WORKSHOP

robocontrol'08

3rd APPLIED ROBOTICS AND COLLABORATIVE SYSTEMS ENGINEERING

with emphasis in

INDUSTRIAL APPLICATIONS AND EDUCATIONAL ENVIRONMENTS

Bauru-SP December 4-5, 2008

VIRTUAL REALITY ESPECIAL SESSION

São Carlos-SP December 3, 2008

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Fractional Calculus: Application in Control and Robotics

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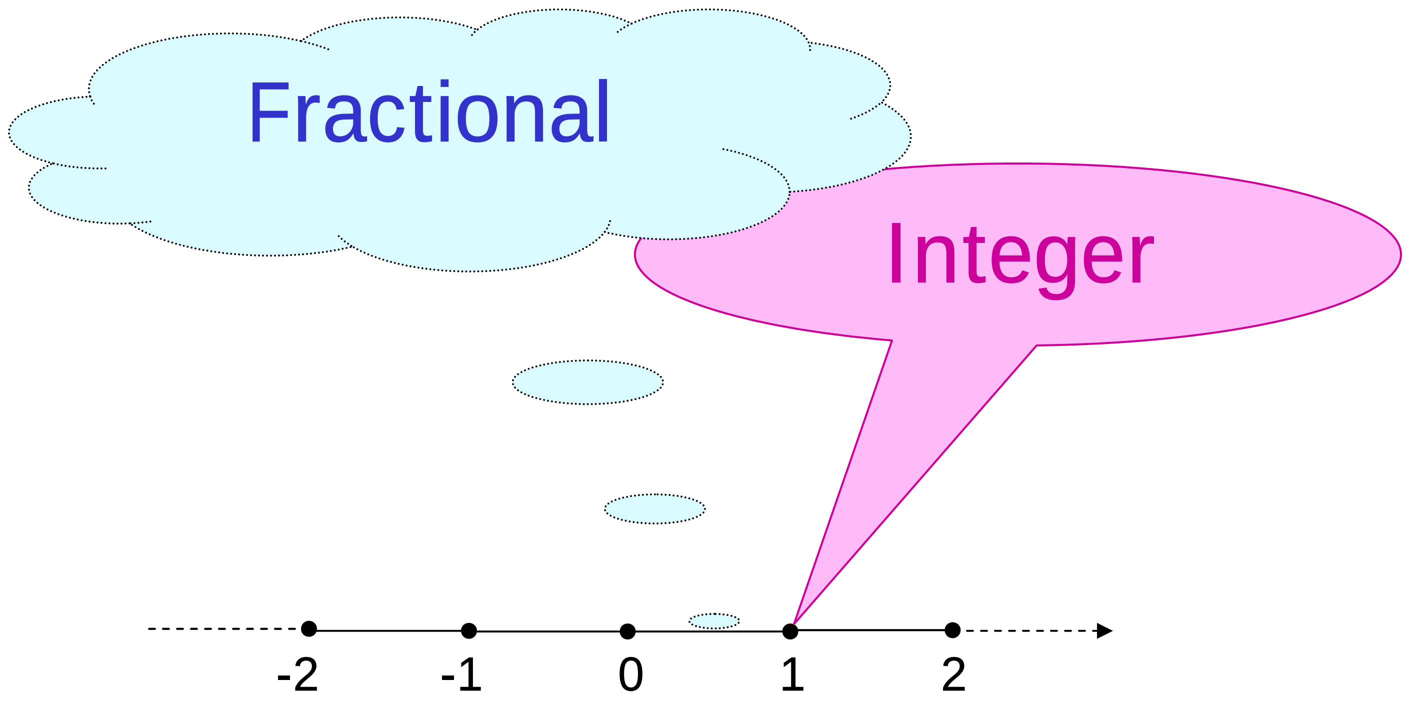
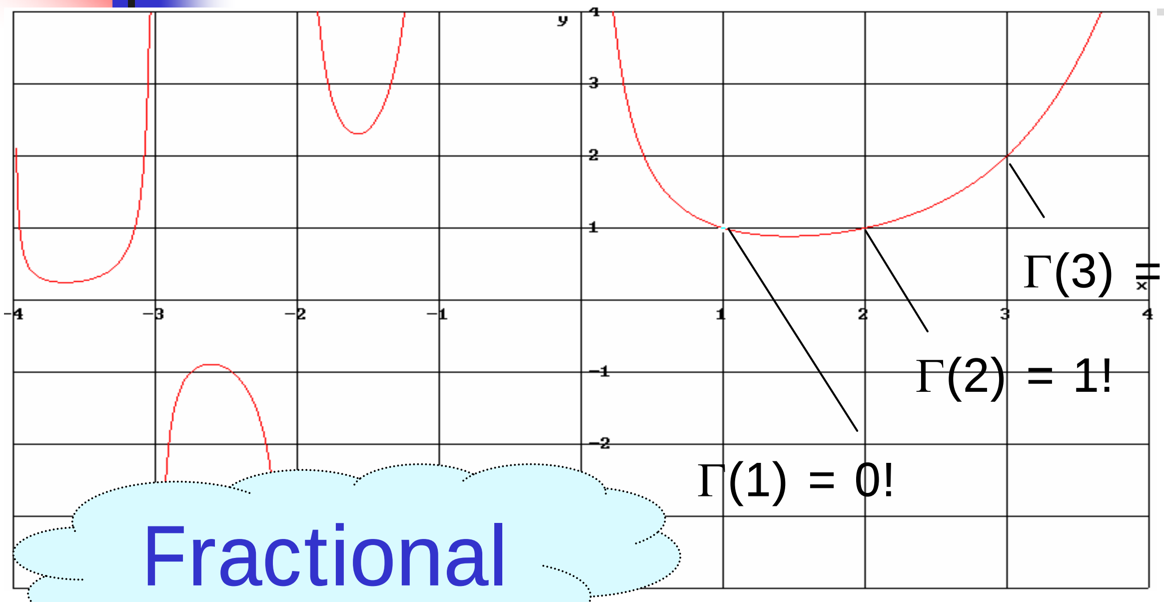
Integer *vs* fractional numbers



Factorial *vs* Gamma function

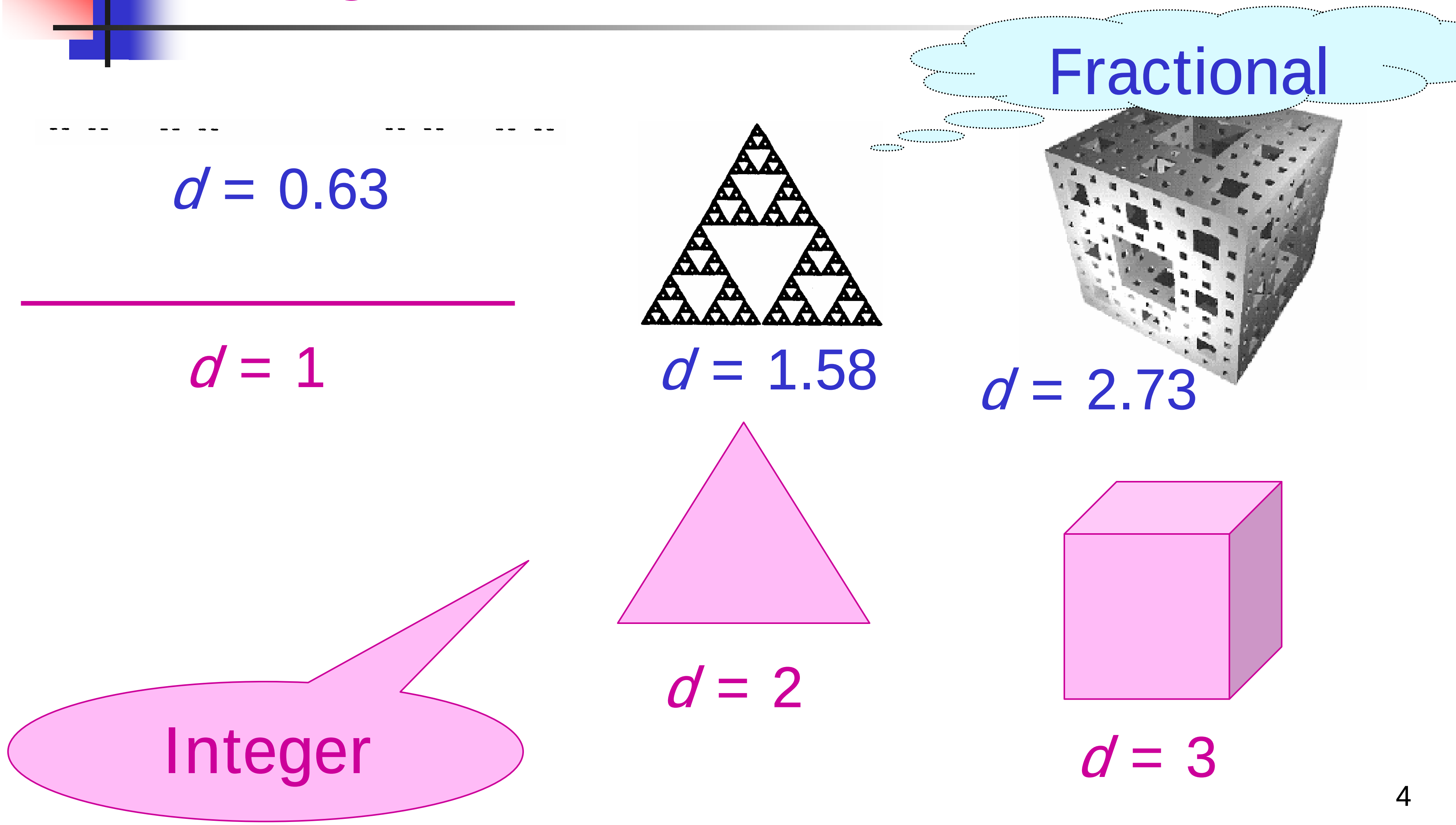


Fractional

Integer

$$\Gamma(n) = \int_0^\infty t^{n-1} e^{-t}\, dt \;\Rightarrow\; \Gamma(n+1) = n(n-1)\cdots 1 = n!$$

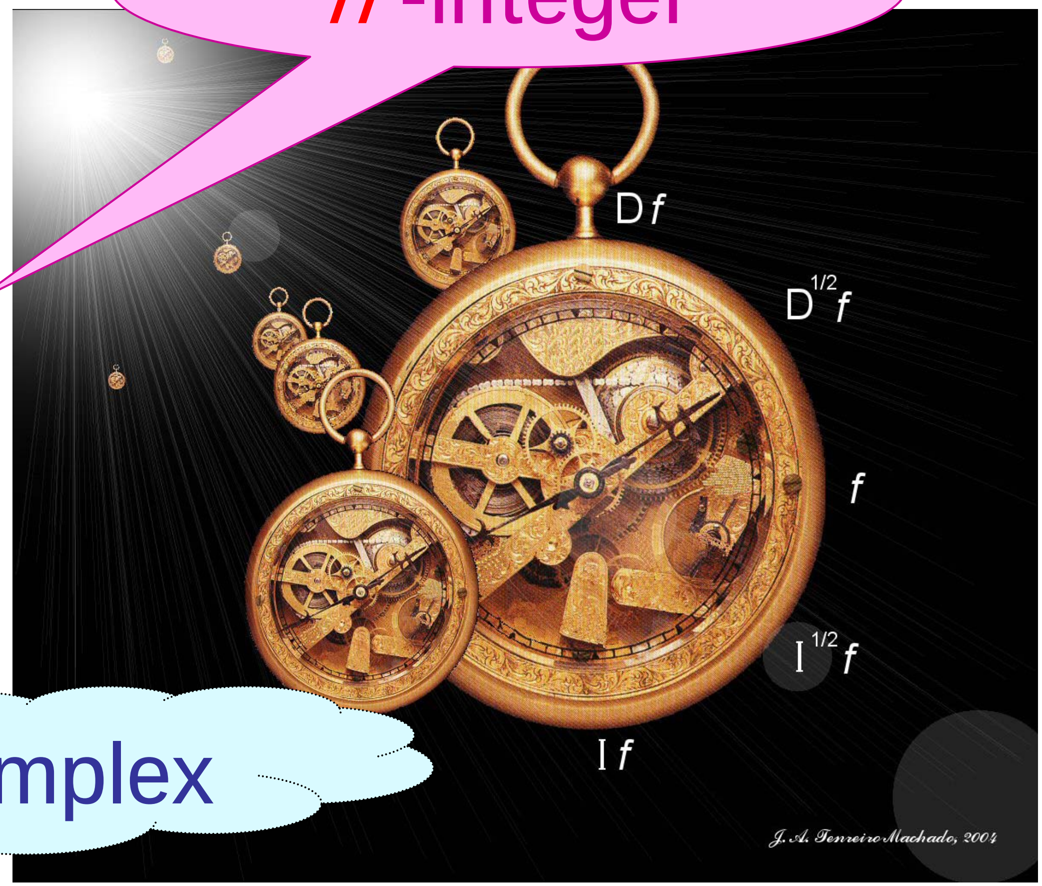
Integer *vs* Fractal dimension



Integer *vs* Fractional derivative

- $D^1(e^{ax}) = a\ e^{ax}$
- $D^2(e^{ax}) = a^2\ e^{ax}$
- $D^3(e^{ax}) = a^3\ e^{ax}$
- ....
- $D^n(e^{ax}) = a^n\ e^{ax}$
- $D^\alpha(e^{ax}) = a^\alpha\ e^{ax}$

n -integer



α - complex

Guillaume de l'Hôpital

(1661–1704)

What is the meaning of $D^{1/2}y$?

1695

intimate connection between derivatives and infinite series

Gottfried Wilhelm Leibniz

(1646–1716)

This is an apparent paradox from which, one day, useful consequences will be drawn...

Fractional Calculus

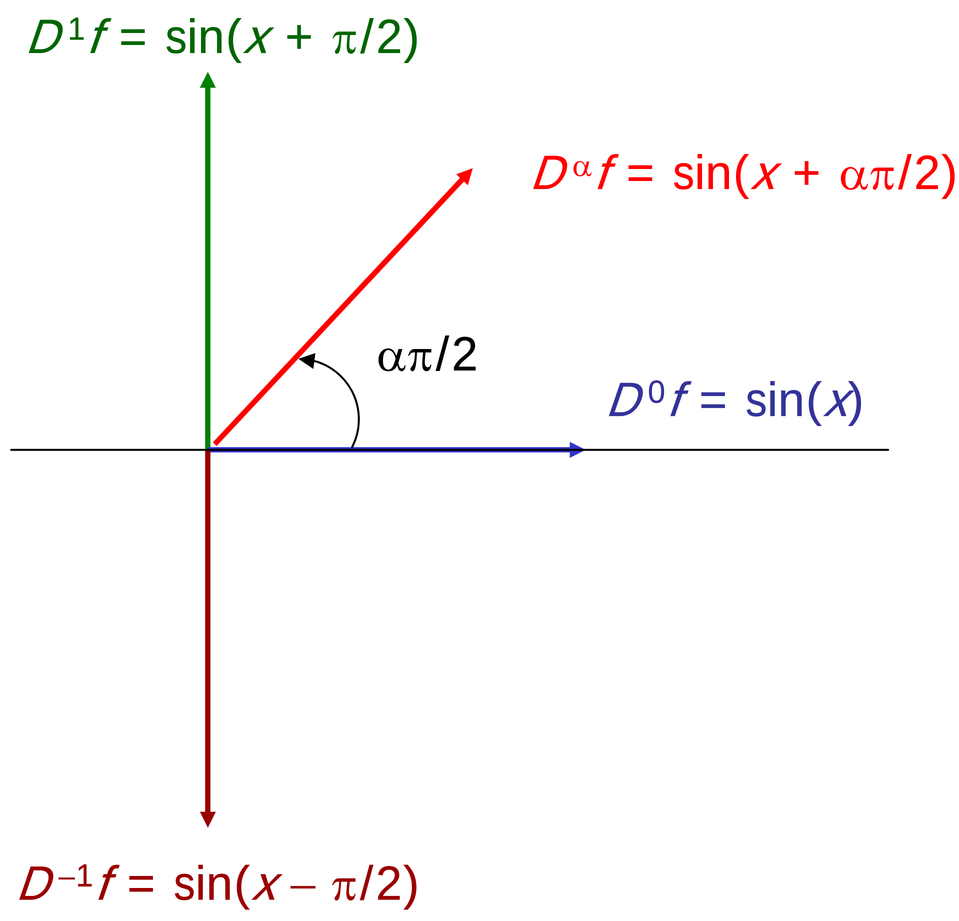
- The name "fractional calculus" is actually a misnomer
- The designation "integration and differentiation of arbitrary order" is more appropriate

Motivation: sin(*ax*) function

- $D^1[\sin(ax)] = a^1 \sin(ax + 1\,\pi/2)$
- $D^2[\sin(ax)] = a^2 \sin(ax + 2\,\pi/2)$
- $D^3[\sin(ax)] = a^3 \sin(ax + 3\,\pi/2)$
- $D^4[\sin(ax)] = a^4 \sin(ax + 4\,\pi/2)$
- ...
- $D^\alpha[\sin(ax)] = a^\alpha \sin(ax + \alpha\,\pi/2)$

Weyl derivative ${}_{-\infty}D_t^\alpha$

Vector interpretation of D^α for the function $f = \sin(x)$



Definitions of fractional derivatives-1

Bernhard Riemann (1826–1866) Joseph Liouville (1809–1882)

Riemann-Liouville definition

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt} \right)^n \int_a^t \frac{f(\tau)d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ n-1 < \alpha < n $$

Grünwald-Letnikov definition

$$ {}_aD_t^{\alpha} f(t) = \lim \frac{1}{h^{\alpha}} \sum_{k=0}^{\left[\frac{t-a}{h}\right]} (-1)^k \binom{\alpha}{k} f(t-kh) $$

$[x]$ – integer part of x

Aleksey Letnikov (1837-1888) Anton Grünwald (1838-1920)

Definitions of fractional derivatives -2

Michele Caputo

Caputo definition

$$ {}_{a}^{C}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{t} \frac{f^{(n)}(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} $$

$$ n-1 \le \alpha < n $$

Pierre-Simon Laplace

(1749-1827)

Laplace definition

$$ D^{\alpha} x(t) = L^{-1}\left\{ s^{\alpha} X(s) - \sum_{k=0}^{n-1} s^{k} D^{\alpha-k-1} x(t)\Big|_{t=0} \right\} $$

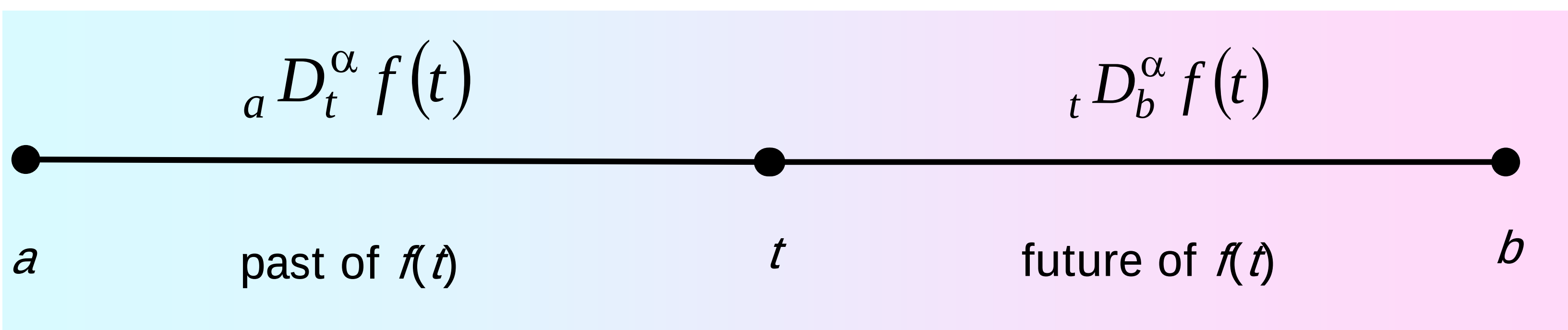
Left and Right fractional derivatives

- Left-sided

$$ {}_aD_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} $$

- Right-sided

$$ {}_tD_b^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\left(-\frac{d}{dt}\right)^n \int_t^b \frac{f(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} $$



Grünwald-Letnikov definition

$$D^1[f(x)] = \lim_{h\to 0} \frac{f(x) - f(x-h)}{h}$$

$$D^2[f(x)] = \lim_{h\to 0} \frac{f(x) - 2f(x-h) + f(x-2h)}{h^2}$$

$$\binom{3}{0} \quad -\binom{3}{1} \quad \binom{3}{2} \quad -\binom{3}{3}$$

$$D^3[f(x)] = \lim_{h\to 0} \frac{f(x) - 3f(x-h) + 3f(x-2h) - f(x-3h)}{h^3}$$

$$(-1)^k \binom{\alpha}{k} = (-1)^k \frac{\Gamma(\alpha+1)}{k!\,\Gamma(\alpha-k+1)}$$

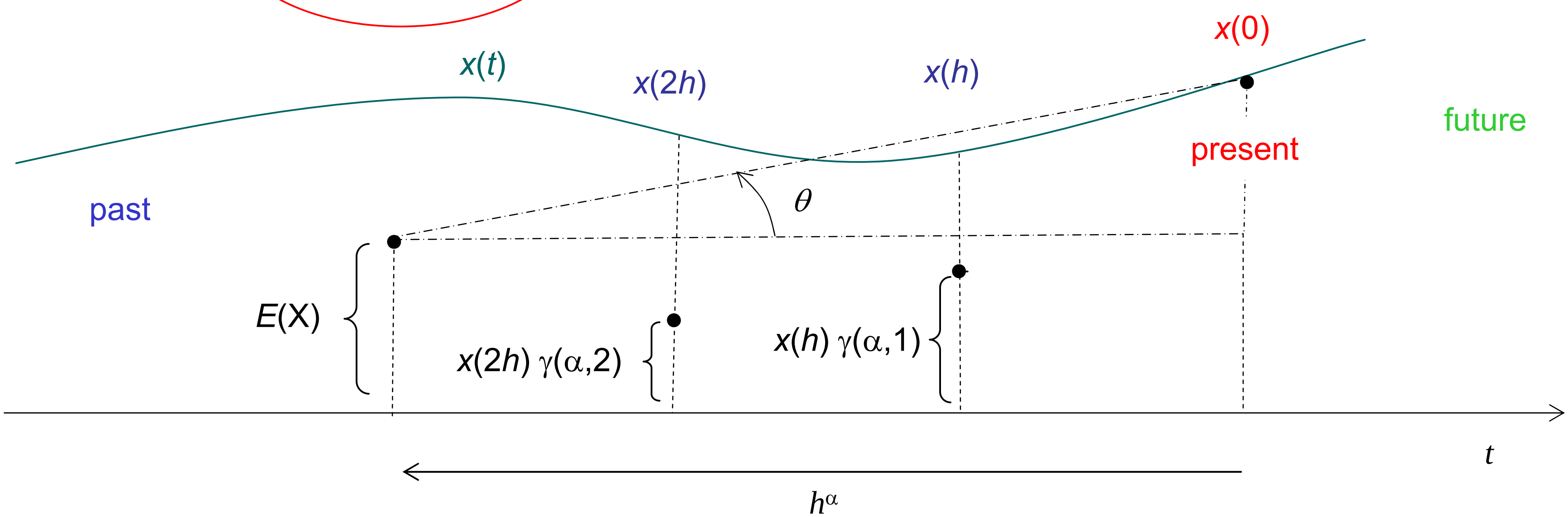
$$D^{1/2}[f(x)] = \lim_{h\to 0} \frac{f(x) - 1/2\, f(x-h) - 1/8\, f(x-2h) - 1/16\, f(x-3h) - \cdots}{h^{1/2}}$$

A probabilistic perspective...

$$D^{1/2}\left[f(x)\right]=\lim_{h\to 0}\frac{f(x)-1/2\,f(x-h)-1/8\,f(x-2h)-1/16\,f(x-3h)-\cdots}{h^{1/2}}$$

$$\frac{1}{2}+\frac{1}{8}+\frac{1}{16}+\cdots=1$$

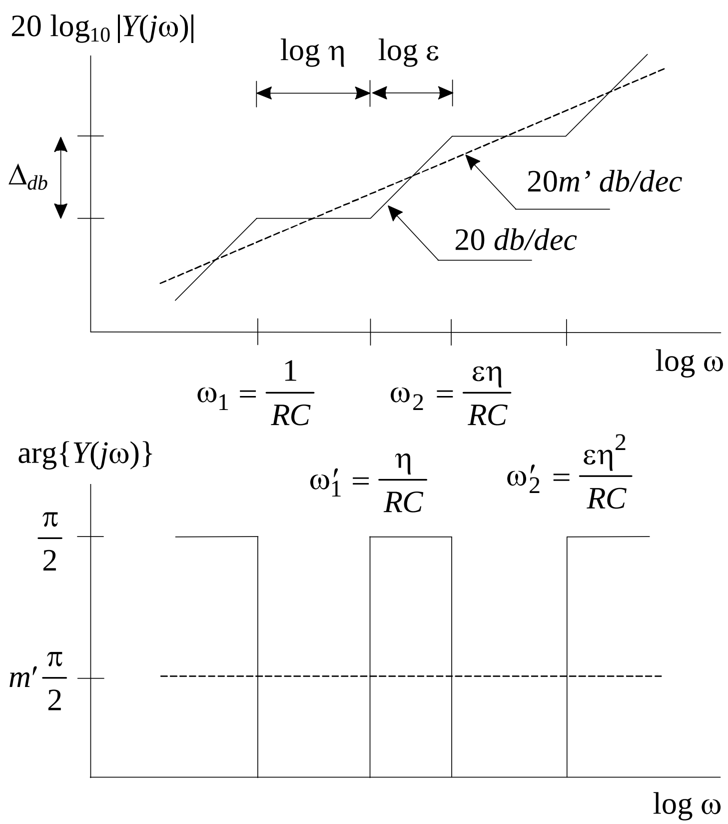
non uniform time variation



Approximations of fractional derivatives

- Two methods:
  - Frequency-based approach
  - Discrete-time approach

Frequency-based FD approximation



- Recursive relationships of pole/zero frequencies:

$$\frac{\omega'_{i+1}}{\omega'_i} = \frac{\omega_{i+1}}{\omega_i} = \varepsilon\eta \qquad \frac{\omega_i}{\omega'_i} = \varepsilon \qquad \frac{\omega'_{i+1}}{\omega_i} = \eta$$

- Average slope:

$$m' = \frac{\log \varepsilon}{\log \varepsilon + \log \eta}$$

- Approach to D^α $(0 < \alpha < 1)$:

$$m' = \alpha$$

Discrete-time FD approximation

Grünwald-Letnikov definition:

$$D^{\alpha}x(t)=\lim_{h\to 0}\left[\frac{1}{h^{\alpha}}\sum_{k=0}^{\infty}(-1)^{k}\frac{\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}x(t-kh)\right]$$

$h\approx T$, T - sampling period:

$$\frac{Z\{D^{\alpha}x(t)\}}{X(z)}\approx\frac{1}{T^{\alpha}}\sum_{k=0}^{\infty}\frac{(-1)^{k}\,\Gamma(\alpha+1)}{k!\;\Gamma(\alpha-k+1)}z^{-k}=\left(\frac{1-z^{-1}}{T}\right)^{\alpha}$$

Fraction approximation

$$\left(\frac{1-z^{-1}}{T}\right)^{1/2}\approx\frac{1}{T^{1/2}}\frac{-\frac{7}{64}z^{-3}+\frac{7}{8}z^{-2}-\frac{7}{4}z^{-1}+1}{-\frac{1}{64}z^{-3}+\frac{3}{8}z^{-2}-\frac{5}{4}z^{-1}+1}$$

Integer *vs* Fractional Mechanics...

- Spring
  - Hooke law

$$F = kx$$

- Viscous friction
  - Newton fluid

$$F = k\dot{x}$$

- Mass
  - Newton 2nd law

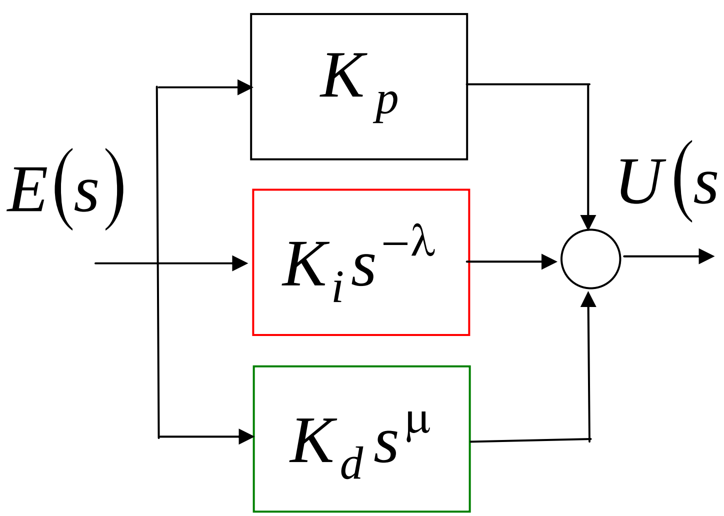
$$F = k\ddot{x}$$

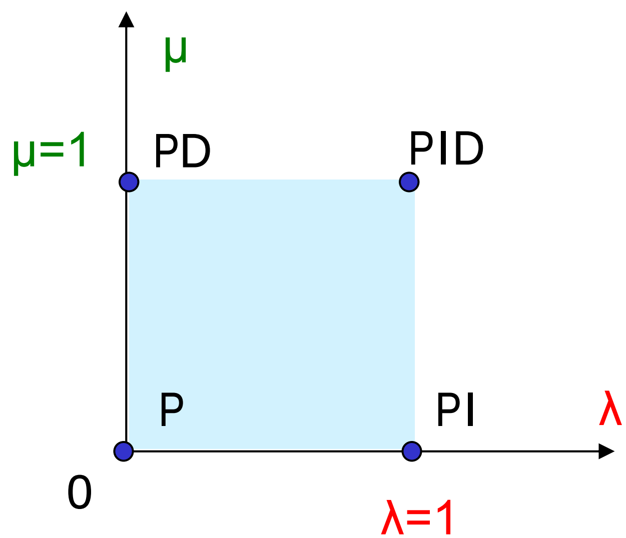
$$F = kx^{(\alpha)}$$

Fractional-Order Controllers

- The fractional-order $PI^{\lambda}D^{\mu}$ controller:

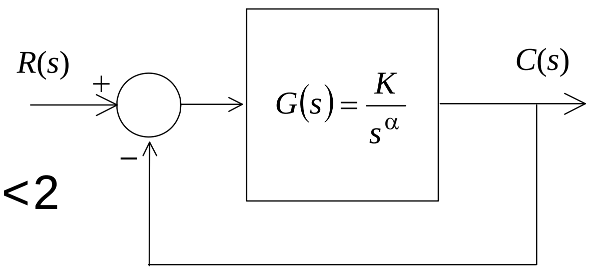
$$C(s) = \frac{U(s)}{E(s)} = K_p + K_i s^{-\lambda} + K_d s^{\mu}$$





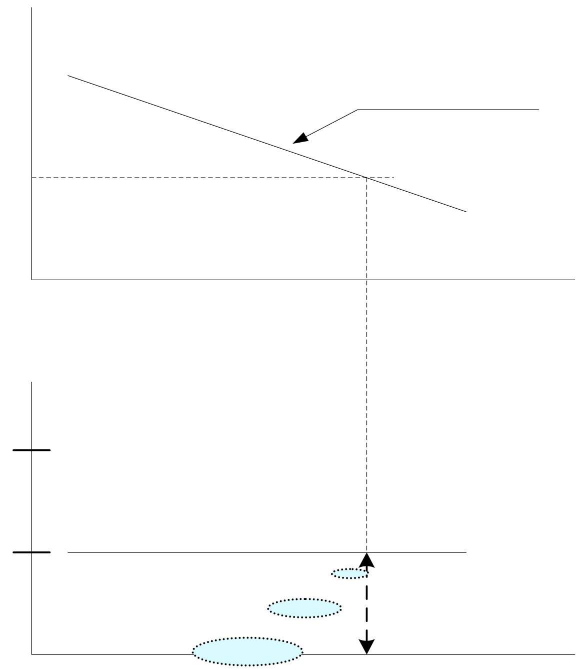
Control robustness

Feedback control system

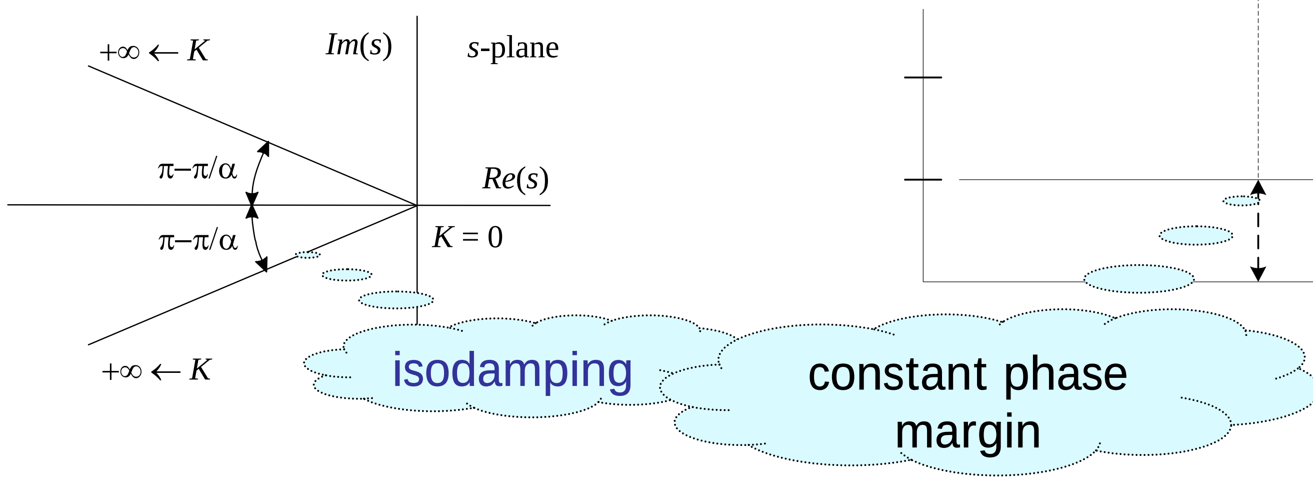


$1<\alpha<2$

Bode diagrams



Root-locus

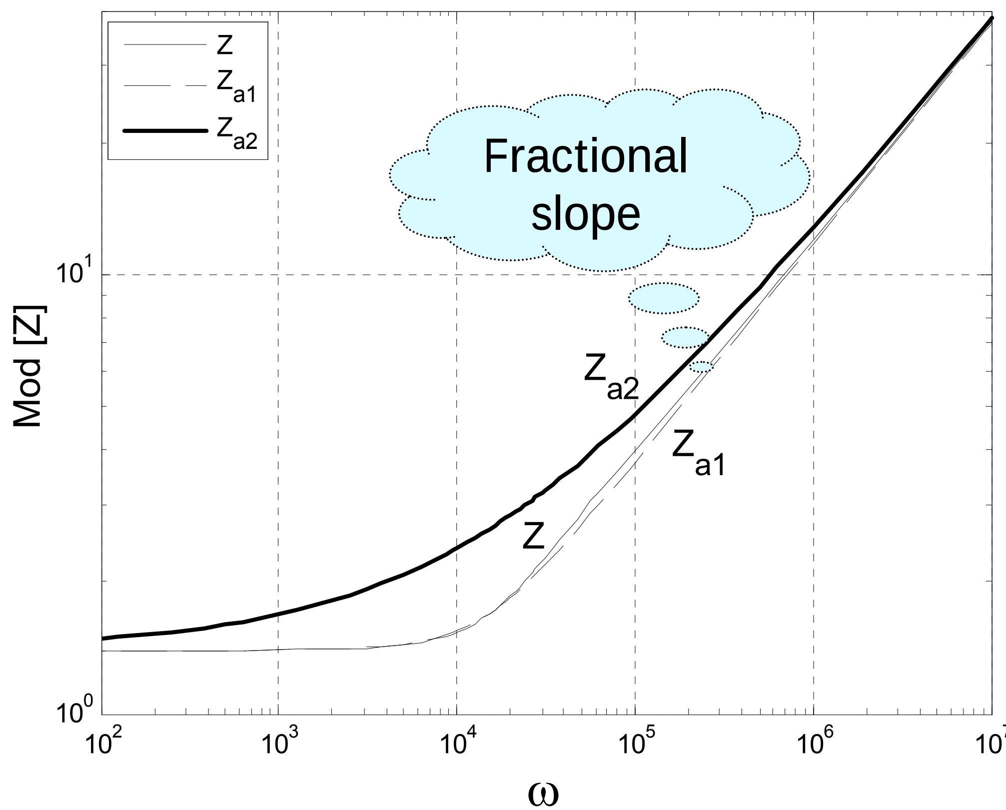


Electromagnetism: Skin effect

$$\tilde{Z} = \frac{q l_0}{2\pi\, r_0 \sigma} \frac{J_0(q r_0)}{J_1(q r_0)}$$

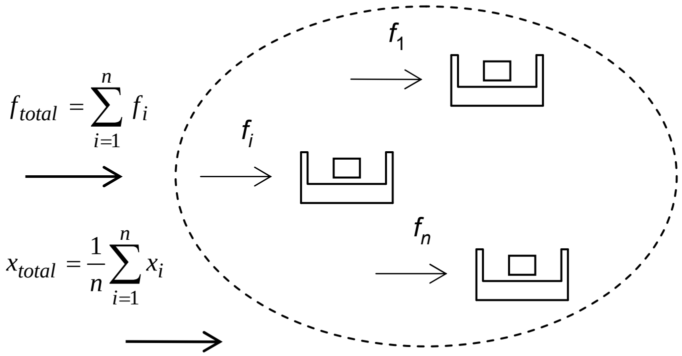
$$\tilde{Z}_{a1} \approx \frac{l_0}{\pi\, r_0^2 \sigma} \left[i\omega \left(\frac{r_0}{2}\right)^2 \mu\sigma + 1 \right]^{1/2}$$

$$\tilde{Z}_{a2} \approx \frac{l_0}{\pi\, r_0^2 \sigma} \left\{ \left[i\omega \left(\frac{r_0}{2}\right)^2 \mu\sigma \right]^{1/2} + 1 \right\}$$

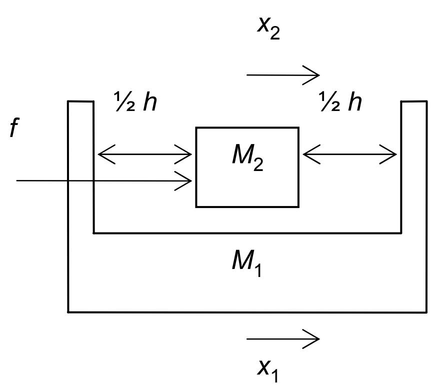


Fractional Dynamics: A Statistical Perspective

- A mechanical system with n identical components having, each one, an independent motion



- Each elemental component consists of two masses M_1 and M_2, with displacement x_1 and x_2, having a backlash h, subjected to impacts under the action of force f



System Simulation and Analysis

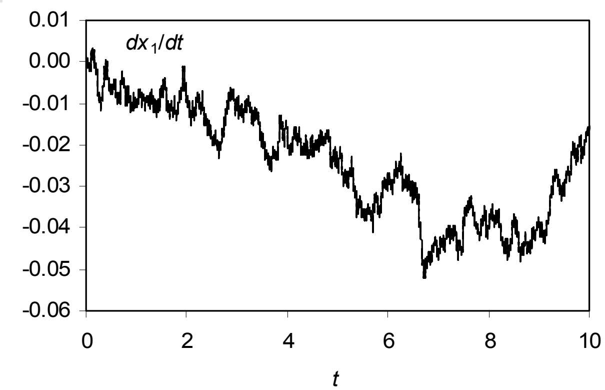
- Each element is driven by:

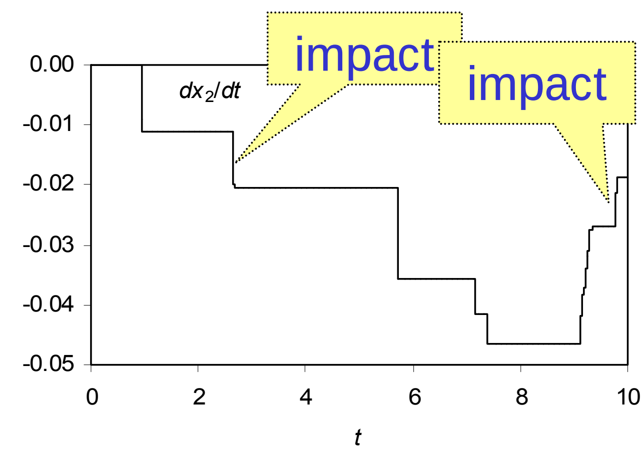
$$f(t) = F_{max}\ rnd(t)$$

- Time history of dx_1/dt, dx_2/dt
- Transfer functions:

$$H_k(j\omega) = \frac{F\{\dot{x}_k(t)\}}{F\{f(t)\}}, k = 1,2$$

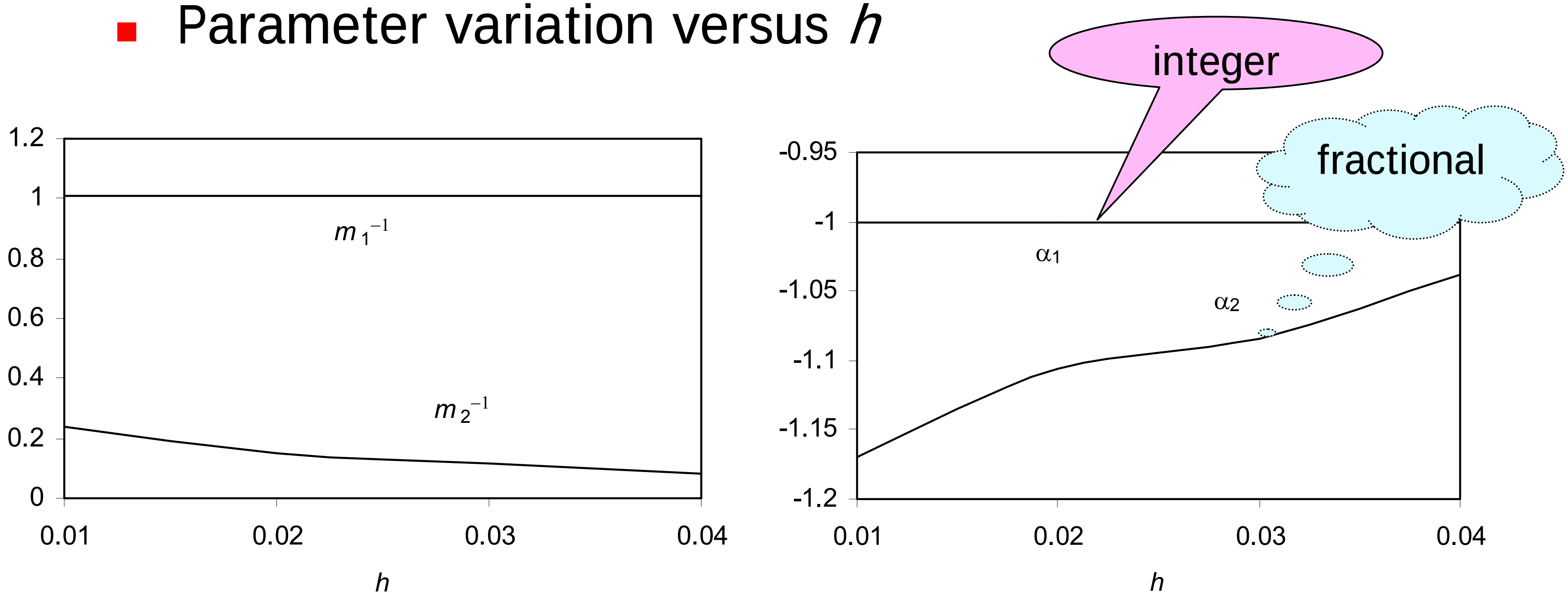
- Approximations: $H_k(j\omega) \approx m_k^{-1}\omega^{\alpha_k}$





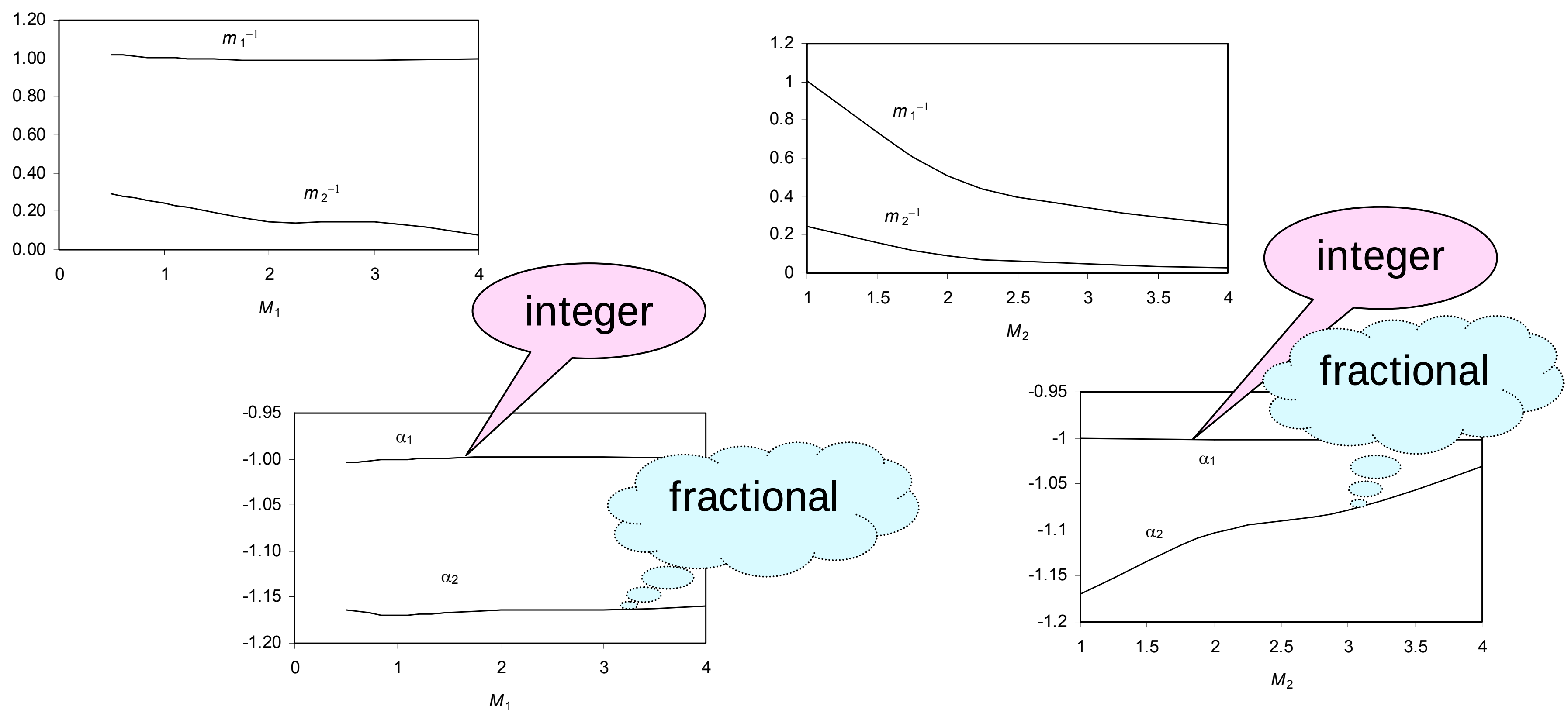
System Simulation and Analysis

- The average transfer functions for $n = 10^3$ elements are smoother
- Parameter variation versus h



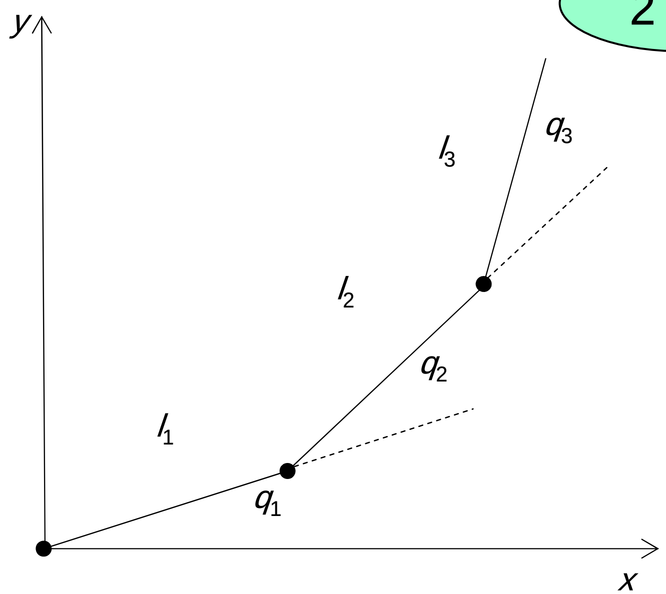
System Simulation and Analysis

- Parameter variation versus M_1 and M_2

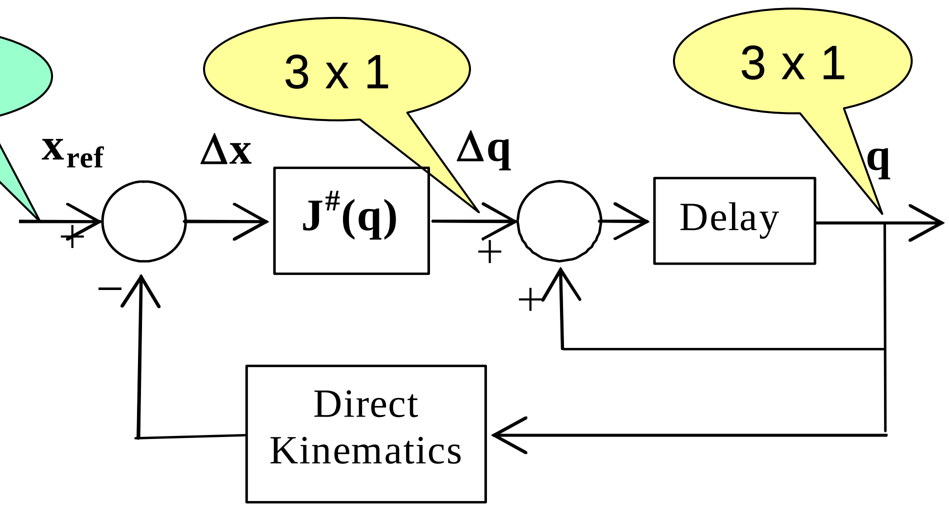


Redundant manipulators: Trajectory planning

- 3*R* planar redundant manipulator



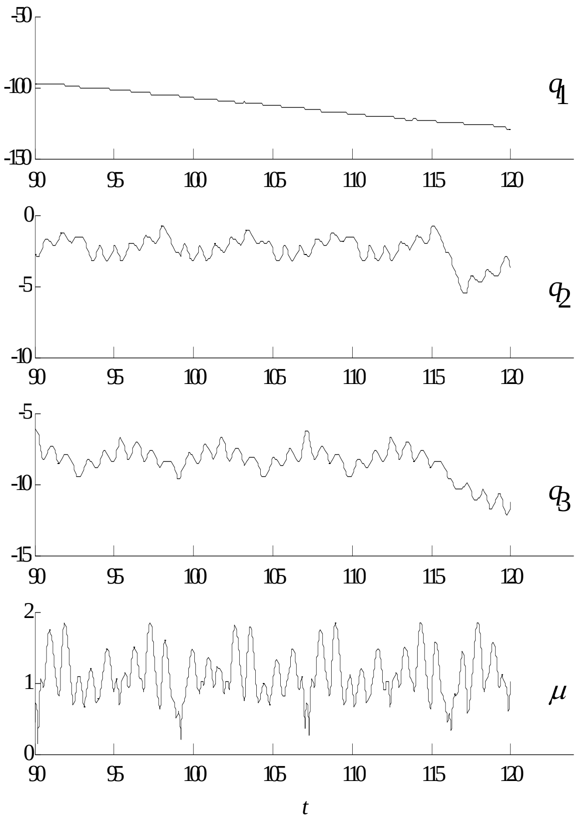
- Computation of joint positions



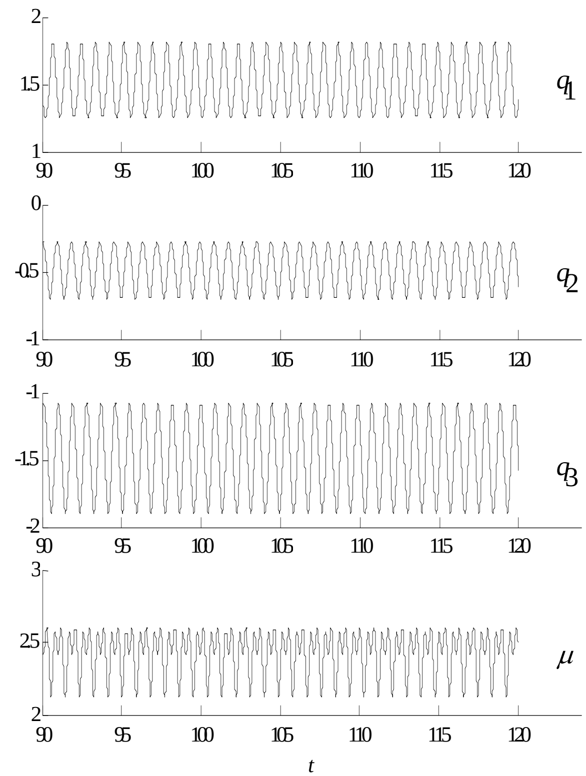
$\mathbf{J}^{\#}$ – generalized inverse

Joint positions and manipulability *vs* time for ρ = 0.5, *r* = {0.6, 2.0}

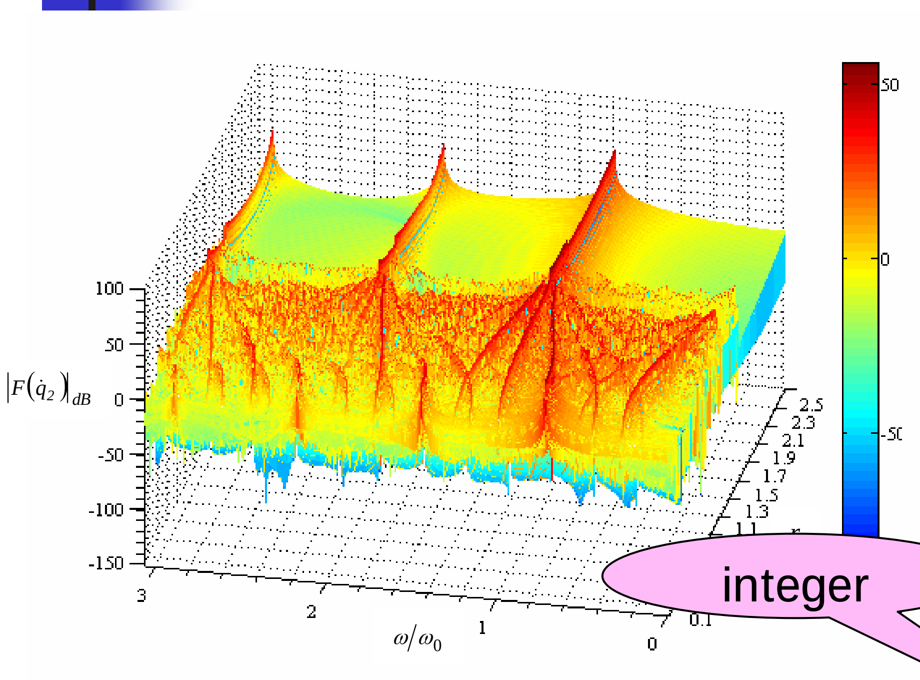
r = 0.6

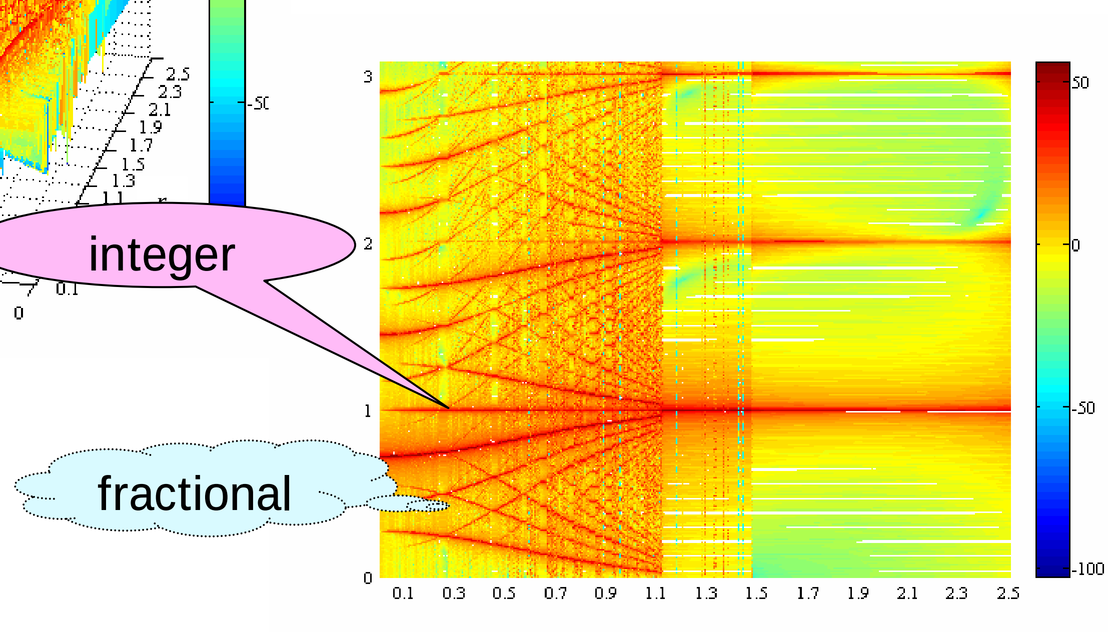


r = 2.0

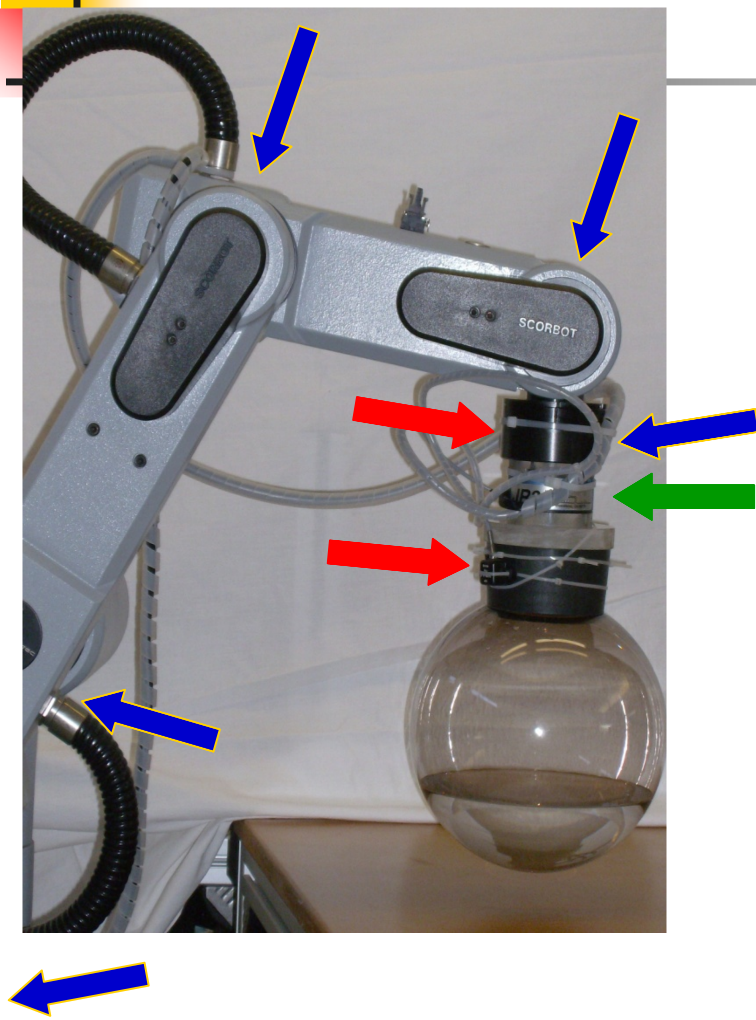


F{dq_2/dt} *vs* (r ,ω/ω_0) for ρ=0.5





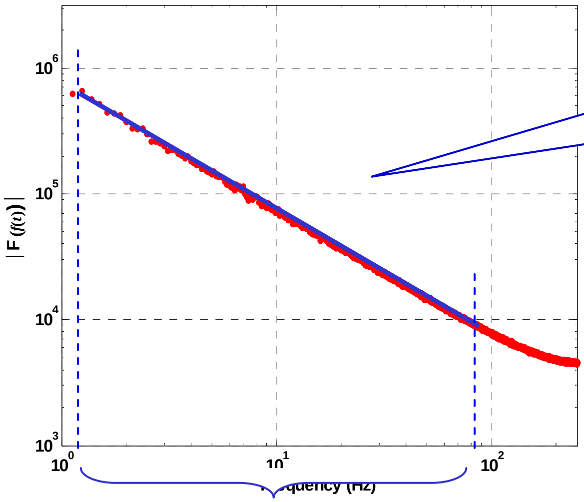
Manipulator vibrations: Impacts and liquid motion



Summary of the captured signals

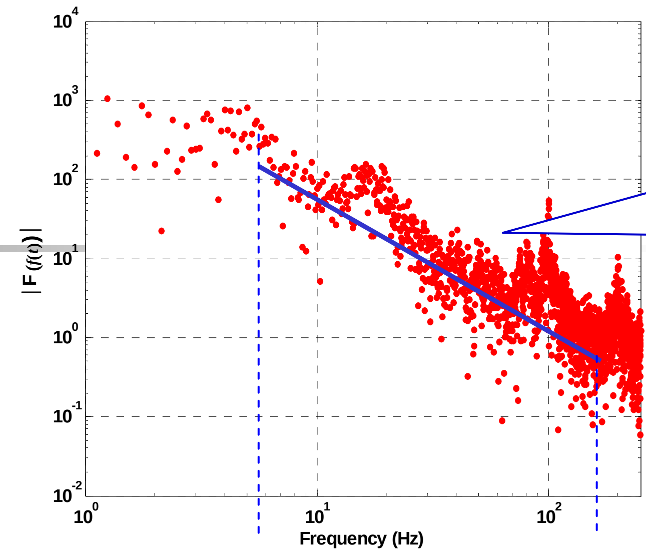
	Axis Positions	Internal signals
	Motors Currents	
	Wrist Forces	External signals
	Wrist Moments	
	Container Accelerations	

Signals in the Frequency Domain: Set of Spectrum Shapes



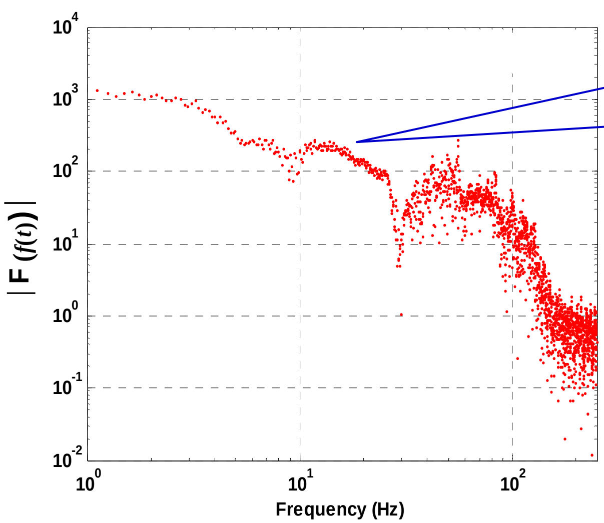
well defined

>= 1 decade



Scattered but partially well defined

>= 1 decade



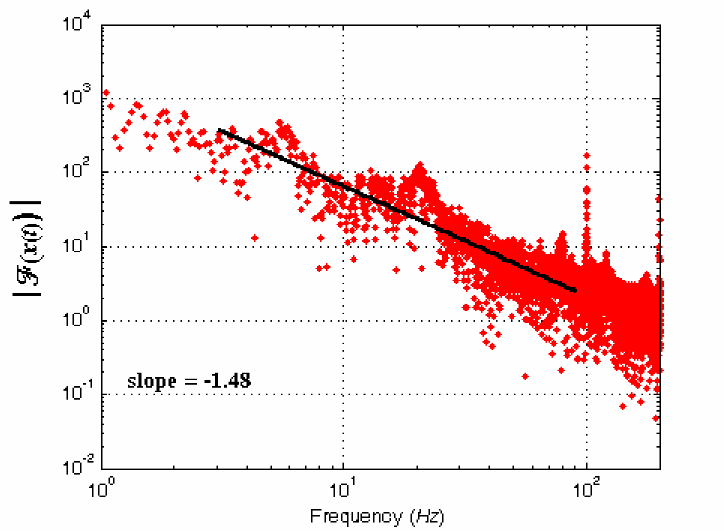
well defined but complicated

Trendline based on a power law approximation

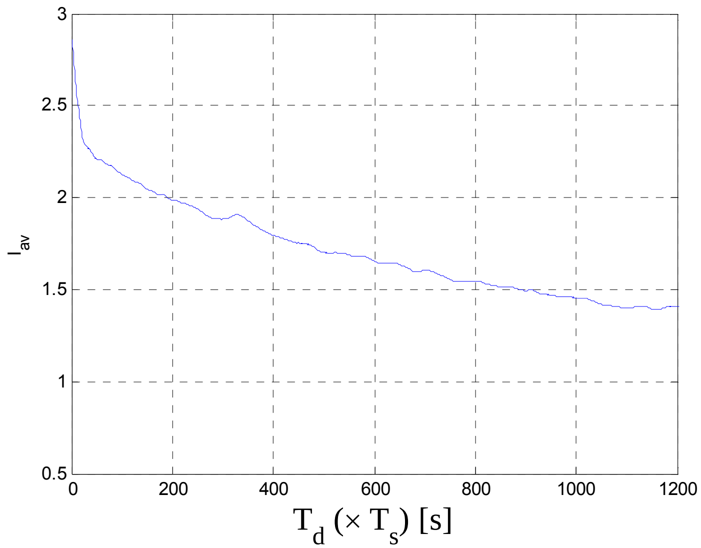
$$\left|\mathrm{F}\left\{f(t)\right\}\right| \approx c\omega^m$$

Pseudo Phase Plane (PPP)

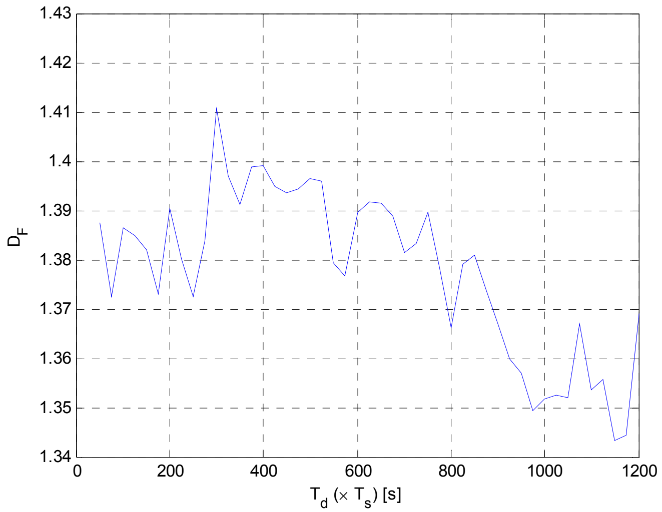
Spectrum of the axis 3 motor current



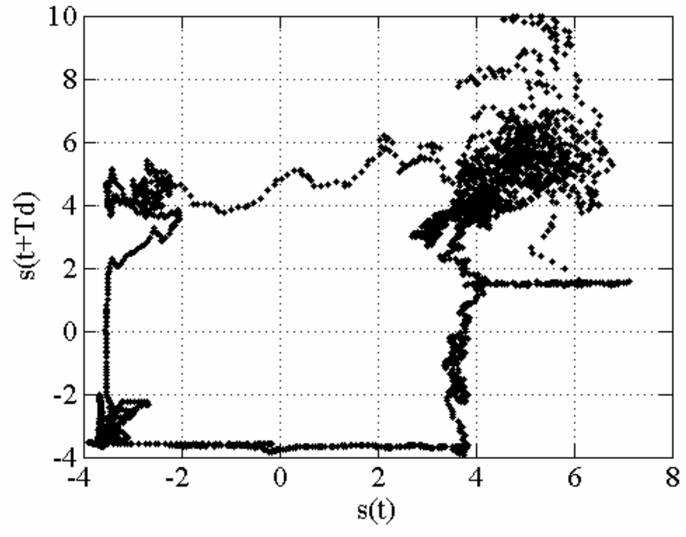
Average mutual info vs time delay



Fractal dimension of PPP vs time delay

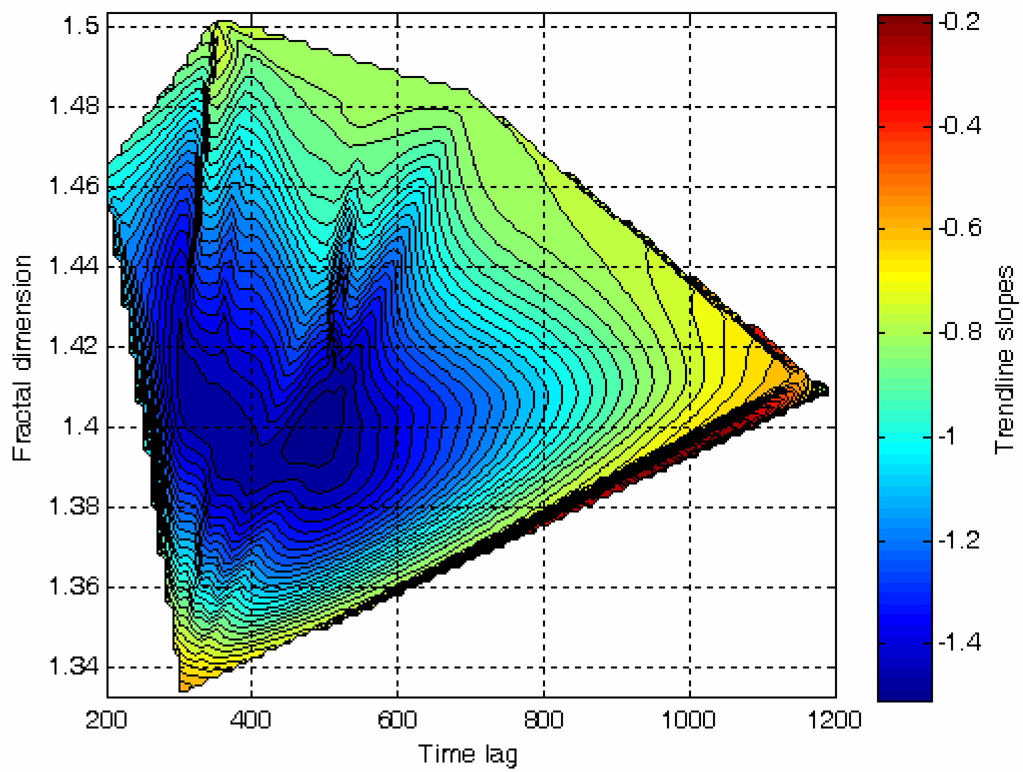


PPP for time delay = 300 samples



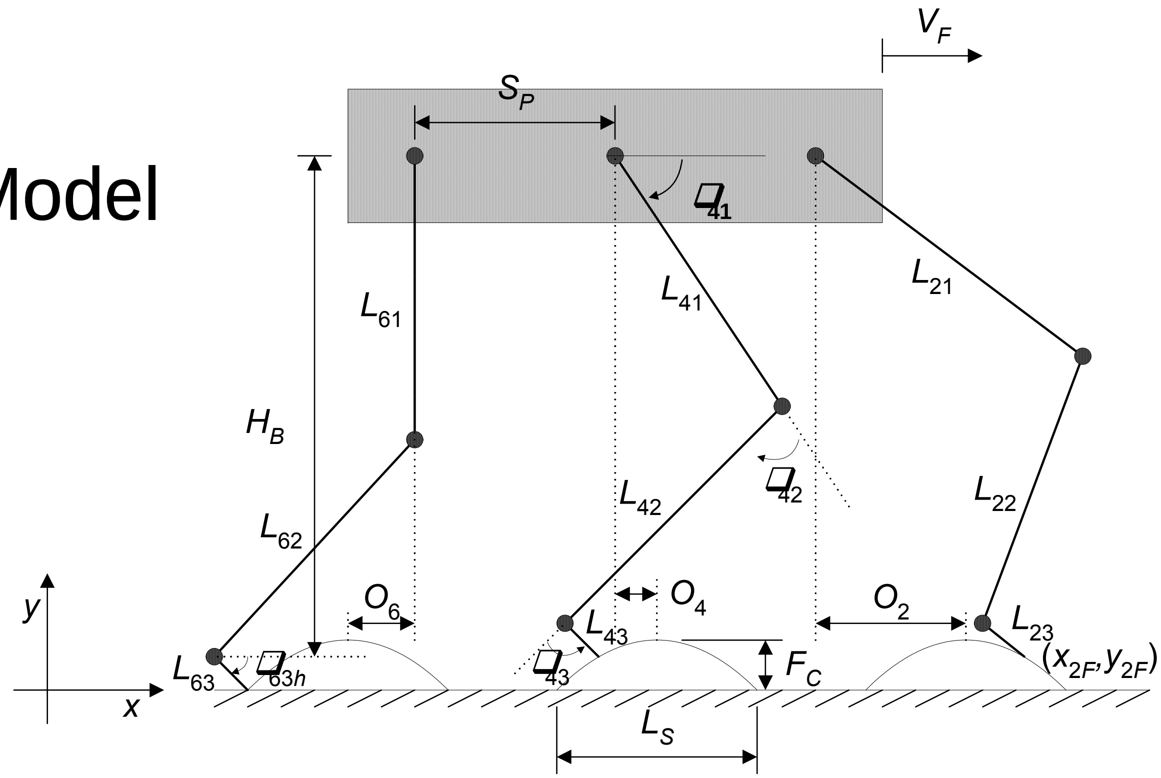
Relationship between descriptions

Trendline slope of the spectra of the electrical motor currents
vs
fractal dimension of PPP and time delay



Hexapod robot: PD$^{\alpha}$ control

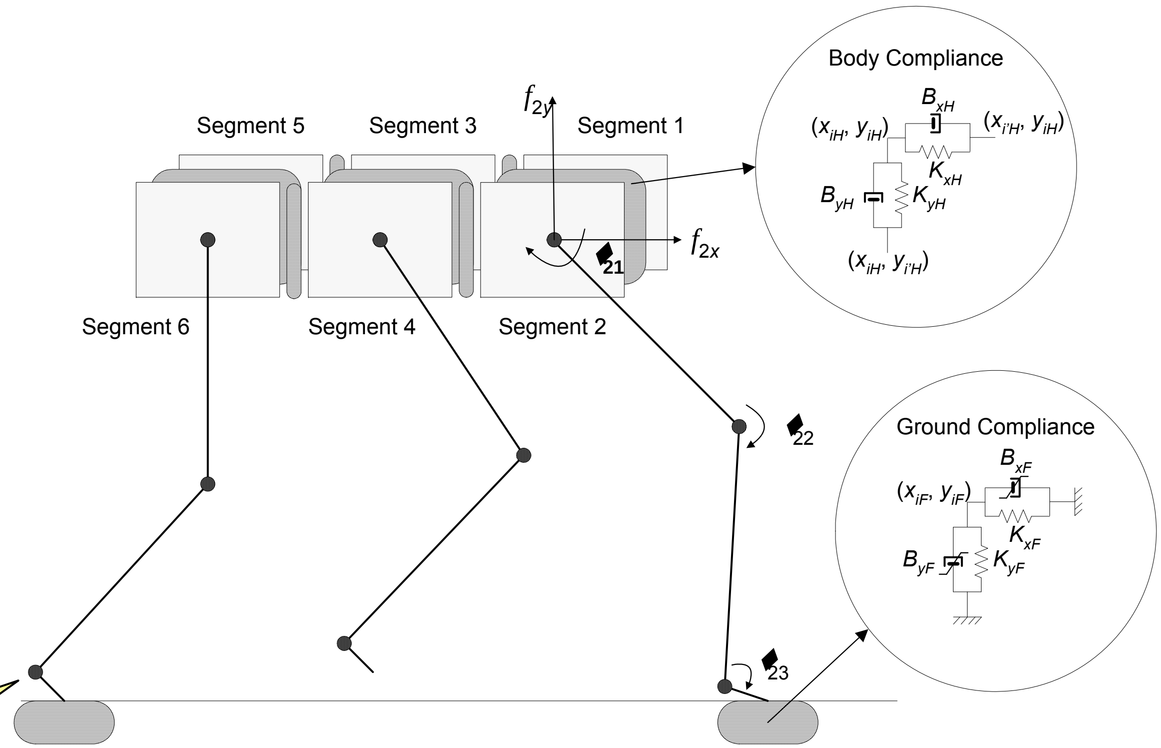
Kinematic Model



Hexapod robot: PD$^{\alpha}$ control

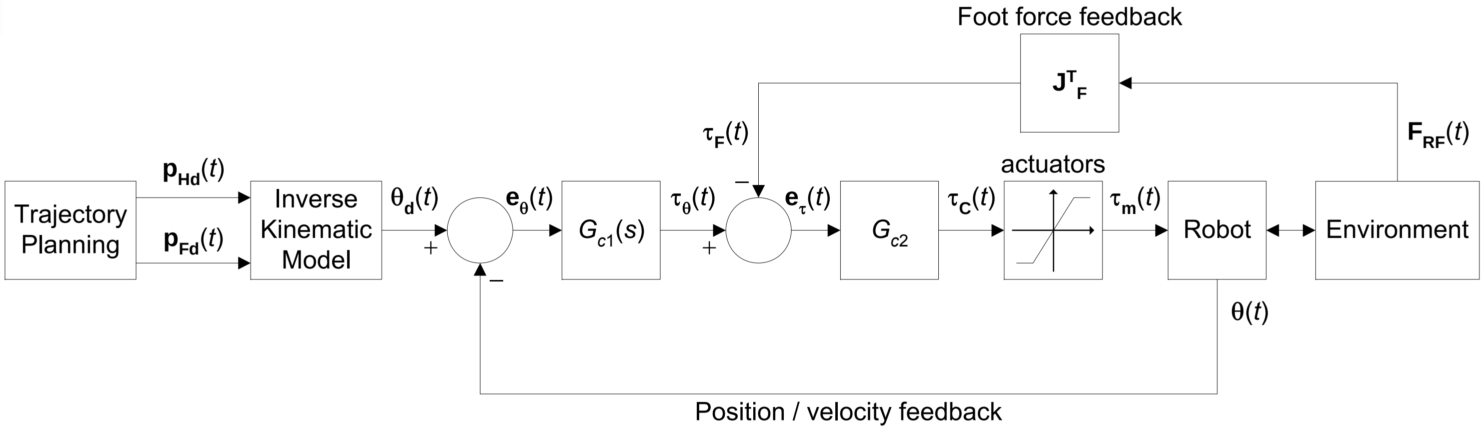
Dynamic Model

- Body compliance
- Ground dynamics
- 3 dof legs



joint 3 actuation:
passive *vs* active

Control architecture



- Outer loop: position feedback
- Inner loop: foot contact force feedback

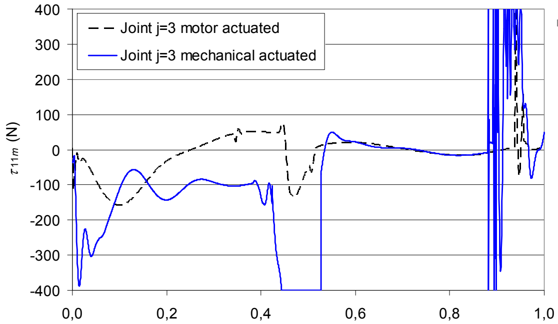
$$G_{C1j}(s) = Kp_{1j} + K\alpha_j s^{\alpha_j}, \quad \alpha_j \in \Re, \quad j = 1,2,3$$

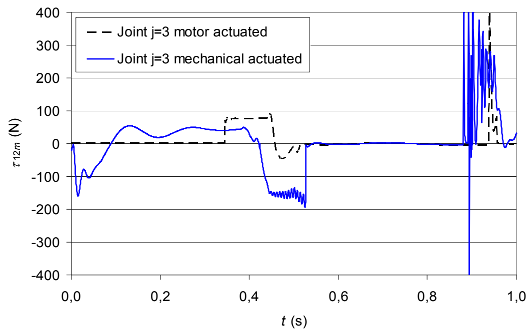
$$G_{C2j}(s) = Kp_{2j}, \quad j = 1,2,3$$

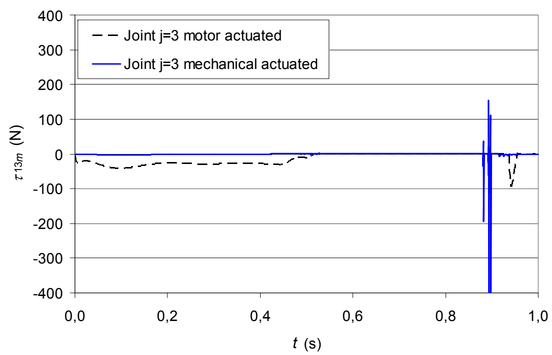
System performance

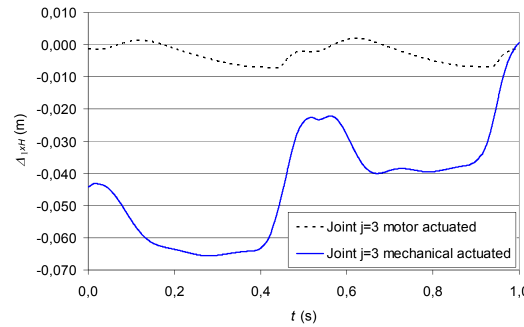
- Analysis of experiments
  - superior performance of the FO controller for $\alpha_j \approx 0.5$
  - best situation when joint 3 is motor actuated
  - comparison of the plots of the leg joint torques τ_{1jm} and of the hip trajectory tracking errors Δ_{1xF} and Δ_{1yF} versus t for both cases under study, when $\alpha_j = 0.5$

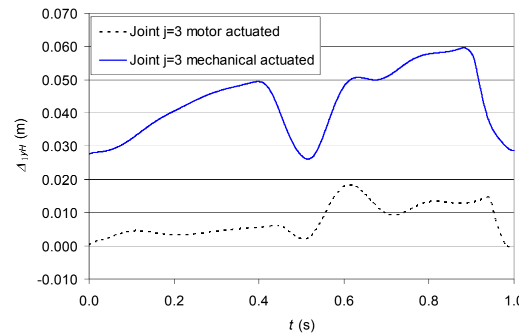
System performance











Conclusions

Only a fraction!

- Fractional models capture phenomena and properties that classical integer-order neglect
- Recent studies encourage the dynamical analysis and control of systems based on FC

FC - Books

- K. B. Oldham, J. Spanier, *The Fractional Calculus*, Academic Press, 1974.
- A. Oustaloup, *La Commande CRONE: Commande Robuste d'Ordre Non Entier*, Editions Hermès, 1991.
- K. S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Fractional Differential Equations*, Wiley & Sons, 1993.
- S. G. Samko, A. A. Kilbas, O. I. Marichev, *Fractional Integrals and Derivatives*. Gordon and Breach, 1993.
- I. Podlubny, *Fractional Differential Equations*, Academic Press, 1999.
- Ed. Jocelyn Sabatier, Om Agrawal, J. Tenreiro Machado (ed), *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering*, Springer, 2007.

FC- Special Issues

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