units are responsible for setting the characteristics of each individual. These sequences and combinations of genes are denominated chromosomes, where the genetic information is stored. Even with low probability, crossing over might cause mutations that can lead to individuals with characteristics more or less competitive in the environment they are in. However, these mutations may result in non significant impact to the individual. Concepts such as chromosome, gene, crossover, mutation, fitness function, population or generation are present in GAs.

The word *genetics* comes from the word *gene*. Genetics is the science that studies the transmission of hereditary characteristics from one generation to another. All the biological information is codified in *genes* that are the basic units of a *chromosome*. These specific characteristics can be *genotypic* (allelic constitution) or *phenotypic* (observable characteristic of appearance or physiology). The same gene may have alternative forms called *alleles*, which means that an allelic variation causes hereditary variation within a species. The chromosome region where a gene is located is called *locus* (plural, *loci*). During replication, which is the crossing over of genetic material between two individuals, the molecular information of the progenitors is carried out to the next generation. Although rare sometimes mutations may occur [20]. In fact, a *mutation* can contribute for genetic variability within a species. The individual's characteristics combined with the environmental conditions determine the basis of natural *selection* process of species, where the strongest ones are always selected. So, the relative probability of survival and reproduction rate of a phenotype or genotype is called *Darwinian fitness,* which evaluates

the ability of each individual to survive and reproduce [20]. All this genetic concepts are present in GAs as their biological basis.

In GAs, the *chromosome* normally represents an individual and a possible solution to the problem, commonly encoded as a string of bits, since computer memory is composed by matrices of bits; in fact, any information can be codified by a sequence of bits. The *genes* are either single bits or short blocks of adjacent bits that encode for a particular element of the chromosome or for a possible solution. An *allele* in a bit string is typically a 0 or a 1, depending on the terminology used to represent it. At each *locus* more alleles are possible when using larger alphabets. *Crossover* typically consists of exchanging genetic material between two single parent's chromosomes. *Mutation* consists of inverting the bit at a randomly chosen *locus* (or, for larger alphabets, replacing a symbol at a randomly chosen *locus* with a randomly chosen new symbol). The majority of genetic algorithms applications use single chromosome individuals, i.e., haploid individuals. Considering a GA using bit strings, the individual *genotype* is simply the configuration, of that individual's chromosome, in bits [9]. An analogy between genetics and genetic algorithms is displayed in Table 3.

Table 3 Correspondence between genetics and genetic algorithms concepts.

Genetics	Genetic Algorithms
Chromosome	Vector or string
Gene	Characteristic
Allele	Value of the characteristic
Locus	Position in the vector
Genotype	Structure or codified vector
Phenotype	Set of parameters, decoded structure

3.1. TYPES OF ENCODING

Encoding is the process of representing the individual genes, using bits, numbers, trees, matrices, lists or other objects that better represent the individual that will depend on the problem itself. Regarding the searching and learning method, the way how candidate solutions are encoded is a central factor for the success of a genetic algorithm. Most GA applications use fixed length, i.e., a fixed order of bit strings to encode candidate solutions. However, recently, many experiments have been performed with other types of encodings. The decision on the most appropriate encoding technique must be taken by the developer. In fact, the developer needs to do the trial, test for errors and adapt the encoding that better fits the problem. One interesting idea is to have a self-adaptable encoding so that the GA could make better use of it [9][21].

The *binary* encoding is one of the most common forms of GAs encoding. Nevertheless, for many applications it is common to use an alphabet with many characters or real numbers to represent chromosomes. These types of encoding forms uses specific approaches by applying encoding characters such as *octal*, *hexadecimal*, *real numbers permutation*, *values*, or *tree* [9].

3.1.1. BINARY ENCODING

In binary encoding each chromosome codifies one binary string and each bit in the sequence represents one characteristic of the solution. Despite that, this string might also represent an integer or real number. Binary encoding varies with problems and allows chromosomes with less number of alleles. According to Holland (1975) [19], binary encoding has the advantage of using less space to store the same data which could result in a better performance. Nevertheless, this type of encoding is not natural to many problems and, sometimes, several modifications must be done after the genetic operations. The most commonly used binary string is the encoded with zeros and ones where the length of the string depends on the required precision. The encoding shown in Figure 33 is an example of binary encoding.

Chromosome 1	1 0 1 1 0 1 0 0 1 0 1 1 0 1
Chromosome 2	0 0 1 0 0 1 1 0 1 1 1 1 1 1

Figure 33 Example of a binary encoding.

3.1.2. MANY-CHARACTER AND REAL-VALUED ENCODINGS

The *octal encoding* is another possible form of encoding that uses octal numbers (0 to 7). In Figure 34 it is possible to observe an example of an octal encoding.

Chromosome 1	1 0 1 3 7 5 5 0 6 7 6 6 0 2
Chromosome 2	7 0 1 3 4 4 2 6 3 5 4 2 1 1

Figure 34 Example of an octal encoding.

The *hexadecimal encoding*, represented in Figure 35, is characterized by the application of decimal numbers (0 to 9) and letters (A to F).

Chromosome 1	1 F C 3 7 A 5 0 6 F 6 B 0 2
Chromosome 2	A 0 F F F C D 3 5 A B 1 0 1

Figure 35 Example of a hexadecimal encoding.

The *real numbers permutation encoding* is useful for ordering problems. Here, each chromosome represents a sequence of real numbers that sometimes has to be modified after the genetic operations so that the consistency of the chromosome is kept and, therefore maintained the sequence validity. The encoding represented in Figure 36 is an example of real numbers permutation encoding.

Chromosome 1	1 0 1 3 7 5 8 0 6 7 6 0 2 9
Chromosome 2	7 0 1 3 4 9 2 6 3 5 4 2 8 1

Figure 36 Example of real numbers permutation encoding.

The *value encoding* is also another form of encoding that consists in a sequence of values that can be anything related with the problem, such as real numbers, characters or other type of objects. Considering an event where complicated values, such as real numbers, are used the application of binary encoding would be very difficult; in this case, the value encoding leads to better results. But sometimes the development of new genetic operators, specific for the problem, is required. In Figure 37 an example of a value encoding is displayed.

Chromosome 1	1.2030 3.01223 4.0034 0.01223
Chromosome 2	ABDRTFGYTDDFGTFFSEERF
Chromosome 2	(blue), (black), (red), (yellow), (green)

Figure 37 Example of a value encoding.

3.1.3. TREE ENCODING

The *tree encoding* is another encoding possibility considering genetic programming evolution. In this case, each chromosome is a tree of some objects such as functions and commands in a programming language. One advantage of tree encoding is the searching of open-ended spaces, although there is the risk of uncontrolled tree growing where many of these very large structured trees might difficult reading.

3.2. GENETIC ALGORITHMS

An algorithm is a sequence of steps that lastly will solve a problem. It is a genetic model inspired method based on searching and finding the best possible solutions. The GA is used to find optimal solutions in a certain *search space* that is the space where all possible solutions are present, and where each point of the search space represents a possible solution. The distance between two sequences of candidate solutions is the number of positions in which the bits, at the corresponding positions, differ from each other. In most cases the next candidate solutions will depend on the previous test sequencing results. Some algorithms assume the existence of some correlation between the quality of the neighbouring candidate solutions. Genetic algorithms assume that high quality parent candidate solutions, from different regions in the space, can be combined via crossover to produce high quality offspring candidate solutions [9].

In GA the populations of chromosomes are processed by successively replacing one population for another. Here, the *fitness function* is a key concept that, in the current population, gives a score (fitness) to each chromosome in order to evaluate how that chromosome solves the problem. The greater the fitness function the higher the probability of an individual being selected for reproduction and remain alive [9]. Therefore, the main objective of this technique is to find the best solutions in all space. However, one of the problems in searching and finding these solutions is the presence of local optimal solutions and the selection of the starting point of search; local optimal solution might not be the global optimal solution leading to bad solutions. Since these types of algorithms are

stochastic, even if the search space has no optimal local there is no guarantee that the GA will find the global optimal solution, but instead a reasonable good solution, i.e., the solution that satisfies the problem specifications [9].

In simple GA operators such as *crossover*, *mutation* and *selection* are used. Moreover, *inversion*, *gene doubling* or *deletion* are also applied operators. So, the selection of the most appropriate operator depends on the problem as well as on the GA developer.

3.3. DESIGNING GENETIC ALGORITHM PRINCIPLES

Before implementing a GA it is important to understand some guidelines in order to create a general search algorithm, i.e., an algorithm of global optimization based on the fitness function properties and on the most common optimization methods.

A purely *deterministic* search may lead to an extremely high quality solution. A *non deterministic* search should be stochastic, though it could have a substantial part of determinism. A purely stochastic method normally is too slow, and is reasonable more efficient deterministic predictions in the most promissory directions.

When approaching GA the criterion of *integrity* is very important. Every solution must have its own codification, the *non redundancy*, codes and solutions should have an univocal matching named *robustness*. Every genetic code produced by genetic operators should have a matching solution, and considering the *perseverance of the characteristics* the descendents must inherit useful parent characteristics. The basic steps to solve a GA problem are the representations of the problem, the calculation of the fitness function, the several variables and parameters involved in the control of the algorithm − population size, crossover rate, and mutation rate − the representation of the results and the way how to finish the algorithm.

3.4. SIMPLE GENETIC ALGORITHM

The basic structure of a GA starts with randomly generating an initial population of individuals that might be possible solutions. Sometimes heuristic knowledge could also be used to create the first population. This first population should be large and diverse to explore all searching spaces, otherwise the algorithm might only explore a small part of the search space and miss a global optimum leading to premature convergence. However, if

the size of the initial population is very large the computational complexity increases taking more time to the algorithm to converge. The algorithm converges when the individuals are so similar that a mutation might be needed to improve the population. Every individual must be evaluated by the fitness function. The next step is to create a new generation of individuals, using the genetic operator selection, crossover and mutation, and repeat it until the new generation is created. The new population should then replace the previous one. This new generation must be evaluated to see if it satisfies the requisites of the problem. In a positive case, the algorithm stops and the solutions are found; otherwise, the algorithm should be restarted with the last population of individuals. In Figure 38 it is represented an example of a flowchart of a simple GA.

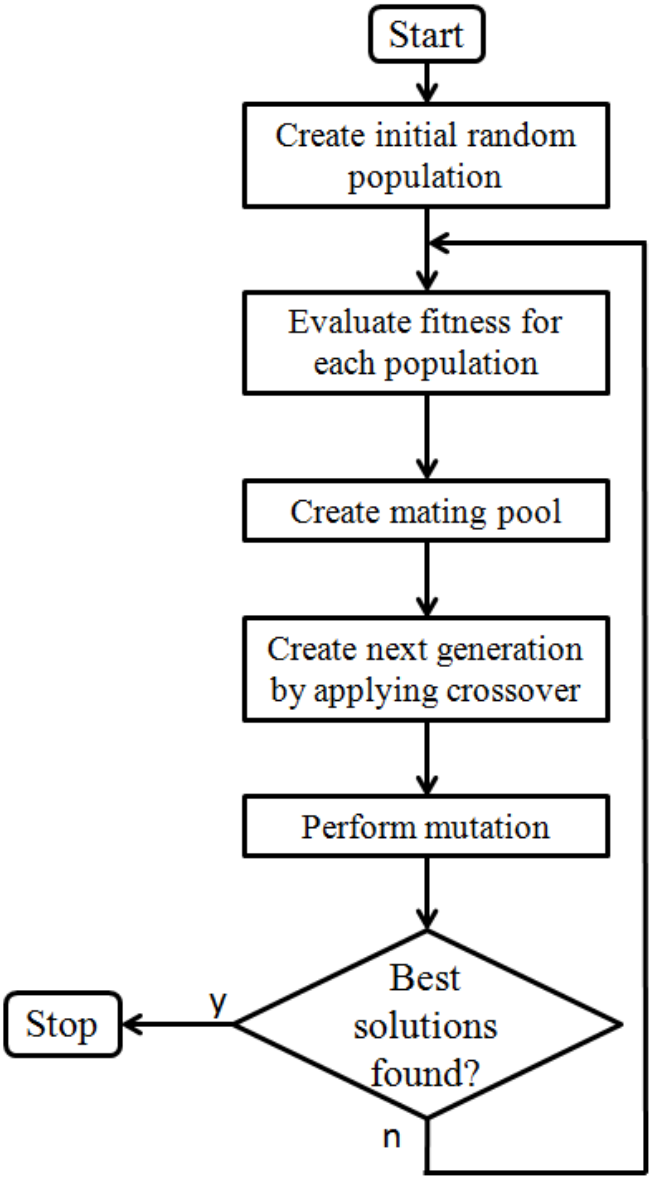


Figure 38 Example of a simple Genetic Algorithm flowchart.

Based on the previously analysed references the main tasks of a genetic algorithm can be:

- The *selection* of individuals for reproduction is the first step. It is a random selection with a certain probability depending on the relative aptitude of the individuals and where the bests are selected for reproduction. The fitness function has an important task in the selection of individuals, in which the better values lead to better individuals.

- The *crossover* is the second step of when building an algorithm. The descendents are created by the individuals that were selected for reproduction. A combination of two chromosomes where subparts of each other are combined, originating two new generation chromosomes. It is important to mention that recombination and mutation can be used to generate new individuals [9][22].

- The *mutation* that is a rare random change of an individual's portion of genetic material. This occurs after the crossover phase due to swap or inversion of one or more genes in the generated chromosome. In every generation the reproduction chromosomes are selected in order to originate new chromosome generations.

- The *replacement* is the last step where the last population is replaced by the *new generation* of individuals or chromosomes. The individuals are evaluated according to their fitness and normally the ones with high values of fitness function are the best solutions and the better candidates to be in the new generation.

The algorithm stops when the *stopping criteria* is settled, i.e., when the solutions found converge to the global optimal solution according to the specifications of the problem and the stopping conditions.

3.5. SELECTION METHODS

The selection is the artificial mechanism of natural selection when finding the most able individuals giving emphasis to the most able ones. This process starts a random selection of the parent's chromosomes in agreement with the fitness function of the individuals.

During the *pressure selection*, which is the ratio between the maximum aptitude and the average aptitude, it is important to select the best individuals. If the pressure is high the intensification grows and the better individuals are favoured; in the case of less pressure

the diversity will be less. The pressure leads the algorithm to improve the aptitude along with generations. The rate of convergence of the GA is highly determined by the magnitude of the pressure that may lead to higher rates of convergence. GAs should be capable of identifying optimal solutions, or at least closer to the optimum, by using an intelligent scheme of pressures. However, if the pressure is too low the rate of convergence will be too slow and therefore the GA will take more time to find the ideal solution. On the other hand, if the pressure is too high a premature convergence can lead to a local optimal solution or even bad solutions [9].

Besides providing pressure selection the selection schemes should also preserve the population diversity by helping to avoid premature convergence. In the complete absence of pressure GA will become a total stochastic process of *random walk* type rendering the selection probability equal for all individuals.

Normally there are two types of selection − the *proportional* and the *ordinal* selection.

In proportional selection individuals are selected based on the comparison of the fitness function values with the fitness function values of other individuals. In the ordinal selection individuals are selected based on their positions in the population rather than on the value of their fitness function. For that, the pressure needs to be independent from the distribution of the population fitness and only dependent on the population ranking. It is also possible to use a scale function to redistribute the fitness population interval to adjust the pressure. The selection has to be balanced with the variation of crossover and mutation. A very strong selection will lead to suboptimal highly able individuals reducing the diversity needed for evolution and progress. A very weak selection will result in a slow evolution.

3.5.1. PROPORTIONAL SELECTION

The *proportional selection or roulette wheel* is a selection directly proportional to the fitness function. Considering a circle divided by *n* regions, that is the size of the population, where the area of each region is proportional to the fitness of the individual. A roulette is placed with *n* cursors equally spaced; after one turn the cursors position indicates the selected individuals. The individuals with greater area will have higher probability of being selected more often (Figure 39).

This type of selection adds the sum of the fitness values for all individuals in the population *T*. Then generates a random number (*r*) in the interval [0, *T*] and travels all the population by adding the values of the fitness until the sum is equal or greater than *r*. The individual whose fitness value passes that limit is the selected one. The first step is only performed once for each population. In this case premature convergence might occur.

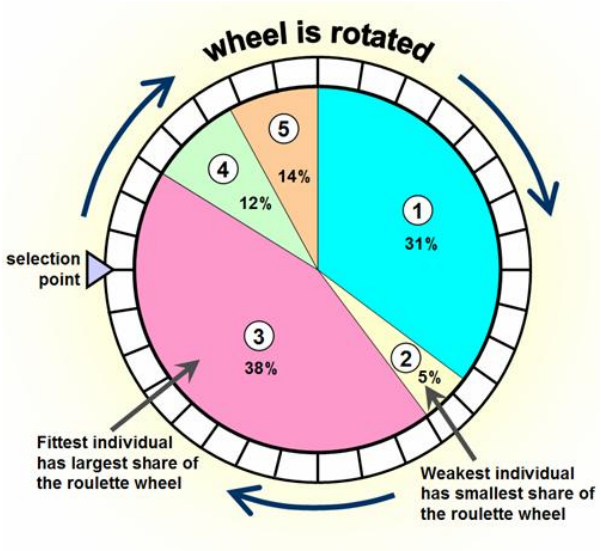


Figure 39 Example of a roulette wheel selection method [23].

3.5.2. RANDOM SELECTION

The *selection* is a method that, as the name indicates, randomly selects one of the parents from the population. Overall, it is more appropriate to break the genetic codes than the roulette selection.

3.5.3. RANK SELECTION

The *rank selection* exists due to the big differences between fitness values in the roulette selection. First the rank selection classifies the population and then gives each individual a fitness value determined by its classification. The worst individual will have a fitness of 1, the second worst 2, etc. so that the best will have their fitness equal to the number of individuals in the population. After this process all individuals have a probability of being selected; however, this method may result in a lower convergence since the best individuals cannot be distinguished from each other. This method reduces the premature convergence giving more importance to the less able.

3.5.4. TOURNAMENT SELECTION

The *tournament selection* creates a selection strategy capable of adjusting the selective pressure by applying a competition by tournament between all individuals of the population. The tournament best individual has the highest fitness and, therefore is the winner. The competitors and winner are placed in the matting pool to repeat the tournament until all reproducers are selected. Further, the group of selected reproducers has an average fitness function higher than the rest of the population. In fact, this fitness function difference introduces the selection pressure needed to improve the next generations in terms of fitness. Thus this method reveals to be more efficient and leads to an optimal solution.

3.5.5. STOCHASTIC SAMPLING SELECTION

The *stochastic sampling selection* method is done by mapping all individuals into contiguous segments of a line. The size of each segment is proportional to the value of the mapped individual fitness. The sizes are normalized and a number i between 0 and $\frac{1}{n}$ is randomly selected, n pointers are pointed to the lines segments at positions $i, i+\frac{1}{n}, i+\frac{2}{n}, \dots, i+\frac{n-1}{n}$. The individuals that own that pointed line segment are then selected. This ensures that the selection of descendents is closer to the deserved in comparison with the roulette selection.

3.6. CROSSOVER

The crossover step is inspired on the biological mechanisms of reproduction where living organism from the same specie mate and give origin to new identical organisms. Crossover is the operator responsible for the recombination of the parent's characteristics during reproduction leading to the inheritance of these features by the new generation. It is considered the most predominant genetic operator, so it is applied with the probability given by the rate of crossing. To achieve this purpose some crossover techniques can be used:

- *One point crossover*. A point of crossing is randomly selected and in that point the genetic information is switched. One part of each parent is given to the child so that it will keep the same chromosome format as parents.

- *Two points crossover*. Based on the same principle as the one point crossover, but instead of switching the information of both parents in two parts, it switches the information in three parts. The child receives two parts from one parent and one part of the chromosome of the other parent.

- *Multi-point crossover*. It is a generalization of one point crossover where several points are chosen to transfer the information to the child, keeping the same principle as the one and the two point crossover.

- *Uniform crossover*. In this case points are not used; instead a probability for each variable is applied to be changed by parents to the child using an auxiliary chromosome as a mask that determines the probability.

- The *three parents crossover* technique compares each bit of the first parent with the second parent and if they are equal the bit passes to the child; if not the bit of the third parent is passed instead.

- The *reduced surrogate crossover* constrains the crossing to insure the creation of new individuals when possible. The implementation is made constraining the crossing to the points where the gene's values are different.

- The s*huffle selection* is related with the uniform crossing. One crossing point is used, but before switching the variables they are randomly disordered in both parents; after recombination the variables are reordered.

- The *precedent preservative crossover (PPX)* keeps the order of the parent's gene precedence throughout a mask similar to the mask used in the uniform crossing. This method selects genes from one parent; in case this gene creates a non-viable child the operator searches for the next gene of the same parent to maintain the child viability.

- In the *partially mapped crossover (PMX)* two randomly crossing points are made in both parents; the resulting subchains genes are integrally inherited by both children. These subchains also determine the relationship mapping between the parent genes with the goal of treating the infeasibility originated by the process.

- The *ordered crossover (OX)* is based on the idea that, two randomly crossing points are made in both parents and the resulting gene subchains are inherited integrally by both children. After the last cut in each chromosome, other parent's genes that are not in the inherited subchain, are then searched to fill the child chromosome.

The frequency at which the crossover is done is called *crossover probability*. If there is no crossing the children are exact copies of their parents. But if crossing occurs the children chromosomes are a mixed of their parent's chromosomes parts. If the probability of crossover is 100%, then all children chromosomes are created by the crossing. A high crossing probability means that new structures are rapidly introduced in the population; if this rate is too high structures with high fitness could be removed and the greater part of the populations will be replaced. If this rate is low the algorithm could become too slow.

3.7. MUTATION

Mutations happen at a very low probability and consist of small changes during the crossing over of the genetic material. These changes may occur in one or more alleles, according to a previously settled probability.

When vectors suffer a low probability mutation algorithm may not stay blocked in an optimal local; this will helps to recover the lost genetic materials acting as an insurance preventing the lost of irreversible genetic material. This is traditionally considered a simple search operator. Crossing finds the best individuals for exploring the actual solution while mutation has the ability to explore the entire search space which is very important to keep the population's genetic diversity. The random insertion of new genetic structures in the population modifies some of their constructive blocks.

There are different ways in which a mutation may occur. In binary representation a simple mutation may consist in inverting the value of a gene with low probability. Normally the probability is in the form of $\frac{1}{L}$ where L is the length of the chromosome. It is also possible to implement operators of *hill-climbing* type, i.e., operators that only make the mutation if the quality of the solution improves. This operator may accelerate the search, yet this must be done with careful since the diversity of the population may be reduced leading the algorithm to converge prematurely to an optimum local. Mutation of one bit consists in inverting one bit from 0 to 1 or the other way around. The different types of mutations are:

- *Mutation by change*. Changes all bits from 1 to 0 and from 0 to 1 based on a mask chromosome.

- *Mutation by interchange*. Two positions are randomly generated and the correspondent bits are swapped.

- *Mutation by inversion*. In this case a position in the chromosome is randomly chosen and the bits next to that position are inverted.

- *Mutation by probability*. The transformation of the chromosome parts is decided by the operator. If the mutation does not take place the result is generated immediately after crossing without any changes. If the mutation takes place, one or more parts of the chromosome are changed. If probability is 100% the entire chromosome is changed; on the other hand, if probability is 0% no changes are made to the chromosome. Mutation normally prevents the algorithm to fall in local optimums. If the mutation frequency is high the genetic algorithm becomes a random search algorithm.

3.8. REPLACEMENT

The replacement is the last step of the entire reproduction cycle.

Two parents are removed from a fixed size population generating two children and only two, of these four members, will pass to the new population. A method must be applied in order to choose which individuals should stay in the new population. The technique must proportionally influence the convergence to the selection operator. There are several techniques for replacing the population in the genetic algorithm:

- The g*eneration type* consists in the creation of *N* children from one population of *N* size to create the new generation that will replace all parents from the old generation. This type of generation implies that an individual can only cross with individuals from the same generation. Other forms of this type of generation are $(\lambda + \mu) - actualization$ and $(\lambda, \mu) - actualization$. Here from one population of parents size μ are created λ children where $\lambda \geq \mu$. The best μ individuals from both populations, parents and children, will create the next generation.

- The s*tationary type* is very radical and consists in inserting new created individuals in the population, contrary to generation actualization where all generation is created in every generation. The insertion of a new individual normally demands the replacement of other member of the population; in this case can be the worst or the oldest member inserting more pressure. Normally the stationary type actualizations use a method based in order, both for selection and replacement.

- The *random replacement* is other method that consists in randomly selecting two individuals in the population and replacing them by new individuals. This technique may be useful to keep the search in small populations since that weak individuals may be introduced in the population.

- The w*eak parents replacement* happens when a weak parent is replaced by a stronger child. From the four individuals, parents and children, only the two more able ones pass to the new population. This process improves the fitness function of the population when used together with a selection method that uses strong parents and weak parents for reproduction.

- The *both parents replacement* is a simple replacement method where the child replaces the parents, so the parents only reproduce once, resulting in a continuous process of moving the genetic material along with generations. However, problems may occur if combined with a selection technique that strongly favours strong parents. In this case, good solutions evolve but are eliminated immediately.

3.9. STOPPING CRITERIA

The algorithm stops when the defined objectives are reached. The stop criteria are:

- The m*aximum number of generations*. If the number of generation hits the maximum number of generations the algorithm stops.

- The *time limit*. If the elapsed time reaches the maximum specified time the algorithm stops.

- The *fitness limit*. The fitness of the best individual, in the actual population, is less or equal to the fitness limit the algorithm stops.

- The g*eneration stall*. If in a number of generations the fitness function does not improve the algorithm stops.

- The *time stall*. If in a defined elapsed time the fitness function does not improve the algorithm stops.

3.10. RESTRICTIONS

To apply the GA to a real and practical problem it is necessary to deal with some restrictions such as financial ones, rules that cannot be broken or market dynamics. Scientific problems may also be subjected to some restrictions such as numeric ones, temperature, light or even time.

Essentially there are two types of restrictions, the *hardconstrains* and the *softconstrains*. The hardconstrains are restrictions that must be completely followed otherwise the solution will not be valid. The softconstrains are desired but may not be followed, for instance speed is always positive and in this case the search space is all positive numbers so there is no need to search negative numbers, leading to a faster algorithm.

The search space is always divided in two regions, the *feasible* and the *non-feasible region*. The feasible region is the part of the search space where the solutions satisfies the restrictions, the rest corresponds to the non-feasible region. Although, invalid solutions should not be completely removed from the search space, thus they may be near an optimal solution as represented in Figure 40.

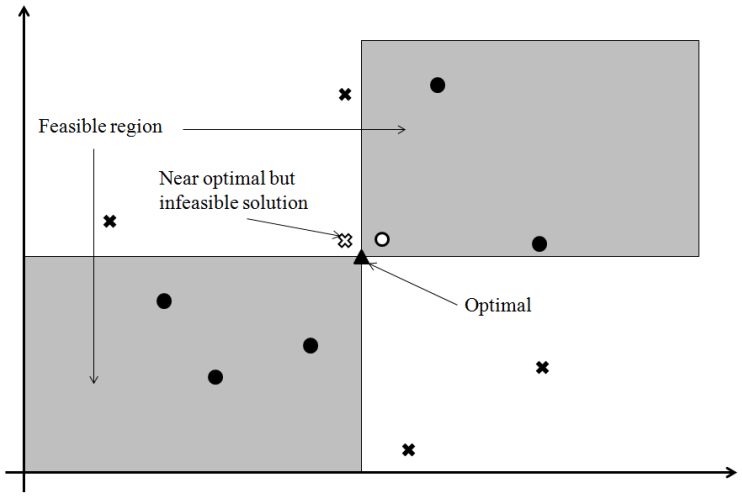


Figure 40 Example of search space with restrictions and feasible region.

3.11. MATLAB GLOBAL OPTIMIZATION TOOLBOX

As previously mentioned, MATLAB is a mathematical computation tool used to develop and simulate numerous scientific and engineering problems. It contains the *Fuzzy Logic Toolbox* plus the Global Optimization Toolbox (GOT) to work with GAs. This toolbox is a collection of functions that includes routines that allows solving optimization problems by using GAs. Those functions are the `ga`, the `gaoptimset` and the `gaoptimget` [24].

The MATLAB version used for this work was the R2011b and the Global Optimization Toolbox was version 6.1.

To solve a minimization or maximization problem using this toolbox it is necessary to define the problem's fitness function to then minimize or maximize. The toolbox itself does not applies the maximization, but instead solves the problem by using the form:

$$\min f(x)$$

So if the intention is to maximize it can be done by minimizing $-f(x)$.

First of all, it is important to mention that the images displayed in this section result from the application and manipulation of the MATLAB toolbox software [24]. Overall, there are two different ways to start the GA toolbox, i.e., this can be done by using the command line or by means of a graphical interface. To use the GA from the command line, the basic syntax presented below must be written, allowing the call of the `ga` function displayed:

```
[x fval] = ga(@fitnessfun, nvars, options)
```

Where:

`@fitnessfun` is a handle to the fitness function.

`nvars` is the number of independent variables for that fitness function.

`options` is a structure that contains options for the genetic algorithm. In case this argument does not pass the default options will be assumed.

Then the results are stored in:

`fval` is the final value of the fitness function.

`x` is the point at which the final value is attained.

The function `gaoptimset` is used to set the options parameters in the genetic algorithm. It is used to set parameters such as population size, restrictions, limits, operators, stopping criteria and the remaining used in genetic algorithms. Once the options are set, it is possible to see the selected ones that will be used by the `ga` function. To do that it is required to use the `gaoptimget` function.

Despite the previously mentioned, to obtain more information regarding the several `ga` function options it is important to analyse the user's manual or to use the command line typing `'help ga'`.

In case the user wants to use the GA with the graphical interface, and without using the command line, the syntax `optimtool ('ga')` − that opens the window shown below (Figure 41) − must be entered.

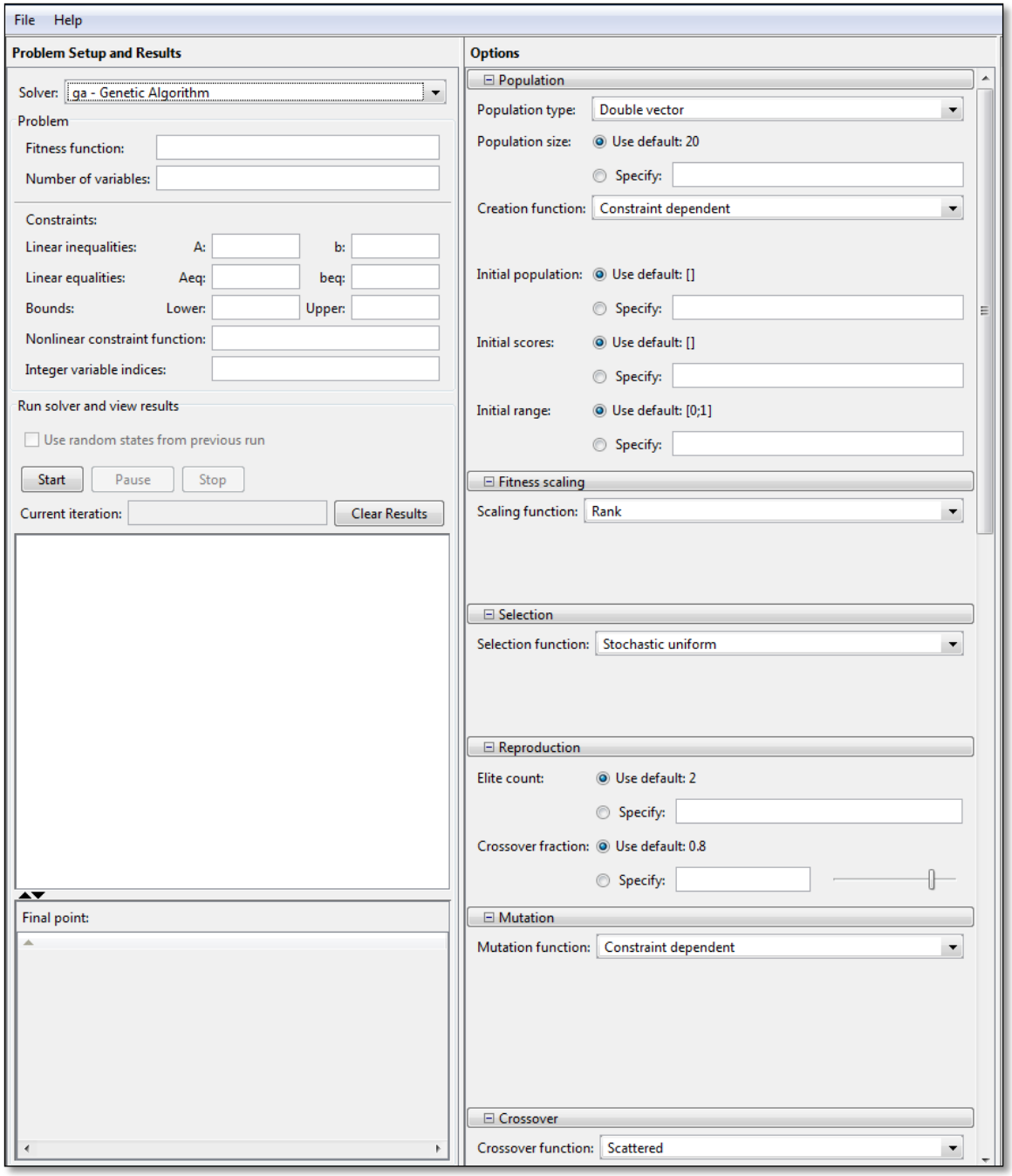


Figure 41 Genetic algorithm solver graphical interface.

4. OPTIMAL TUNING OF FUZZY PID CONTROLLERS

This chapter presents the simulation results for the Fuzzy Logic PID controller tuned with GAs (FL-PID). Simulations were also performed for the PID controller tuned with ZN step-response method (PID-ZN) and for the PID controller tuned with GAs method (PID-GA). In addition to these tuning methods, an optimal tuning method named Zhuang Atherton (ZA) setpoint method was also applied for comparison purposes. The mentioned controllers were tested with six benchmark systems with different dynamics and characteristics.

The main purpose of this work is to develop a FL-PID controller that uses GAs to achieve the parameters optimization, and thus reducing the system step response error to a minimum. For that, several performance indices were adopted in the simulations. Moreover, the step responses and the performance indices results were compared in order to evaluate the effectiveness of the GAs in tuning the PID and Fuzzy PID controllers.

4.1. BENCHMARK SYSTEMS

The proposed Fuzzy controller intends to be used for controlling several types of systems without the need of changing the controller itself, but only their parameters in order to

control that particular system. These parameters should then be tuned with GAs to find the optimal values.

The FL-PID controller must be compared with other type of control to evaluate its performance. The chosen controller used for comparison purposes was the classical PID controller. To compare both controllers' performances, six benchmark systems with distinct characteristics are proposed:

•System 1: First order system with delay [25]:

$$G(s) = \frac{0.014683e^{-8s}}{s + 0.02732} \tag{28}$$

• System 2: Fast and slow mode system [26]:

$$G(s) = \frac{100}{(s+10)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right) \tag{29}$$

• System 3: Fourth order system [26]:

$$G(s) = \frac{1}{(1+s)(1+s0.5)(1+s0.5^2)(1+s0.5^3)} \tag{30}$$

• System 4: Fourth order system with three zeros [25]:

$$G(s) = \frac{0.050561(s-4)(s-0.2667)(s+0.03333)}{(s+3.931)(s+0.04015)(s^2+0.1881s+0.01139)} \tag{31}$$

• System 5: Inverted pendulum system [27]:

$$G(s) = \frac{7.407s}{s^3 + 0.26s^2 - 36.296s - 7.26} \tag{32}$$

• System 6: Time varying system with the following differential equation [28]:

$$\ddot{y}(t) + e^{-0.2t}\dot{y}(t) + e^{-5t}\sin(2t+6)y(t) = u(t) \tag{33}$$

The systems were controlled using both types of control (PID and FL-PID). Firstly, systems were controlled with PID-ZN and PID controller tuned with Zhuang and Atherton method (PID-ZA). It is important to mention that these methods cannot be used for systems 5 and 6 since they cannot be approximately modelled by a first order plus time delay (FOPDT) model. Later, systems were controlled using PID-GA and FL-PID controllers.

To approach the simulation to a real control system, a non-linearity was implemented on the controller output. The saturation levels were set equal for both PID and FL-PID controllers, so they can be compared with each other.

4.2. PERFORMANCE INDICES

The error measurement allows the evaluation on how the system tuning is being evaluated. If the tuning is correctly performed, the error is low; however, if the tuning is poor, the error value is higher. Bearing this in mind, and considering the system response, the performance indices can be achieved by measuring the system step response error.

There are several performance indices errors [29]. Nevertheless, the work focus on four, namely the Integral of Absolute Error (IAE), the Integral of Square of the Error (ISE), the Integral Time multiplied by the Absolute Error (ITAE) and the Integral of Time multiplied by the Square Error (ITSE).

The IAE criterion measures the absolute error deviation from the reference along the time and the mathematical expression is as follows [4]:

$$IAE = \int_0^T |e(t)| \, \mathrm{dt} \tag{34}$$

The main disadvantage of the ISE criterion is that it amplifies large errors and consequently, leads to closed-loop oscillation. The mathematical representation is as follows [4]:

$$ISE = \int_0^T e^2(t) \, \mathrm{dt} \tag{35}$$

To minimize the initial error impact of the step response, and to amplify the error in steady-state the ITAE was also used. This performance index is the one that provides the best selectivity and whose mathematical expression is [4][29]:

$$ITAE = \int_0^T t|e(t)| \, \mathrm{dt} \tag{36}$$

The other used criterion was the ITSE, similar to the ITAE criterion, whose mathematical expression is [29]:

$$ITSE = \int_0^T te^2(t)\,\mathrm{dt} \tag{37}$$

In all performance indices expressions the parameter T is the simulation time and $e = r - y$ is the error, as the difference between the reference signal (step) and the output response.

4.3. PID CONTROLLER

The controller used to compare with the FL-PID was the classical PID and the SIMULINK diagram used was a simple PID with a low-pass filter on the derivative part, without disturbances for simplicity purposes (Figure 42). To simulate a real control system, a saturation was implemented after the controller which expression is as follows:

$$f(e) = \begin{cases} T \; if \; e > T \\ e \; if \; -T \leq e \leq T \\ -T \; if \; e < -T \end{cases} \tag{38}$$

This model accepts gains (K_p, K_i, K_d) as inputs for the PID block in accordance to ZN, ZA or GA methods and returns IAE, ISE, ITAE and ITSE performance indices.

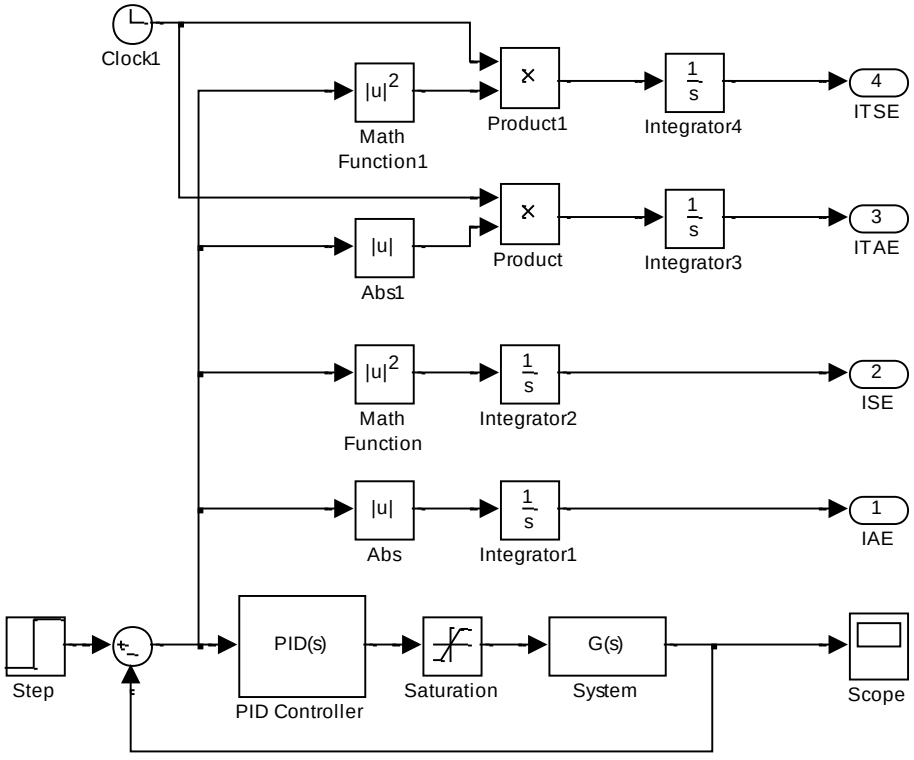


Figure 42 Block diagram of the PID control system.

As previously shown, the expression of a classical PID controller can be given as:

$$u(t) = K_p \left[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right] \tag{39}$$

or

$$u(t) = K_p * e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \tag{40}$$

Where:

$$K_p = K_p \tag{41}$$

$$K_i = \frac{K_p}{T_i} \tag{42}$$

$$K_d = K_p * T_d \tag{43}$$

The parameters K_p, K_i and K_d are the proportional, integral and derivative gains, respectively. These gains are the variables subjected for tuning in the PID controller.

For practical applications, it is common to apply a first order low-pass filter in the derivative part to reduce the noise that is produced in the derivative part [30][28]. The transfer function of the PID with the filter in derivative part can be represented as follows:

$$G(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{s T_d}{1 + \frac{s T_d}{N}} \right) \tag{44}$$

The first order low-pass filter has a coefficient N, which is normally set to $N \geq 10$ if both controllers, (40) and (44), should have similar responses. In this work we set $N = 100$.

To obtain the PID gains using ZN and ZA tuning methods the system should be represented as an approximation of a FOPDT model whose transfer function is [30]:

$$G(s) = \frac{K e^{-sL}}{1 + sT} \tag{45}$$

where K, L and T are the system gain, time delay and time constant, respectively.

Dingyu Xue *et al.* (2007) [28] proposed a MATLAB function `(opt_app)` that identifies the system and approximates it to a FOPDT model. The MATLAB syntax that finds the FOPDT model parameters is shown below [28]:

```
Gr = opt_app(G,0,1,1);
[n,d] = tfdata(Gr,'v');
k = dcgain(Gr);
T = d(1)/d(2);
L = Gr.ioDelay;
```

The ZN gains calculation was done using a MATLAB function `(ziegler)` also proposed by Dingyu Xue *et al.* (2007) [28], based on the ZN step response and frequency response methods (Table 4). The function receives the transfer model parameters and returns the PID parameters. The function syntax is as follows:

```
% ZN step response method
[Gc,Kp,Ti,Td] = ziegler(3,[k L T N])
Kp = Kp;
Ki = Kp/Ti;
Kd = Kp*Td;
```

Table 4 PID controller parameters obtained from the ZN methods [28].

	Step response			Frequency response		
Control type	K_p	T_i	T_d	K_p	T_i	T_d
P	$1/a$			$0.5K_u$		
PI	$0.9/a$	$3L$		$0.4K_u$	$0.8T_u$	
PID	$1.2/a$	$2L$	$L/2$	$0.6K_u$	$0.5T_u$	$0.125T_u$

Where:

$$a = KL/T \tag{46}$$

The parameters K_u and T_u given in Table 4 are obtained when the system is in the limit of stability, i.e. oscillating, and known as the *ultimate gain* and the *ultimate period*, respectively.

Table 5 PID controller parameters using ZA setpoint method [28].

Range of L/T	$0.1-1$			$1.1-2$		
Criterion	ISE	ITSE	IST^{2}E	ISE	ITSE	IST^{2}E
a_1	1.048	1.042	0.968	1.154	1.142	1.061
b_1	-0.897	-0.897	-0.904	-0.567	-0.579	-0.583
a_2	1.195	0.987	0.977	1.047	0.919	0.892
b_2	-0.368	-0.238	-0.253	-0.220	-0.172	-0.165
a_3	0.489	0.385	0.316	0.490	0.384	0.315
b_3	0.888	0.906	0.892	0.708	0.839	0.832

To find the PID gains using the ZA tuning methods another function `(optpid)` was used [28]. The PID parameters are calculated as follows:

$$K_p = \frac{a_1}{K}\left(\frac{L}{T}\right)^{b_1} \tag{47}$$

$$T_i = \frac{T}{a_2 + b_2\left(\frac{L}{T}\right)} \tag{48}$$

$$T_d = a_3 T\left(\frac{L}{T}\right)^{b_3} \tag{49}$$

Where for different ratio $\frac{L}{T}$, the coefficients (a, b) are defined in Table 5.

The ZA setpoint method uses three performance indices to calculate the PID parameters, the ISE, the ITSE and the Integral Squared Time-squared weighted Error (IST^{2}E) [28]. However, in this work it was decided to use only the ITSE criterion. The statement syntax to find the PID parameters is represented below:

```
% ZA setpoint method using ITSE criterion
[Gc kp Ti Td]= optpid(3, 1, [k L T N 2]);
Kp=kp;
Ki=kp/Ti;
Kd=kp*Td;
```

4.4. FUZZY LOGIC PID CONTROLLER

Regarding the FL-PID controller, it was used a Fuzzy Logic PD+I controller whose diagram is represented in Figure 43 and the expression is as follows [31]:

$$u(t) = \left[f\left(K_e * e(t), K_{ce} * ce(t)\right) + K_{ie} * ie(t)\right] * K_u \tag{50}$$

where $f(.)$ is a nonlinear function of inputs, error (e) and derivative of error (ce).

To test if the FL-PID controller is properly designed it is possible to use the PID controller gains results from the ZN method, but with the FL-PID controller linear approximation. Then both step responses must be exactly the same [31].

The linear approximation of the fuzzy PID diagram is [31]:

$$u(t) = [K_e * e(t) + K_{ce} * ce(t) + K_{ie} * ie(t)] * K_u \tag{51}$$

$$u(t) = K_e * K_u * \left[e(t) + \frac{K_{ce}}{K_e} * ce(t) + \frac{K_{ie}}{K_e} * ie(t)\right]; K_e \neq 0 \tag{52}$$

Where:

$e(t) = Error$

$ce(t) = Change\ on\ Error$

$ie(t) = Integral\ of\ Error$

$K_e = Gain\ of\ Error$

$K_{ce} = Gain\ of\ Change\ on\ Error$

$K_{ie} = Gain\ of\ Integral\ of\ Error$

$K_u = Gain\ of\ Output$

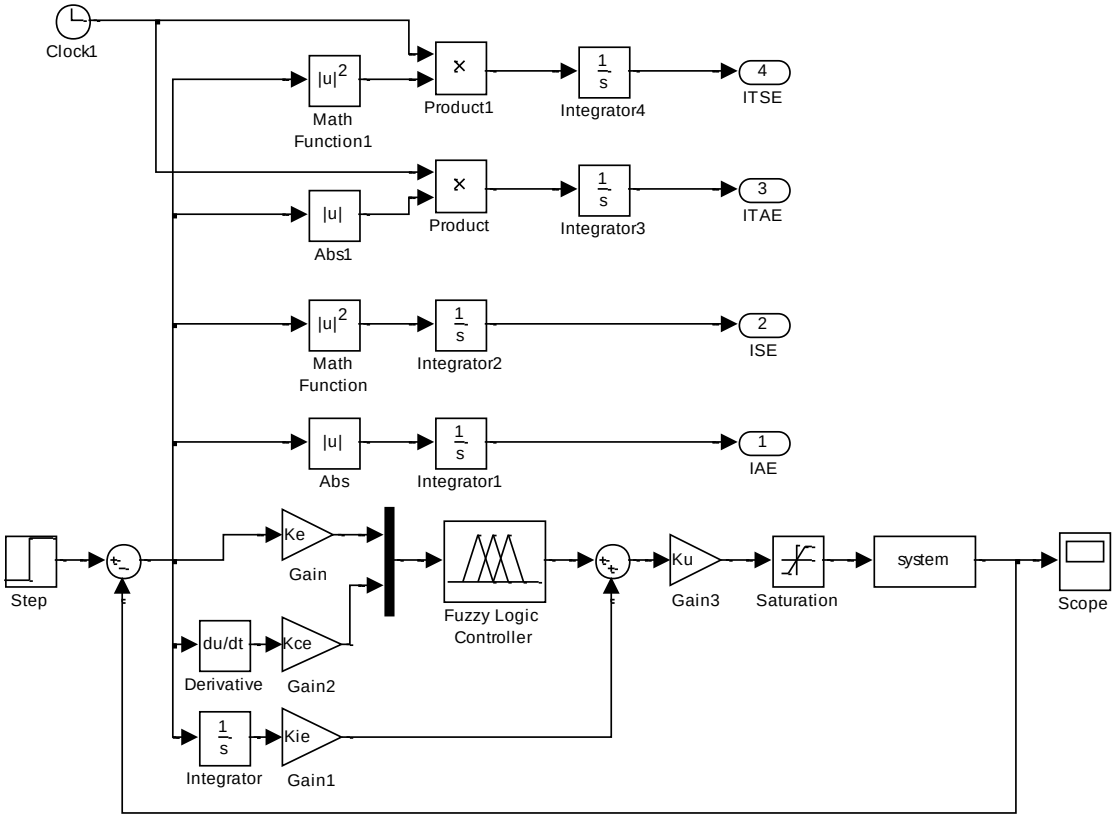


Figure 43 Block Diagram of the Fuzzy PD+I control system.

The FL-PID controller linear approximation implies that the rules-base should act as a summation. The input and output sets have to be triangular functions and 50% overlapped, i.e., the sets should be crossed at $\mu = 0.5$, for the AND connective must be used the algebraic product, the output of the rule base must be the AND product of all inputs and the COG defuzzification method must be used [31]. This controller has four gains instead of three $-K_e$, K_{ce}, K_{ie} and K_u- thus, this controller has four degrees of freedom against three in the classical PID. The K_e should be non-zero and must be set as the maximum error in the fuzzy controller. The other three gains are calculated as follows:

$$K_{ce} = K_e * \frac{K_d}{K_p} \tag{53}$$

$$K_{ie} = K_e * \frac{K_i}{K_p} \tag{54}$$

$$K_u = \frac{K_p}{K_e} \tag{55}$$

The SIMULINK model presented above (Figure 43) is used for all the displayed systems and is based on a Mandani fuzzy inference mechanism.

Seven linguistic terms were used for both inputs (error and change on error) and output and since the terms are 7, the rules used for all the systems are 49 and are listed in Table 6.

Table 6 Fuzzy rule-base for output.

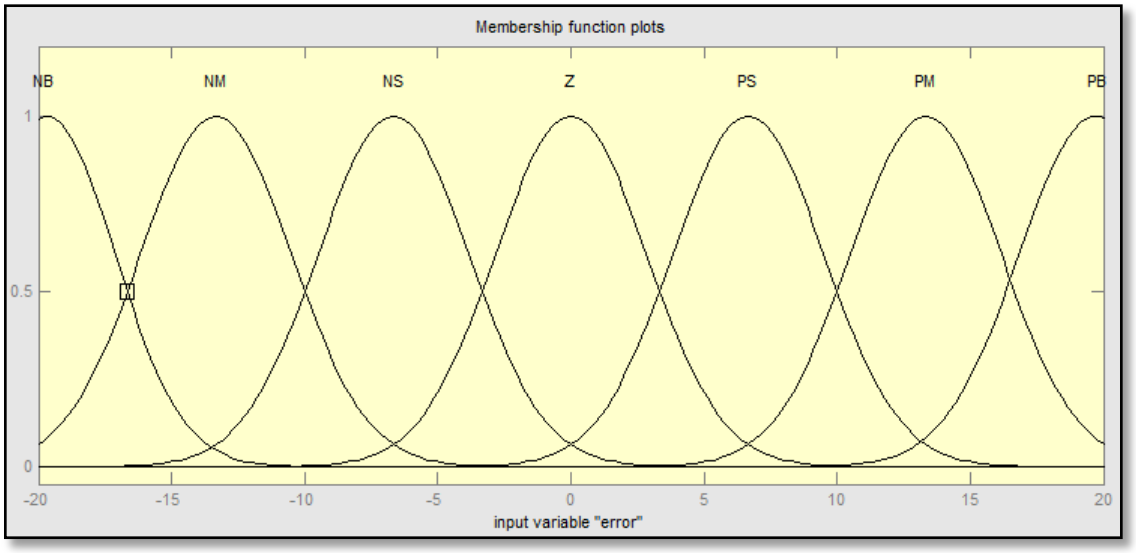
$u(t)$		$e(t)$						
		NB	**NM**	**NS**	**Z**	**PS**	**PM**	**PB**
$\frac{d}{dt}e(t)$	**NB**	NB	NB	NB	NB	NM	NS	Z
	NM	NB	NB	NB	NM	NS	Z	PS
	NS	NB	NB	NM	NS	Z	PS	PM
	Z	NB	NM	NS	Z	PS	PM	PB
	PS	NM	NS	Z	PS	PM	PB	PB
	PM	NS	Z	PS	PM	PB	PB	PB
	PB	Z	PS	PM	PB	PB	PB	PB

Before choosing the membership functions, some tests were made in order to check which function better fits the proposed problems. Simulations, using triangular functions and Gaussian functions, were made. The results with Gaussian functions revealed to be better for the majority of systems. Therefore, the Gaussian membership functions were chosen for all the linguistic variables (error, change on error and output).

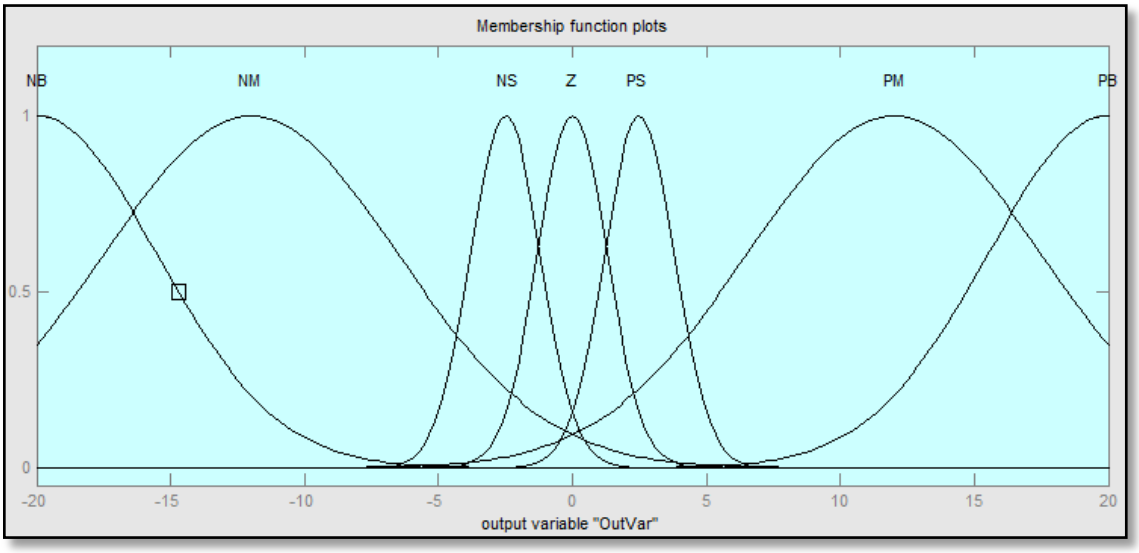
In order to find the input and output membership function limits that better fit all the proposed systems some tests were also done. The best results were achieved for the linguistic variables limits between ± 20 for both inputs and output. The distribution of the membership functions along the universe of discourse were chosen empirically and were set as in Figure 44. For this fuzzy inference mechanism the corresponding surface is shown in Figure 45.

Another parameter from the Mandani inference mechanism, that was changed from its default value, is the antecedents of rules AND method, that was set to use the *product (*)* operator instead of the *minimum* $(min[\mu_{A^1}(x), \mu_{A^2}(x), \dots, \mu_{A^L}(x)])$ operator. The remaining parameters were left as default. The OR method for the antecedents rules, uses the default *maximum* $(max[\mu_{A^1}(x), \mu_{A^2}(x), \dots, \mu_{A^L}(x)])$ operator. The rules' weight is the same for all rules and was left as default, i.e., equal to one. The implication method of rules

uses the default *minimum* operator. The aggregation of rules uses the default *maximum* operator and the defuzzification method to find the crisp output value is the default COG. These parameters were set as displayed in Figure 46.



(a)



(b)

Figure 44 Gaussian membership functions of inputs (a) and output (b).

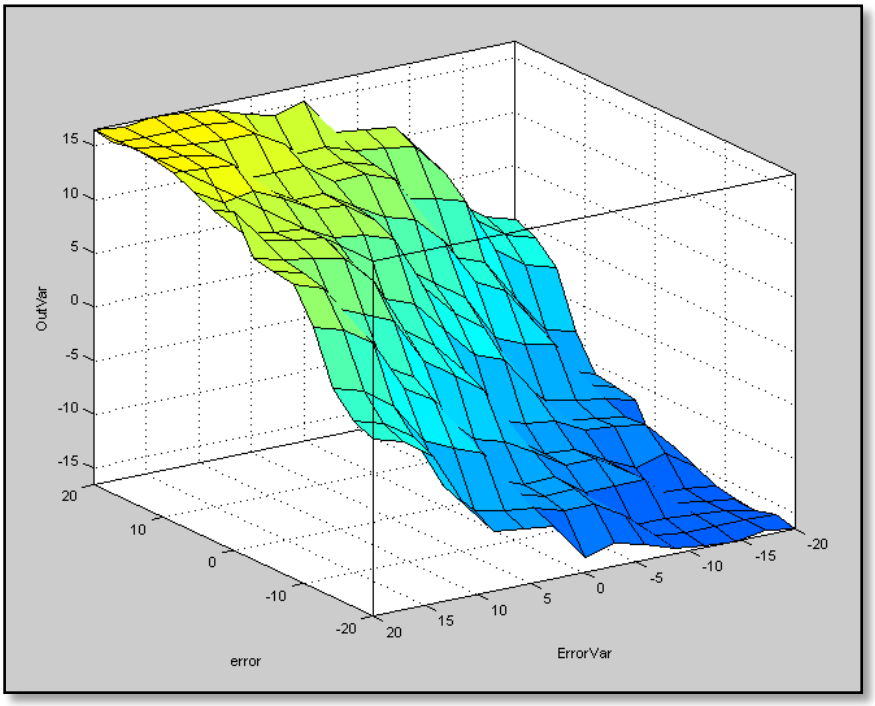


Figure 45 Output control surface.

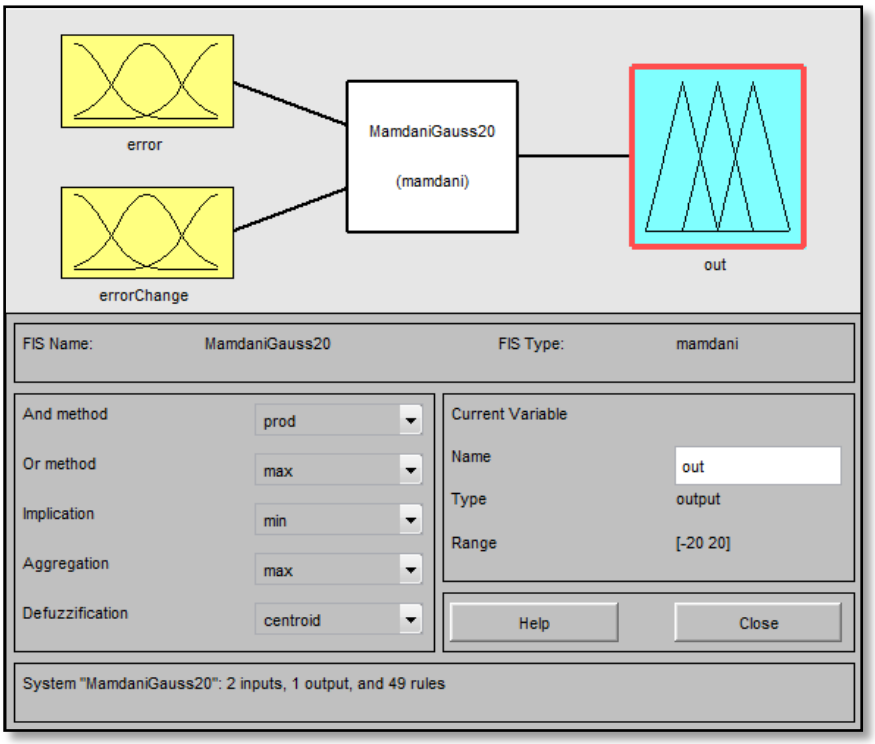


Figure 46 Fuzzy inference system.

4.5. STRUCTURE OF THE GENETIC ALGORITHM

GAs were chosen to find the optimal parameters for the PID and FL-PID controllers; for that a MATLAB program was created. This program uses the function `ga` available in the MATLAB GOT [24]. The function that allows changing the `ga` parameters is the `gaoptimset`. The changed parameters are: the number of generations, the population initial range, the population size, the creation function, the selection function, the crossover function, the mutation function, the tolerance function and the initial population; the remaining parameters were left as default. To create the initial population the function `gacreationlinearfeasible` was used; this is the default creation function when there are constrains. An individual was inserted to the initial population − the resulting gains of the ZN tuning, except for systems 5 and 6 where there are no results. The crossover function used is the `crossoverscattered`, and the mutation function used is the `mutationadaptfeasible`.

The stopping criterion, number of generations, was changed to 70 generations instead of the default value of 100 generations. This number of generations was chosen because after some tests with both numbers of generations the fitness results were very similar in all the cases. Despite this, it is important to mention that for some cases, 30 generations are sufficient. Overall, 100 generations do not significantly improve the fitness functions, and the costs of processing time do not compensate the improvement. The processing time for 100 generations is approximately 30% more than the time to process 70 generations. In fact, in some cases the algorithm started to converge far before the 70 generations. Since the purpose of this work is to develop a FL-PID that covers the largest number of systems possible, 70 generations seems to be a good commitment between the algorithm performance and the systems response. Despite this stopping criterion, the algorithm should stop after a certain number of generations, if the fitness does not improve, even if the number of generations stopping limit is not reached. Thus, the GA option tolerance function value for the stall generations limit was changed to 0 and the stall number of generations was changed to 20.

The number of chromosomes are denoted as population size and defined as 30 individuals. Initially, some tests were performed with 50 individuals but the processing time was very high and does not justify the cost regarding the results, i.e., the fitness values were very similar. During the first tests, the algorithm showed that it is preferable to have more

generations rather than more individuals. The tournament selection method used was the `selectiontournament` that increases some pressure in the algorithm, since the global optimal should be near the results of the ZN methods. This function uses four randomly chosen individuals. Since the number of best individuals that migrate to the new generation is two, the elite-count was set on 2 individuals. Some plots were selected to show information such as the current best solution, the mean and current best individual, while the algorithm is running. A summary with the GA parameters used in all simulations are presented in Table 7.

Table 7 GA parameters used in all simulations.

	PID	FL-PID
Number of variables	3	4
Individuals	30	
Generations limit	70	
Stall generations limit	20	
Tolerance function value	0	
Elite-count	2	
Creation function	`gacreationlinearfeasible`	
Crossover function	`crossoverscattered`	
Mutation function	`mutationadaptfeasible`	
Selection function	`selectiontournament`	
Plot functions	`gaplotbestf ; gaplotbestindiv`	

The GA options syntax is listed below:

```
options = gaoptimset('PlotFcns',...
{@gaplotbestf,@gaplotbestindiv},...
'SelectionFcn',@selectiontournament,...
'CreationFcn',@ gacreationlinearfeasible,...
'CrossoverFcn',@crossoverscattered,...
'MutationFcn',@mutationadaptfeasible,...
'StallGenLimit',20,...
'TolFun',0,...
'EliteCount',2,...
'Generations',70,...
'PopulationSize',30);
```

The lower bound for the variables is zero since the gains should be positive. The initial population is defined for each system individually, the values differ from every system, and these are the results of the ZN method for that particular system. The syntax examples displayed below are from System 1; nevertheless, it is important to mention that the procedure is the same for the remaining systems. An example of the code, that uses the `ga` function to find the optimal gains for System 1 PID-GA and IAE criterion, is shown below:

```
vars=3;
options.PopInitRange=[0 0 0;15 1 40];
options.InitialPopulation=[Kp_ZN Ki_ZN Kd_ZN];
[X,FVAL] = ga(@Fit_01_IAE,vars,[],[],[],[],[0 0 0
0],[15 1 40],[],options)
```

Another example that uses the `ga` function to find the optimal gains for System 1 FL-PID and IAE criterion is shown below:

```
vars=4;
Ke_ini=20;
Ku_ini=Kp_ZN/Ke_ini;
Kce_ini=Kd_ZN/Ku_ini;
Kie_ini=Ki_ZN/Ku_ini;
options.PopInitRange=[0 0 0 0;30 100 2 2];
options.InitialPopulation=[Ke_ini Kce_ini Kie_ini
Ku_ini];
[X,FVAL] = ga(@Fit_01_IAE,vars,[],[],[],[],[0 0 0
0],[30 100 2 2],[],options)
```

Regarding the fitness function, it receives the variables (gains) and returns the fitness value for the current variables. According to the number of variables, 3 or 4, the function processes the PID or the FL-PID control system, respectively. The SIMULINK model is called and returns the four performance indexes (fitness values) calculated in the system.

An example of the fitness function code for System 1 and IAE criterion is displayed below:

```
function J=Fit_01_IAE(K)
if length(K) < 4
  % PID-GA
  Kp=K(1);
  Ki=K(2);
  Kd=K(3);
  [T X IAE ISE ITAE ITSE]=sim('PID_Sys_01.mdl');
else
  % FL-PID
  Ke=K(1);
  Kce=K(2);
  Kie=K(3);
  Ku=K(4);
  [T X IAE ISE ITAE ITSE]=sim('Fuz_Sys_01.mdl');
end
J=IAE(end);
```

The PC used has installed the Windows7 64 bit Home Premium Service Pack 1 operating system. The CPU is an Intel Core Duo P7450 running at 2.13 GHz with 4.00 GB of RAM. The MATLAB version edition installed is the R2011b 64-bit and the Global Optimization Toolbox (which includes Genetic Algorithms) is the version 6.1.

4.6. SIMULATION AND RESULTS

The simulation and results for the systems response controlled with PID and FL-PID controllers are presented in this section. It is important to mention that, to have results with statistical value, the GA was performed 10 times and the best result was selected. In the first four systems, the PID-ZN and PID-ZA were used and compared with PID-GA and FL-PID. As already mentioned, Systems 5 and 6 were not controlled using the PID-ZN and PID-ZA. The system step responses are presented and the performance indices displayed in order to compare and discuss the obtained results.

4.6.1. SYSTEM 1

System 1 uses a first order system with a delay; it was proposed by Nina *et al.* (2008) [25], and its transfer function is represented as follows:

$$G(s) = \frac{0.014683e^{-8s}}{s + 0.02732} \tag{56}$$

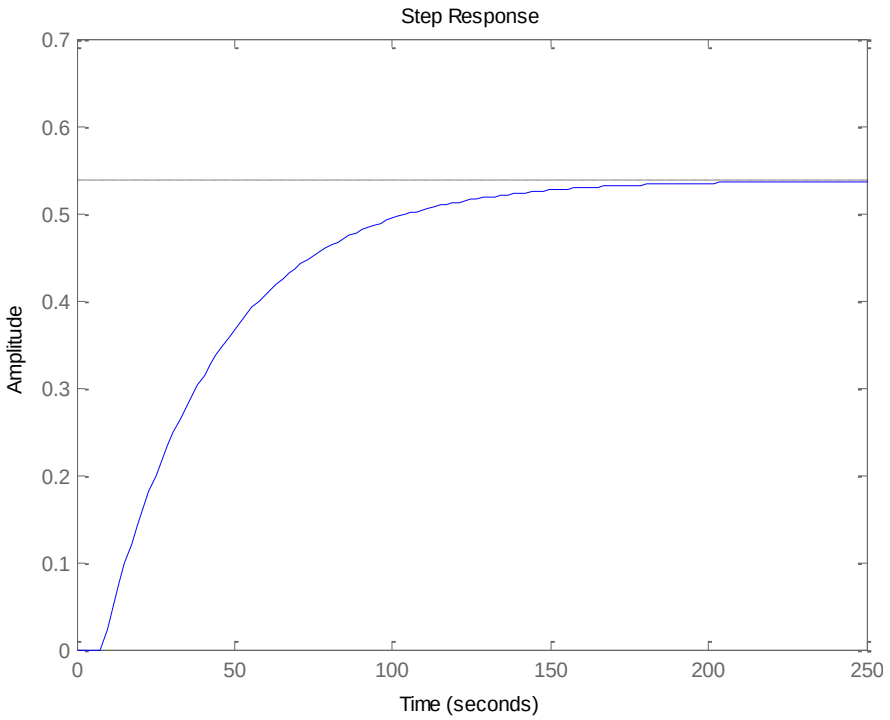


Figure 47 Step response of System 1.

The system uncontrolled step response is shown in Figure 47. According to the system dynamics response, the step time used for the simulations was $\Delta T = 0.01\,s$ and the simulation time was $T = 150\,s$, which are reasonable values and covers the four simulations (PID-ZN, PID-ZA, PID-GA and FL-PID). The controller output saturation

limits were set to ± 10. The initial population and upper bounds used in the GAs are presented in Table 8.

Table 8 GA upper bounds and initial population used for System 1 simulations.

	PID			FUZZY PID			
	K_p	K_i	K_d	K_e	K_{ce}	K_{ie}	K_u
Initial Population	12.0295	0.8853	40.8636	20	67.9391	1.4719	0.6015
Upper Bounds	20	1	45	30	80	2	1

The GA fitness evolution, for the 70 generations, for the IAE, ISE, ITAE and ITSE criteria for PID-GA and FL-PID controllers are shown in Figure 48. The fitness evolution of System 1 converged (meaning that it has approximately the same fitness value of the last generation) in all the tested criteria before the 40th generation. It is notorious that regarding the PID-GA, this convergence for the IAE and ISE criteria occurred right after 10 generations. In the case of the PID, and comparing both controllers, the GA required less generations to converge the fitness to the final value.

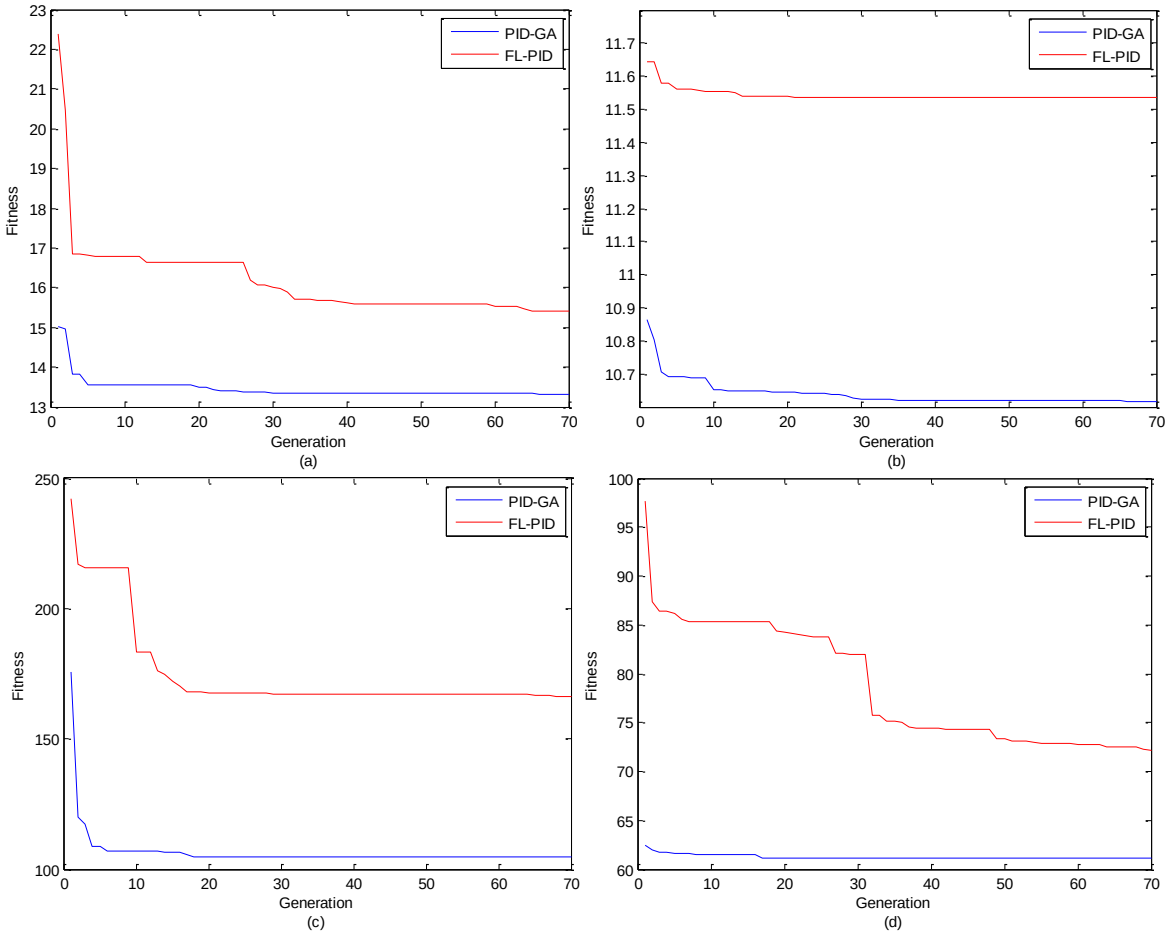


Figure 48 Evolution of best fitness for the IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria regarding System 1.

The obtained results for the PID-ZN, PID-ZA, PID-GA and FL-PID, such as gains, performance indices, percent overshoot (M_p), rise time (T_r) and settling time (T_s) are shown in Tables 9 and 10 for the FL-PID and PID controllers, respectively.

Table 9 Results of Fuzzy PID tuned with GAs for System 1.

	Criterion	Error	Gains				Time specifications		
		J	K_e	K_{ce}	K_{ie}	K_u	$M_p(\%)$	$T_r(s)$	$T_s(s)$
FL-PID	IAE	13.9626	10.4330	26.3018	0.2219	0.6219	1.2055	9.2874	20.5870
	ISE	11.5351	19.9986	60.1656	0.3693	0.4788	5.1205	8.6278	>150
	ITAE	166.3534	20.8174	65.8669	0.3591	0.3462	1.0232	13.4562	26.3099
	ITSE	72.1393	11.6940	44.5881	0.2560	0.5957	4.8667	9.0776	123.4329

Table 10 Results of PID tuned with ZN, ZA and GAs methods for System 1.

	Criterion	Error	Gains			Time specifications		
		J	K_p	K_i	K_d	$M_p(\%)$	$T_r(s)$	$T_s(s)$
ZN	IAE	22.5635	12.0295	0.8853	40.8636	76.1792	6.0196	66.4432
	ISE	16.1235						
	ITAE	391.0852						
	ITSE	202.4182						
ZA	IAE	16.7984	8.8327	0.2719	35.4354	16.7889	6.2088	83.9584
	ISE	10.9634						
	ITAE	311.7143						
	ITSE	72.4907						
GA	IAE	13.3135	7.3085	0.1420	23.6208	2.6730	8.2203	35.5794
	ISE	10.6158	9.8262	0.1754	36.3355	14.9986	6.0327	62.9333
	ITAE	104.5882	7.8950	0.1502	25.6243	7.2207	7.1484	35.9486
	ITSE	61.0970	8.7909	0.1627	31.1613	12.4503	6.4141	44.4589

The step responses of System 1 for the four controllers (PID-ZN, PID-ZA, PID-GA and FL-PID) and for indices IAE, ISE, ITAE and ITSE are shown in Figure 49. It is possible to state that the system step response presents worse results when controlled with the PID-ZN controller, and is notorious the big overshoot when comparing with the other controllers. In fact, the error was higher in all the measured criteria using this controller. The step responses analysis showed that this controller presents the worst results, except for the rise time, that is the best for all simulations, although, for ISE criterion the difference is not significant when comparing with the PID-GA. The FL-PID presents the lowest overshoot in all criteria, and the fastest settling time for IAE and ITAE criteria. The rise time of the FL-PID is the slowest of all, but not significantly different from the other controllers, except in the case of the ITAE criterion. In fact, this characteristic is the reason the fitness value is not so good has the PID-GA, which has the best fitness in all criteria. The FL-PID

settling time for ISE and ITSE criteria is the highest observed, having an oscillation in the steady-state. It is important to note that the ISE and ITSE criteria penalises more the initial error than the steady-state error, and this is the reason of this oscillation for these criteria.

Overall, it is possible to state that for this system, the FL-PID presents lower overshoot for all criteria and lower settling time in the IAE and ITAE criteria. Even though, the error criteria are not the best, only the PID-ZN presents worse results. The PID-GA controller presents the most satisfactory results comparing with the others PID controllers (PID-ZN and PID-ZA).

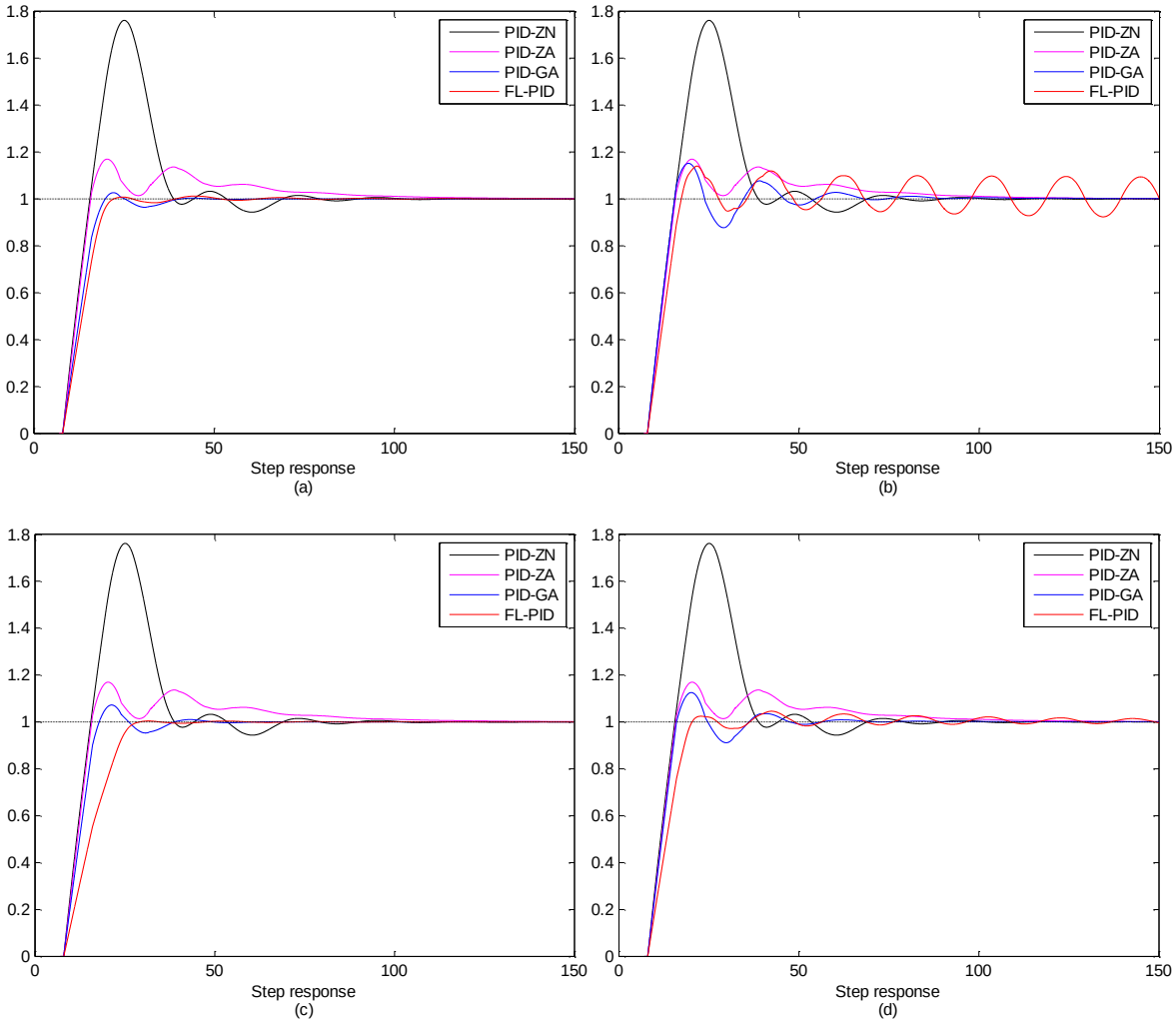


Figure 49 Step responses of System 1 according to IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria.

4.6.2. SYSTEM 2

System 2 uses a fast and slow mode system and was proposed by C. Thammarat *et al.* (2007) [26], and the transfer function is represented as follows:

$$G(s) = \frac{100}{(s+10)^2}\left(\frac{1}{s+1} + \frac{0.5}{s+0.05}\right) \tag{57}$$

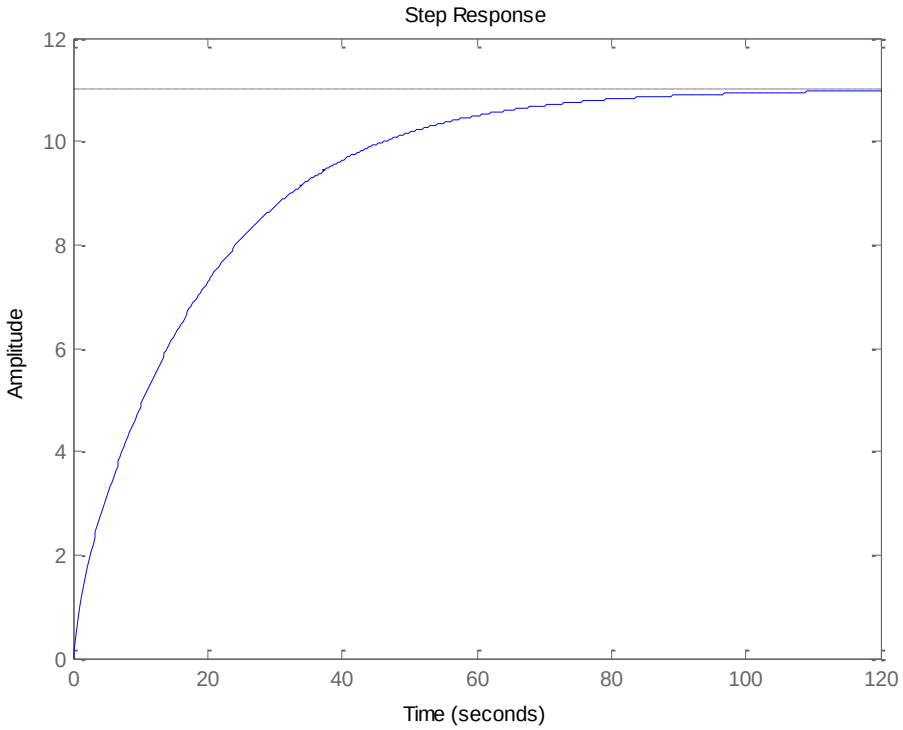


Figure 50 Step response of System 2.

The system uncontrolled step response is shown in Figure 50. The step time used for the simulations was $\Delta T = 0.01\ s$ and the simulation time was $T = 5\ s$, which are reasonable values and covers the four simulations (PID-ZN, PID-ZA, PID-GA and FL-PID). The controller output saturation limits were set to ± 10. The initial population and upper bounds used in the GAs are presented in Table 11.

Table 11 GA upper bounds and initial population used for System 2 simulations.

	PID			FUZZY PID			
Gains	K_p	K_i	K_d	K_e	K_{ce}	K_{ie}	K_u
Initial Population	11.7007	34.1900	1.0011	20	1.7111	58.4409	0.5850
Upper Bounds	70	5	10	40	5	60	10

The GA fitness evolution, for the 70 generations, for IAE, ISE, ITAE and ITSE criteria for PID-GA and FL-PID controllers are shown in Figure 51. The fitness evolution of System 2

converged before 10 generations for all criteria and in both controllers. The number of generations required to converge in both controllers are similar in all cases.

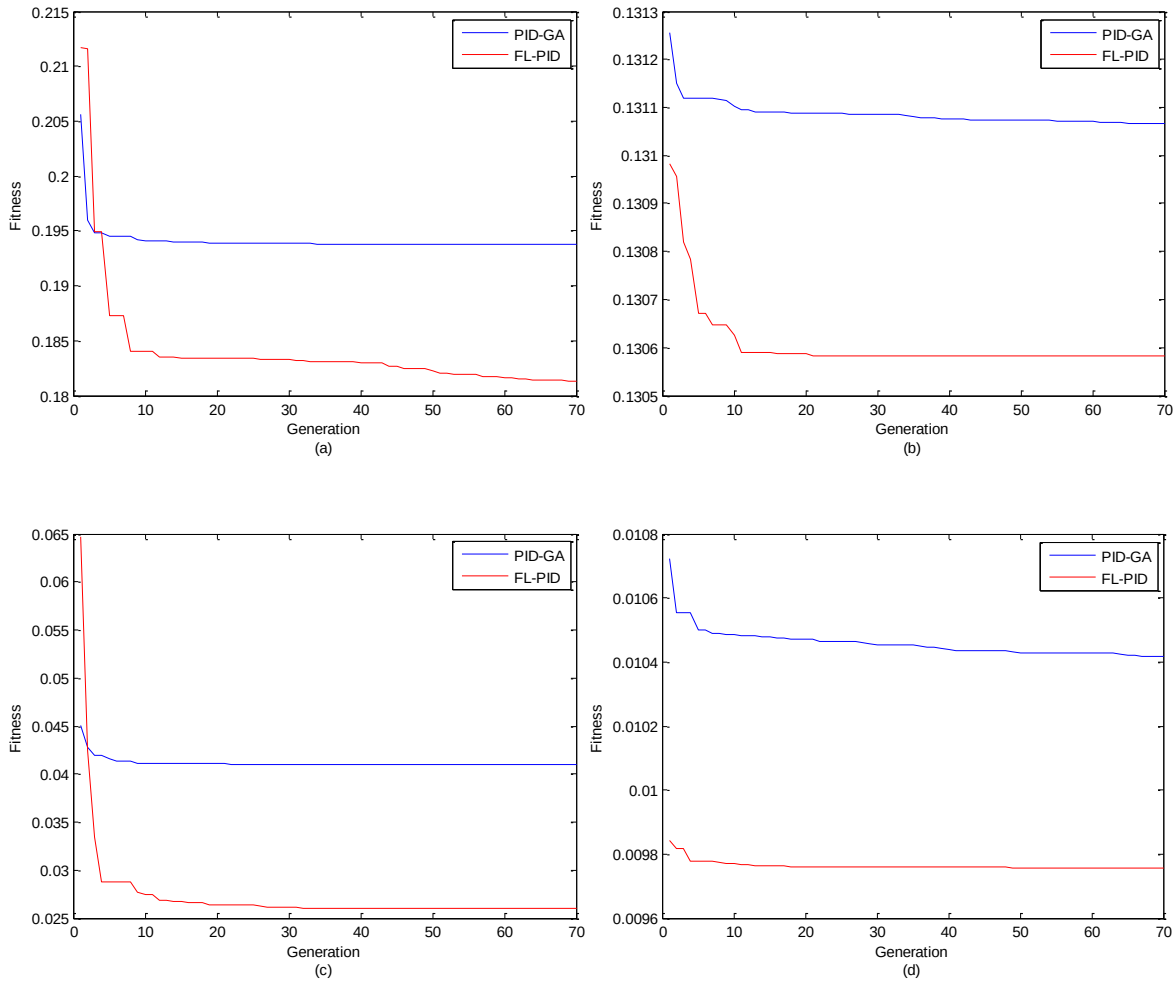


Figure 51 Evolution of best fitness for the IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria regarding System 2.

The obtained results for the PID-ZN, PID-ZA, PID-GA and FL-PID, such as gains, performance indices, percent overshoot (M_p), rise time (T_r) and settling time (T_s) are shown in Tables 12 and 13 for the FL-PID and PID controllers, respectively.

Table 12 Results of Fuzzy PID tuned with GAs for System 2.

	Criterion	Error	Gains				Time specifications		
		J	K_e	K_{ce}	K_{ie}	K_u	$M_p(\%)$	$T_r(s)$	$T_s(s)$
FL-PID	**IAE**	0.1813	28.8210	2.6536	0.7620	1.5682	2.6727	0.1551	0.6230
	ISE	0.1306	26.5711	2.3094	0.9621	9.3268	1.0385	0.1481	>5
	ITAE	0.0260	15.8725	1.5510	0.3868	3.0192	1.2151	0.1605	0.5693
	ITSE	0.0098	20.0447	2.0265	0.5693	2.5504	2.5381	0.1508	0.5943

Table 13 Results of PID tuned with ZN, ZA and GAs methods for System 2.

	Criterion	Error	Gains			Time specifications		
		J	K_p	K_i	K_d	$M_p(\%)$	$T_r(s)$	$T_s(s)$
ZN	**IAE**	0.3137	11.7007	34.1900	1.0011	54.8041	0.1377	1.0934
	ISE	0.1869						
	ITAE	0.0982						
	ITSE	0.0332						
ZA	**IAE**	0.3303	6.3133	0.4099	0.8918	0	0.3297	2.1734
	ISE	0.1640						
	ITAE	0.2211						
	ITSE	0.0208						
GA	**IAE**	0.1938	40.4850	1.1655	3.8210	4.4551	0.1484	0.9504
	ISE	0.1311	39.2095	2.1186	3.4777	5.3982	0.1449	0.9861
	ITAE	0.0410	42.9914	1.0016	4.6676	2.0130	0.1649	0.8765
	ITSE	0.0104	49.2967	1.3481	4.6900	4.1245	0.1489	1.1165

The system step responses for the four controllers (PID-ZN, PID-ZA, PID-GA and FL-PID) and for the indices IAE, ISE, ITAE and ITSE are shown in Figure 52. Regarding the results for System 2, the system step responses also presents worse results when controlled with the PID-ZN controller; in fact the error was also higher for all the measured error criteria. The step response analysis presented worse results, except for the rise time, that was better for all simulations. The FL-PID presents better overshoot in all simulations, around half the overshoot of the PID-GA and PID-ZA controllers. The rise time of the four controllers is similar, except for the PID-ZA, that is more than the double of the rise time of the other controllers. The best results in error measurement criteria was achieved by the FL-PID controller, despite by a small margin when comparing with the PID-GA.

Overall, it is possible to state that for this system, all controllers had similar responses, except for the PID-ZN, that presents a big overshoot comparing with the other controllers. Even though, with similar responses, in general the FL-PID presents better results.

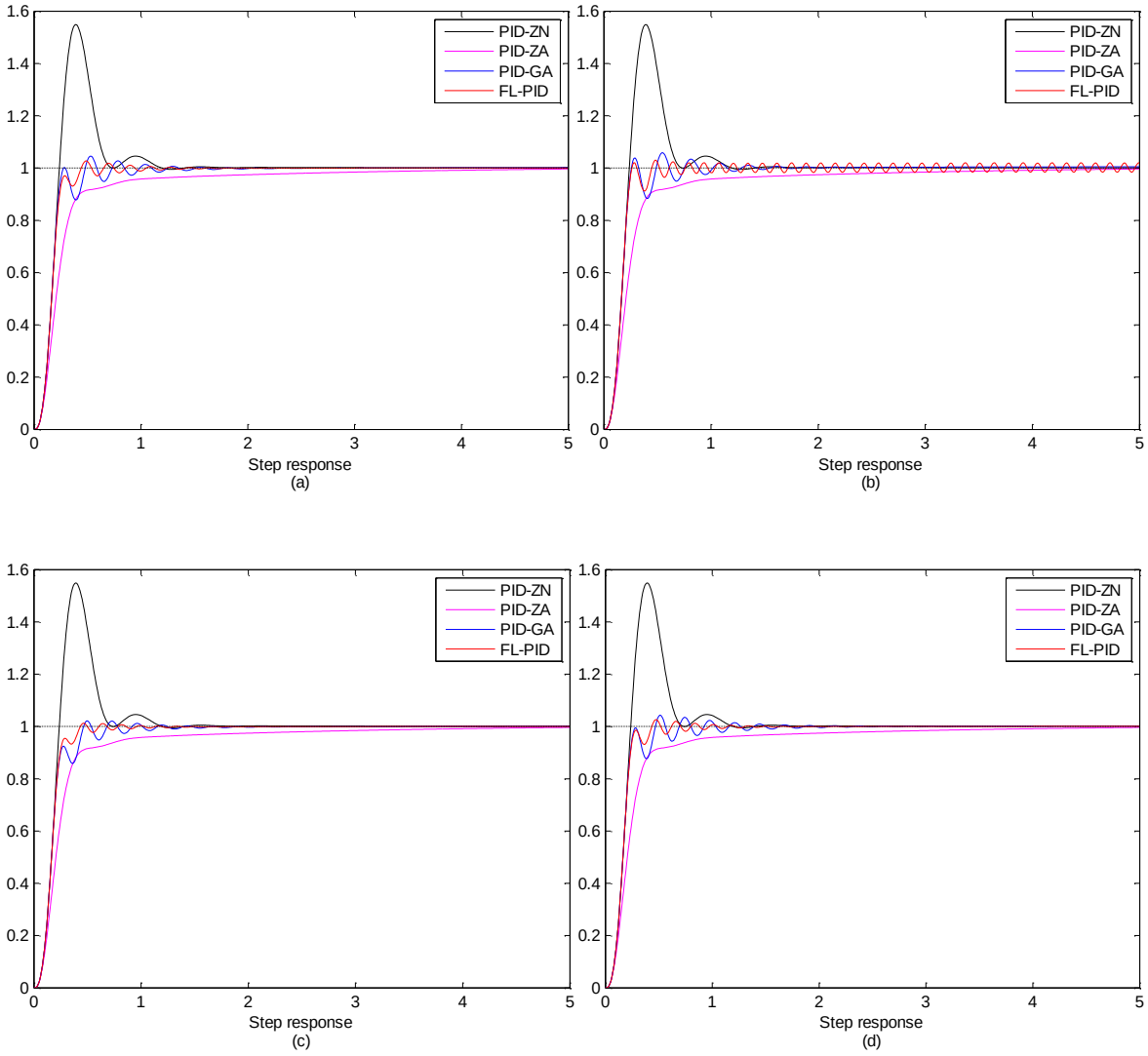


Figure 52 Step responses of System 2 according to IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria.

4.6.3. SYSTEM 3

System 3 uses a four order system and was proposed by C. Thammarat *et al.* (2007) [26], and the transfer function is as follows:

$$G(s) = \frac{1}{(1+s)(1+s0.5)(1+s0.5^2)(1+s0.5^3)} \tag{58}$$

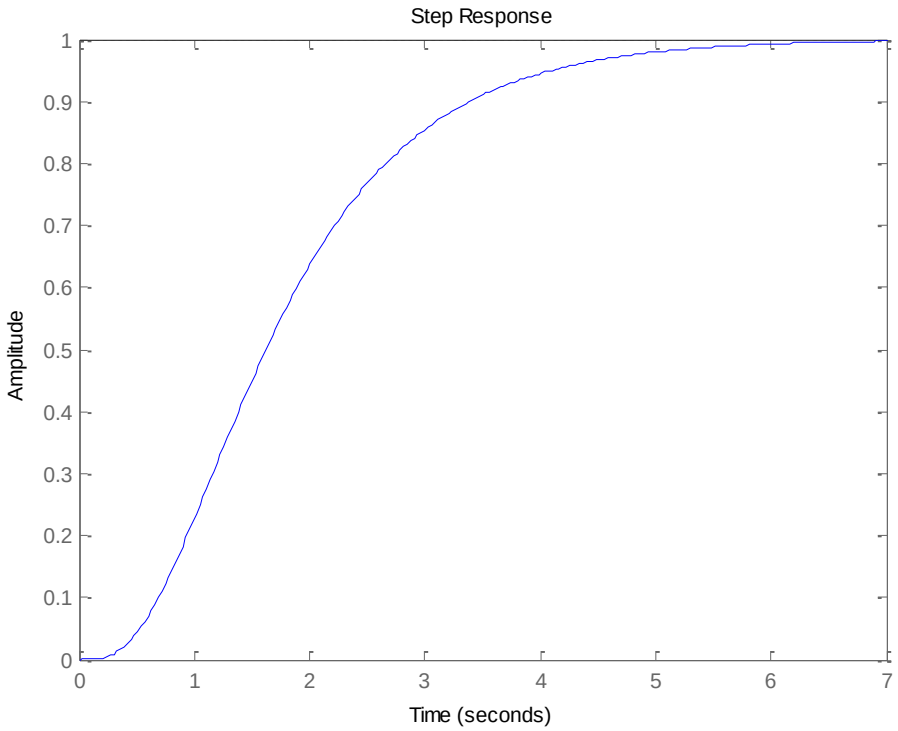


Figure 53 Step response of System 3.

The system uncontrolled step response is shown in Figure 53. According to the system dynamics response, the step time used for the simulations was $\Delta T = 0.005\ s$ and the simulation time was $T = 10\ s$, which are reasonable values and covers the four simulations (PID-ZN, PID-ZA, PID-GA and FL-PID). The controller output saturation limits were set to ± 10. The initial population and upper bounds used in the GAs are presented in Table 14.

Table 14 GA upper bounds and initial population used for System 3 simulations.

	PID			FUZZY PID			
	K_p	K_i	K_d	K_e	K_{ce}	K_{ie}	K_u
Initial Population	2.2938	1.7295	0.7605	20.000	6.6313	15.0799	0.1147
Upper Bounds	20	5	10	30	10	20	1

The GA fitness evolution, for the 70 generations, for the IAE, ISE, ITAE and ITSE criteria for PID-GA and FL-PID controllers are shown in Figure 54. In System 3, the fitness value converged in all criteria before approximately 20 generations for both controllers. The number of generations needed to converge are similar in all cases.

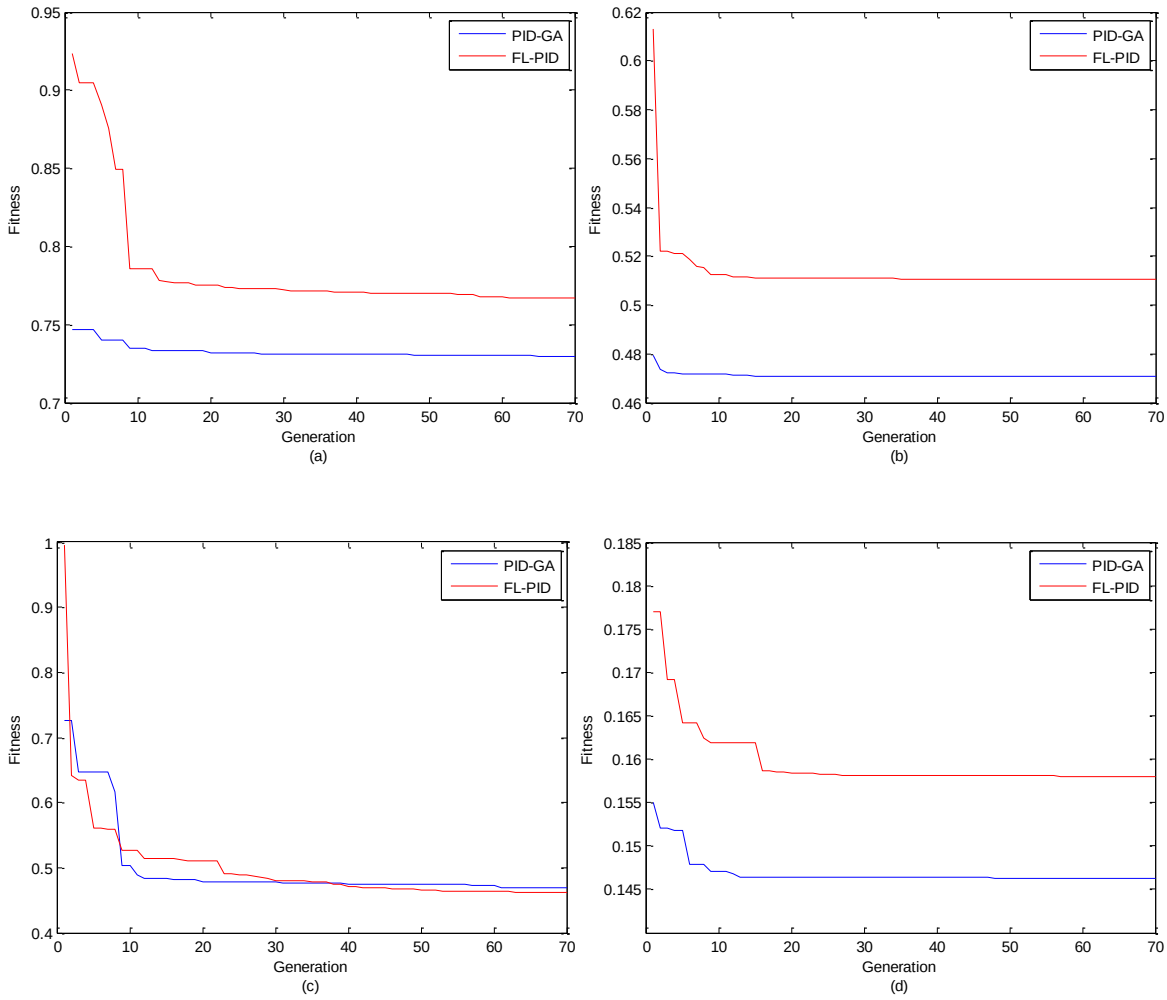


Figure 54 Evolution of best fitness for the IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria regarding System 3.

The obtained results for the PID-ZN, PID-ZA, PID-GA and FL-PID, such as gains, performance indices, percent overshoot (M_p), rise time (T_r) and settling time (T_s) are shown in Tables 15 and 16 for the FL-PID and PID controllers, respectively.

Table 15 Results of Fuzzy PID tuned with GAs for System 3.

	Criterion	Error	Gains				Time specifications		
		J	K_e	K_{ce}	K_{ie}	K_u	$M_p(\%)$	$T_r(s)$	$T_s(s)$
FL-PID	**IAE**	0.7667	12.2924	5.6405	3.0024	0.4825	3.5904	0.6228	3.7046
	ISE	0.5106	19.6130	9.6729	11.0075	0.5620	28.3641	0.4229	>10
	ITAE	0.4619	8.7223	4.4969	2.2998	0.5709	1.9291	0.7059	2.2265
	ITSE	0.1580	9.6231	4.7088	2.2796	0.7245	5.9657	0.5493	5.1601

Table 16 Results of PID tuned with ZN, ZA and GAs methods for System 3.

	Criterion	Error	Gains			Time specifications		
		J	K_p	K_i	K_d	$M_p(\%)$	$T_r(s)$	$T_s(s)$
ZN	**IAE**	1.1330	2.2938	1.7295	0.7605	18.8131	0.8856	3.9073
	ISE	0.7066						
	ITAE	1.1158						
	ITSE	0.3490						
ZA	**IAE**	1.2170	1.8739	1.4820	0.6534	15.5804	1.0309	4.4601
	ISE	0.7541						
	ITAE	1.3339						
	ITSE	0.3935						
GA	**IAE**	0.7299	7.1659	1.5191	3.2616	4.3141	0.5533	3.3321
	ISE	0.4709	9.9292	2.2709	4.4336	12.9374	0.4578	6.0390
	ITAE	0.4689	6.7521	1.4556	3.1404	3.2883	0.5795	3.3448
	ITSE	0.1463	7.8301	1.7342	3.4527	6.6100	0.5171	3.9531

The system step responses for the four controllers (PID-ZN, PID-ZA, PID-GA and FL-PID) and for the indices IAE, ISE, ITAE and ITSE are shown in Figure 55. Regarding the results of System 3, it is possible to state that the system step response presents worse results when controlled with the PID-ZN controller; except, when compared with the overshoot of the FL-PID (ISE criterion), and the settling time of the PID-GA (ISE and ITSE criteria) and FL-PID (ISE criterion). The FL-PID presents lower overshoot in all simulations, except for the ISE criterion, which presents the worst overshoot of all controllers. The rise time has differences in all controllers but nothing significant, nevertheless, the PID-GA and FL-PID has lower rise times than the PID-ZN and PID-ZA controllers. In terms of settling time the differences are not notorious, except in the FL-PID controller for criteria ISE, where the output presents an oscillatory response in steady-state.

Overall, it is possible to state that for this system the FL-PID and the PID-GA has better results, but the differences between these two controllers are not significant, even though, the PID-GA has slightly better responses. The other two controllers, the PID-ZN and the PID-ZA controllers, presents worse results.

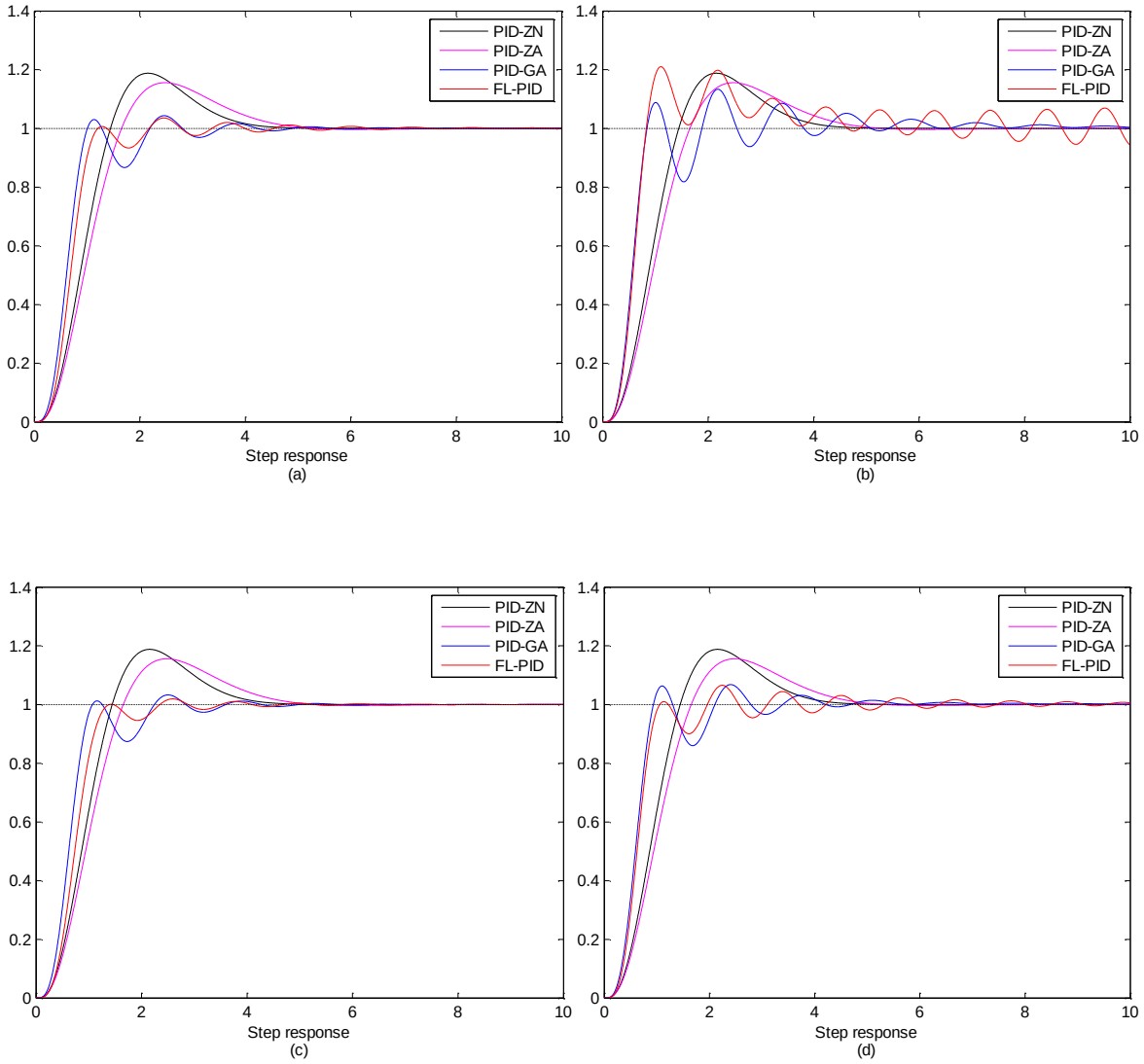


Figure 55 Step responses of System 3 according to IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria.

4.6.4. SYSTEM 4

System 4 uses a tank with stirrer with a heating and cooling model system. It was proposed by Nina *et al.* (2008) [25], and its transfer function is represented as follows:

$$G(s) = \frac{0.050561(s-4)(s-0.2667)(s+0.03333)}{(s+3.931)(s+0.04015)(s^2+0.1881s+0.01139)} \tag{59}$$

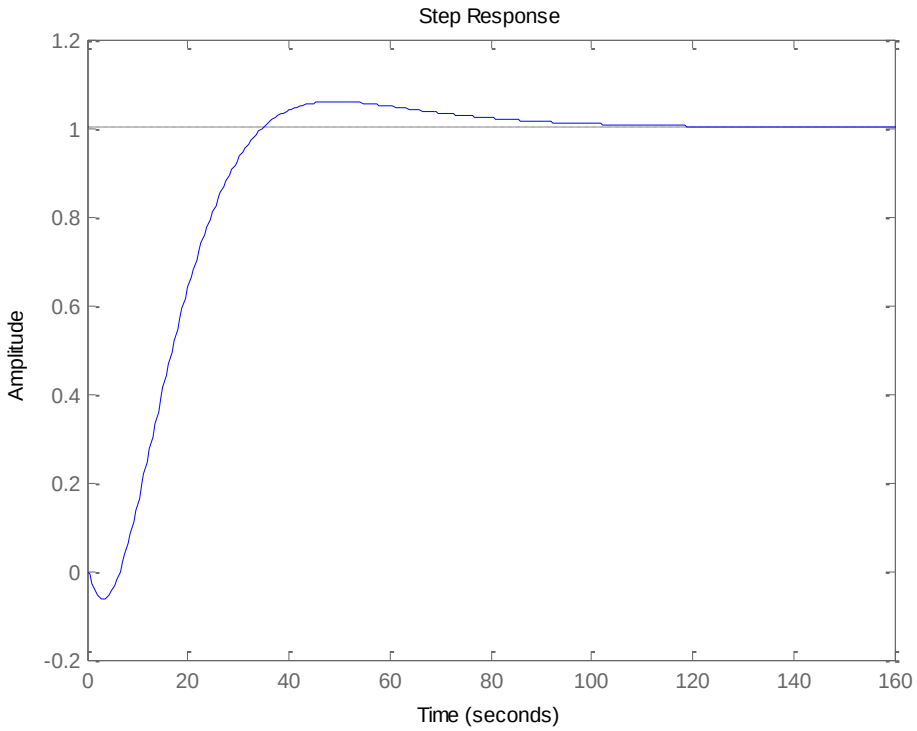


Figure 56 Step response of System 4.

The system uncontrolled step response is shown in Figure 56. According to the system dynamics response, the step time used for the simulations was $\Delta T = 0.01\,s$ and the simulation time was $T = 100\,s$, which are reasonable values and covers the four simulations (PID-ZN, PID-ZA, PID-GA and FL-PID). The controller output saturation limits were set to ± 10. The initial population and upper bounds used in the GAs are presented in Table 17.

Table 17 GA upper bounds and initial population used for System 4 simulations.

	PID			FUZZY PID			
	K_p	K_i	K_d	K_e	K_{ce}	K_{ie}	K_u
Initial Population	1.0850	0.0561	5.2430	20	96.6483	1.0347	0.0542
Upper Bounds	5	1	20	30	120	2	1

The GA fitness evolution, for the 70 generations, for the IAE, ISE, ITAE and ITSE criteria for PID-GA and FL-PID controllers are shown in Figure 57. In System 4, the fitness converged in all criteria approximately before 30 generations, except in the PID-GA for the IAE criterion, that required more generations, around 55.

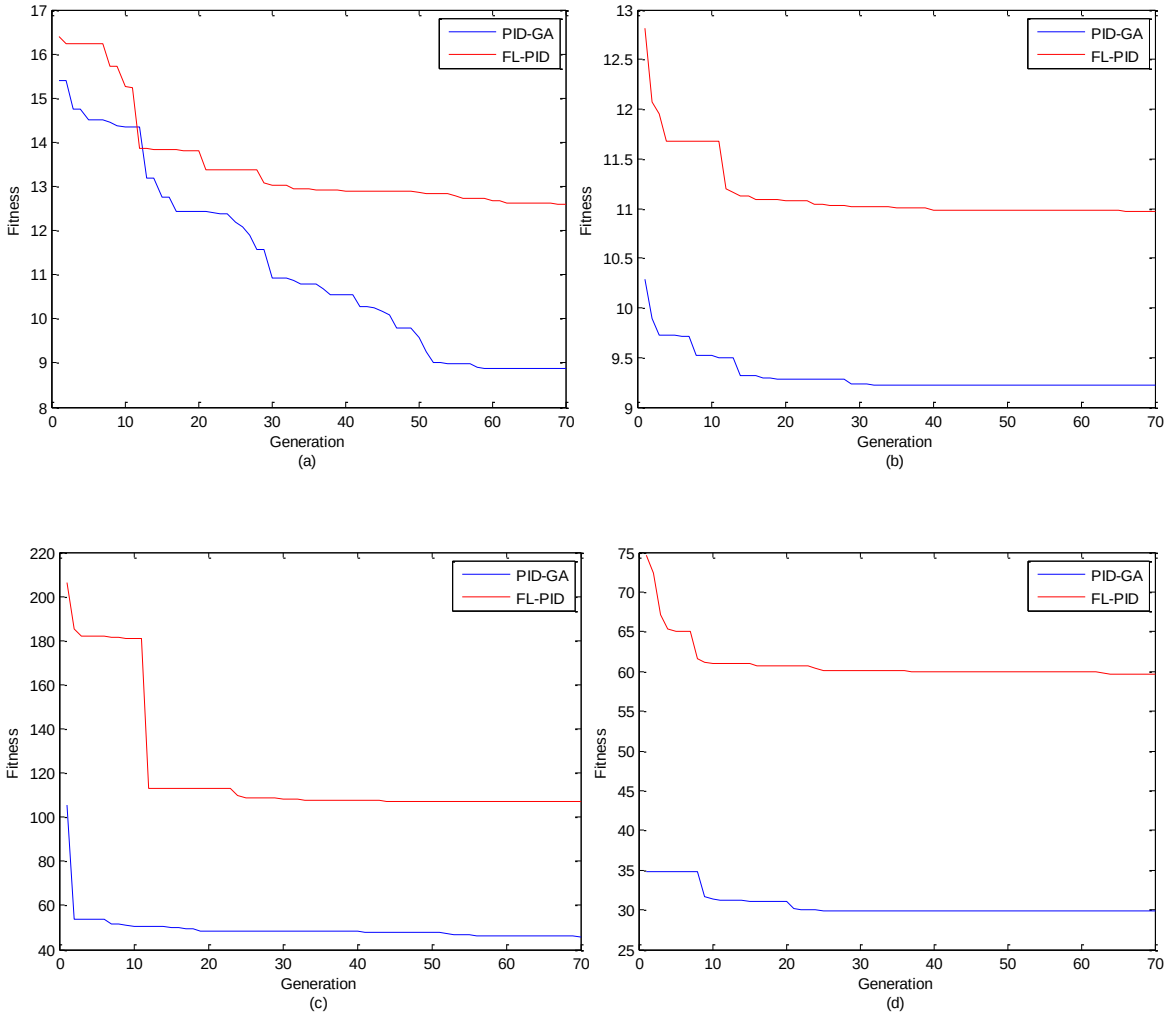


Figure 57 Evolution of best fitness for the IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria regarding System 4.

The obtained results for the PID-ZN, PID-ZA, PID-GA and FL-PID, such as gains, performance indices, percent overshoot (M_p), rise time (T_r) and settling time (T_s) are shown in Tables 18 and 19 for the FL-PID and PID controllers, respectively.

Table 18 Results of Fuzzy PID tuned with GAs for System 4.

	Criterion	Error	Gains				Time specifications		
		J	K_e	K_{ce}	K_{ie}	K_u	$M_p(\%)$	$T_r(s)$	$T_s(s)$
FL-PID	**IAE**	12.7164	17.4441	72.0042	0.5209	0.1439	5.2835	6.5787	>100
	ISE	10.7838	18.4292	113.8933	0.7225	0.1522	10.6630	10.8474	>100
	ITAE	107.1201	20.0203	79.5582	0.7451	0.1110	11.3715	8.1889	28.0419
	ITSE	59.6902	19.1328	106.0468	0.6950	0.1237	1.5838	12.2189	>100

Table 19 Results of PID tuned with ZN, ZA and GAs methods for System 4.

	Criterion	Error	Gains			Time specifications		
		J	K_p	K_i	K_d	$M_p(\%)$	$T_r(s)$	$T_s(s)$
ZN	**IAE**	17.4025	1.0850	0.0561	5.2430	0	17.0648	33.8687
	ISE	12.4572						
	ITAE	241.7287						
	ITSE	82.4615						
ZA	**IAE**	19.9938	1.0899	0.1002	5.0118	24.1498	11.0077	80.8138
	ISE	13.1307						
	ITAE	348.6541						
	ITSE	107.1933						
GA	**IAE**	8.9877	3.1179	0.1170	13.8706	3.7243	3.0272	17.1222
	ISE	9.2227	2.7958	0.1036	14.4219	1.3840	9.9544	17.7573
	ITAE	45.9068	3.1652	0.1164	14.1389	4.6208	2.8356	21.5776
	ITSE	29.8276	3.1140	0.1139	14.4956	3.0473	2.9027	20.8411

The system step responses for the four controllers (PID-ZN, PID-ZA, PID-GA and FL-PID) and for the indices IAE, ISE, ITAE and ITSE are shown in Figure 58. Looking at these graphics is possible to state that the step responses of the four controllers are distinct, for instance the PID-ZN rise time is notoriously higher than the other three controllers, although, this controller presents the lower overshoot. The step responses analysis showed that the error measurement has worse results with the PID-ZN controller, except for the ISE criterion when compared with the PID-ZA controller, that presented the worst error in this criterion. The PID-GA has the lowest error in all criteria compared with the other controllers, followed by the FL-PID controller, even though, in the ITAE and in the ITSE criteria the difference is almost the double, when comparing these two controllers tuned with the GAs. The FL-PID presents oscillation in steady-state, only with ITAE criterion it shows no oscillation. The lowest overshoot is achieved by the PID-ZN controller in all criteria, while the worst overshoot was presented by the PID-ZA. The worst results in terms of settling time was achieved by FL-PID controller, except in the ITAE criterion when compared with the PID-ZN and the PID-ZA, mostly motivated by the oscillation in steady-state. Nevertheless, in the ITAE criterion, the FL-PID presents satisfactory responses.

Overall, it is possible to state that for this system the FL-PID does not present satisfactory results in terms of steady-state response when comparing with the other controllers. Regarding overshoot, the FL-PID is lower than the PID-ZA in all criteria, and lower than the PID-GA in the ITSE criterion. In this system the FL-PID probably needs to be better

tuned to improve performance, especially in oscillation. This improvement could be done by changing other parameters in the fuzzy logic inference mechanism, for instance, the membership functions shape and limits and/or adapting the rules-base.

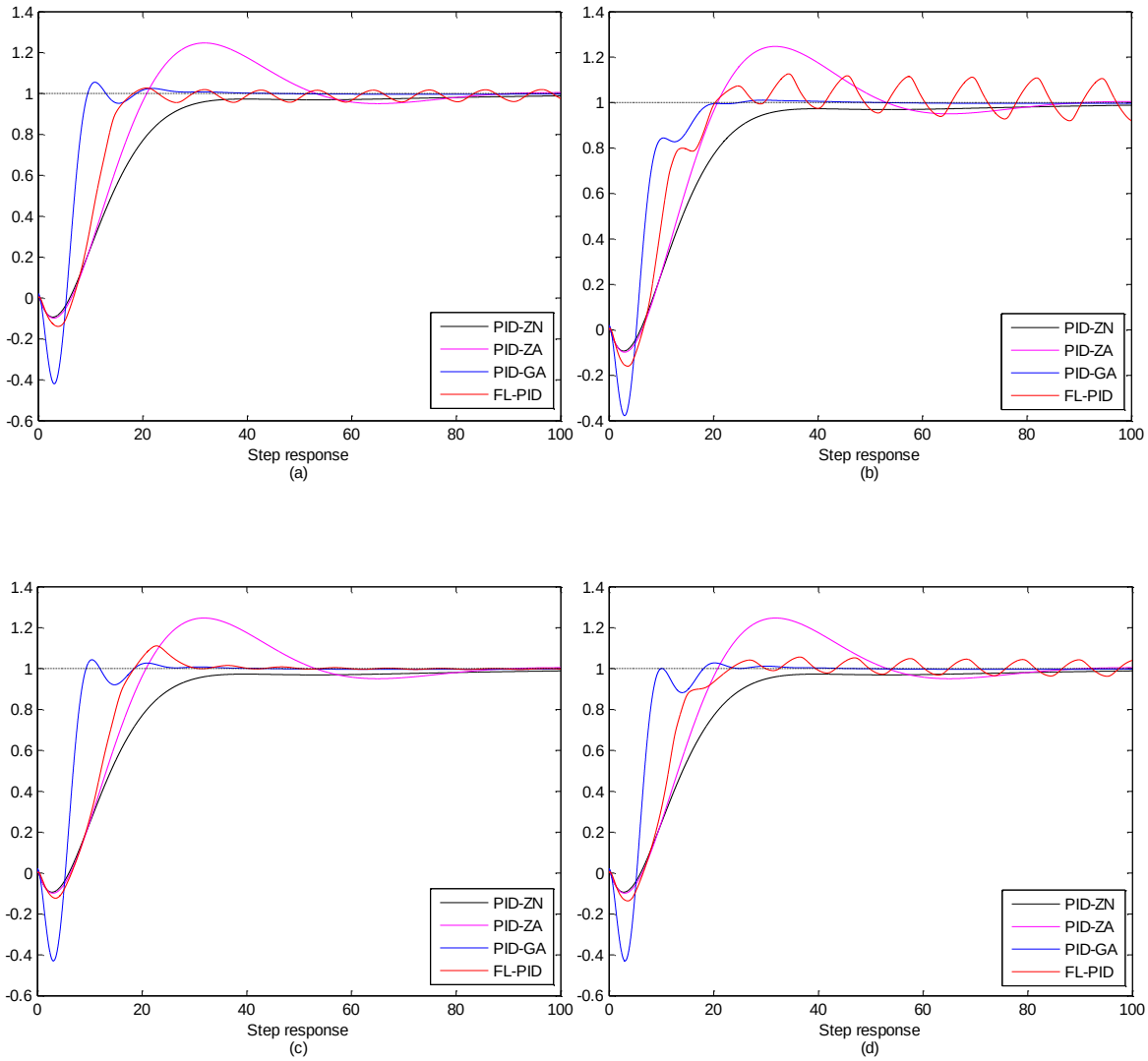


Figure 58 Step responses of System 4 according to IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria.

4.6.5. SYSTEM 5

System 5 uses an inverted pendulum proposed by Mahadi Hasan *et al.* (2012) [27], whose transfer function is represented by:

$$G(s) = \frac{7.407s}{s^3 + 0.26s^2 - 36.296s - 7.26} \tag{60}$$

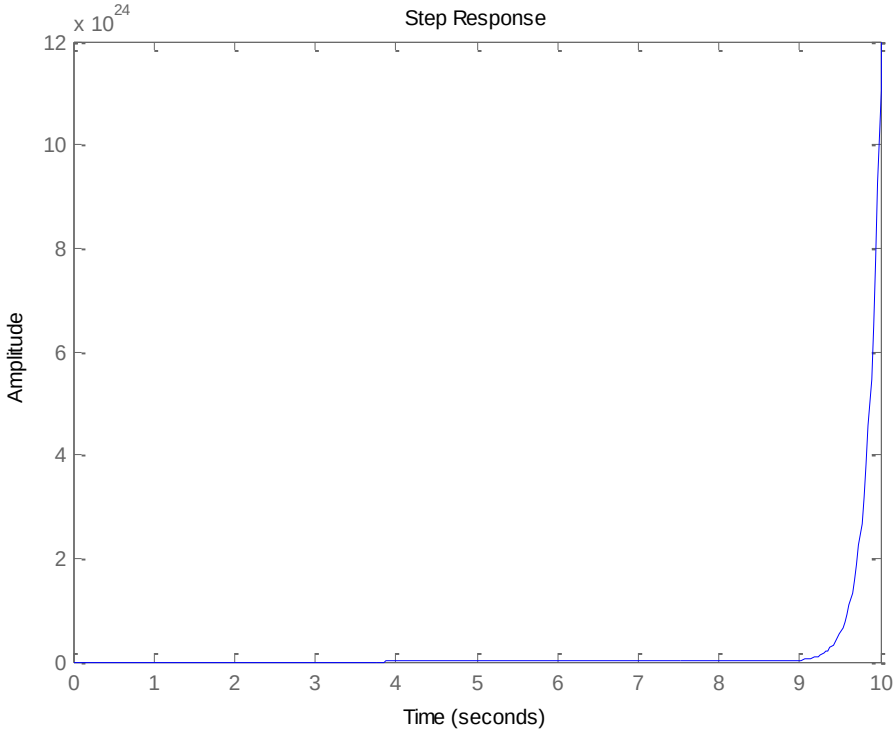


Figure 59 Step response of System 5.

The system uncontrolled step response is shown in Figure 59. The step time used for the simulations was $\Delta T = 0.001\ s$ and the simulation time was $T = 5\ s$, which are reasonable values and covers the two simulations (PID-GA and FL-PID). The controller output saturation limits were set to $\pm\ 10$. The initial population and upper bounds used in the GAs are presented in Table 20.

Table 20 GA upper bounds and initial population used for System 5 simulations.

	PID			FUZZY PID			
	K_p	K_i	K_d	K_e	K_{ce}	K_{ie}	K_u
Initial Population	-	-	-	-	-	-	-
Upper Bounds	350	15	30	40	5	1	100

The GA fitness evolution, for the 70 generations, for the IAE, ISE, ITAE and ITSE criteria for PID-GA and FL-PID controllers are shown in Figure 60. The fitness evolution in

System 5, in general, converged in all criteria before around 10 generations in both controllers. The number of generations required to converge are similar, but in the case of the FL-PID the number of generations required to converge are a little bit less than the PID-GA controller.

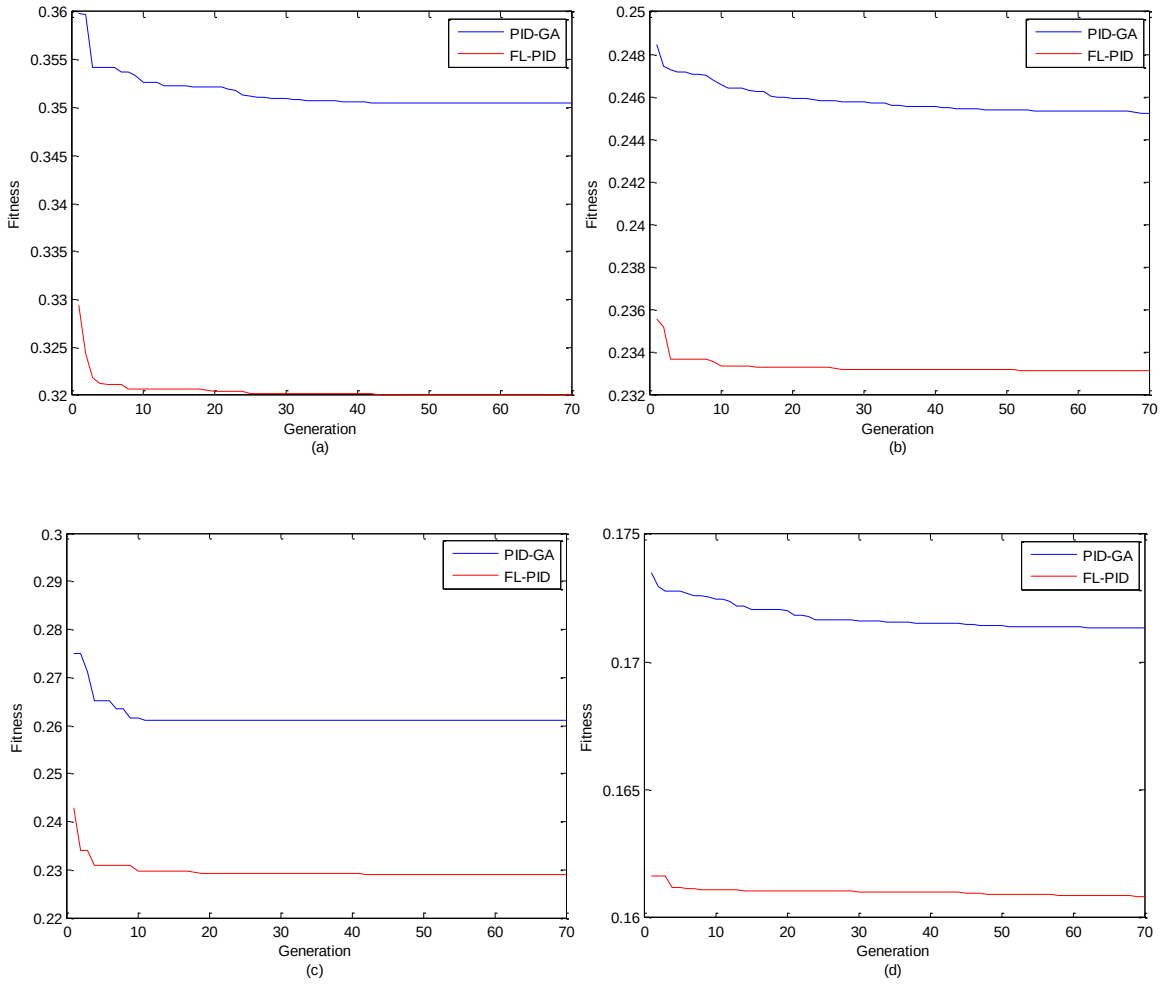


Figure 60 Evolution of best fitness for the IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria regarding System 5.

The obtained results for the PID-ZN, PID-ZA, PID-GA and FL-PID, such as gains, performance indices, percent overshoot (M_p), rise time (T_r) and settling time (T_s) are shown in Tables 21 and 22 for the FL-PID and PID controllers, respectively.

Table 21 Results of Fuzzy PID tuned with GAs for System 5.

	Criterion	Error	Gains				Time specifications		
		J	K_e	K_{ce}	K_{ie}	K_u	$M_p(\%)$	$T_r(s)$	$T_s(s)$
FL-PID	**IAE**	0.3201	19.1309	1.6321	0.1213	77.4072	0.2414	0.1520	1.4556
	ISE	0.2332	29.3575	2.2943	0.0065	46.2671	6.8308	0.1420	1.4286
	ITAE	0.2291	20.2984	1.6832	0.0973	86.0909	3.6755	0.1460	1.4453
	ITSE	0.1608	12.5003	0.9620	0.0333	50.8648	8.1566	0.1410	1.4200

Table 22 Results of PID tuned with GAs method for System 5.

	Criterion	Error	Gains			Time specifications		
		J	K_p	K_i	K_d	$M_p(\%)$	$T_r(s)$	$T_s(s)$
GA	**IAE**	0.3504	268.5237	10.2484	24.3540	2.7311	0.1510	1.5018
	ISE	0.2453	249.6788	0.0178	21.3981	8.0222	0.1410	1.4764
	ITAE	0.2611	237.2882	9.5052	20.8583	5.2902	0.1440	1.4893
	ITSE	0.1713	294.4720	0.0270	25.0687	8.9230	0.1400	1.4743

The system responses for the two controllers (PID-GA and FL-PID) and for the indices IAE, ISE, ITAE and ITSE are shown in Figure 61. By observing System 5 responses, with both controllers, it is possible to state that it presents worse results when controlled with the PID-GA controller, except for the rise time, that is similar to that of the FL-PID controller in all criteria, nevertheless, the rise time of the PID-GA is marginally lower. The FL-PID presents lower overshoot, lower settling time, better error response in all simulations. It is important to mention that in all criteria, the PID-GA cannot reach the reference value of 1, presenting an error, while in the case of the FL-PID this error is null, i.e., it can reach the desired reference value (Figure 61).

Overall, it is possible to state that for this system the FL-PID has the better results. Definitely this is a case where the FL-PID has more advantages and presents more satisfactory results than the conventional PID controller.

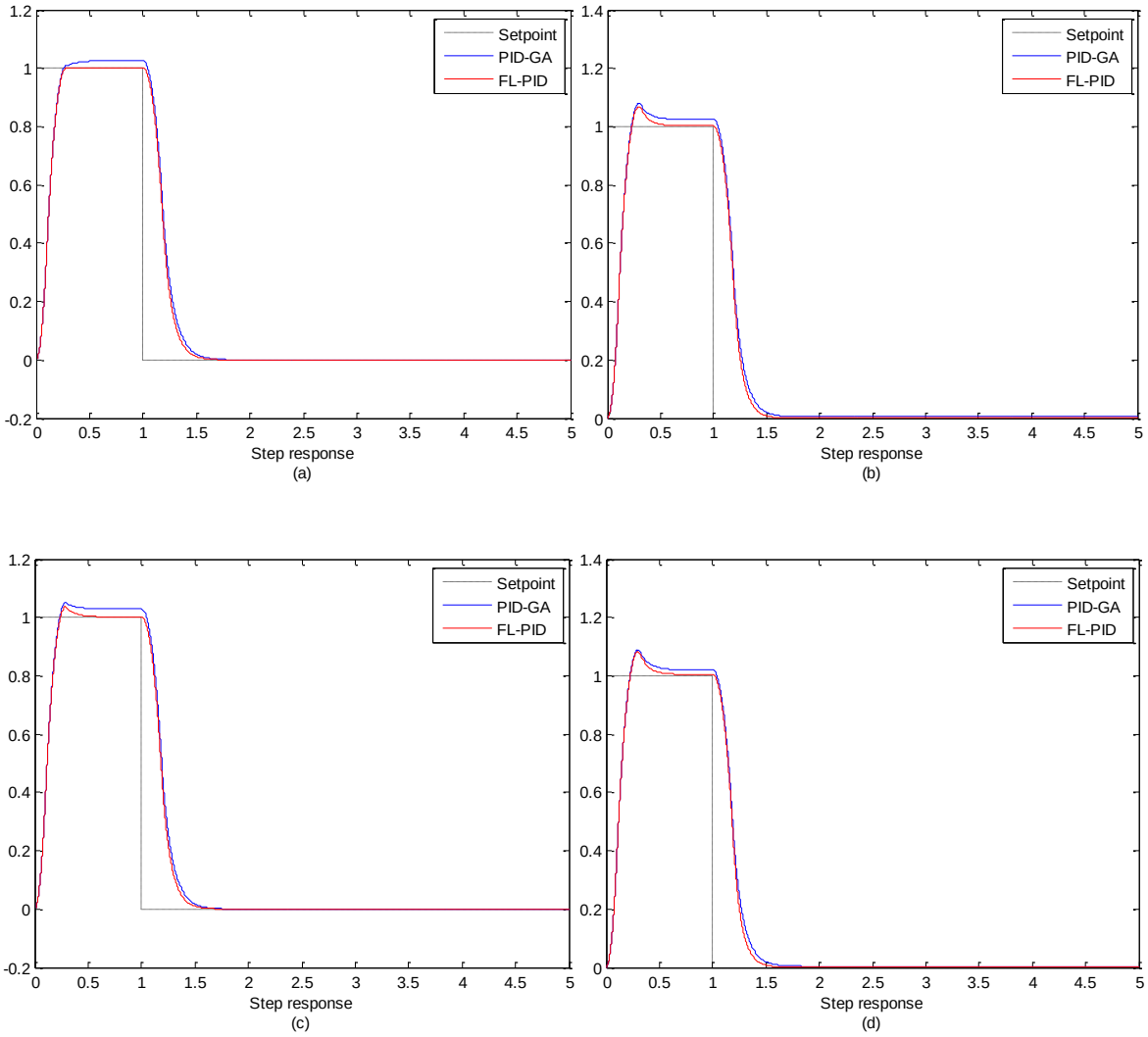


Figure 61 1 second impulse responses of System 5 according to IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria.

4.6.6. SYSTEM 6

System 6 uses a nonlinear time varying system that was proposed by Xue *et al.* (2007) [28], whose state space representation is:

$$\begin{cases} \dot{x_1}(t) = x_2(t) \\ \dot{x_2}(t) = -e^{-0.2t}x_2(t) - e^{-5t}\sin(2t+6)\,x_1(t) + u(t) \end{cases} \tag{61}$$

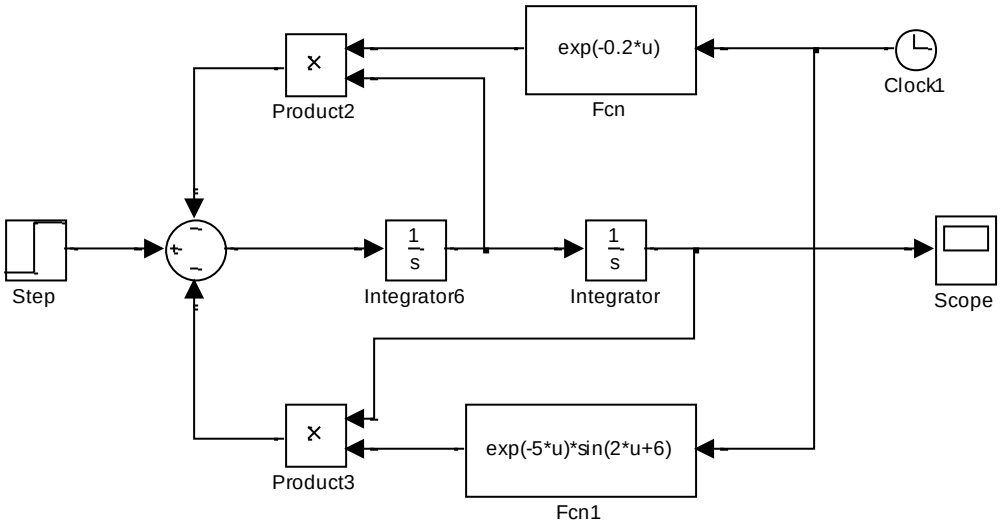


Figure 62 SIMULINK block diagram of System 6.

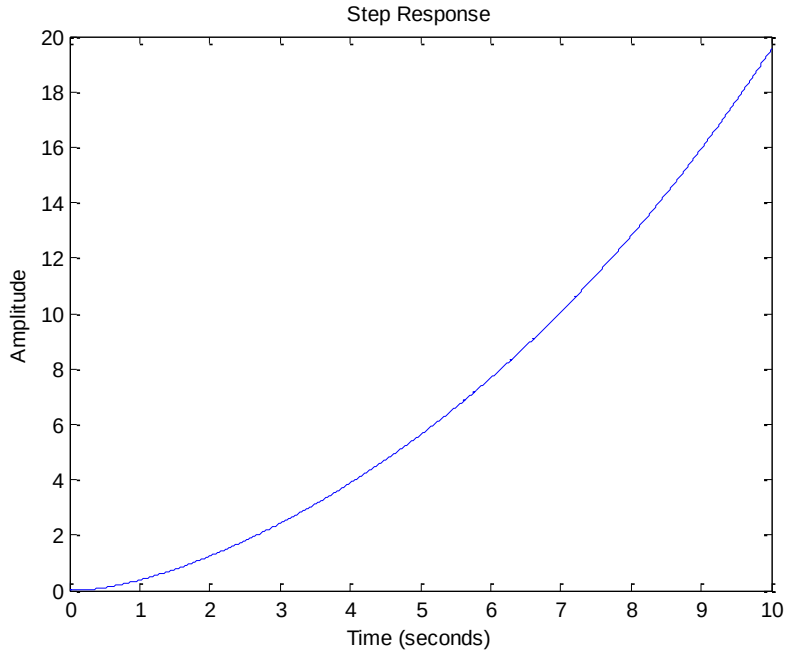


Figure 63 Step response of System 6.

The SIMULINK block diagram that represents System 6 is displayed in Figure 62 and the respective uncontrolled step response is shown in Figure 63. The step time used for the simulations was $\Delta T = 0.01\ s$ and the simulation time was $T = 5\ s$, which are reasonable values and covers the two simulations (PID-GA and FL-PID). The controller output saturation limits were set to $\pm\ 10$. The initial population and upper bounds used in the GAs are presented in Table 23.

Table 23 GA upper bounds and initial population used for System 6 simulations.

	PID			FUZZY PID			
	K_p	K_i	K_d	K_e	K_{ce}	K_{ie}	K_u
Initial Population	-	-	-	-	-	-	-
Upper Bounds	200	200	200	200	200	200	200

The GA fitness evolution, for the 70 generations, for the IAE, ISE, ITAE and ITSE criteria for PID-GA and FL-PID controllers are shown in Figure 64. In System 6 the fitness converged in all criteria approximately before 50 generations.

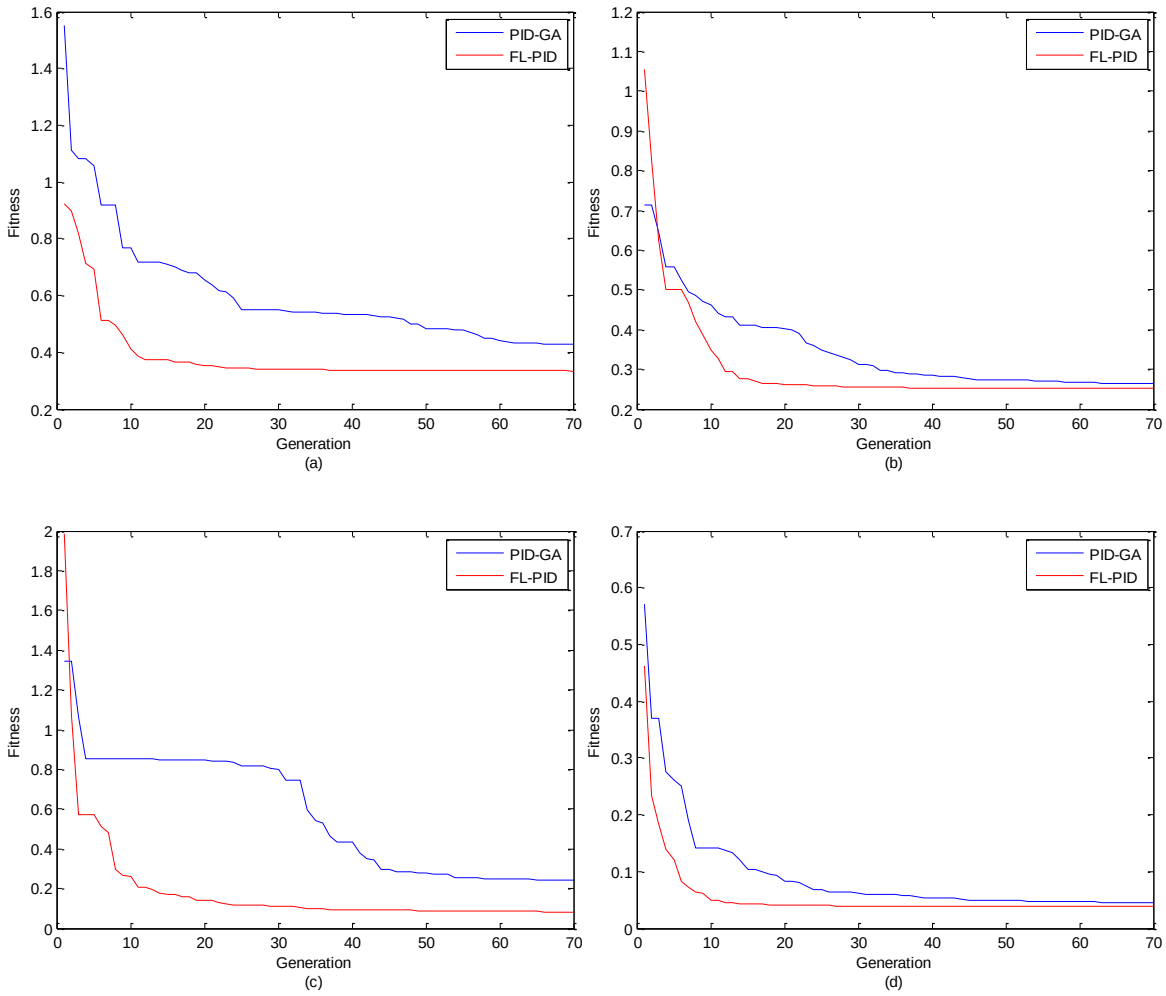


Figure 64 Evolution of best fitness for the IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria regarding System 6.

The obtained results for the PID-ZN, PID-ZA, PID-GA and FL-PID, such as gains, performance indices, percent overshoot (M_p), rise time (T_r) and settling time (T_s) are shown in Tables 24 and 25 for the FL-PID and PID controllers, respectively.

Table 24 Results of Fuzzy PID tuned with GAs for System 6.

	Criterion	Error	Gains				time specifications		
		J	K_e	K_{ce}	K_{ie}	K_u	$M_p(\%)$	$T_r(s)$	$T_s(s)$
FL-PID	**IAE**	0.3332	9.1703	1.5443	0.0269	8.9210	0.4080	0.3671	0.5894
	ISE	0.2527	5.0747	0.6545	0.0207	11.7548	6.3193	0.3456	0.8494
	ITAE	0.0837	10.2721	1.5760	0.1172	13.3333	0.4490	0.3585	0.5764
	ITSE	0.0381	7.6341	1.0113	0.0527	11.9326	5.2029	0.3471	0.8102

Table 25 Results of PID tuned with GAs methods for System 6.

	Criterion	Error	Gains			time specifications		
		J	K_p	K_i	K_d	$M_p(\%)$	$T_r(s)$	$T_s(s)$
GA	**IAE**	0.4272	15.2599	0.0402	4.2555	5.4990	0.5238	1.5253
	ISE	0.2629	23.1950	0.0781	4.1531	12.3975	0.3880	1.2556
	ITAE	0.2424	7.6059	$9.7656e^{-4}$	3.1847	2.9757	0.8196	2.0571
	ITSE	0.0446	21.8549	0.1123	4.6887	8.1311	0.4245	1.3077

The step responses of System 6 controlled with the two controllers (PID-GA and FL-PID) and for indices IAE, ISE, ITAE and ITSE are shown in Figure 65. Observing these graphics and the step responses analysis, we can confirm that the responses are very different, the FL-PID presents a notorious better step response in all analysis performed. Analysing the step responses, we can say that the FL-PID presents lower overshoot, lower rise time, lower settling time and lower error. This is the case of another system where the FL-PID is definitely a better choice, obtaining better results in all response characteristics and for all error criteria.

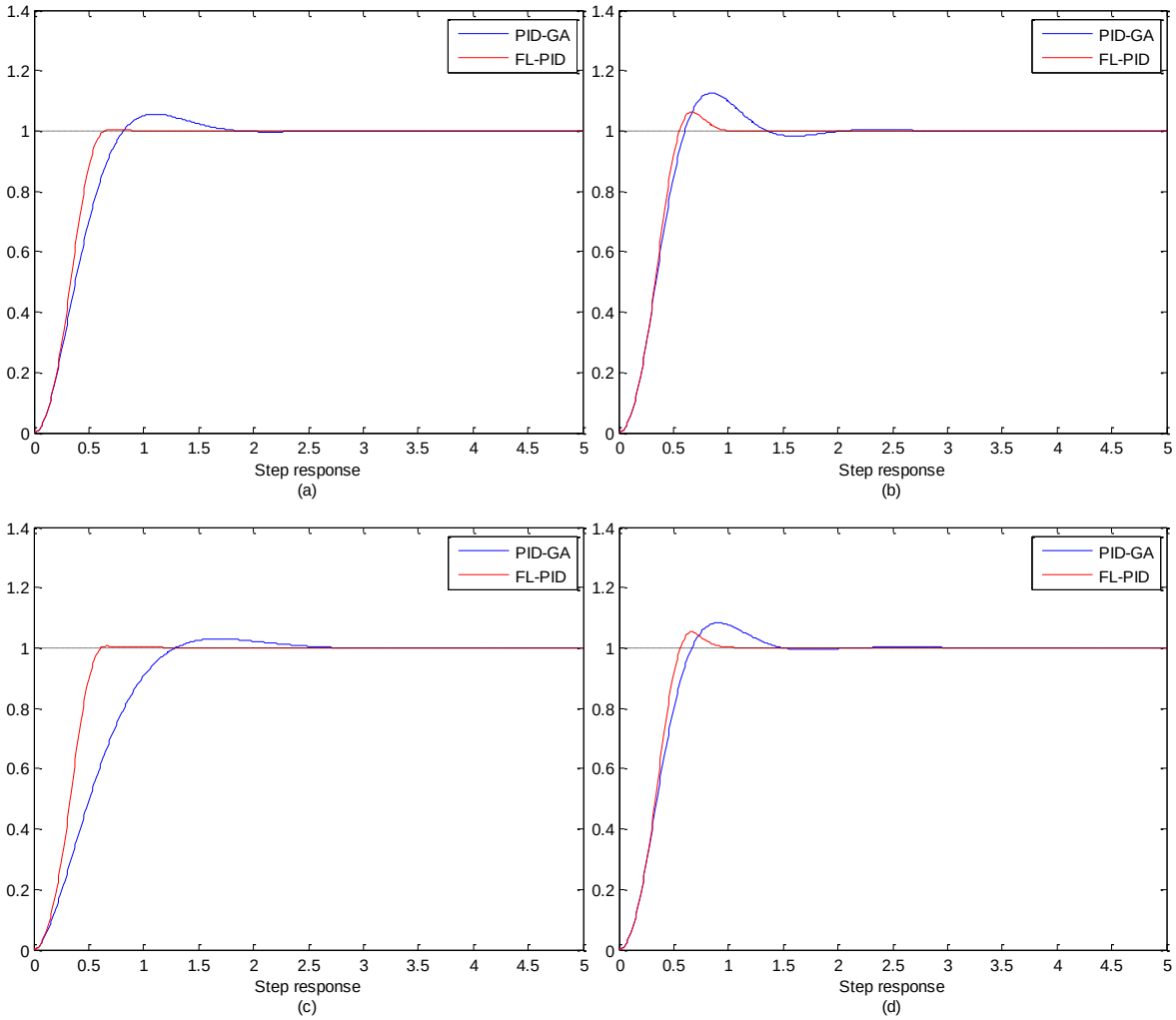


Figure 65 Step responses of System 6 according to IAE (a), ISE (b), ITAE (c) and ITSE (d) criteria.

4.7. CONTROLLERS RESULTS AND DISCUSSION

According to the obtained results, the PID controller tuned with GAs revealed better step responses in comparison to the other PID tuning methods tested, namely the ZN and the ZA methods. In fact, the PID controller tuned with ZN method have shown the worst step responses. Therefore, it is possible to state that the GAs are the best PID tuning technique tested along this project. Nevertheless, when comparing the PID with the FL-PID, both tuned with GAs, in many cases the FL-PID becomes a more advantageous controller; this was observed for more complex systems such as Systems 2, 5 and 6. In fact, even in System 1, which is a simple first order system with delay, the FL-PID controller also proved better results in terms of overshoot. Is important to mention that in the majority of the simulations, the overshoot of the FL-PID is the lowest, especially with IAE and ITAE

criteria. The results also showed that the best responses of PID-GA and FL-PID were obtained with IAE and ITAE criteria, with the ITAE having a small advantage, while ISE and ITSE criteria generate more overshoot and more oscillating responses.

Observing the generation evolution graphics, it is possible to state that, for all systems, GA converged before the last generation (70^{th} generation) which means that the GA seems to be properly designed and that it can be used to tune different systems controlled with PID and FL-PID controllers in a satisfactory manner. Overall, in the experiments of this work, the GA presented the best results than the other tuning methods used, either for the PID and for the FL-PID controllers. In general, the repeatability of the GA is very good in both controllers, but in the case of the PID the convergence is better in terms of number of generations, i.e., less generations required to converge. This could be explained by the fact that the chromosome has less genes (variables) to tune in the case of the PID. In fact it has three genes (K_p, K_i and K_d), while the FL-PID had four genes (K_e, K_{ce}, K_{ie} and K_u).

Regarding the two controllers tuned with GAs, FL-PID revealed a better performance in more complex systems, which might indicate that for more complex systems, and systems with more nonlinear characteristics, this type of controllers could be the election controller. Nevertheless, the PID controller is a very useful controller for more simple systems; in fact, a large percentage of real systems are simple and PID obtains satisfactory results for this type of systems. PID has the advantage to be more simple and easy to implement comparing with the FL-PID. On the other hand if the purpose is to develop a controller capable of control either simple and/or complex systems, then the developing of a FLC could be a good choice, once this type of controllers has much more degrees of freedom than the PID.

5. CONCLUSIONS

Ideally, the GAs would be able to tune all fuzzy controller parameters so that the tuning would be totally automatic taking advantage of all its potential. In this work, a FL-PID controller was developed tuned with GAs in order to control the majority of the available systems. An advantage of the fuzzy controllers is their simplicity to design since they use verbal language.

The objective was accomplished and the results are satisfactory, considering the time and the resources available. A key factor of this project is that, instead of a developing a tool to optimal tune FL-PID, it was developed a tool to tune both FL-PID and PID.

The study revealed that in general, to perform controllers' tuning in closed-loop, the best error criteria of the four indices used in this work, are the IAE and ITAE, even though the ITAE revealed even better results. These criteria showed lower overshoot, faster settling times and consequently less oscillating responses.

The GAs tuning method proved to be the best method to tune either PID or FL-PID controllers, while the heuristic ZN method revealed to be the worst method used in this study. GAs are definitely the best tuning method of all the methods presented in this work.

The FL-PID stands out from the other controllers when applied in more complex systems and revealed to be better than the PID when compared with the ZN and the ZA tuning methods. Even though, it obtained better responses in more simple systems in some situations when compared with the PID-GA. When choosing a controller, the requisites of the problem must be taken in account. If the objective is to develop a controller that might be able to control systems with distinct dynamics, then a FL-PID tuned with GAs must be considered, since it is relatively easy to implement and very effective.

Having in mind the work that has been performed it would be interesting to use more real and complex systems to see if the FL-PID improve significantly over the classical PID. To achieve this, a good approach is to let the GAs also tune other parameters in the fuzzy controller, like the parameters and type of membership functions, inputs and output limits and other fuzzy inference mechanism parameters that might make the difference, even for the type of systems used in this work. In a future work the rules-base could also be explored in order to improve the FL-PID controller, therefore, obtain better responses than the PID-GA, either in more complex or more simple systems.

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