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STATISTICAL MODELLING OF ROBOT MANIPULATORS

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Abstract: A novel approach to the modelling of robot manipulators is presented. Usually, system descriptions are based on a set of differential equations which, due to their nature, lead to very precise results and strategies but, in general, require laborious computations. This motivates the need of alternative methods based on other mathematical concepts. The proposed statistical model gives clear guidelines towards the robot kinematic and dynamic optimization and the definition of rest and active manipulation regions. Furthermore, the results point out ideal properties for the actuators namely as "position and acceleration" devices instead of "velocity" machines. This observation is of utmost importance as it gives a basis to new mechanical robot manipulator structures, with performances close to biomechanical systems.

Introduction

In the last years the area of application of robot manipulators has widened to encompass a large range of industrial and scientific applications. However, many of them pose challenging issues to the available industrial manipulators. The relatively poor performance of today's industrial manipulators when compared with the human arm, motivated an extensive research towards the development of better mechanical structures and actuators. The present paper deals with the problem of the manipulator performance optimization. Considerable research in this area has already been done on the kinematics [1-2] and dynamics [3-5]. From those studies it is clear that a simple optimization criterion comprising both the kinematics and dynamics is still lacking. In this paper we address this problem, and our presentation is organized as follows: in section two the robot manipulator model and the associated optimization criterion are discussed, in section three a new model which leads to a natural optimization criterion is formulated and applied to a 2R robot manipulator, and finally, in section four, future developments of this work are discussed and some conclusions are drawn.

On the Statistical Modelling of Robot Manipulators

The classical modelling of robot manipulators is well known. For the kinematics a set of equations relating the joint space and the operational space, can be found to be of the form:

$$q = \alpha(p) \quad (1a)$$
$$\dot{q} = \theta(p,\dot{p}) \quad (1b)$$
$$\ddot{q} = \Phi(p,\dot{p},\ddot{p}) \quad (1c)$$

where $\{q,\dot{q},\ddot{q}\}$ ($\{p,\dot{p},\ddot{p}\}$) are the n-vectors of positions, velocities and accelerations in the joint (operational) space for a n degrees of freedom (d.o.f.) manipulator. Associated with the kinematic model we have the statics model, that relates the operational space forces Γ with the joint actuator torques T:

$$T = J(q)^T \, \Gamma \quad (2)$$

where J(q) is the jacobian matrix corresponding to the differential relationship $\dot{p}=J(q)\dot{q}$.

The dynamics is described by a nonlinear matrix differential equation:

$$T = I(q)\ddot{q}+C(q,\dot{q})+G(q) \quad (3)$$

having $T_I=I(q)\ddot{q}$, $T_C=C(q,\dot{q})$ and $T_G=G(q)$ for the n-vectors of inertial, Coriolis/centripetal and gravitational torques.

Based on these equations considerable research has been done on issues such as manipulator structure optimization [1-5] and path planning algorithms [6-7]. However, a more sound consideration of the whole theme reveals that these methods are far from achieving a comprehensive formulation. This observation motivates the re-evaluation of the approachs in use. In fact, expressions (1)-(3) show that the plethora of variables and parameters involved, gives rise to a cumbersome work both in the analysis and design stages. The gigantic number of possible combinations of values indicates that, in order to overcome implementation problems, alternative concepts are required. Statistics is a mathematical strategy well adapted to this type of problem. If with this method, we lose the "certainty" of the deterministic model, we gain a simpler and more intuitive viewpoint. This approach has already been used by other researchers [8-9] in some restricted classes of problems. In the sequel we refer to the new approach, as the statistical model [10-11] to stress the contrast with the standard method.

Our modelling procedure comprises:

- The statistical description of a set of input variables, that is variables that are free to change independently.
- The statistical description of a set of output variables, that is, variables that are functions of the previous ones.
- A set of parameters which are to be optimized in the design stage.

The above definition allows a considerable freedom in the choice of each set. In the present case, the distribution of the relevant variables through the three referred sets is established as follows:

- $\{p,\dot{p},\ddot{p}\}$ act as input variables of the kinematic system. This option enables a definition of the required kinematic performances on the operational space which are more natural to the designer.
- $\{q,\dot{q},\ddot{q}\}$ act as output variables of the kinematic system, but play the role of input variables set in the dynamic model. In this way we arrive at a relationship between kinematics and dynamics in a form amenable to performance optimization criteria as defined in the sequel.
- The set of dynamic output variables consists of the required joint torques {T}.
- The parameter set consists of link lengths, masses and inertias.

In other words, we are stating that in the kinematics (dynamics), p, $\dot{p}$ and $\ddot{p}$ (q, $\dot{q}$ and $\ddot{q}$) are considered as independent random variables, its probability density functions (p.d.f.'s) being similar to the histograms of a long run sampling, while q, $\dot{q}$, $\ddot{q}$ (T) are the corresponding random dependent variables. The statistical description of the involved variables, does not consider the (implicit) time variable. In this way, variables that are related through the time derivative operator are considered independent of each other.

Let us now adopt the 2R joint-actuated robot manipulator (Fig. 1) as the support for the development and implementation of the new modelling concepts. In the next sub-section we begin by introducing our approach in the kinematic case. In the second sub-section we shall analyse the dynamic case and in the third sub-section we investigate the properties of the overall (i.e. kinematics + dynamics) system.

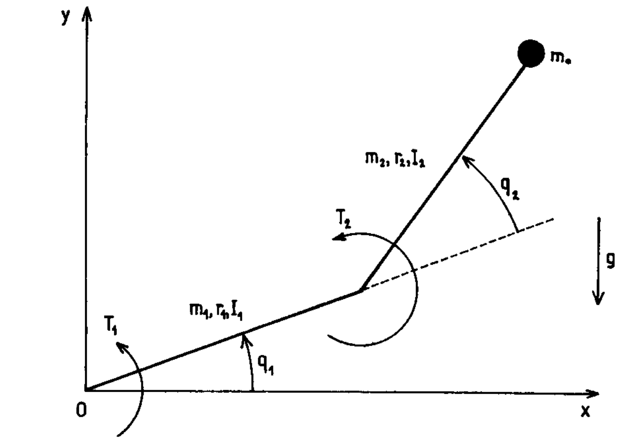

Fig. 1 The 2R joint-actuated robot manipulator

<u>A Statistical Model for the Kinematics of the 2R Joint-Actuated Manipulator</u>

The set of kinematic input variables consists of position, velocity and acceleration that our prototype manipulator is required to perform in the operational space. Therefore, it is necessary to characterize them in statistical terms, namely by defining appropriate p.d.f.'s for each variable. As there is no a priori knowledge about the typical behaviour we start with some reasonable assumptions namely, for the position variable $p=[x,y]^T$ we consider a bidimensional uniform p.d.f.

$$f_P(p)=\begin{cases} C & \text{if } (r_1-r_2)^2 \le x^2+y^2 \le (r_1+r_2)^2 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

with $C=1/\{\pi[(r_1+r_2)^2-(r_1-r_2)^2]\}$.

In the sequel we will see how to modify the input p.d.f. in order that the kinematic performances are optimized. It is also necessary to define the p.d.f.'s for velocity and acceleration. By the same above arguments, we decided to use bidimensional Gaussian p.d.f.'s with zero mean

$$f_{\dot{P}}(\dot{p})=EXP[-(\dot{x}^2+\dot{y}^2)/(2\sigma_{\dot{P}}^2)]/(2\pi\sigma_{\dot{P}}^2) \quad (5)$$
$$f_{\ddot{P}}(\ddot{p})=EXP[-(\ddot{x}^2+\ddot{y}^2)/(2\sigma_{\ddot{P}}^2)]/(2\pi\sigma_{\ddot{P}}^2) \quad (6)$$

Moreover, using these p.d.f.'s we impose some useful properties, such as:

- The random variables position, velocity and acceleration in the operational space are independent of each other.
- The velocity and acceleration vectors are made of two independent components, that is $\dot{x}$ ($\ddot{x}$) is independent of $\dot{y}$ ($\ddot{y}$).

The "excitation" of the (inverse) kinematic system produces output random variables q, $\dot{q}$ and $\ddot{q}$, with p.d.f.'s which are related to the previous ones by:

$$f_Q(q) = J_P f_P(p) \quad (7a)$$
$$f_{Q\dot{Q}}(q,\dot{q}) = J_V f_{P\dot{P}}(p,\dot{p}) \quad (7b)$$
$$f_{Q\dot{Q}\ddot{Q}}(q,\dot{q},\ddot{q}) = J_A f_{P\dot{P}\ddot{P}}(p,\dot{p},\ddot{p}) \quad (7c)$$

where the jacobians J_P, J_V and J_A are:

$$J_P=\frac{\partial(p)}{\partial(q)}=r_1r_2S_2 \quad (8a)$$

$$J_V=\frac{\partial(p,\dot{p})}{\partial(q,\dot{q})}=J_P(r_1r_2S_2) \quad (8b)$$

TABLE 1 Numerical values of the 2R manipulator

R_1=0.05 m, R_2=0.0389 m, r_1=0.3 m, r_2=0.3 m
m_1=2.16 kg, m_2=1.68 kg, m_0=0 kg
$I_i=m_i(r_i^2/12+R_i^2/4)$, i=1,2

$$J_A=\frac{\partial(p,\dot{p},\ddot{p})}{\partial(q,\dot{q},\ddot{q})}=J_V(r_1r_2S_2) \quad (8c)$$

Each of the expressions (7) is made of two distinct factors:

- Weighting factors - J_P, J_V and J_A - which depend solely on the system kinematic properties
- The "excitation" p.d.f.'s - $f_P(p)$, $f_{P\dot{P}}(p,\dot{p})$ and $f_{P\dot{P}\ddot{P}}(p,\dot{p},\ddot{p})$ - which are a measure of the task requirements.

These factors can be interpreted in a system theoretic framework. The jacobians characterize the system intrinsic properties, while the excitation p.d.f.'s correspond to the system response to the input variables.

Bearing these facts in mind, several experiments were performed, having:

- The total link length constant, L=0.6.
- Seven robot configurations with ratios $\mu=r_1/r_2$ equal to 0.4, 0.6, 0.8, 1, 1.2, 1.4 and 1.6, respectively.
- Operational space categories corresponding to nine distinct requirements of velocity and acceleration:
  1. $\sigma_{\dot{P}}=0.1$ $\sigma_{\ddot{P}}=0.1$, 4. $\sigma_{\dot{P}}=1$ $\sigma_{\ddot{P}}=0.1$, 7. $\sigma_{\dot{P}}=10$ $\sigma_{\ddot{P}}=0.1$
  2. $\sigma_{\dot{P}}=0.1$ $\sigma_{\ddot{P}}=1$, 5. $\sigma_{\dot{P}}=1$ $\sigma_{\ddot{P}}=1$, 8. $\sigma_{\dot{P}}=10$ $\sigma_{\ddot{P}}=1$
  3. $\sigma_{\dot{P}}=0.1$ $\sigma_{\ddot{P}}=10$, 6. $\sigma_{\dot{P}}=1$ $\sigma_{\ddot{P}}=10$, 9. $\sigma_{\dot{P}}=10$ $\sigma_{\ddot{P}}=10$
- Excitation of the kinematic system with a numerical random sample of 4000 operational space variables obeying the p.d.f.'s (4)-(6).
- Analysis of the resulting histograms of the output variables amplitude. In order to simplify matters, only marginal p.d.f.'s were considered.

After a large number of experiments using the numerical set of parameters depicted in Table 1, we concluded that the shape of the resulting p.d.f.'s varied significantly from variable to variable, but all of them showed symmetry around zero. For this reason, and in order to characterize the resulting histograms by a scalar index, we decided to adopt for this index the difference between the 97.5% and 2.5% percentiles that is, the 95%-inter-percentile range. The resulting histograms are condensed through this index and depicted in Fig. 2. We can observe in the majority of the charts a minimum about μ=1 yet, this conclusion can be easily inferred from (7). In fact, for symmetrical histograms about zero on the x-axis, with a peak on that point, a larger value of the jacobian corresponds to a smaller dispersion of the random variable. This, in turn, means average smaller amplitude requirements posed to that variable. Therefore, we have found an optimization criterion which is based on the new statistical modelling concepts.

As the maximization of J_P, J_V and J_A requires the same steps, we have for:

$$L = r_1+r_2 \quad (9a)$$
$$\mu = r_1/r_2 \quad (9b)$$

that a maximum occurs when:

$$\mu = 1 \quad (10a)$$
$$q_2 = \pi/2 \quad (10b)$$

which coincide with the results obtained (using the classical approach) in previous studies [1,3]. Furthermore, our optimization criteria enables additional conclusions:

- Because J_P, J_V and J_A are consecutive powers of $r_1r_2S_2$, we see that for a given deviation from the

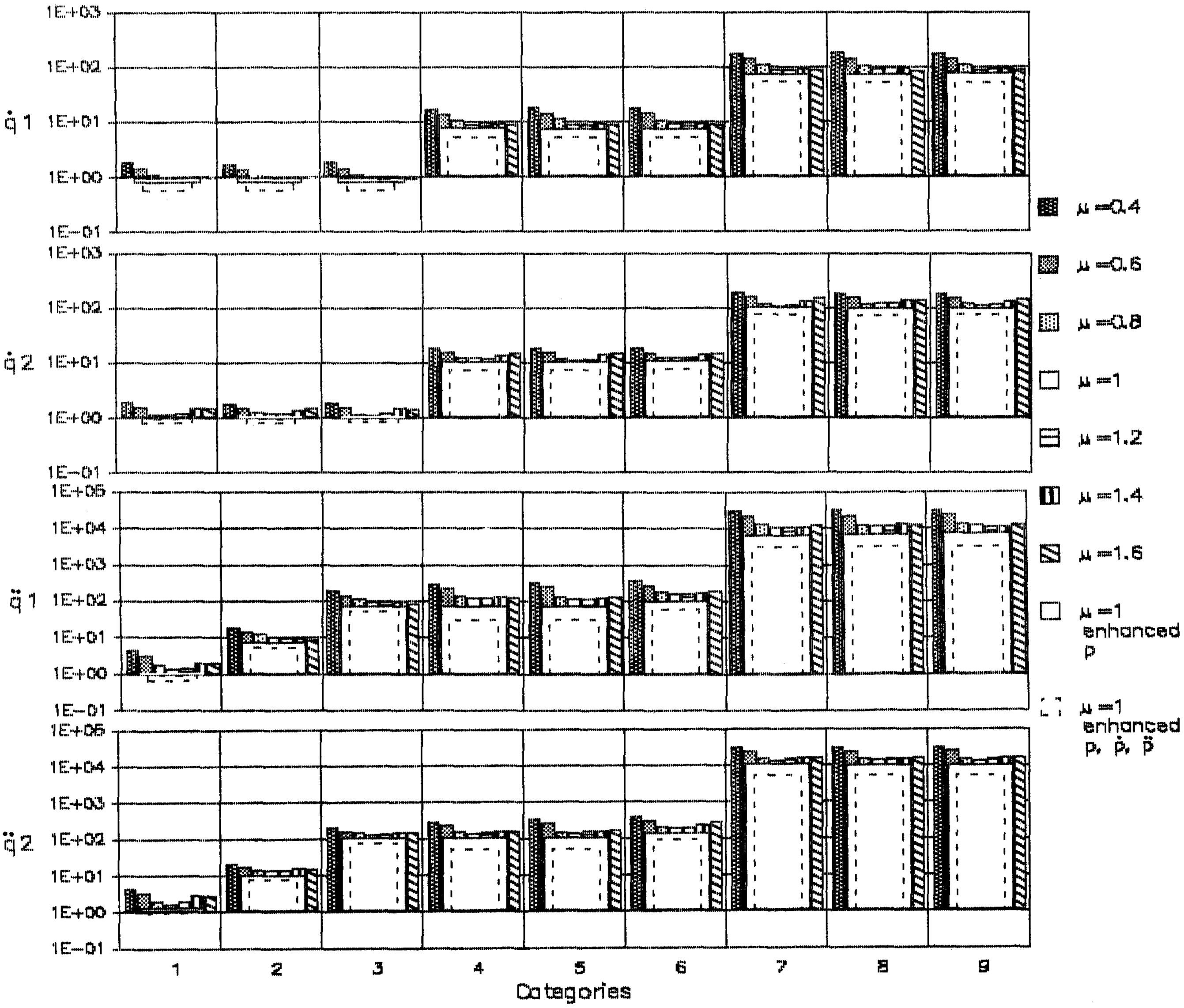

Fig. 2 Comparison chart for the 2R joint-actuated robot kinematic performances. The narrow columns correspond to seven geometric configurations "excited" with p.d.f.'s (4), (5) and (6). The wider columns correspond to the optimum geometric $\mu=1$ "excited" with the enhanced p.d.f. (12) and (5)-(6) for the solid borders and enhanced p.d.f.'s (12), (14) and (15) for the dotted borders.

optimal values (10) we have an increasingly degradation of the optimization criterion with the powers of $r_1 r_2 S_2$. In other words, for a given deviation we have, by increasing order of sensitivity, position, velocity and acceleration.

- Due to (2) a kinematic optimization is equivalent to a static optimization.
- If further optimization is desired, then the next step will be the selection of an optimum "excitation" p.d.f.. This second step of optimization will define, in a statistical sense, an optimum kinematic class for the manipulator trajectories. Obviously, we can find a multitude of different p.d.f's obeying (10); nevertheless, for the subsequent study a particular choice is of minor importance. Consequently, we decided to adopt the following family of position p.d.f.'s in the operational space (with $K \geq 1$):

$$f_P(x,y)=cons*\{1-[(x^2+y^2-r_1^2-r_2^2)/(2r_1r_2)]^2\}^{(K-1)/2} \quad (11)$$

which, in the joint space, corresponds to:

$$f_Q(q_1,q_2)=constant*S_2^K \quad (12)$$

As extreme cases, we have that for K=1 it becomes the uniform p.d.f. (4), while for K->∞ we get Dirac type optimum p.d.f. ($\delta(.)$) in the sense of (10):

$$f_P(x,y)=\delta[x^2+y^2-(r_1^2+r_2^2)] \quad (13a)$$
$$f_Q(q_1,q_2)=1/2[\delta(q_2+\pi/2)+\delta(q_2-\pi/2)] \quad (13b)$$

As far as velocity and acceleration are concerned we can see that the kinematic study does not point out any special class of p.d.f.'s. Nevertheless, these variables are negatively affected by the position deviation from the optimum configuration $q_2=\pi/2$. Therefore, we decided to study the system behaviour both for performance requirements described by p.d.f.'s (12), (5) and (6) and for the alternative situation corresponding to p.d.f. (12) associated with the "enhanced" q_2-dependent velocity and acceleration p.d.f.'s:

$$f_{\dot{P}}(\dot{p},q_2)=EXP\{-(\dot{x}^2+\dot{y}^2)/[2\sigma_{\dot{P}}^2(q_2)]\}/[2\pi\sigma_{\dot{P}}^2(q_2)] \quad (14a)$$

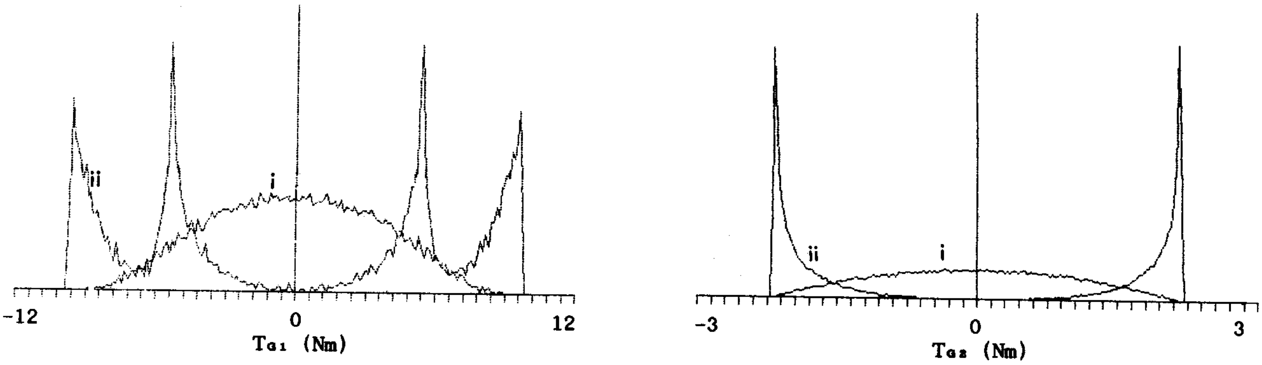

Fig. 3 Histograms of the joint-actuated robot torques for the gravitational terms T_G with "excitation" p.d.f.'s: i. $f_q(q_1,q_2)=constant*(S_1S_{12})^3$ ii. $f_q(q_1,q_2)=constant*(C_1C_{12})^3$.

$$\sigma_{\dot{p}}(q_2)=\begin{cases}2\sigma_{\dot{p}}|q_2|/\pi & \text{if } 0<|q_2|\le\pi/2\\ 2\sigma_{\dot{p}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\le\pi\end{cases} \qquad (14b)$$

$$f_{\ddot{p}}(\ddot{p},q_2)=EXP[-(\ddot{x}^2+\ddot{y}^2)/[2\sigma_{\ddot{p}}^2(q_2)]]/[2\pi\sigma_{\ddot{p}}^2(q_2)] \qquad (15a)$$

$$\sigma_{\ddot{p}}(q_2)=\begin{cases}2\sigma_{\ddot{p}}|q_2|/\pi & \text{if } 0<|q_2|\le\pi/2\\ 2\sigma_{\ddot{p}}|q_2-\pi|/\pi & \text{if } \pi/2<|q_2|\le\pi\end{cases} \qquad (15b)$$

To test numerically the above conjectures, the previous results for $\mu=1$ are compared with a new case using $\mu=1$ and K=3 in (11)-(12). This has revealed a remarkable performance improvement as shown in Fig. 2, particularly for velocity-dependent requirements.

A Statistical Model for the Dynamics of the 2R Joint-Actuated Manipulator

The statistical description of the dynamics requires steps similar to those adopted in the kinematics, namely:

- Characterisation of the input variables ($q,\dot{q}$ and $\ddot{q}$) through appropriate p.d.f.'s.
- "Stimulation" of the system behaviour through numerical experiments.
- Analysis of the histograms of the output variables (T).

However, a preliminary observation shows that the dynamic study is much more complex than the kinematic one. Due to this reason, and in order to gain a deeper insight for the subsequent study we decided to consider, in a first stage, as dynamic output variables, the components of the joint torques, that is the gravitational, Coriolis/centripetal and inertial torques. Based on this preliminary analysis we then consider the total joint torques. In the first stage we have:

$$f_G(q) = J_G f_q(q) \qquad (16a)$$
$$f_C(q,\dot{q}) = J_C f_{q\dot{q}}(q,\dot{q}) \qquad (16b)$$
$$f_I(q,\dot{q},\ddot{q}) = J_I f_{q\dot{q}\ddot{q}}(q,\dot{q},\ddot{q}) \qquad (16c)$$

where:

$$J_G=\frac{\partial(q)}{\partial(T_G)}=[(m_1/2+m_2+m_0)(m_2/2+m_0)g^2r_1r_2S_1S_{12}]^{-1} \qquad (17a)$$

$$J_C=\frac{\partial(q,\dot{q})}{\partial(T_G,T_C)}=J_G\{[2(m_2/2+m_0)r_1r_2S_2]^2\dot{q}_1\dot{q}_{12}\}^{-1} \qquad (17b)$$

$$J_I=\frac{\partial(q,\dot{q},\ddot{q})}{\partial(T_G,T_C,T_I)}=J_C\{[(m_1/4+m_2+m_0)r_1^2+I_1+(m_2/2+m_0)r_1r_2C_2][(m_2/4+m_0)r_2^2+I_2]-[(m_2/4+m_0)r_2^2+I_2+(m_2/2+m_0)r_1r_2C_2](m_2/2+m_0)r_1r_2C_2\}^{-1} \qquad (17c)$$

Unlike the kinematic situation, where the optimization was similar for all the jacobians, now their effects differ according to each dynamic term. Analysing the jacobians (17) we conclude that:

- The maximizing of J_G stipulates that q_1 and q_{12} should have p.d.f.'s with maxima at 0 or π. The observation of histograms resulting from "excitation" p.d.f.'s obeying these conditions showed an interesting result. As expected the (symmetrical) histograms resembled Dirac pulses; however, those peaks were located at non-zero values. In fact, the plots showed sharp symmetrical peaks located at the maxima (positive and negative) values attained by the gravitational torques. This means that, for this case, the optimization procedure must adopt an inverse strategy, that is to say we must minimize J_G (Fig. 3).
- The maximizing of J_C implies that q_2 must have a p.d.f. with a maximum on 0 or π. Numerical experiments showed that in this case the resulting histograms of the Coriolis/centripetal terms tended, as desired, towards a Dirac on zero.
- The analytical expression of J_I is more complex. Nevertheless, its analysis revealed a maximizing condition similar to the previous one (i.e. q_2 should have a p.d.f. with maxima at 0 or π).

Therefore, we may say that J_G defines a "rest region" while J_C and J_I define an "active region" of operation.

Now, we can proceed to the second stage, that is, the study of the (total) dynamics. The direct application of our optimizing method to the dynamics would require the mathematical and numerical treatment of 3n-dimensional p.d.f.'s. In order to avoid this intricate analysis, we decided to integrate the (partial) conclusions pointed out in the first stage (i.e. the guidelines resulting from the separate study of T_G, T_C and T_I) in the formulation of our present investigation. In this sense we decided to "excite" the dynamics with four different position p.d.f.'s (having K=3):

$$f_q(q_1,q_2)=constant*S_2^K \qquad (18)$$
$$f_q(q_1,q_2)=constant*(S_1S_2)^K \qquad (19)$$
$$f_q(q_1,q_2)=constant*(S_1S_{12})^K \qquad (20)$$
$$f_q(q_1,q_2)=constant*C_2^K \qquad (21)$$

which are suggested by the optimization of the kinematics, a compromise between kinematics and gravitational torques, the gravitational torques, and the Coriolis/centripetal and inertial torques, respectively. Due to the non-existence of optimization guidelines on $\dot{q}$ and $\ddot{q}$, we decided to consider two gaussian "excitation" p.d.f.'s (i=1,2):

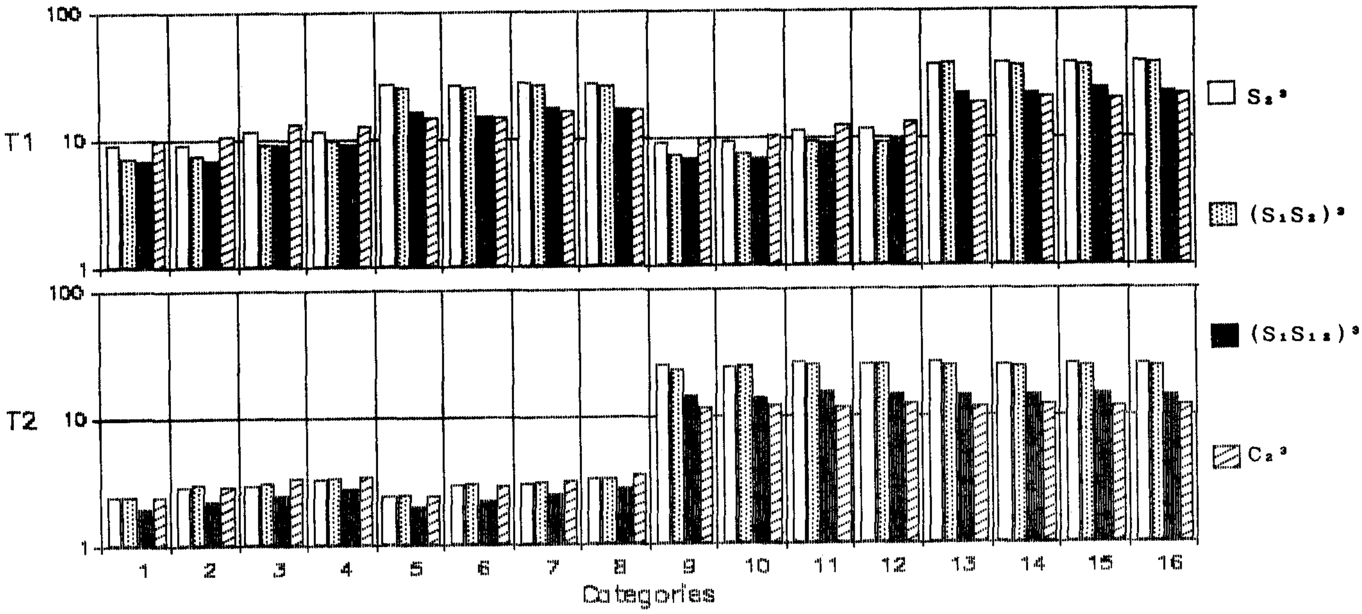

Fig. 4 Comparison charts for the 2R joint-actuated robot dynamic performances with $\mu=1$ when subjected to "excitation" p.d.f.'s:
1st column: $f_q(q_1,q_2)=\text{constant}*S_2{}^3$ 3rd column: $f_q(q_1,q_2)=\text{constant}*(S_1S_{12})^3$
2nd column: $f_q(q_1,q_2)=\text{constant}*(S_1S_2)^3$ 4th column: $f_q(q_1,q_2)=\text{constant}*C_2{}^3$

$$f_{\dot{q}_i}(\dot{q}_i)=\text{EXP}[-\dot{q}_i{}^2/(2\sigma_{\dot{q}_i}{}^2)]/(2\pi\sigma_{\dot{q}_i}{}^2) \quad (22)$$
$$f_{\ddot{q}_i}(\ddot{q}_i)=\text{EXP}[-\ddot{q}_i{}^2/(2\sigma_{\ddot{q}_i}{}^2)]/(2\pi\sigma_{\ddot{q}_i}{}^2) \quad (23)$$

and the following sixteen different categories:

1. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=0.1$
2. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=10$
3. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=0.1$
4. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=0.1$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=10$
5. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=0.1$
6. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=10$
7. $\sigma_{\dot{q}_1}=0.1$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=0.1$
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13. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=0.1$
14. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=0.1$, $\sigma_{\ddot{q}_2}=10$
15. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=0.1$
16. $\sigma_{\dot{q}_1}=10$, $\sigma_{\dot{q}_2}=10$, $\sigma_{\ddot{q}_1}=10$, $\sigma_{\ddot{q}_2}=10$

Figure 4 depicts the results of T_1 and T_2 when the 95% index is applied to the corresponding histograms. These charts revealed several important properties:

- T_1 (T_2) depends strongly on $\dot{q}_2$ ($\dot{q}_1$).
- The joint torques (T) have low sensitivity, in a statistical sense, to acceleration ($\ddot{q}$) requirements.
- The suggestions pointed out by the first stage are compatible with these last results. In fact, for "rest" (or "non-active") requirements, p.d.f. (20) is the more appropriate, while for the "active" (or "non-rest") situation p.d.f. (21) is the optimal.

The Statistical Description of the Total System

Up to now we discussed the kinematics and dynamics separately however, in the real manipulator these systems can not be divided. In other words, the study of a robot manipulator, must integrate both systems. Therefore, the statistical description of the total system (i.e. both the kinematics and dynamics) will have cross-coupling effects and its influence must be evaluated. To test these effects, the total system was numerically "excited" through random samples according to p.d.f.'s (18)-(21). These position p.d.f.'s combined with the two alternative velocity and acceleration p.d.f.'s (5)-(6) or (14)-(15) (more precisely we are using their equivalent p.d.f.'s defined on the operational space), reveal that (Fig. 5):

- For low velocity and acceleration requirements (category 1), the 95% index gives almost similar results for all p.d.f.'s, because the gravitational torques predominate.
- Velocity requirements ($\dot{p}$) have a much stronger influence than acceleration requirements ($\ddot{p}$).
- Kinematic effects prevail over the dynamic ones and, therefore, the best results come from the "kinematic-dependent" p.d.f.'s (18) and (19).

In conclusion, the statistical analysis shows that the kinematics and dynamics have different effects upon the robot system. As shown, mechanical manipulators are much more sensitive to velocity requirements than to acceleration requirements. These facts indicate that we are dealing with "position and acceleration machines" rather than "velocity machines". Although obvious, this aspect has been somewhat overlooked. Moreover, it points out that the usual robot actuators, which are developments of standard "velocity machines" are not well adapted to robotic applications. Alternative solutions, such as muscle like "position and acceleration" actuators [12-15] will allow more efficient robot structures [16].

Conclusions

A new method for modelling robot manipulators is presented. Usually, system descriptions are based on a set of differential equations which, due to their nature lead to very precise results and strategies but, on the other hand, can be very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The proposed statistical model is a step in that direction which has been shown to present interesting properties and to enable new procedures. It provides a framework giving clear guidelines towards the robot

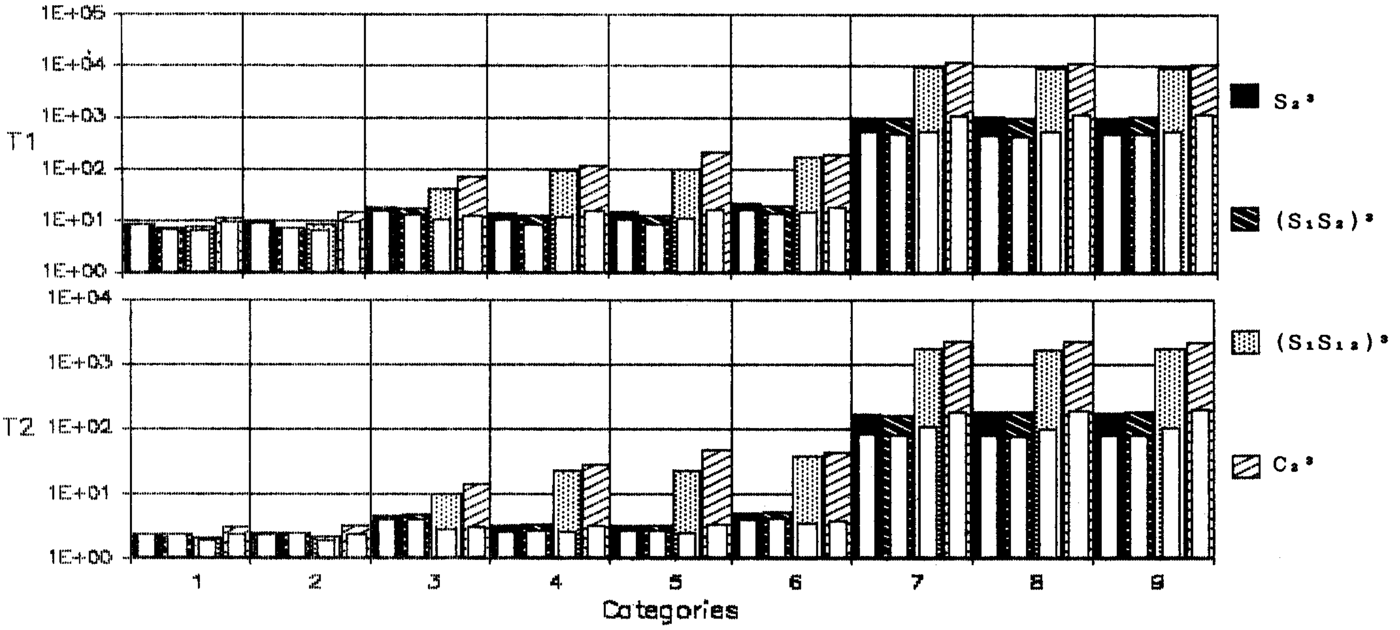

Fig. 5 Comparison chart for the 2R joint-actuated robot overall (kinematic + dynamic) performances with $\mu=1$ when subjected to "excitation" p.d.f.'s:
1st column: $f_Q(q_1,q_2)=\text{constant}*S_2{}^3$ 3rd column: $f_Q(q_1,q_2)=\text{constant}*(S_1S_{12})^3$
2nd column: $f_Q(q_1,q_2)=\text{constant}*(S_1S_2)^3$ 4th column: $f_Q(q_1,q_2)=\text{constant}*C_2{}^3$
The back columns correspond to p.d.f.'s (5)-(6) and the front white columns correspond to the enhanced p.d.f.'s (14)-(15).

structure optimization. As a result, the manipulator design procedure, both kinematic and dynamic, leads to simple and intuitive conclusions. Previously proposed schemes gave similar results for the simple kinematic case; unfortunately, they are difficult to apply in the (more complex) dynamic stage. With the proposed method both situations appear as natural and clear extensions of a common and systematic methodology. Furthermore, the inherent use of histograms allows not only fast calculation procedures but, above all, the use of experimental data; consequently, complex dynamic modelling exercises can be avoided. On the other hand, the results pointing out some characteristics of the trajectory planning block, defining optimal rest and active regions, and ideal-actuator properties, as "position and acceleration" devices instead of "velocity" machines. This observation is of utmost importance as it gives a clear basis to new mechanical robot manipulator structures, with performances close to the muscle-actuated biological systems.

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