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ON THE STATISTICAL ANALYSIS OF MECHANICAL MANIPULATORS

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Abstract- A novel approach to the modelling of manipulators is presented. Usually, system descriptions are based on differential equations which lead to very precise results but need laborious computations. The statistical modelling overcomes these problems and gives clear guidelines towards performance optimization.

I. INTRODUCTION

The performances of the human arm induced an extensive research towards the design of efficient mechanical manipulators [1-3]. However, a simple optimization criterion comprising all the relevant phenomena is still lacking. In this paper we address this problem as follows. Sections two and three discuss the modelling problem and the principles of the statistical method respectively, and section four presents the conclusions.

II. ON THE STATISTICAL MODELLING OF ROBOT MANIPULATORS

The modelling of manipulators using the formalism of classical mechanics leads to a set of differential equations which describe the kinematics, the statics and the dynamics. However, such descriptions involve a multitude of factors which give rise to complex analysis and design procedures. On the other hand, statistics is a mathematical strategy well adapted to this type of problem. In this line of thought, we may consider the statistical description of both the input and output variables (i.v.'s and o.v.'s) as well as a third set consisting on the parameters which are to be optimized.

In the case of a manipulator the vectors of positions, velocities and accelerations in the operational space $\{p,\dot{p},\ddot{p}\}$ act as i.v.'s of the kinematic system. The vectors of positions, velocities and accelerations in the joint space $\{q,\dot{q},\ddot{q}\}$ act as o.v.'s of the kinematic system, but play the role of i.v.'s in the dynamic model. The vector of joint torques $\{T\}$ corresponds to the o.v.'s of the dynamic system and the set of parameters consists of link lengths, masses and inertias. The statistical description of these variables, does not consider the time variable. Therefore, variables that are related through the time derivative operator are considered independent of each other.

III. A STATISTICAL MODEL OF THE 2R ROBOT MANIPULATOR

In statistical terms the kinematics relates the p.d.f.'s of $\{p,\dot{p},\ddot{p}\}$ and $\{q,\dot{q},\ddot{q}\}$ through the expressions:

$$f_q(q) = |J_P| f_P(p) \quad (1a)$$

$$f_{q\dot{q}}(q,\dot{q}) = |J_V| f_{p\dot{p}}(p,\dot{p}) \quad (1b)$$

$$f_{q\dot{q}\ddot{q}}(q,\dot{q},\ddot{q}) = |J_A| f_{p\dot{p}\ddot{p}}(p,\dot{p},\ddot{p}) \quad (1c)$$

Each of the p.d.f.'s of the o.v.'s $(f_q, f_{q\dot{q}}, f_{q\dot{q}\ddot{q}})$ consists on two factors namely, the jacobians (J_P, J_V, J_A) which depend solely on the system kinematic properties and the p.d.f.'s of the i.v.'s $(f_P, f_{p\dot{p}}, f_{p\dot{p}\ddot{p}})$ which are a measure of the task requirements. Bearing these facts in mind, we decided to excite through a numerical random sample different structures of a 2R manipulator having the same total dimension $L=r_1+r_2$ but distinct ratios $\mu=r_1/r_2$ of link lengths (r_i, $i=1,2$). These experiments were performed for several categories of requirements of the i.v.'s and the results were analysed through the histograms of the marginal p.d.f.'s for the o.v.'s. The numerical results reveal the high stress imposed on $\dot{q}$ and $\ddot{q}$ by requirements on $\dot{p}$ while being much less sensitive to requirements on $\ddot{p}$ [4,5]. Furthermore, the results show a minimum for mechanical structures with $\mu=1$ yet, this conclusion can also be inferred from expression (1). Indeed, for symmetrical histograms about zero on the horizontal axis, having a peak on that point, a larger value of the jacobian corresponds to a smaller dispersion of the random variable. This, in turn, means average smaller amplitude requirements posed to that variable. Applying this optimization criterion to the jacobians and noting that they are consecutive powers of $r_1 r_2 S_2$, then its maximization requires the same steps. Therefore, for:

$$L = r_1+r_2, \quad \mu = r_1/r_2 \quad (2)$$

a maximum occurs when:

$$\mu = 1 \quad (3a)$$

$$q_2 = \pi/2 \quad (3b)$$

Moreover, our method shows that further optimization can be achieved through the selection of adequate p.d.f.'s for the i.v.'s. This second step of optimization defines, in a statistical sense, an optimum kinematic class for the manipulator trajectories. In this sense, (3a) and (3b) represent the optimal kinematic conditions for the robot structure and for the trajectories, respectively. These conjectures were tested numerically and the results revealed a remarkable improvement particularly for $\dot{p}$ requirements.

The statistical study of both the kinematic and dynamic systems requires steps similar to those adopted previously, namely characterisation of the p.d.f.'s of the i.v.'s $\{p,\dot{p},\ddot{p}\}$, "stimulation" of the system through numerical experiments and analysis of the histograms of the o.v.'s $\{T\}$. The statistical experiments [4,5] reveal that low requirements on $\dot{p}$ and $\ddot{p}$ gives almost similar results because the gravitational torques predominate. On the other hand, high requirements of $\dot{p}$ have a much stronger influence than high requirements of $\ddot{p}$ and therefore, kinematics prevails over the dynamics. Therefore, standard "joint-actuated" robot structures are not well adapted to manipulating tasks and alternative solutions using "muscle-like" actuators and mechanical levers will allow superior performances.

IV. CONCLUSIONS

The statistical modelling of mechanical manipulators is presented. This method provides a framework giving clear guidelines towards the optimization of the robot structure and the definition of better trajectory planning algorithms. The inherent use of histograms allows not only fast calculation procedures but, above all, the use of numerical data; consequently, complex modelling exercises can be avoided. The results give a net basis towards better robotic structures, with performances similar to those revealed by the biological systems.

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