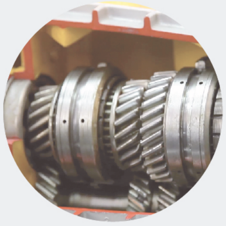
INSTITUTO SUPERIOR DE ENGENHARIA DO PORTO
MESTRADO EM ENGENHARIA E GESTÃO INDUSTRIAL






ALGORITMO DE AVALIAÇÃO DA ESTABILIDADE DE UMA PALETE

JOÃO CORREIA DE ARAÚJO
novembro de 2022

POLITÉCNICO
DO PORTO

ALGORITHM FOR EVALUATION OF STABILITY OF THE LOAD IN A PALLET

João Correia de Araújo

2022

Instituto Superior de Engenharia do Porto

Departamento de Engenharia Mecânica

isep

P.PORTO

ALGORITHM FOR EVALUATION OF STABILITY OF THE LOAD IN A PALLET

João Correia de Araújo

Student n.º 1170455

Dissertation submitted to Instituto Superior de Engenharia do Porto to fulfill the requirements to obtain the Master´s degree in Engenharia e Gestão Industrial, carried out under the supervision of Doctor António Galrão and co-supervision of Professor Elsa Silva.

2022

Instituto Superior de Engenharia do Porto

Departamento de Engenharia Mecânica

P.PORTO

ACKNOWLEDGMENTS

For the writing of this master's thesis, I had the support of a lot of persons, to whom I wish to express my gratitude since without their help both the realization of the project in which I was involved as well as the dissertation of this thesis would have been substantially more difficult.

Therefore, I want to start by thanking my family, because without them I would never have been able to get this far in my academic path.

To Engineer António Galrão Ramos, firstly for the availability and dedication showed in accepting to guide me in this project and for his constant cooperation, guiding my work throughout the entire project, providing enriching contributions both to the project itself, sharing ideas with me that allowed me to better develop the same, as for the writing of this thesis.

I would also like to thank Engineer Manuel Pereira Lopes for his support throughout the master's degree and, in the context of its finalization, I would like to highlight the guidance and assistance given to guide me towards a project that was totally in line with my objectives and ambitions.

A special thanks to Engineer Elsa Silva, as she was together with engineer António Galrão Ramos, a very important pillar for me to take this project forward.

To all the professors with whom I contacted throughout my academic training, from elementary school to the master's, for all the knowledge they passed on to me, allowing me to grow not only intellectually, but also personally.

To the friends who accompanied me throughout these five years of academic attendance, among which I would like to highlight my colleagues Jorge Domingues, João Viana, and José Ribeiro, who were always present, with all the support, affection and understanding, helping to overcome all the most difficult moments, including those encountered throughout this project.

To all, thank you very much!

purposely blank page

ABSTRACT

The present dissertation was developed to present as a final proof to obtain the master's degree in the Master's degree in Industrial Engineering and Management, from the Instituto Superior de Engenharia do Porto and aims to describe the project carried out in this sense, presenting and analyzing its development and also the results obtained.

The project in question is part of a much more complex project to be developed at INESC TEC, located in Porto. In this sense, only the initial phase of this project was developed, which aimed to create an algorithm capable of evaluating the stability of the load present on a pallet.

For this, in the first step was carried out an analysis of a previously provided C++ project, which aims to best arrange a set of boxes in a container. In this sense, a document was previously provided with the characteristics of the set of boxes to be arranged in the container, as well as the characteristics of the container itself. Based on the information given in this document, the best organization of this set of boxes in the container was then idealized, knowing that not all the boxes would fit in it and ensuring that the organization idealized is viable in practice, that is that there are no problems like overlapping of boxes.

After this analysis, the second phase of the project started, in which an algorithm capable of analyzing the organization defined for the set of boxes arranged inside the container to evaluate its stability was developed in C++, applying the support polygon concept, that is, assessing whether the boxes inside the container were stable following the support polygon methodology or not.

Finally, in a third phase, this algorithm was applied and the results obtained were analyzed to prove its viability, leading to the conclusion that the algorithm effectively identified correctly when the set of boxes arranged inside the container was stable and when it wasn´t.

KEYWORDS

Stability, Palletization, Robotization, Automation, Algorithm

purposely blank page

INDEX

purposely blank page

FIGURES INDEX

purposely blank page

TABLES INDEX

purposely blank page

LISTS OF ACRONYMS AND SYMBOLS

List of Acronyms

ISEP	Instituto Superior de Engenharia do Porto
P.Porto	Instituto Politécnico do Porto
INESC TEC	Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência
CSCMP	Council of Supply Chain Management Professionals
PPP	Pallet Packaging Problems
DPPP	Distributor's Pallet Packaging Problems
PPPP	Producer's Pallet Packaging Problems
LIFO	Last in First Out
CIM	Computer Integrated Manufacturing
HMI	Human Machine Interface
SCARA	Selective Compliance Assembly Robot Arm

Listo f Symbols

$L_{TOUCHABLE}$	Touchable Lenght	cm
$H_{TOUCHABLE}$	Touchable Height	cm

purposely blank page

1. INTRODUCTION

This chapter is intended to introduce not only the context but also the content of this dissertation. In this sense, it begins with the contextualization of the project, also explaining the relevance of its development for the industry nowadays. After it is presented the explanation of the question that led to the emergence of this project and of what objectives are intended to be achieved with it, or in other words, what are the problems to be addressed with its development. This chapter also includes the justification of the methods applied throughout the course of the project. Finally, the structure of the present work is also presented in this chapter, to facilitate its reading and simplify the organization of the document, making easier the search for pieces of information in this dissertation.

1.1. Contextualization and relevance

This project can be considered a starting point for a much more complex project to be developed at INESC TEC. The global project fits fully into the aims of the current industry with the transformation to Industry 4.0, seeking to automate and automatize as much as possible all industrial processes. In this way, it is undoubtedly pertinent to invest in the development of methodologies that could lead to making warehouse operations also more autonomous and automated, to turn these operations faster, more efficient, and more effective, thus eliminating the need for human work in these sectors, which is replaced by programmed robots.

However, there is still a long way to go reached/achieve this goal, being the work presented in this dissertation is just the tip of the iceberg that will start this whole project. Even so, the work carried out and presented in this dissertation is an essential basis for the development of the remaining project, being fundamental for the continuity of the project.

1.2. Research question and objectives

Since, as already mentioned, this project is the starting point for a much more complex project, it is important to understand the relationship between them. Regarding the overall project, the main objective is to make palletizing operations fully autonomous and automated, using robots for that. In this sense, it is necessary to create the code to be applied to these robots to ensure that they perform the palletizing operations correctly. For this, it first needed to study a series of aspects, to guarantee that the robots effectively carried out the palletization operations in the best possible way, not only regarding the occupation of the pallet space but also the palletization time and many other aspects. In this sense, one of the aspects that need to be studied is the stability of the load. In terms of this aspect, it is essential to ensure that the load on the pallet is stable, as it is not worth building a pallet very fast and very well-occupied if it is not stable. Thereby, this phase of the process will focus on finding a solution that can guarantee that the load on the pallets is stable.

Therefore, the objectives for this phase of the project are:

- Analysis of the code already developed in C++ for this project, to perfectly understand all the files and functions already developed, to be able to proceed with the project assertively;
- Development of an algorithm in C++ that can assess the stability of the load on the pallets;

– Proof of the correct functioning of this algorithm.

1.3. Methodological options

For the different objectives and stages of this project, different methodologies were applied. Starting with the analysis of the code previously developed in C++, was undoubtedly the most time-consuming phase of the project since there were already more than 1500 lines of code, distributed over 10 files. Even so, all the files were analyzed, following a logical order between them, considering the existing relationships between the files. Within each file, all lines were carefully analyzed, line after line, making notes regarding the functionality of each line. In this sense, a lot of online research was carried out to understand all the elements (symbols, functions, etc …) present throughout the code, using the most diverse online sources for that.

Regarding the creation of an algorithm capable of evaluating the stability of the load present in the pallets, it was developed using the program "Visual Studio 2019" and the C++ code language. This program was chosen not only because it is recognized as one of the best tools for programming in any type of language, but also because all the persons involved in the project were already familiar with it. As for the language selected for the development of the code, C++ was chosen because the code previously provided was also in this language, and taking into account that the algorithm would be inserted in that code, it would only make sense to develop the algorithm in this language. In terms of code development, a lot of online research was also carried out at this stage, to solve a series of problems that arose during its development.

Finally, in terms of proving the correct functionality of the developed algorithm, a set of instances with different possible arrangements of boxes in pallets were created for this purpose. After their creation, the algorithm was then applied to them to prove that the results obtained were the expected, that is, that the algorithm indicated that the load was stable in the case of stable arrangements and that it was not stable when applied to the unstable arrangements.

1.4. Work structure

This dissertation was organized to facilitate the reader's understanding of the project exposed in it. Thus, it is divided into five chapters.

Chapter 1 is intended for the introduction, including the analysis of the contextualization and relevance of this project, the explanation of the research question and the objectives established, the justification of the methodological options, and the presentation of the work structure.

Chapter 2 contains the bibliographic review of the dissertation, in which all the concepts of special interest are exposed. Considering the context in which this project is inserted, the presentation and explanation of all of them are essential for the perception of the remaining dissertation.

Chapter 3, in its turn, includes a description of the methods used in the development of the project and an explanation of their application in the various phases of the work.

In chapter 4, in addition to exposing the results achieved in the project, a critical analysis of these same results is also carried out.

Finally, in chapter 5 comes the conclusion of the dissertation, in which a critical reflection on the project is presented, being also analyzed the limitations found and some possible future works suggested.

2. BIBLIOGRAPHIC REVIEW

2.1. Palletization

First of all, it is important to start by realizing that this whole project was developed to assist and improve logistics management services, so it is, therefore, necessary to understand correctly what logistics management is.

According to the CSCMP, Logistics Management consists not only of the efficient and effective planning, implementation, and control of supply chain flows but also of all necessary activities for storing goods, services, and related information between the initial point of the supply chain – suppliers – and the final point – consumers – to ensure customer requirements (*CSCMP*, 2010).

Logistics management develops all its activity around the supply chain, which in turn can be defined as the system that includes the entire development of a product, from its initial phase of obtaining raw materials to the final phase in which products are delivered to end customers (Min & Zhou, 2002). In this sense, a supply chain can be divided into two sectors, the first one includes suppliers and the producers themselves, being coordination between the two elements of this sector necessary to ensure efficient and effective management of raw materials and production control. The second sector includes Distributors and Retailers, together they guarantee the distribution of products, starting from the producers and following to the final consumers (Beamon, 1998).

As an essential element in supply chains, pallets appear. These were already used as a tool in the industry in the 30’s decade of the last century, but only in the following decade, with the invention of forklifts, they began to be more widely used, since when use pallets were combined with forklifts, material handling became much faster. Since then, its use at industrial level has always increased. Nowadays, they are already used in all industries, being an important element in all transportation operations, since the transport of raw materials from suppliers, to the transport of final products to consumers (Yam, 2009).

It is in this context that the palletization problem arises, which consists of choosing the best combination to place a set of products, which can be boxes or any other objects, on pallets, structuring the various layers of the pallet to achieve a certain objective (Dos Santos, 1995).

But to talk about palletization problems, it is better to introduce the Cutting and Packaging Problems first.

Currently and increasingly, stock control and good organization in distribution operations are essential for a company's success. However, companies already started paying attention to these factors a long time ago to improve their competitiveness and it was in this context that the Cutting and Packaging Problems appeared. Although it is present in the industry for many years, it was not until the beginning of the 60's decade of the last century that a mathematical approach appeared for the first time to try to solve Cutting and Packaging Problems. (Gilmore & Gomory, 1963) were responsible for this first mathematical approach and since then numerous studies have been carried out in this area, considering the advantages that the industry can obtain from optimizing the solutions found for these problems.

First of all, it is essential to understand that in these problems it is necessary to deal with 2 sets of data, one of the sets consisting of larger objects and the other set consisting of smaller objects, to pack the smaller objects in the larger ones, so can be considered that this is a problem that aims to analyze the possible geometric combinations of smaller objects in larger objects and obtain the best one. In this sense, if there are spaces in the larger objects to which there are no smaller objects allocated, those spaces will be considered waste (Dyckhoff, 1990).

However, not all Cutting and Packaging Problems are identical, so it is important to distinguish them. (Dyckhoff, 1990) decided to divide the Cutting and Packaging Problems. However, (Dyckhoff, 1990) didn´t consider that only one factor would be enough to divide these problems into different groups, considering instead that they could be distinguished in terms of four factors: dimension; task type; classification of larger objects; classification of smaller objects. For each of these factors (Dyckhoff, 1990) elaborated several categories, so that each Problem could be placed in a category that was suitable for it. (Dyckhoff, 1990) also establishes a symbol for each category of the various factors, so that each problem could be represented by 4 symbols, one equivalent to each factor, to elaborate the classification of the problem with these 4 symbols.

The table below shows the symbols selected by (Dyckhoff, 1990) for each category of the various factors.

Table 1 - Dyckhoff Symbols

Factor	Category	Symbol
Dimension	One-dimensional	1
	Two-dimensional	2
	Three-dimensional	3
	N-dimensional	N
Task Type	Larger objects and the selection of small objects	B
	Smaller objects and the selection of whole objects	V
Classification of Larger Objects	Single Object	O
	Similar objects	I
	Distinct objects	D
Classification of Smaller Objects	Reduced Quantity	F
	High quantity and variety	M
	High Quantity and Low Variety	R
	All similar objects	C

It is important at this point to understand what (Dyckhoff, 1990) refers to when he talks about smaller and larger objects. For (Dyckhoff, 1990) we have smaller objects that are objects to be allocated within the larger ones, in the case of packing problems, while in the case of cutting problems, in an analog form, the smaller objects are the objects that must be obtained from the cutting of the larger objects.

Heading now to the analysis of the various categories distinguished by (Dyckhoff, 1990) for this factor, this is, for the task type, once this is the factor that evaluated the relationship between larger and smaller objects, (Dyckhoff, 1990) presented only two categories: larger objects and the selection of smaller objects (B); Smaller objects and the selection of larger objects (V). Category B includes all problems in which we have an excess of smaller objects in relation to larger objects, so the problem is to choose which of the smaller objects to allocate to the larger ones. Within category V, there are problems in which the number of larger objects is more than enough for the number of smaller objects, so the objective then becomes to find the best possible allocation of smaller objects to the larger ones, to save as many larger objects as possible.

Looking now at the classification of larger objects, three categories were defined for this factor: Single object (O); Similar objects (I); Distinct objects (D). Category O reunites all problems in which only a single large object is used. Category I includes all problems in which there is more than one larger object, but all of them have similar characteristics, thus leaving category D for problems in which, even though there is more than one large object, they have different characteristics (Dyckhoff, 1990).

Finally, for the classification of smaller objects, four categories were considered: Reduced quantity (F); High Quantity and Variety (M); High Quantity and Low Variety (R); Similar objects (C). In this way, we have introduced in category F all problems in which the number of small objects is reduced. Category M includes problems that deal with a large number of small objects, with the specificity of having a great variety in the characteristics of these objects, while, on the other hand, category R reunite all problems that similarly deal also with a large number of small objects, but in which the variety of their characteristics is reduced. Finally, there is also category C which, similarly to the last two, also deals with a large number of small objects, but this time all these objects have identical characteristics, this is, all objects are equal (Dyckhoff, 1990).

After analyzing all these factors for a given problem, it is possible to obtain its final classification. The problem classification will then be based on a four-letter code, each corresponding to a factor, in the following order (Dyckhoff, 1990):

- First letter of the code: Symbol of the category in which the problem is included in the Dimension factor;
- Second letter of the code: Symbol of the category in which the problem is inserted in the Type of Task factor;
- Third letter of the code: Symbol of the category in which the problem is included in the Large Object Classification factor;
- Fourth letter of the code: Symbol of the category that the problem belongs to in the Small Objects Classification factor;

However (Dyckhoff, 1990) was not the only author to try to classify the problems of cutting and packing and more works in this area appeared later. In 2004, (Poldi, 2004) presented a new method for classifying cutting and packing problems, this time evaluating only one factor, the dimension, presenting, however, 6 categories for this factor: one-dimensional; two-dimensional; three-dimensional; multidimensional; 1.5-dimensional; 2.5-dimensional.

Although the first four categories are easily understood, the same does not happen with the last two.

In the case of Problems in which only one dimension is taken into account, then they are included in the One-Dimensional category. Within this category are introduced problems such as cutting paper rolls into cross sections. The Two-Dimensional category, as the name implies, includes problems in which two dimensions are analyzed, as can be seen, for example, in a problem of cutting glass plates. The Three-Dimensional category, in turn, gathers all problems in which three dimensions are taken into account, as in problems of cutting foam blocks (Faccio, 2008). The Multidimensional category, in turn, includes all problems in which more than three dimensions are involved, as is the case, for example, of a problem in which, in addition to paying attention to length, height and width, time is also taken into account (Dyckhoff, 1990).

Turning now attention to the last two categories, these are not so easy to understand only by their names. In this sense, it is therefore important to explain carefully that problems included in these categories are the ones that have variable dimensions. Thus, the 1.5-dimensional category includes problems in which two dimensions are dealt with, but one of them is variable, while the 2.5-dimensional category reunites problems in which three dimensions are taken into account, being one of them also variable (Poldi, 2004).

Later also (Wäscher et al., 2007) presented another classification method for these problems, based on the classification method already presented by (Dyckhoff, 1990), but introduced some changes to it. In this sense, these authors chose to divide these problems into three classes: Basic problems; Intermediate Problems; Refined Problems.

For the first class, which is Basic Problems, it is first important to realize that (Wäscher et al., 2007) made a series of conversions with the terms used by (Dyckhoff, 1990). In this sense, the problems of “Larger objects and the selection of small objects” came to be seen as problems of “Output Maximization”, while the problems of “Smaller objects and the selection of integer objects” came to be called “Input Minimization” problems. In the case of maximizing output, this objective is applied when facing a scenario in which the number of pallets is not enough for the total number of products that we want to accommodate in them, so the objective is then to maximize the number of products to introduce on the pallets. On the other hand, input minimization is appliable when the number of available pallets is greater than the number necessary to accommodate all the products, so what is sought is to use the minimum number of pallets possible, which eventually can lead to reducing the number of trips that companies need to make, thus leading to a reduction in distribution expenses (Wäscher et al., 2007). As for the classification of smaller objects, these authors only considered the existence of “Identical objects”, “Strongly heterogeneous objects” or “Weakly heterogeneous objects”. In this sense, within Basic Problems class (Wäscher et al., 2007) considered that the problems could be divided into 6 categories:

- Identical Item Packing (includes all output maximization problems where smaller objects are identical);
- Placement problem (groups all output maximization problems where smaller objects are little heterogeneous);
- Knapsack problem (encompasses the totality of output maximization problems where smaller objects are strongly heterogeneous);
- Cutting Stock Problem (includes input minimization problems in which smaller objects are little heterogeneous);
- Bin Packing problem (groups input minimization problems where smaller objects are very heterogeneous);

- Open-dimensional problem (contain all input minimization problems dealing with a variable dimension, that is, all problems belonging to the 1.5-dimensional and 2.5-dimensional categories defined by (Poldi, 2004)).

Moving on to the second class, that of Intermediate Problems, there aren´t more problem categories created in this class. What happens in this class is the improvement of the classification of problems, once at this stage the evaluation of larger objects, which hasn´t been done in the Basic Problems class, is now introduced. In this sense, at this stage, an analysis must be carried out on the larger objects present in the problems and considering this analysis the problems must be reclassified. As an example, if in the Basic Problems class, it was defined that a problem was a "Placement Problem" and if it turns out that there are several larger objects in this problem, all of them identical, then it becomes a "(Multiple) Identical Large Object Placement Problem". It is important to mention that in this class there is an exception, in the case of "Identical Item Packing Problem" because the authors consider that this classification does not need to be reassessed and this class does not undergo any change (Wäscher et al., 2007).

Finally, we have the class of Refined Problems, in which once again a reassessment of the classification of problems must be carried out, this time adding a description of the number of dimensions that the problem deals with to its classification. Recovering the example from the previous paragraph, a "(Multiple) Identical Large Object Placement Problem " could become a "Three-dimensional (Multiple) Identical Large Object Placement Problem" at the end of this class (Wäscher et al., 2007).

So, we can conclude that there is a great diversity within the Cutting and Packaging Problems, which consequently leads also to the arise of a series of different methodologies to classify and group them.

In the case of the problems that will be dealt with in this project, they are part of a group of problems generally called PPP. This type of problem is characterized by having two sets of objects, the pallets and the items to be placed on the pallets, being the objective of this type of problem optimize the allocation of items to the pallets, trying to allocate the largest possible number of items to each pallet.

This group of problems is, however, divided into two subclasses: the DPPP and the PPPP (Bischoff & Ratcliff, 1995a). The difference between these two subclasses lies in the fact that in the case of PPPP the items to be allocated to the pallets are all identical, while in the case of the DPPP there is variety in the items to be allocated to the pallets, therefore not all of them are similar. Bearing this in mind, it is easy to see that the studies of DPPP's present a much higher complexity than that of PPPP's (Barbosa, 2018). For this same reason, it is currently reasonable to consider that it is possible to obtain solutions, using exact algorithms, for the vast majority of PPPP´s in reduced computational times (Silva et al., 2016). On the other hand, the same is not true in the case of DPPP-type problems, for which, due to their complexity, there is still great difficulty in obtaining acceptable solutions in acceptable computational times (Bischoff & Ratcliff, 1995a).

This is a type of problem that presents many restrictions, among which some of the most important is, without any doubt, ensuring that in the idealized solution there is no overlap of products, that is, there aren´t two products to place in the same position on the pallet and ensure that the products are fully within the limits of the pallet.

Although many methods that respect these restrictions have already been presented, many other restrictions aren´t usually taken into account when theoretically idealizing solutions for this type of problem, so the idealized solutions often end up not being valid in practice (Bischoff & Ratcliff, 1995b) (Bortfeldt & Gehring, 2001). One of these restrictions that are usually not taken into account is the need to ensure the stability of the load on the pallet, and this topic will be addressed later (Ramos et al., 2016).

To solve this type of problem, many studies have been carried out in recent years, in the most varied areas, from Mathematics to Engineering, through Management and Computer Science and many others. This led to the creation of multiple methods to obtain solutions for these problems, but once these are very complex problems and there is a huge variety within this type of problems from case to case, the solutions found were never optimal, usually appearing only as a response to a particular case (Dos Santos, 1995).

Nowadays pallets have such an important role in the efficiency of supply chains that more and more companies are studying the best way to use pallets (McGuire, 2015).

However, pallets do not only bring advantages, but also some disadvantages in their use. In this sense, (McGuire, 2015) defined what were the advantages and disadvantages that could be verified with the use of pallets, and among all of them, those in the table below can be highlighted:

Table 2 - Advantages and Disadvantages of Pallets

Advantages	Disadvantages
Great Availability	High Weight
Allows the transport of large volumes of materials at once	May lead to reduced storage capacity
Allows to keep a large amount of materials together	Requires the purchase of own lifting equipment
Reduces the risk of accidents that can occur when transporting and lifting materials	Can be too wide for doors
Great flexibility	Are not suitable for storing small materials and small quantities
Globally standardized	
Allows for better storage of materials	

As stated in the table above, one of the advantages of using pallets is the fact that they are globally standardized. In Europe, the most used pallets have dimensions of 800cm in length and 1200cm in width, being used the name “Europallet” for such pallets.

So the majority of companies generally use the same type of pallets, but the management of the pallets isn´t similar in all those companies. This is because companies can choose two different types of pallet management. Some companies choose to acquire their pallets and later manage them independently, with their maintenance being, of course, their sole responsibility. On the other hand, we have companies that choose to use pallets that are owned by others, and there are companies specialized in the supply of pallets, such as, for example, Pooling Partners, in which case the responsibility for the maintenance of the pallets passes then to these same companies (Arcanjo, 2017).

Also, regarding palletization systems, companies can opt for different systems. In this case, there are currently 4 main types of palletization systems that companies can employ, each with its distinct characteristics (Foltz, 2004).

It will be presented these 4 palletization systems, describing how each one work according to (Foltz, 2004) and (Filipe et al., 2013).

Starting with the manual palletization system, is the system in which palletization is carried out only with human intervention, with workers being responsible for, without the aid of any machine, transport objects from a certain location or platform, such as a conveyor belt, to the pallet and there palletize them (Foltz, 2004).

In this way, the entire responsibility for palletization falls on the workers who carry it out, so they must previously know/plan how many objects will be placed on the pallets and also how many will be placed on each layer, thereof in addition to ensuring a correct transport of objects the workers also have to guarantee a correct palletization, in which stability and all the necessary conditions for the pallet are ensured (Foltz, 2004).

This system, however, has some limitations, such as the fact that it only makes sense for companies where the production volume is relatively low, otherwise a lot of time would be lost to carry out the palletization of all products if only this system is used, which in the case of companies that work with short delivery times is unfeasible. Another limitation is the fact that the objects for palletization cannot have a very high weight, since in that case the workers would not be able to transport them without the aid of any tool (Filipe et al., 2013).

Even though this isn´t a limitation, the fact that this system requires workers to do very repetitive and tiring work is also one of the factors that leads some companies to move away from this system. Conditions like this are not only undesirable for the health of the collaborators, but can even lead to workers not making the most correct palletization decisions, or even, in the worst case scenario, being injured due to accidents or even the appearance of chronic injuries over time, which will consequently increase health insurance costs (Foltz, 2004).

On the other hand, this system has some characteristics that allow it to stand out in certain points, such as, for example, the fact that it is very flexible to the variation in size, shape, and even mass of objects since these variations would not present major complications to the workers, who, as long as the size and mass of the objects are within their physical capabilities, don´t have any difficulty in moving different objects (Foltz, 2004).

As an advantage of this type of system, there is the fact that it is economical, it doesn't require a very high initial investment for the purchase of equipment for lifting and transporting objects and it is not very expensive over time, because with this system the only cost that practically exists lies in the labor. The fact that it does not require a large area to carry out the palletization is also an advantage of this system, as it also does not require a secure area like some other systems (Foltz, 2004).

Moving on to conventional palletization systems, these are characterized by, in the first phase, transporting the objects to the pallet area through a transmission system, such as conveyor belts, followed, in the second phase, by the palletization of these same objectives, being this same palletization carried out with the aid of a pneumatic system. These pneumatic systems have an advantage over manual systems, as they allow the system to have no limitations regarding the weight of objects, being able to palletize high-weight objects (Foltz, 2004).

It is important to mention that these systems are generally designed to be able to palletize several different types of objects, being stored in their memory how to act with each type of object, and, as such, before starting the palletization it is only necessary to configure the system for the correct characteristics and dimensions of objects (Foltz, 2004).

However, not all conventional palletization systems are built identically. So, inside the conventional palletization systems, there are systems with huge differences as can be seen with the Floor Level Infeed and High Level Infeed systems (Foltz, 2004).

There`s the Floor Level Infeed systems, in which the entire palletization process, from the transport of objects using the transmission system to the palletization itself, takes place at ground level. In this way, in this type of system, the entire process begins with the transport of the objects to a platform, where they are pushed by a pneumatic tool to the position previously assigned to them. This step is repeated for all objects on a layer until all objects on that layer are in the corresponding position. Once the layer is finished, these objects are transferred from that platform to the pallet, placing it on top of the previous layer (or in the case of the first layer on top of the base of the pallet). This step is also repeated continuously until all the layers of the pallet are already arranged in it. Then the palletization process is completed and the pallets are transported to their storage area (Popple, 2009)(Gauss & Sellito, 2017)(Foltz, 2004).

As for the High Level Infeed systems, they, in their turn, are characterized by having two levels along the palletization system. In this type of system, the process is very similar to that performed in Floor Level Infeed systems, starting with the transport of objects, which is aided by a transmission system, to a platform on which, using a pneumatic tool, the objects are guided to the position on the layer that was previously defined to them, repeating this for all objects until the layer is finished. It is then, at this moment, that the difference between this type and the Floor Level Infeed is verified, because in these systems, after the layer is finished, happens a transfer of this same layer from the platform on which it was built to another platform, that performs upward and descendant movements, in which the pallet is located. This second platform then moves to reach the level of the platform where the layers are built, so that the layer can then be deposited on top of the previous one. This step is then repeated for all layers until the pallet is complete. Once the pallet is complete, it is moved to ground level, where it is then taken to the storage area (Foltz, 2004)(Foltz, 2004).

Of course, each of these types of conventional palletization systems has its advantages and disadvantages, which are described in the following table (Foltz, 2004)(Filipe et al., 2013)(Popple, 2009):

Table 3 - Advantages and disadvantages of different types of conventional palletization systems

Floor Level Infeed		High Level Infeed	
Advantages	**Disadvantages**	**Advantages**	**Disadvantages**
The process is carried out at the operator's view, facilitating its control	High initial investment	Can handle even heavier objects than Floor Level Infeed systems	High initial investment
Flexible system, being able to work with objects of different sizes and weights	Occupying a large area	Flexible system, being able to work with objects of different sizes and weight	Occupying a large area
Faster palletization	Requires a skilled worker in the system as it is complex	Faster palletizing	Requires a skilled worker in the system as it is complex
Reduces forklift operations		Reduces forklift operations	Part of the process takes place at a level superior to human vision, making it more difficult to detect errors
In the event of a breakdown or accident, repairs are carried out at ground level			Repairs may be necessary at a high level, requiring stairs to access certain parts of the system
			Demand for 2 or more interface panels

This completes the description of conventional palletizing systems, so we move on to Gantry Robot Palletization systems. In these systems, palletization is performed using a Gantry robot, which is a robot that can be programmed to move to any position within a given frame. This frame, in turn, can be made up of several lines, each one intended for a given product, so that the robot can carry out the palletization of various products and pallets can even be built with different products. It should be noted that all these lines of the frame were then joined in a specific one that will be destined for the area where the pallets that received the products were placed (Foltz, 2004).

As with other palletization systems, this one also has its advantages and disadvantages. Starting with its advantages, it is worth noting (Foltz, 2004):

- Allow the possibility of carrying out the palletization of several pallets simultaneously;
- Also allow the possibility of including different products on the same pallet;
- Be viable for palletizing high-weight products;
- High Autonomy.

Regarding their disadvantages, stand out (Foltz, 2004):

- High implementation cost, the technology used in this system being relatively expensive;
- High complexity in the implementation, being the installation processes of the frame, the robot, and all their programming highly time-consuming;
- Occupation of a large area in the warehouses;
- Low speed.

Finally, we also have hybrid palletization systems. These are the palletization systems in which a combination of the aforementioned palletization systems is sought, to combine different advantages of each of them (Foltz, 2004).

However, these systems continue to present some disadvantages, such as being expensive, as the combination of two or more palletization systems is never a cheap process, which leads to companies only carrying out these combinations when they find guarantees that they will allow great capital gains and consequently monetize the investment. The fact that they generally require a large area is another of the disadvantages that these systems present, although normally the area they occupy is even less than the area that the combined systems would occupy, which, in this point of view, can also, therefore, be seen as an advantage (Foltz, 2004).

These systems also have the benefit of allowing the palletization of several pallets simultaneously (Foltz, 2004).

There are multiple examples of this type of system, among which we have the hybrid palletization system Shuttle Car, which consists of a system that combines the characteristics of conventional palletization systems with transmission systems, to allow the existence of several lines of transport of products to the palletization area, each line destined for a specific product, so that the palletization of the various products can be carried out. We also have the hybrid system Rotary Table, which is nothing less than a system very similar to the Shuttle Car, but instead of using transmission systems to transport the products, it uses a rotary table (Foltz, 2004).

2.2. Static stability of the load

As mentioned before, many of the algorithms created to solve palletization problems end up not seeing their viability proven when they are practically used, since they do not consider a series of restrictions/conditions that occur in the real world (Ramos et al., 2016).

Among all these conditions, one of the most important is stability, since in most of these algorithms the pallets are structured without paying any attention to whether they end up collapsing at the end, which obviously must be avoided (Ramos et al., 2016).

But it is important to recognize that stability is different when the load is stopped and when it is in motion, so there are two different stability variants. Static or vertical stability is the one verified in the context of a stationary load, as happens during the palletization of the pallets. On the other hand, dynamic or horizontal stability is the one verified when something is in motion, as is the case with pallets when they are being transported (Ramos et al., 2016) (Bortfeldt & Wäscher, 2013).

In the particular case of this project, the stability that should be studied was static stability, so what was intended would be to ensure the conditions of static mechanical balance in the structuring of the pallets (Ramos et al., 2016).

In this sense, to ensure static stability, three different methods emerge (Ramos et al., 2016):

- Full base support

In this perspective, all objects must have their base fully in contact either with the base of the pallet or with the upper face of other objects also included in the pallet (Ramos et al., 2016).

This method has a great advantage over the other two as it is the only one that fully guarantees static stability (Ramos et al., 2016).

However, this method doesn´t allow the meeting of a good solution in terms of maximizing the use of available space on the pallet, as this method will greatly restrict the positions that objects can occupy on the pallet, leading to high waste of space (Bischoff & Ratcliff, 1995b).

- Partial base support

In this scenario, one of two possible conditions would be required: the base of the products is in full contact with the base of the pallet; at least a certain percentage of the base of the products is in contact with the support face of other products also accommodated on the same pallet. In the case of this second condition, this percentage can take several values, with different authors defending different percentages, with some authors such as (Christensen & Rousøe, 2009) defend that this percentage should never be less than 80%, but, on the other hand, some authors such as (Mack et al., 2004) consider that a minimum value of 55% is sufficient. In either case, this percentage must be defined before applying the method (Ramos et al., 2016).

- Approach to static mechanical balance

This approach also argues that it is only necessary to comply with one of two possible conditions, allowing, as in the last approach, the possibility of the entire base of the product being in contact with the base of the pallet or the possibility that the product's center of gravity is located above the contact surface of the products that support it (Ramos et al., 2016).

These are the 3 methods that can be used in the search to ensure static stability on the pallets.

What, however, often happens is that the researchers end up opting for the simplest approach, applying the method of requiring full base support, which, as already mentioned, isn´t the most suitable, as it leads to a high waste of space available on the pallet, since by requiring the base of all products to be 100% supported by other products or by the base of the pallet/container, it will greatly limit the possible combinations, preventing combinations that would allow a better occupancy of the pallet and that consequently would allow the accommodation of more products while still maintaining the stability of the pallet (Ramos et al., 2016).

In the case of container loading problems, with which can be made an obvious analogy since a container loading problem is similar to a pallet loading problem, the first time stability was analyzed in a more complex way was by (De Castro Silva et al., 2003). These authors chose, at the time, to divide their analysis of this condition into two stages. In this sense, in the first stage, it was verified whether the objects met at least one of three possible conditions: they were in full contact with the base of the container; their center of gravity was located above one of the objects that supports it; their center of gravity, not being located above one of the objects that supports it, is included in a vertical plane that connects two of its supporting objects, although these objects are not in contact with each other. In the second stage, the moments of the objects were studied, because if it were found that the sum of the moments of all the objects present in the container was zero, only then could be considered that the load was stable. Although this was undoubtedly an interesting and very avant-garde approach at the time, it had a big drawback that was that the authors did not present any algorithm to ensure the fulfillment of the two stages, so this approach was limited to the theoretical plane (Ramos et al., 2016).

Later (Ramos et al., 2016) also wanted to analyze this condition more deeply, presenting an approach that considered the conditions of static mechanical equilibrium for rigid bodies. To this end, it was considered Newton's first and third laws.

Starting with the first, this says that whenever an object is stationary or in uniform motion in a straight line, it always remains in that state unless forces are exerted on it that make it change. So, whenever an object is stationary and no force is exercised on it, then it will remain stationary. The same condition is applicable for objects that are in uniform motion in a straight line, that would remain in this state if there isn´t any force exercised on them. In this sense, also if there is a set of forces being applied on an object, but the resultant force is zero, then the object will not change its state, remaining stationary if it was stationary, or in uniform motion in a straight line if it was previously already performing this type of movement.

The third law states that for any action that takes place there is always an equal reaction in the opposite direction, this is, when a given object exercises a force on another object, that object will also exercise a similar force on the first in response (Hibbeler, 2010).

In this sense, the conditions for the static equilibrium of objects would then be that the resultant force applied to them is zero and that, in the same way, the moment is also zero, thus guaranteeing both translational and rotational equilibrium (Hibbeler, 2010) (Ramos et al., 2016).

Bearing in mind that the objective is to ensure the static stability of the load and that the load is made up of a set of objects that move independently of each other, to ensure the stability of the entire load it is necessary to ensure the stability of each one of the objects that constitute it, so it is needed to ensure that the two conditions mentioned above are met for all objects (Ramos et al., 2016).

Still following Newton's laws, it is considered that an object can be simplified by summarizing it in its center of mass, which can also be called a point of mass, and the coordinates of this point must be properly defined. We also know that in most cases the center of mass of an object is in the same position as its center of gravity. So, in the case of objects that have a heterogeneous distribution of their mass throughout their volume, this process of determining where the point of mass is located is more difficult, but in the case of an object whose mass is homogeneously distributed throughout the body, it is considered the point of mass is located exactly at the geometric center of the body. In this sense, to simplify, (Ramos et al., 2016) decided to assume that they would always work with boxes with the mass evenly distributed over their body, thus assuming for all boxes that their point of mass was the geometric center of the body.

In this way, a certain box t will be characterized by (Ramos et al., 2016):

- Be located at position (x_t, y_t, z_t)- note that these coordinates correspond to the lower left rear corner of the box, and the origin of the coordinate system is also located in the lower left rear corner of the container;
- Length equal to d_t;
- Width equal to w_t;
- Height equal to h_t;
- Have S as your support polygon (this concept will be explained extensively later);
- Have P as the point at which the resultant force exerted on the box acts, being its coordinates (x_p, y_p, z_p);

(Ramos et al., 2016), based on everything that has been exposed previously and, on their investigation, they concluded that for a box to be considered stable, one of three situations would have to be verified:

- Its base is in full contact with the base of the container, that is, z_t=0;
- The point P is above one of the support boxes, that is, if considering that a support box has coordinates (x_s, y_s, z_s), of length d_s, width w_s, and height h_s, exist any support box that makes $x_s \leq x_p \leq x_s+d_s$, $y_s \leq y_p \leq y_s+d_s$ e $z_p=z_s+h_s$;
- The projection of point P onto the (0,0, z_t) plane, which consists of a plane parallel to the base of the container and coincident with the base of the box t, would have to be inside or on the boundary of S.

At this stage, it is then necessary to understand what the support polygon is and how to build it. But for that first it is important to understand that there is a set of two types of intersections that can be verified between rectangles (this being the shape of the bases of the boxes), being the following (Ramos et al., 2016) (Edelsbrunner & Maurer, 1981):

- One of the rectangles contains the back left point of the other;
- The left edge of one of the rectangles intersects the back edge of the other.

Therefore, (Ramos et al., 2016) concluded that if we had two rectangles, b and p, for which the coordinates of the lower left points are (x_b, y_b) and (XP, y_p) and the coordinates of the upper right points are (x_b+d_b,y_b+w_b) and (x_p+d_p,y_p+w_p), then the vertices of their intersection area would have the following coordinates:

- Lower left vertex: max (x_b, x_p), max (y_b, y_p)
- Lower right vertex: min (x_b+d_b, x_p+d_p), max (y_b, y_p)
- Top left vertex: max (x_b, x_p), min (y_b+w_b, y_p+w_p)
- Top right vertex: min (x_b+d_b, x_p+d_p), min (y_b+w_b, y_p+w_p)

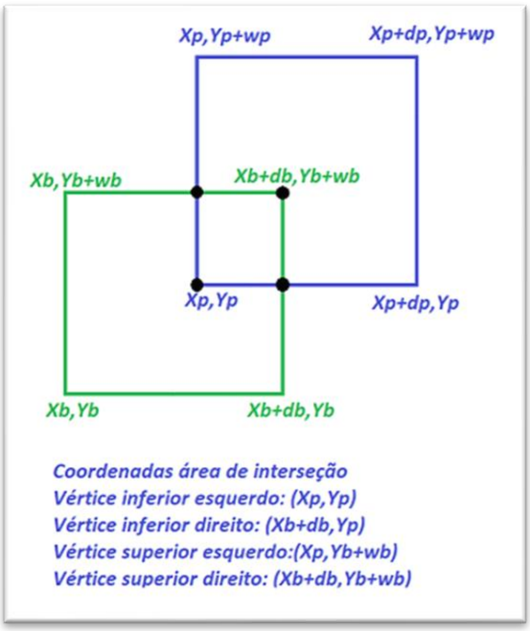


Figure 1 - Intersection area example

This is how the intersection areas between the lower bases of the boxes and the upper bases of their support boxes are calculated. This will be the first phase of the construction of the support polygons, and then must be calculated the coordinates of the vertices of all areas of intersection between a box and its support boxes (Ramos et al., 2016).

Then, must be looked at all these vertices and choose the one that has a lower Y value (if there are two vertices with an equal value, choose the one that has a lower X value, that is, the one that finds leftmost), defining that vertex with V0. Then, looking at the remaining vertices must be selected the one that forms the smallest angle in the anti-clockwise direction in relation to V0, and must only be analyzed the vertices with which V0 makes positive angles, thus selecting a new vertex. This last step must then be repeated, always based on the last selected vertex, until the vertex to be selected is V0. Finally, all these vertices are then joined, thus forming the support polygon (Ramos et al., 2016).

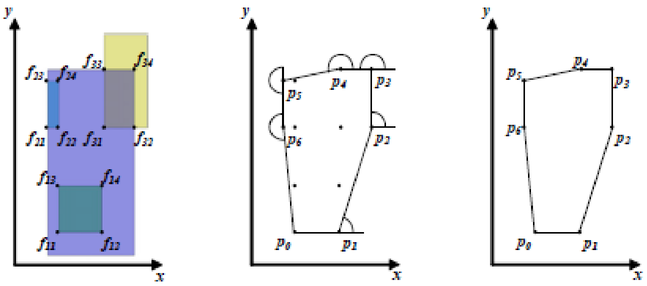


Figure 2 - Support Polygon Construction

Source : (Ramos et al., 2016)

Having constructed the support polygon, it can then be verified whether the projection of point P on the (0,0, z_t) plane is or is not within or on the boundary of that same polygon. To this end, the strategy that must be followed is to analyze whether point P is outside some of the edges of the polygon because if this is the case, this means that point P is located outside the polygon (Haines, 1994) (Shimrat, 1962).

To verify whether this condition is met or not, the following method passes through for each edge of the polygon to create two vectors, one that connects the initial vertex of the margin to the final vertex, and another that connects the initial vertex to the point P. Having these two vectors created, the angle formed between them must then be analyzed, using as a starting point the vector that connects the initial vertex of the margin to the point P, that is, if the other vector is located counterclockwise then the angle formed will be positive and if the other vector is located clockwise then the angle is negative. In this way, if it is verified for all margins that the angle formed is negative, then it can be concluded that the point P is inside the support polygon, whereas, consequently, if for some of the margins there is a positive angle then it is already known that point P is located outside the polygon (Ramos et al., 2016).

2.3. Load Sequencing

Load sequencing is another factor that must be considered in the process of structuring a pallet, and it´s necessary to study the best sequence to accommodate the products on the pallets. This study, however, can only be done after having determined how many and which products are to be placed on the pallet, as well as what position they will occupy, so before defining this sequencing, the statistical stability of the load and also the restrictions on the efficiency of the load loading operations, because only with the analysis of these two factors can we determine the best positions for each product and only then proceed to the generation of the best sequencing (Bischoff & Ratcliff, 1995b) (Ngoi et al., 1994) (Ramos et al., 2016).

Evidently, during the process of idealizing the load loading sequence, it is necessary to pay attention to the existence of loading and/or unloading restrictions, which can lead to a totally different sequencing (Ramos et al., 2016).

The cargo loading sequence is also closely related to the ease of the loading process, as the effort that will be required to carry out loading and unloading depends substantially on the sequence that is established (Ramos et al., 2016).

Regarding possible loading restrictions that may occur, one of the most common concerns is the fact that on the same pallet can be disposed products that are to be distributed to different customers, so it is, therefore, necessary that each customer's products remain next to each other and organized so that when unloading a customer's products it is not necessary to touch the other customers' products, thus avoiding unnecessary operations such as the reorganization of the pallet each time the unloading takes place in a customer, which is also taken into account when analyzing the ease of the loading process because if reorganization operations are carried out this increases the effort made. It was concluded that when the load is organized so that the unloading process does not require reorganizations, then the unloading process will be easier (Fuellerer et al., 2010) (Moura & Oliveira, 2007).

In a situation in which it is known that on a single pallet will be products that are to be distributed to more than one customer, when defining the load sequencing must then be paid attention to the sequence of the customers, that is, which sequence in which distribution to customers is carried out, or in other words, the sequence in which customers are served. In this sense, a LIFO strategy is usually chosen, that is, products that are intended for the last customers to be served are introduced first on the pallet, thus ensuring that these products do not interfere with the discharges to the first customers, thus not being necessary reorganization operations. By following this type of strategy, it is ensured that in the process of unloading a given box, there were no boxes either in front or on top of it, as all the boxes that occupied these positions had already been unloaded (Christensen & Rousøe, 2009). Another strategy that is sometimes also used in this type of situation is to evaluate the maximum reach, which is nothing more than a parameter that represents the reach of the worker's arm that performs the loading/unloading process or the equipment responsible for this process, then using the value of this parameter to define the maximum distance up to which a customer's products can be accommodated, starting from the previous customer's virtual plan, that is, the distance to which a customer's products can be accommodated measurement starts in the previous customer's virtual plan. In this logic, it is important to realize that the virtual plane of a customer is an ideological plane parallel to the front of the pallet and that is located coincident with the front face of the box closest to the front of the pallet of that customer (Junqueira et al., 2011).

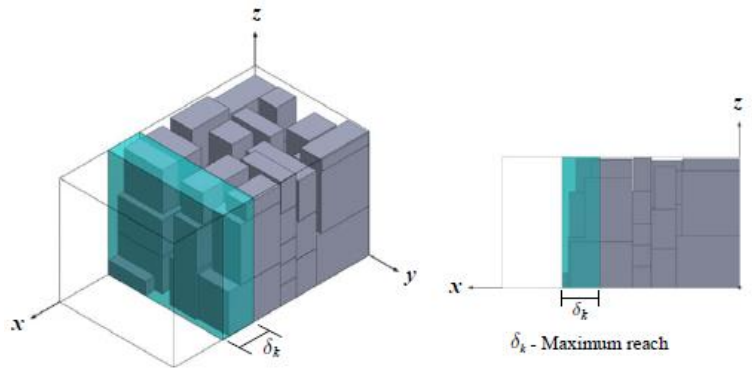


Figure 3 - Maximum reach

Source:(Ramos et al., 2016)

Another parameter that is also interesting to analyze from this perspective is the cost of unloading. According to (Liu et al., 2011) this parameter represents the effort required to unload a box from a container, and in the case of this project, it makes more sense to talk about the effort required to unload a product from a pallet. To be able to evaluate this parameter, (Liu et al., 2011) considered that its value should be directly proportional to the number of boxes that would need to be moved to unload that box, that is, the cost of unloading a particular box is then directly proportional to the number of boxes that would have to be unloaded and then loaded again to be able to unload that same box. In this sense, (Liu et al., 2011) considered that the best way to quantify this parameter would be using the invisible and untouchable rule, according to which to calculate the cost of unloading a box, it is necessary to check how many boxes, that have not yet been delivered, fall under one of two conditions:

- They are positioned in front of that same box;
- They make that same box untouchable by the worker in charge of the distribution process, thus making it impossible for him to carry out his task.

To be able to evaluate this last condition, that is, to be able to determine whether a box is untouchable or not, it is necessary to analyze two variables (Ramos et al., 2016):

- $L_{TOUCHABLE}$- distance between the tip of the feet of the worker who is carrying out the distribution and the front face of the same box;
- $H_{TOUCHABLE}$- distance between the base where the worker's feet are placed and the underside of the box.

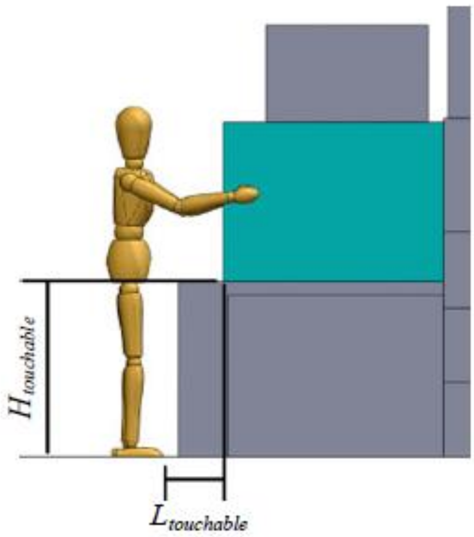


Figure 4 - Touchable Height and Touchable Lenght

Source:(Ramos et al., 2016)

Based on these variables, (Liu et al., 2011) then argued that a box should be considered untouchable if it does not meet any of the following conditions:

- $L_{TOUCHABLE}+H_{TOUCHABLE} \leq$ (worker height + worker arm size)
- $L_{TOUCHABLE}\leq$ min [(worker height + worker arm size - height at which the underside of the same box is located); worker arm size]

2.4. Loading Operations Efficiency

Due to the continuous growth of the world population and the spread of an increasingly consumerist culture, the distribution sector has become chaotic. As such, many studies have been carried out to try to optimize all processes in this sector. One of the areas subject to much analysis and evolution is the structuring/storage of cargo in pallets and/or containers because it was realized that a better accommodation of the products would allow better use of the available space, making each pallet/ container carry more products, which in turn would lead to greater efficiency in the distribution system. However, it is currently not enough to better organize the products in containers and/or pallets to solve the problems that occur in the distribution, and it is also necessary that the performance at the level of the warehouses and distribution center be better, increasing the organization and speed of processes, so that bottlenecks do not occur at these points in the supply chain (Ramos et al., 2016).

To this end, one of the aspects to focus on is the reduction of the loading and unloading time of products, which will lead to great savings in time and, consequently, to the avoidance of bottlenecks. Generally, to measure the speed with which this type of activity is carried out, the measure used is the number of products loaded/unloaded per hour per man. To increase this number, factors such as the use of more efficient loading/unloading methods and also good storage of the products are important aids, because if a pallet has a bad accommodation of its products, when unloading it can be necessary to unload a certain product to move others that would not be unloaded, making the whole process slower and much less efficient (Ramos et al., 2016).

It is, however, necessary to realize that the loading/unloading processes are not always performed in the same way, several techniques can be used, which can be grouped into 3 categories (Ramos et al., 2016):

- Manual Handling– The products are loaded/unloaded manually, that is, the workers carry out the loading/unloading by lifting and transporting the products by themselves (Ramos et al., 2016). This method has, on the one hand, the advantage of being cheap, as it does not require investment in equipment, but on the other hand, it also has major disadvantages, such as, for example, being slow and presenting major inconveniences at an ergonomic level, since the activities of lifting and transporting cargo can sometimes be very physically demanding and can even sometimes lead to injuries, resulting from workers' fatigue, repetitive movements, among other possible causes (Bhattacharya & McGlothlin, 2012).
- Use of handling equipment – In this case, the loading/unloading of products is already done using equipment that helps in these processes, such as forklifts or other similar equipment (Ramos et al., 2016). This method is substantially more expensive than the previous one since investment is necessary for the acquisition of equipment, however, it substantially increases the speed of operations. However, this method has yet another disadvantage in relation to manual handling, since it requires a much larger area to carry out the operations.
- Automated Systems – These systems are programmed to carry out the loading/unloading process of the products in a totally or partially autonomous way. An example of these systems is robot arms or even complete robots programmed with the function of loading and unloading materials (Ramos et al., 2016). Among the great advantages of this type of system are its speed and autonomy. As for the disadvantages, without a doubt, the ones that stand out are the investment needed to install this type of system, as well as the complexity of their use.

A constant to all these systems is that they all require that there is no obstruction at the time of loading so that the product can be placed in the pre-defined place, both vertically and horizontally. As such, whenever a new article is loaded, either in a container or on a pallet, it must be previously checked that there is no impediment to its placement in its place, this is, if boxes have not already been fully or partially loaded in that place. Considering that the article intended to load must be deposited in the position (X_A, Y_A, Z_A), has a length equal to C_A, a width equal to L_A, and a height equal to H_A, it is assumed that this same article can be loaded in a given container/ pallet in which T_B articles are already found if for all these articles the following condition is verified (Ramos et al., 2016):

$$(X_B+C_B \leq X_A) \vee (Y_A+L_A \leq Y_B) \vee (Y_B+L_B \leq Y_A) \vee (Z_A+H_A \leq Z_B)$$

If this condition is met for all articles subsequently loaded, then it can be said that all these articles will not interfere with the loading (Ramos et al., 2016).

On the other hand, when loading a given article, should also be considered the articles to be loaded later. In this way, assuming that there are still T_E articles to be loaded, for all of them you should check whether the following condition is met:

$$(X_E+C_E \leq X_A) \vee (X_A+C_A \leq X_E) \vee (Y_A+L_A \leq Y_E) \vee (Y_E+L_E \leq Y_A) \vee (Z_E \geq Z_A+H_A)$$

If this condition is always respected, this means that the loading of that same article will not interfere with the loading of the articles that remain to be loaded (Ramos et al., 2016).

There are also different possible methods for how to arrange the articles during loading and/or unloading activities, three of which stand out:

- Layer-by-Layer- This method indicates that articles should be loaded and/or unloaded one layer at a time, obviously starting from the bottom up. The major constraint of this method is that sometimes the layers are longer than arm´s reach of the workers responsible for these activities, so the layers must be divided so that loading is always carried out without exceeding the reach of the arms (Ramos et al., 2016), as shown in the following figure:

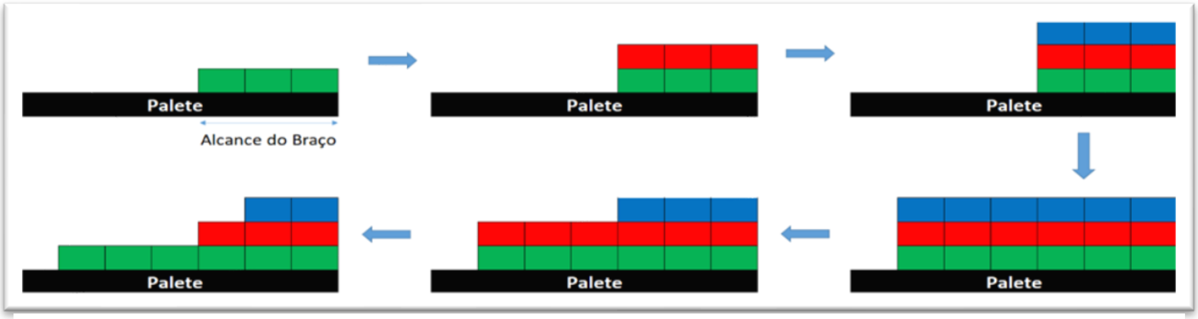


Figure 5 - Layer-by-Layer Loading

- In the Form of a Pyramid- When following this method, the articles must always be loaded and/or unloaded in a diagonal logic, thus forming a kind of pyramid, as can be seen in figure 6. In this sense, when using this method, the loading is initiated by the article that is supposed to occupy the position located lowest and more to the back of the container/pallet and is finished with the article that must occupy the position located highest and more to the front of it (Ramos et al., 2016).

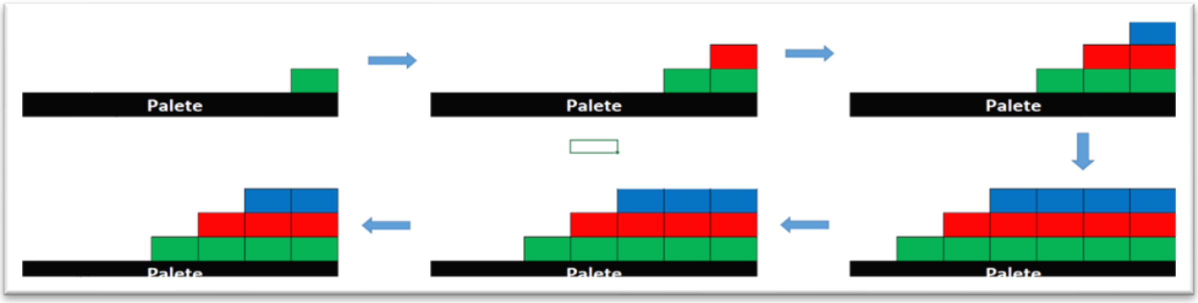


Figure 6 - Loading in the form of a Pyramid

- Wall construction approach- Following this method, the loading begins with the article that must be positioned in the lower left rear corner of the container or pallet, then moves on to the loading of the articles that are on its right side and then for those that occupy the positions above themselves, then repeating this order continuously. This method has a great advantage over the others, the fact that it allows workers to always be close to the articles when they are going to lift them, which from an ergonomic point of view is quite advantageous, helping to avoid fatigue and possible injuries, which, in turn, it contributes to increasing the efficiency of loading/unloading operations (Ramos et al., 2016).

It is known, however, that the efficiency of loading and/or unloading operations is not only affected by this type of situation but is also affected by many other factors, such as the weight of the items to be loaded, the need to dispose and/or reach articles that are higher than the worker who is responsible for these activities, among many others (Ramos et al., 2016).

In this sense, many variables can be used to try to measure the efficiency of these operations. One of the most used is the average arm´s length. The option to use this variable to quantify the efficiency of loading and/or unloading operations is often given because it is an easy-to-measure geometric metric, thus facilitating the process of quantifying efficiency (Ramos et al., 2016).

To determine this variable, the arm length must be measured for all U loaded/unloaded articles, so that its average can then be calculated. To measure the length of the arm for a given article u, the procedure is to measure the distance between the front face of the same article and the front face of the article that is under the most advanced article, this is, it must be analyzing the articles that are located under the article under analysis and checking which one has the front face closest to the front of the pallet/container, so that the distance between the front face of that article and the front face of the article can then be measured in analysis. The articles that are located under the article under analysis are characterized by falling into the following equation (Ramos et al., 2016):

$$(X_u<X_b+C_b) \wedge (X_b<X_u+C_u) \wedge (Y_u<Y_b+L_b) \wedge (Y_b<Y_u+W_u) \wedge (Z_u \geq Z_b+H_b)$$

(Being the set of all other items present on the pallet/container, excluding the item under analysis)

It is, however, important to note that when analyzing the articles that are located below the article being analyzed and verifying that for none of them the front face is closer to the front of the pallet/container than the front face of the article being analyzed so we consider the arm length to be 0 (Ramos et al., 2016).

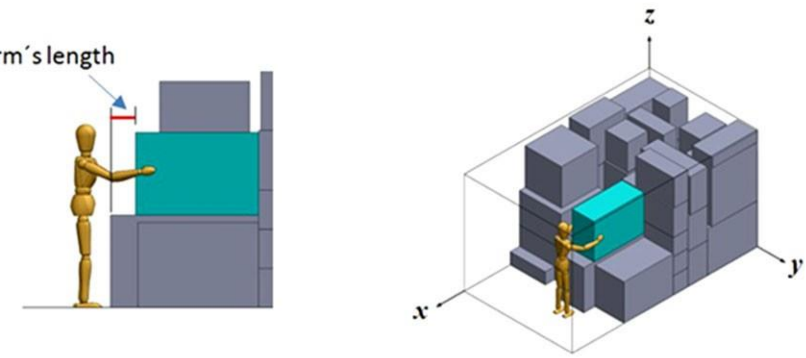


Figure 7 - Arm Length

Source:(Ramos et al., 2016)

2.5. Robotization

In response to growing consumerism worldwide, the industry felt the need to improve all its processes, in order not only to achieve much higher levels of production but also to improve the relationship with customers, thus needing to increase processes speed and improve distribution procedures. In this sense, in recent years, renovation at the industry level has been constant, with numerous innovations that make industrial processes faster, more efficient, and safer. In this way, the world is currently facing a new industrial revolution, to transition to a new generation of much more automated and autonomous industry, called Industry 4.0 (Vaidya et al., 2018)(Kiran, 2019).

This transition to Industry 4.0 aims to greatly modify processes at an industrial level, through the implementation of new techniques, such as the use of “Clouds” for information-sharing processes and the robotization of production processes (Vaidya et al., 2018).

In this logic, there is a constant search for automation and automatization as a means to make industrial processes faster, more efficient, and safer, since by combining these two concepts it is possible to achieve goals such as (SHASTRI, 2010)(Gupta et al., 2016):

- Increase production, which is achieved through greater accuracy and repeatability of the processes that the more autonomous and automated methods present;
- Decrease the number of accidents at work, using technologies that avoid the need for workers to be in contact with dangerous environments or to perform risky procedures that may cause short and/or long-term injuries;
- The decrease in production costs, for several reasons, such as the lower need for labor;
- Improve the working conditions of employees, avoiding, as mentioned above, their contact with dangerous activities and reducing their effort with heavy and tiring operations;
- Increased production quality, achieved through the use of more precise techniques and tools, reducing the occurrence of errors, especially with regard to human errors;
- Improvement of control operations, carrying out a tighter control and less subjective to human analysis.

So, these are the advantages that can result from the automatization and automation of industrial processes, thus is easy to understand the reason for their incessant demand.

It is, therefore, first, interesting to understand what automation really is. According to (Silveira & Lima, 2003) automation embraces all the methods used to automate the execution of tasks, substituting in this sense the work of manpower for computational elements. In this way, we can consider that automation is the property that gives the performance of activities independence from human work, allowing activities to be performed independently through the use of a series of tools, such as machines and robots, which eliminate the need for human involvement (Silveira & Lima, 2003).

We can then, in a way, consider that the concept of automation is quite broad, so (Groover, 2019) then decided to separate automation into 4 large sets:

- Support systems for manual production;

As much as automation and automatization are increasingly constant in the industry today, the presence of human labor is still essential for the functioning of a factory/organization, let alone for the programming and maintenance of machines and equipment (Groover, 2019).

However, there are still situations in which human labor cannot be dispensed with at all, such as in the execution of tasks whose automatization is too difficult or expensive, and in the production of unique products or of which only small quantities are required, making it not worth it to invest in the automation of their production processes. It is, therefore, in this type of case that the most we could establish are support systems for manual production (Groover, 2019).

- Semi-automatic systems;

These are systems in which there is a conciliation between human work and automated work, with a part of the production operations performed by labor and another part performed by automated tools (Groover, 2019).

- Automatic systems;

These systems are characterized by extremes, being, on the one hand, those that need less support from the workforce, making them very attractive from this point of view, but also, on the other hand, those whose implementation is more expensive, which is a major obstacle to its propagation (Groover, 2019).

Focusing on the positive side of these systems, these are the ones in which the help of employees is less necessary, with the entire production process being carried out automatically (Groover, 2019).

Within this set, however, we find quite different systems, having (Bayer et al., 2011) divided them into 3 categories: fixed automation, programmable automation, and flexible automation.

Starting with fixed automation, this is verified in production lines designed to produce a single product. This is the ideal type of automation to implement in a company that produces large quantities of a single product, allowing in these cases to achieve very high production rates. However, it has the major disadvantage of orienting production to just one product, because if the sale of that product goes into decline and if it is now desirable to change the product or even advance to the production of a different product, then the entire production line will go into disuse, so it is a risk to bet on this type of automation (Bayer et al., 2011).

Moving on to programmable automation, this occurs in turn when a single piece of equipment can produce a wide range of products, being only necessary to program them to follow the production steps of each one of them. If, on the one hand, fixed automation was used when wanting to produce large quantities of a certain product, on the other hand, programmable automation is ideal to use when wanting to produce reduced quantities of several different products (Bayer et al., 2011).

Finally, flexible automation, also called CIM, is found when dealing with equipment that can be programmed to produce a range of products that have a limited set of configurations, thus the range of products that this equipment can manufacture is less than the ones with programmable automation. In this way, can be said that this is a compromise solution between the two automation mentioned above, being ideal for the production of products in medium amounts (Bayer et al., 2011).

- Robotic systems.

They embrace systems that are characterized by being able to be reprogrammed according to production needs, being able to be executing a given operation in a certain way, and then being reprogrammed to execute it differently or even to execute different operations. They are also known for being able to analyze information from the environment independently, which is normally possible due to the integration of sensors in them (Ollero et al., 2006).

These systems are ideal for activities with repetitive work cycles, for performing tasks in a risky environment, as well as for operations involving the manipulation of personal objects, and for continuous operations (Groover, 2019).

Often interconnected with the concept of automation, the concept of automatization arises, which, contrary to what one might think, is already very old, dating back to the end of the 18th century, when it was one of the factors that led to the emergence of the first industrial revolution, leading to a reform in the methods and tools used at the industrial level, leaving the need to resort solely to labor (Filipe et al., 2013).

Since then, automatization has been growing exponentially and the invention of robots has greatly contributed to this. Although the word “Robot” first appeared in 1922 in Czechoslovakia as a definition for “forced labor”, it was not until four decades later, in 1961, that the world saw the first industrial robot. This robot, which was given the name Unimate, already had two completely revolutionary characteristics at the time, such as the fact that it has its own tools and also the fact that it can be reprogrammed according to existing needs, allowing reprogramming to run different functions. Until now, there have been many developments in robotics, making it possible today for all sectors of the industry to turn to robotics to look for robots for a huge set of applications. Nowadays, not only there are different robots capable of responding to an increasing range of applications, but also current robots have greater flexibility, which can be reprogrammed to make an extended set of functions (Noberto Pires, 2002).

Regarding platforms that allow interaction between robots and the workers who are responsible for their operation and control, there has been a great evolution, being increasingly intuitive and simple. These platforms normally go through HMI software, which is software that allows operators to supervise the activities of robots. In this sense, this software is already associated with a series of sensors that facilitate the control of the robot's operations, even communicating instantly with them, to ensure the correct functioning and realization of the tasks (Groover, 2019).

But before all this, it is important to understand what a robot really is. According to the ISO 8373 standard, any programmable mechanism that presents some degree of autonomy to perform activities such as movement, manipulation or positioning can be considered a robot (ISO - International Organization for Standardization, 2021).

It is essential to understand that robot and automaton are then distinct concepts because the concept of automaton is broader. An automaton is any machine or tool capable of performing one or more tasks without the need for assistance from external sources, having its own energy source, and performing these tasks independently. In this way, the robot concept is encompassed by the automaton concept, so any robot is also an automaton. The opposite, however, can no longer be concluded, it cannot be said that all automatons are robots (Nof, 2009).

But the robots themselves can also be separated into categories according to their properties, characteristics, functions, etc…. One can, for example, separate robots between collaborative and non-collaborative. Collaborative robots are those that work together with employees, not replacing them, but helping them in their functions, with constant interactions between robots and employees. On the other hand, non-collaborators are those who work separately from employees, with no interaction between employees and robots, even for the employees' own safety, so they are usually even surrounded by barriers that prevent employees from approaching them. This is because in the case of collaborative robots, when employees perform some movement that could put them in physical danger, there are sensors that detected these movements, leading to the immediate stop of operations by the robots, while for non-collaborative robots none of this happens (Djuric et al., 2016)(Franklin et al., 2020).

The concept of industrial robot is a little more restricted, being defined in ISO 8373 as a manipulator that is automatically controlled, with the ability to perform multiple tasks, that can be reprogrammed in that regard. The ISO 8373 standard also mentions that regarding their programming, they can be programmed in a set of three or more axes, and in terms of their positioning, they can either be fixed, permanently located in a certain location, or mobile. It is also defined by the ISO 8373 standard that the industrial robot concept, in addition to the manipulator itself, also includes its controller and the platforms that allow its programming (ISO - International Organization for Standardization, 2021).

Regarding its design, most industrial robots have a structure similar to the upper limbs of a human being, because often these are intended for functions that were later performed by humans using their upper limbs, functions such as picking up and carrying objects. In this logic, the first joints of a robot are generally equivalent to the shoulder and elbow of the human being, being responsible for positioning, while the joints of the robot's wrist respond to the functions that the human being's wrist performs, being then in charge of object orientation, being normally assisted by the terminal-object, or, in the case of the human beings, by their hands. To be able to work at all angles around them, that is, to be able to operate in 360º, like the human being, industrial robots generally have 6 axes. However, even so, the movements of industrial robots are generally limited, since they are not able to overcome certain movement limitations, which are called singularities. Singularity is the name given to the loss of mobility of the joints, which then limits the movement of robots. These singulars must then be calculated before using the robots, to previously formulate an idea of what movements they will be unable to perform (Groover, 2019).

Regarding the wrist of industrial robots, they are characterized by being able to perform 3 types of movements (Groover, 2019):

- Pitch, which corresponds to the rotation of the wrist vertically;
- Yaw, corresponding to horizontal rotation;
- Roll, refers to the rotation around the arm.

Bearing in mind these 3 types of movements, industrial robots’ wrists can assume two distinct configurations (Filipe et al., 2013):

- Pitch-Yaw-Roll, corresponding to the wrist that most resembles the human fist;
- Roll-Pitch-Roll or spherical wrist, although it is not so similar to the human fist, it has characteristics that make its use generally more advantageous, being as such the most used in the industry, although they present as a great disadvantage to show some problems concerning singularities.

All this diversity in the components of industrial robots means that currently there is also a wide diversity in the configurations that can be used in these robots. In this sense, the International Federation of Robotics decided to group robots into different groups according to their mechanical structure, thus dividing robots into 5 categories (Reddy Vemula, 2015):

- Linear Robots, ideal for material handling, machining, or component assembly operations, being this the category that includes Gantry robots and Cartesian robots;
- Cylindrical robots (RPP), although falling into disuse, were generally used for activities related to material handling and component assembly;
- SCARA robots (RRP), suitable for assembly functions, especially for those where vertical operations are required;
- Articulated or anthropomorphic robots (RRR) are suitable for a series of tasks, including the most used at an industrial level;

- Parallel robots are distinguished from all others by having closed rather than open kinematic chains, which allows them to have light structures and high accelerations in movements, which makes them ideal for use in packaging functions (Maya et al., 2013).

Turning now to the analysis of the advantages that can be achieved with the use of robotic systems instead of manual systems or any other types of systems, these are undoubtedly numerous. Among all of them (Gupta et al., 2016), (SHASTRI, 2010) and (Filipe et al., 2013) highlight the following:

- Increase in production, as the characteristics of these systems generally allow companies to obtain greater production capacity;
- Process speed, which makes it easier to meet delivery dates, even in the case of orders with a short delivery time;
- Waste reduction, as these systems are normally programmed to perform the various operations most efficiently and effectively, thus avoiding waste;
- Increase in specialized employment, with the use of these systems being a strong incentive to hire skilled labor capable of working with them;
- Improvement of the quality of work, being generally associated with these systems a high precision, in addition to the fact that due to the autonomy that they have, human errors are eliminated;
- Increase in work safety, achieved through the allocation of these systems to high-risk tasks, which due to their high precision means that the number of work accidents is reduced;
- Improvements in workers' health, because through the allocation of these systems to risky tasks, workers are exempted from performing tasks that could cause them injuries in the short or long term;
- Reduction in insurance costs, achieved through the reasons explained in the two previous points;
- Flexibility, achieved because robots can be reprogrammed to perform different tasks;
- Reduction of labor costs, achieved by reducing the need for labor and by reducing waste;
- Increase in the ability to face complex operations and/or involving products of high complexity.

Regarding the disadvantages of these same systems, the same authors highlighted the following (Filipe et al., 2013) (SHASTRI, 2010)(Gupta et al., 2016):

- Unemployment, caused by the fact that these systems often occupy posts that were later occupied by labor;
- Initial investment, being very expensive to purchase and install this type of system;
- Emergence of a new type of work accident, which includes man-machine collisions and mechanical accidents;
- Increase in expenses with specialized labor, necessary to work with these systems, to which sometimes is added the expenses associated with their training so that they can interact correctly with the systems and keep up to date;

Bearing in mind that one of the disadvantages of implementing this type of system is, as mentioned above, the emergence of a new type of accident, measures were idealized to counteract the occurrence of those. Among these measures can be highlighted (Filipe et al., 2013):

- Implementation of barriers, so that workers do not enter the robot's operating area;

- Implementation of emergency buttons, which make the robots stop immediately whenever something goes wrong;
- Placement of signs, to warn workers of potential hazards.

As for the applications in which robots can be implemented, the range is increasingly widened. However, a constant that is generally verified is that they are implemented with one of two objectives, either to carry out tasks that were previously performed by operators and proved to be very tiring, repetitive, or dangerous or as a way of making the processes faster and more efficient (Gupta et al., 2016). One of the areas in which industrial robots are used to achieve both objectives is palletizing. In this way, through its implementation, workers do not need to carry out repetitive work that often requires a lot of effort, such as the transport of products/materials, and it also allows the entire palletization process to become faster (Filipe et al., 2013).

In this sense, there are currently several different methods for palletizing (Abdou & Lee, 1992):

- Manual palletization, in which all operations from the transport of objects to their placement on the pallets are carried out by employees;
- Mechanical palletization;
- Robotic palletization.

It is within the latter that flexible robotic palletizing systems emerge. These systems are used to transport objects from a certain location, usually shelves or conveyors, to another location, in this case, even pallets. To this end, when implementing this type of system, it is first necessary to analyze what will be the maximum load that the robot will be required to carry, to choose a suitable robot capable of carrying this same load. Likewise, the terminal organ must also be chosen in advance and the objects that will be transported by the robot must be properly analyzed, to choose the most appropriate terminal organ. In addition, a series of sensors and/or actuators must also be coupled to the systems, which facilitate the control of the activities of the systems, making these elements essential for the efficiency of the activities performed by the robots (Filipe et al., 2013).

In this way, these systems were able to revolutionize palletizing, due to a series of characteristics that made them more attractive than any of the palletizing solutions used until then, such as (Dzitac & Mazid, 2008):

- High speed in operations, thus allowing faster palletization, thus allowing greater production, and also responding to orders with short delivery times;
- High flexibility, acquired through the typical flexibility of robots, achieved through the possibility of reprogramming them;
- High precision in operations;
- Ability to carry objects of all types of shapes and weights.

In this area of palletizing, one of the aspects of robots that is most important to analyze, having already been mentioned above, is the terminal organs. As already mentioned, it is essential that before implementing a robotic palletizing system, the type of terminal unit that is most suitable for the operations that the robot is intended to perform is chosen, that is, for the objects that it is intended to transport. Thus, for the selection of the terminal organ that will be used, the objects that the robot is intended to transport must be analyzed, studying their mass, volume, shape, and sensitivity (Popple, 2009).

Currently, several types of terminal organs can be installed in industrial robots, among which the following can be highlighted as being some of the most used (Popple, 2009):

- Jaw
- Unilateral fork
- Friction
- Vacuum

Each of these manipulators has very different characteristics, so a summary of the attributes and functionalities of each is presented below.

Starting with the mandibles, these are formed by claws that are capable of grasping objects, with the specificity of grasping them from below. For this, the surface where the products are to be collected must have small fixings, such as space between the rollers of a conveyor belt or fins on a table, so that the jaws claws then pass through this space and manage to grab the objects underneath. This type of manipulator is also characterized by having supports on the side or above to better support the load they carry, together with the claws that then grab the items from below. It is important to highlight a disadvantage of using this type of manipulator when unloading the products, when dropping them there is a small fall, and therefore these may not be the most suitable manipulators for fragile objects (Popple, 2009)(Groover, 2019)(Gupta et al., 2016).

Moving on to the unilateral forks, these are then distinguished from the jaws, as, as indicated in the name, they have only forks on one side and not on both. In the same way, the jaws also grip the products from below, so they also need to have protrusions on the surfaces where they are going to collect the products. They also generally have support from above to be attached to the support of the products while they are being loaded. As a great advantage in relation to the jaws, they have the fact that they place the products with a much superior smoothness, because in the process of dropping them, what happens is that there is a plate that then forces the products to land smoothly while the fork it just retreats back, thus dropping the products (Popple, 2009)(Groover, 2019)(Gupta et al., 2016).

Next, the method of handling the products using friction. The manipulator is made up of bilateral plates that descend to the height at which the products are and where they grip the products on both sides using friction. Later, to drop the products, the bilateral plates open, disappearing the friction effect, thus leading to the products being dropped in the right place in a smooth way (Popple, 2009)(Groover, 2019)(Gupta et al., 2016).

Finally, the method of handling products using suction is undoubtedly the easiest to explain. With this type of manipulator, the process boils down to the lowering of the manipulator to the height of the products, where a vacuum will be generated between the suction cups present in the manipulators and the products, thus grabbing the products from above, then lifting them and transporting them to the intended location. When at the intended location, the manipulator only must descend to the defined position, where the vacuum effect will be turned off, thus releasing the products (Popple, 2009)(Groover, 2019)(Gupta et al., 2016).

To reconcile the advantages of each of these types of manipulators, some manipulators combine more than one of the methods presented above, for example, manipulators with unilateral forks and at the same time suction systems can be found (Groover, 2019).

Considering the description presented of these various manipulator options, it is easy to see that although there may be some similarities between them, there are also many differences. In this sense, in terms of similarities, it is worth noting the fact that all these manipulators have a similar useful loading area, which means that the maximum number of objects they can load is similar for all. Another similarity lies in the fact that the rotation capacity is identical for all these manipulators, which is a very important feature because if the manipulator has restrictions regarding its rotation capacity, it may not be able to place the objects on the pallet in the desired orientation (Gupta et al., 2016).

Turning now to the major differences found between these four types of manipulators, two can also be highlighted. The first one is in favor of manipulators that use suction, as only these do not need space to release objects, that is, in all other manipulators, both jaws and unilateral forks and friction manipulators, to drop objects there is an opening of the forks or plates to the sides, so when working with these manipulators to put an object in a certain position there must be a certain space to the sides, to allow the manipulators to open in that direction. When using suction manipulators, the objects are only held from above, so to put the objects down it is only necessary to let them go, there is no opening action. Regarding this aspect, it is also important to note that for both the jaws and the manipulators that use friction, space is required for both sides of the object, while in the case of unilateral forks, space is only needed on one side, the one where the forks will open (Gupta et al., 2016).

The second major difference between these types of manipulators is the fact that only the friction and suction manipulators have grip strength, not the same for the jaws and unilateral forks. This is because grip force is the force responsible for suspending a given object in the air and only in the case of friction and suction manipulators happens suspension of materials, while in jaws and unilateral forks the materials are loaded from below, so there is no such grip force (Gupta et al., 2016).

This variety in the range of end-piece options that can be mounted on industrial robots is yet another reason for expanding the use of industrial robots because with all this variety it is always possible to find an ideal end-piece for whatever operations desired the robots to do it. This is also true in the palletizing area, where after an analysis of the shape and mass of the objects to be loaded, it is always possible to find a type of terminal organ that will serve perfectly to transport these same objects (Popple, 2009).

The type of terminal organ to implement in the robot depends on a lot from case to case. For example, in the case of palletizing boxes, neither the use of unilateral forks, nor of jaws, nor of friction manipulators would be the most appropriate strategies, as they could damage the boxes and make the palletization process more difficult because when opening and closing to drop the boxes, they could collide with the boxes next to them. In this sense, the option for suction manipulators can be shown to be the most appropriate, which through its suction process holds and transport the boxes, avoiding collisions with any boxes around, being only necessary to lift and put the boxes down, without any opening and closing movements.

It is important to note that when suction manipulators are implemented as the robot's terminal organ, several details must be paid attention to:

- Number of suction cups: the number of suction cups used must be defined according to the size and mass of the objects to be transported;

- Position of suction cups: likewise, the distribution of suction cups must be analyzed according to the size, shape, and mass distribution of the objects to be transported;
- Number of vacuum generators: for the suction cups to work correctly, they must be combined with vacuum generators. In this sense, the number of vacuum generators to be installed must also be defined. A correct strategy to follow in this regard is to install a number of vacuum generators similar to the number of suction cups, to separately ensure the vacuum conditions in each suction cup, thus making it easier to control each of the suction cups. In this way, it will be possible to ensure similar conditions in each suction cup, avoiding transport imbalances that could affect transport or even cause the materials to fall (Filipe et al., 2013).

Other aspects of industrial robots that are also of great interest to analyze are the inclusion of sensors and/or actuators, which are used to facilitate the control of the activities carried out by the robots.

In this sense, in the case of robots intended for palletizing operations, optical sensors are often introduced to detect falling objects during transport. They are made up of an emitter and a receiver, the emitter being responsible for sending a beam of light with a high refractive index, which must follow to the position where the objects should be found, carrying out the refraction when encountering them and returning towards the receiver. In this way, when the beam arrives back at the receiver, it can be seen that the object is in the correct position and has not fallen, because in the event of a fall there would be no object in that position, so the beam would disperse and the receiver would not receive any signal (Filipe et al., 2013).

3. METHODS AND APPLICATIONS

3.1. Problem Presentation

To explain the development of this project, it is first necessary to understand the context in which it is inserted.

As already explained in the introduction of this dissertation, this project appears inserted in the context of the transition to industry 4.0, in which the maximum automation and automatization of all industrial processes are sought. In this sense, the storage sector is one of the areas where there is still a long way to go, since most processes in this sector still depend on human strength and are therefore not very automated and autonomous. However, this transition obviously must be done carefully, as it is necessary to guarantee that when automating processes, through the application of robots, for example, the processes will be carried out with a degree of efficiency and effectiveness at least equal to that observed when these same processes were carried out using only human labor, so when introducing changes in processes it is important to ensure their improvement.

Being the storage sector one of the more important of the value chain of all products, it is easy to understand why this transition, instead of being rushed, must be consummated very carefully. This is not, however, the only reason that justifies a careful and extensive work in this area. There are often useless operations in warehouses that do not add any value to the products, being this also a point that justifies the huge necessity for improvement in this sector. In this sense, the unnecessary movements that often occur, the unnecessary transports, and even the excessive waits should be highlighted as aspects that need to be treated.

So, it becomes obvious that before making the transition from human labor to robots, it is necessary to ensure that these robots will perform all the processes in the most efficient and effective way possible, reducing waste as much as possible.

Regarding this sector, it is essential to point out that it involves a series of processes, some of which are very different from product to product. Yet one of the processes that is used for almost all products is palletizing. This is because since its invention, the use of pallets, as already mentioned in the literature review, has grown exponentially, being currently used to transport a large diversity of products and raw materials. In this sense, palletizing is a very important process in the storage and transport of materials in many warehouses, being a lot of time spent on its operations.

Even so, this is one of the processes that still depends a lot on human labor. As such, its operations are often influenced by the subjectivity of its operators. This same factor is verified, for example, in the movements carried out in palletizing operations, that vary from operator to operator, often presenting unnecessary movements and transports, opting for longer paths than the ideal. Also, regarding the accommodation of products on the pallets, there is a presence of subjectivity of the operators, which leads to the waste of space on the pallets. This happens because many times when the products are accommodated on the pallets, many empty spaces are left between boxes, not maximizing the occupation of the pallets, thus generating great waste, which can lead to the need to use unnecessary pallets, this is, to the need to use a number of pallets higher than necessary. The subjectivity of operators in palletizing often also means that the accommodation carried out is not ideal in terms of cargo security, not considering many restrictions that should be taken into account, not only for the safety of employees but also for the safety of the products.

In this sense, all these factors must be taken into account and carefully studied in the process of automation and automatization of the palletizing processes, with the goal of elimination of human subjectivity, thus ensuring that palletizing is always done with the greatest efficiency and effectiveness. The project to be developed at INESC TEC aims to pay attention to all these factors, being the objective of that project the creation of a code that would allow the robots used in palletizing operations to, in addition to being autonomous, do all those operations in the most effective and efficient way.

Therefore, one of the factors that is important to analyze is the stability of the load arranged on the pallets, being this the question that is at the bottom of this first phase of the project.

However, the choice of this factor to be analyzed in the first phase of this project was not random. This is a very important factor in palletizing processes, which if not considered could lead to the arise of many problems. Among these problems is, without a doubt, the collapse of the load, since if the stability of all the products placed on the pallets is not guaranteed, they will inevitably become unbalanced, leading to the fall of part, or even total, of the load. Because of this collapse, an even bigger problem can arise, which is the possible damage to the products. In any case, whether the products end up being damaged or not, if this factor is not considered, the palletizing processes will take much longer, since after the load has collapsed, it will have to be placed on the pallets again, losing up a lot of time. In the case of product damage, the time lost will be even greater, as it will be necessary to repair the products or even produce new ones, thus leading to a great waste of time, being spent a lot of time in operations that do not add any value. In addition to the waste of time that will come from repairing or producing new products, there will also be wasted labor and raw materials in unnecessary operations, which will be used in these operations.

It is then concluded that stability is a fundamental factor in the accommodation of products on pallets. In this sense, to make the palletizing process autonomous and automatic, one of the first aspects that must be verified is the stability of the load, because if the load is not stable, the rest of the process will be useless, because sooner or later the load will eventually collapse, leading to the need to redo the palletizing process. Therefore, if the palletization process is automated, first, a palletization plan must be designed, in which the stability of each of the products and, consequently, of the entire load is guaranteed, so that the remaining factors of the palletization could be analyzed later on, once it makes no sense to analyze the other factors without first analyzing the stability. It is then clear why this was the factor studied in the first phase of this project because only after having fully studied all aspects of this factor does it make sense to move on to the next phase.

In this first phase of the project, the aim is to create an algorithm that would allow the verification of the stability of the load placed on the pallets. As such, first, it is needed to idealize the arrangement of the products on the pallet, so that the developed algorithm can then be applied, evaluating whether or not this arrangement will be stable.

In this sense, if the algorithm returns that the generated arrangement is effectively stable, then it will proceed to the analysis of the remaining factors, and only after having verified all of them will the idealized arrangement be applied. In other words, a disposition of the products is first conceived, but it is not immediately applied. First, this arrangement will be analyzed in terms of all factors, starting with stability, and only if the analysis of all factors proves that this is the best possible arrangement will it be put into practice.

If the algorithm evaluates that the idealized arrangement is not stable, then this arrangement is abandoned and another arrangement must be idealized since it would make no sense to proceed with this arrangement because if it were put into practice, the load would collapse sooner or later.

As already mentioned, the automation and automatization of the palletizing process ultimately aim to reach the point where all palletization processes are carried out autonomously by robots. In this sense, these robots should be able to, based on the set of products that are indicated to them that are intended to be placed on the pallets, devise the best arrangement of the products on the pallets by themselves, without any human help, respecting all relevant factors, proceeding to the palletization operations only after find the optimal arrangement. As such, these robots must not only be capable of grasping, transporting, and disposing the products but also of autonomously devising product arrangements on the pallets, taking into account the dimensions of the pallets and the dimensions and characteristics of the products. Allied with this, they must also be capable to evaluate the dispositions devised by themselves, to guarantee that the disposition of the products that will be carried out is effectively the best, being totally safe in terms of stability.

It is then necessary to create a set of codes to apply in these same robots that will enable them to fulfill these functions. It is in this sense that arises the need to develop the code created in this project, which is a code that, when inserted in robots, will enable them to assess whether or not the idealized dispositions/arrangements are stable.

This concludes the presentation of the problem that led to the development of this phase of the project, being now time to analyze its various stages, which are presented in the following subchapter.

3.2. Project Development

As already explained, the development of this project will be divided into several phases, this being only the initial one. In each of these phases, each of the factors that influenced the palletization must be studied, being this first phase the one where it is intended to study the stability of the load.

However, this initial phase can also be subdivided into several stages, that were covered to be able to successfully complete it. These stages and their order are shown in the flowchart in figure 8.

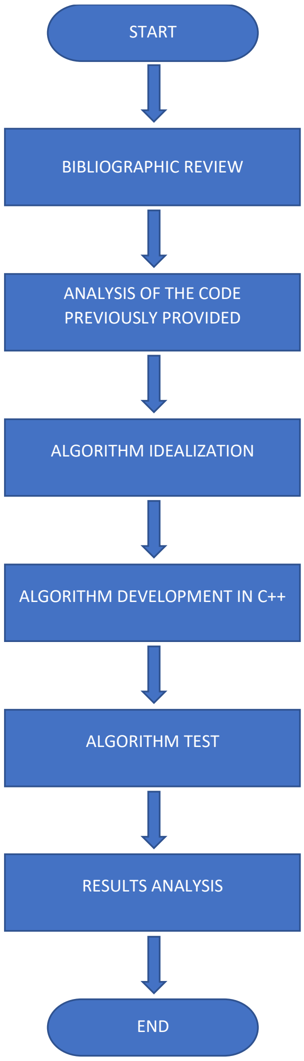


Figure 8 - Project Development Flowchart

As can be seen from the flowchart, in the first stage bibliographic research on this topic was carried out, studying all the details of the stability of the load on pallets, as well as the best methods to guarantee it. This step would serve as a basis for all the rest of the work, being the fulfillment of this stage essential to have conditions to then move forward with the creation of the algorithm.

However, there was still another step to do before moving on to the development of the algorithm. At the beginning of this phase of the project, a code was already provided as a base, which was, among other functions, capable of reading "notepad" type files in which the dispositions of products on pallets were presented, which is needed to then be able to evaluate if they are stable or not. In this sense, before the development of the algorithm capable of evaluating the stability of the load in pallets, it was still necessary to first analyze all this code provided as a basis, not only to understand all its functionalities but also because, as the algorithm that was to be developed would be included in this code, it was necessary to understand the best way to fit the algorithm developed in this code, using the variables and functions already present in it.

After analyzing this code, the next stage was the idealization of the algorithm for the evaluation of stability, applying to this end the concepts studied in the first stage and the contents visualized in the second. At this stage, it was not intended to idealize the structure of the code that would represent the algorithm, but the structure of the algorithm itself, defining all its actions, step by step, and idealizing the methods used to perform these actions, not yet thinking about the code to represent/perform them.

Having idealized the algorithm, it was necessary to move on to its development in C++ language. At this stage, the idealized algorithm would then be transcript to C++ language, using, for this purpose, the available features of this language and applying variables and functions already created in the code provided as a basis. To complete this phase, it was also necessary to carry out a lot of online research, to discover the commands in C++ language necessary to put into practice the actions idealized for the algorithm. This happened due to the inevitable lack of knowledge of how to perform certain actions in language C++, leading then to the need to resort to online research to find ways to perform them. In this sense, it was, for example, necessary in many situations to investigate what were the functions available in C++ libraries that allowed performing certain actions that were necessary for the development of the code.

One of the aspects of the code that took a lot of time to be developed, being therefore worth mentioning, was the creation of the convex hull. Initially, the best option seemed to develop an own approach to this topic. However, the various attempts made to try to create a code capable of correctly creating the convex hull all failed, once none of them was able to generate the correct convex hull in every situation. It was then decided to search online for an already created code that would be capable of creating the convex hull correctly in the most diverse situations. A series of codes of this kind were quickly found, but this did not complete the work, because it wasn´t possible to simply copy the codes found online and include them in the algorithm. It was necessary to study how it would be included, preparing the algorithm code already developed for its reception and adapting the code found online to be included in the algorithm code. This process took a long time, but in the end, the algorithm was able to correctly generate the convex hull in all possible situations.

As well as the convex hull, all other procedures that were part of the algorithm were also transformed into C++ code at this stage of the project, obtaining at the end the C++ code representative of the algorithm capable of evaluating the stability of the load on the pallets.

Having finished the development of the algorithm in C++, all that remained was to apply it, to prove its efficiency and effectiveness. For this, a series of instances were provided. This set of instances included both instances in which the load on the pallets was unstable, as well as instances in which the load allocated on the pallets was stable. In this sense, when applied, the algorithm should correctly evaluate when the load was stable and when the load was unstable.

After applying the algorithm to all the instances provided, the results obtained were then analyzed, verifying whether or not they were the intended ones. The execution time of the developed code was also analyzed in this stage, to understand if, in addition to being effective, the algorithm created was also efficient, evaluating all situations correctly and quickly.

The finish of this analysis marks, as can be seen in the flowchart, the end of this phase of the project. Was successfully created an algorithm capable of evaluating the stability of load in pallets, thus meeting the conditions to proceed to the next phase of the project, in which another factor that compromises the palletizing operations will be analyzed.

3.3. Algorithm Idealization

One of the objectives of this project was, as already mentioned, the development of an algorithm that would allow the evaluation of whether the projection of the center of gravity of the boxes, in the plane that includes the support polygon of these boxes, was or not within these same support polygons, allowing like that the analysis of whether or not the boxes were stable. This happens because the condition imposed to confirm the stability of the boxes is that the projection of the center of gravity of the boxes needs to be within its support polygon.

For this purpose, a code in C++ was developed and tested, but before was necessary to idealize the algorithm that should be created to evaluate this condition and later transcript to C++ language.

It was known that the first step of this algorithm should be create the support polygon of that same box, to verify if the projection of its center of gravity fits within that support polygon afterwards.

So, in the first phase of this algorithm would be necessary to create the support polygon. But to create the support polygon, it would be first needed to know which points would constitute it. As such, it would be required to discover the coordinates of these points. To do so, the first step would be gathering a set of points that would join all the vertices of both the box in question, this is, the box whose stability was being analyzed, as well as the boxes that support it, being this the set of vertices that could be used to form the support polygon.

Having established this set, in the next phase would be necessary to define which, among this set, would be the vertices that would really belong to the support polygon. For this, it would be necessary to create a convex hull algorithm, similar to “Jarvis March”. In this sense, an algorithm of this kind should be devised.

After the application of this algorithm, it would be expected that the vertices of the support polygon would already be defined. Having the vertices, it would be necessary to unite them to form the polygon. For this, it would be necessary to form straight to unite the different vertices.

After the creation of all the support polygon straights, all that would still be remaining to do would be to evaluate whether or not the projection of the center of gravity of the box is inside the support polygon. Considering the rule presented in the literature review, it is known that the conditions for considering that the projection is inside or outside the polygon are the following:

1) If the projection is inside all the straights, then the projection is considered to be inside the polygon;
2) If there is at least one straight for which the projection is found to lie outside of itself, then the projection is said to be outside the polygon.

Considering this, it is obvious that would be necessary to check if the projection is outside or inside each of the straights. To do so, the process explained in the literature review would be applied.

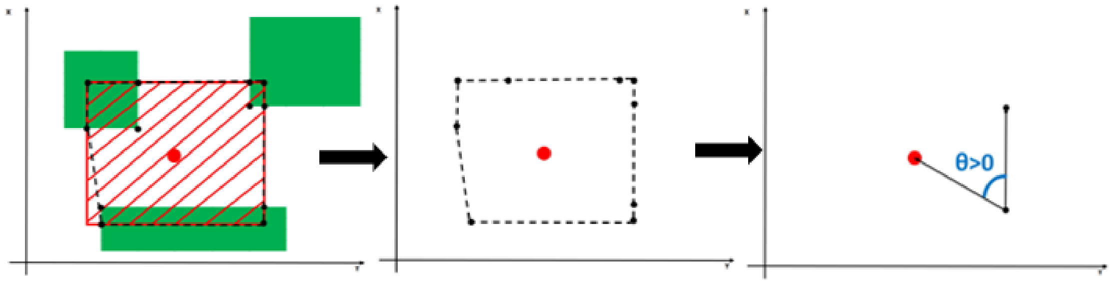


Figure 9 - Process to evaluate if a point is inside or outside the support polygon

If the angle formed is positive, it means that the projection is inside. On the other hand, if the angle formed is negative, it means that the point is outside the straight, and automatically it can be concluded that the projection of the center of gravity is not inside the support polygon.

This would be the strategy to be followed, thus checking for each of the polygon's straight whether or not the projection is inside them and, consequently, within the support polygon, being, like that, able to conclude if the box is stable.

Subsequently, to assess the stability of the entire load, since, for the load to be completely stable, each of the boxes must also be stable, the algorithm would check the stability of each one of the boxes that constitute the load. In this sense, to consider the load stable, it must be verified that all the boxes are stable. On the other hand, if only one box is not stable, that would already be enough to consider that the load is not stable.

Having explained the algorithm idealization process, it is now time to move on to the presentation of its development process, using in C++ language.

3.4. Algorithm Development in C++

After the idealization of the algorithm to evaluate the stability of the load on the pallets, it was necessary to convert it into code, in C++ language, so that it could be included in robots in the future, thus giving those robots the ability to autonomously assess whether the arrangement they had previously idealized for the products on the pallets is stable or not.

This process was, however, quite complex, being necessary to go through several phases until the algorithm was fully transcribed into C++ language.

It was necessary to create two separate functions, one that would evaluate the stability of only one box and another that would apply this last function to all the boxes on the pallet, thus allowing the stability of the entire load to be evaluated because if all the boxes are stable, the load can also be considered to be completely stable.

Starting with the function that would evaluate the stability of an individual box, this one was named “Poligono”. It is a Boolean function, which will return the result “True” if the box is stable or “False” if the box does not meet the necessary conditions to be considered stable. As for the requirements for its execution, this function needs a set of 8 parameters to be executed. These parameters correspond to:

1) Initial x coordinate of the box whose stability is to be analyzed;
2) Final x coordinate of the box whose stability is to be analyzed;
3) Initial y coordinate of the box whose stability is to be analyzed;

4) Final y coordinate of the box whose stability is to be analyzed;
5) Initial z coordinate of the box whose stability is to be analyzed;
6) Final z coordinate of the box whose stability is to be analyzed;
7) Dimension Y(Width) of the container where this box is located;
8) List with the arrangement of boxes in that same container;

Moving now to the structure of this function, to make its explanation not too tedious, it was summarized in the following flowchart:

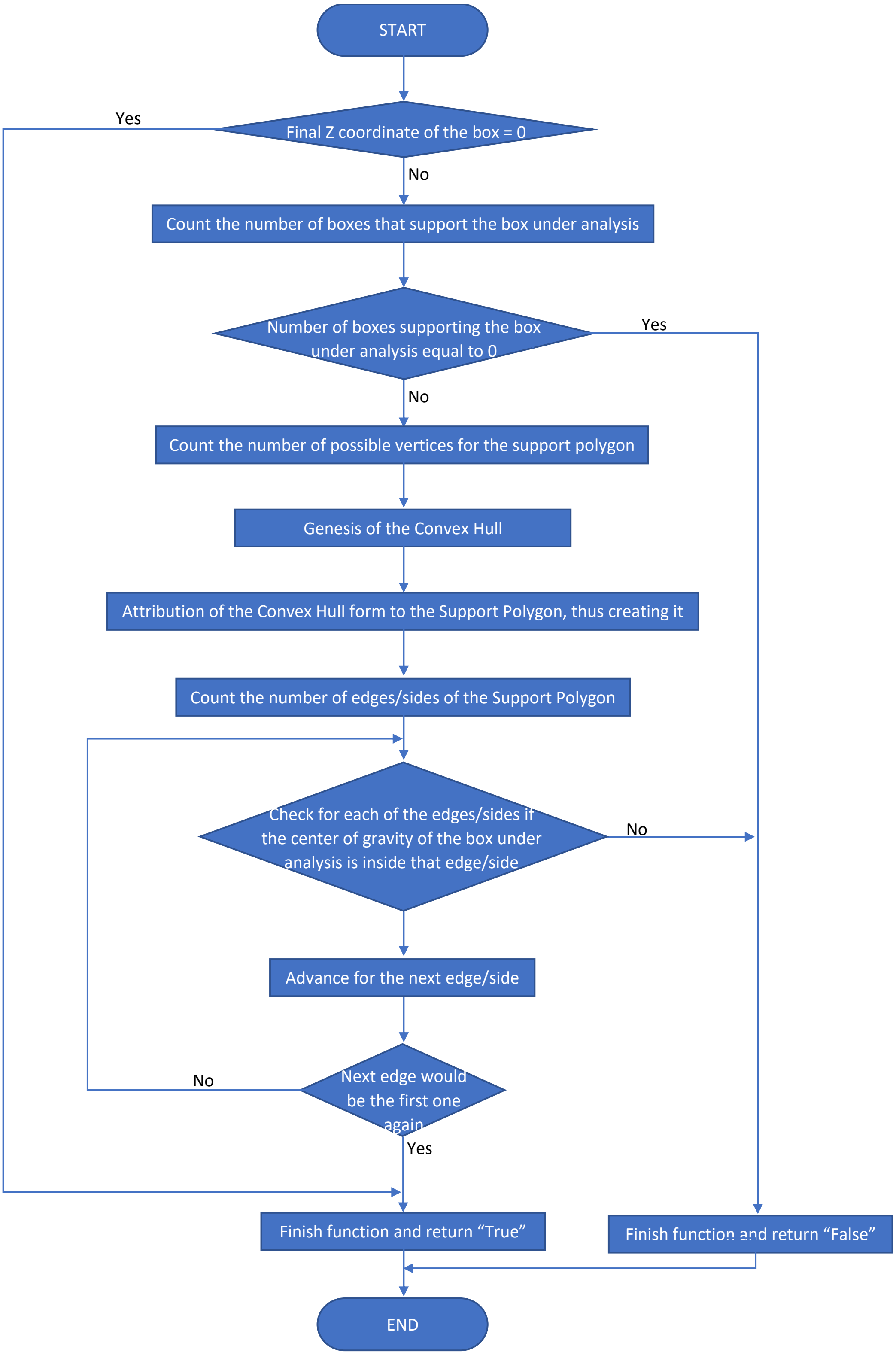


Figure 10 - Flowchart with the explanation of function "Poligono"

In addition to the "Poligono" function, as previously mentioned, another function was created. This function was created with the name "VerificacaoPoligonoTotal" and is also a Boolean function because this function was created to evaluate the stability of the load, returning as a result "true" if it is verified that the entire load is stable or "false" in the in case the load is not stable. This function was only created after the creation of the function "Poligono" because the function

“Poligono” would be included in its structure, so it would not make any sense to create this function first.

As for the parameters required by this function, there are only two, an integer value and a list of arrangements. In terms of the integer value, it must correspond to the length of the container, while the list must contain the positioning of the various boxes that are inside that container.

Once the parameters are provided, the function can then be executed, being his operation summarized in the following flowchart:

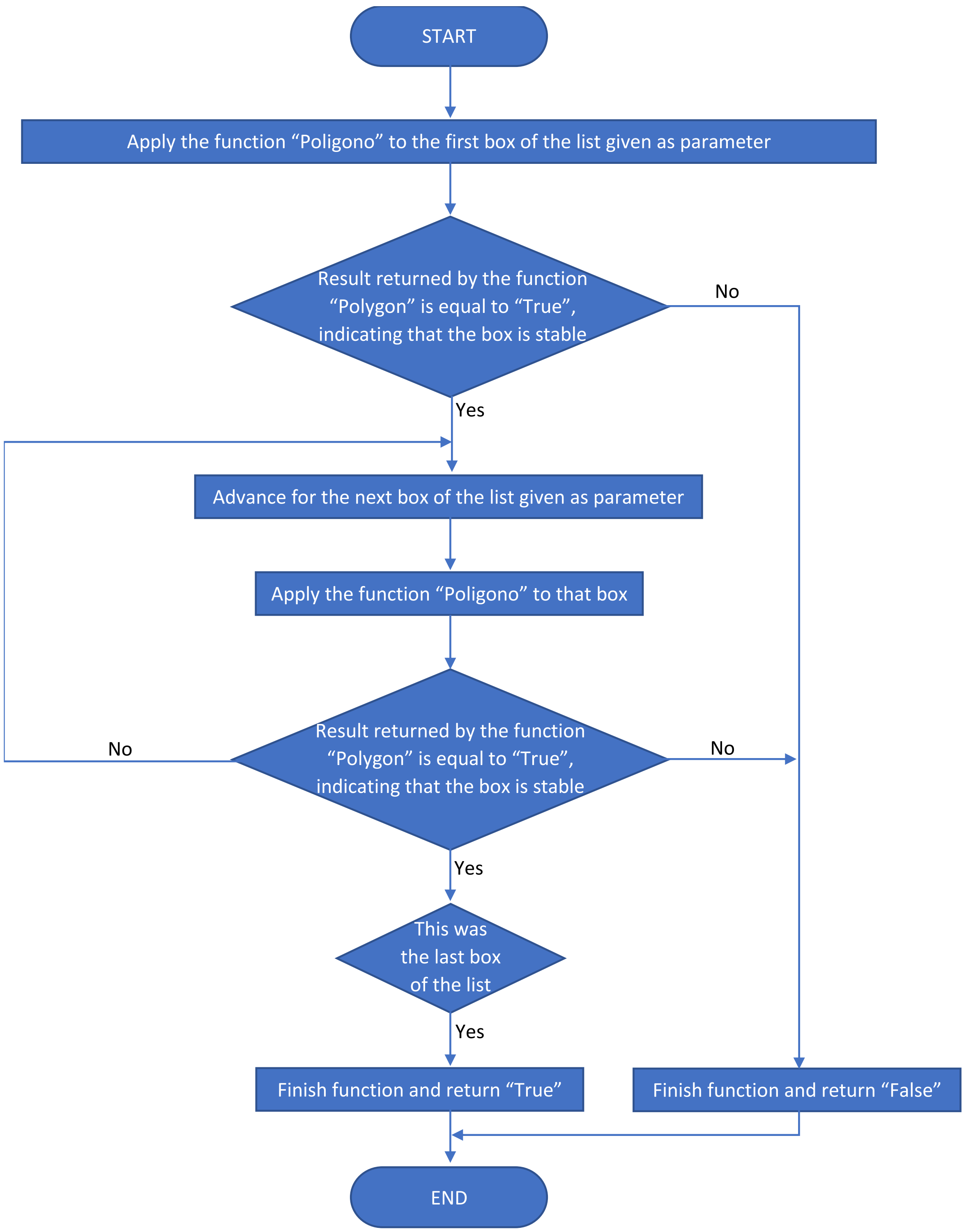


Figure 11 - Flowchart with the explanation of function "VerificacaoPoligonoTotal"

Once created and defined, all that remains to do is to apply the functions, to be able to carry out the stability evaluation of the solutions presented to the program.

Bearing in mind that the objective is to evaluate the stability of the entire load, the only function that would be used is “VerificacaoPoligonoTotal”, even do the function “Poligono” is also indirectly used, once it is part of the structure of “VerificacaoPoligonoTotal”.

4. RESULTS AND DISCUSSION

After the creation of the algorithm in C++ language, it was then applied to a series of instances, to evaluate its effectiveness and efficiency. For that, it was analyzed not only if the algorithm returned the intended result in terms of the evaluation of the stability of the load on the pallet, but also the execution time of the algorithm until returning the evaluation result.

Concerning the instances used to evaluate the functioning of the algorithm, they are divided into four sets:

1) BR;
2) RunByOccupation;
3) RunByVolume;
4) RunByWeight;

However, the first of these four sets is very different from the rest. This is because the first set is intended solely to verify the correct functioning of the created algorithm. In this sense, only three instances are included in this set, each one presenting a different arrangement of boxes on a single pallet. Further on, in subchapter 4.1, these arrangements will be explained and the results of applying the algorithm will be analyzed.

As for the remaining three sets, their focus was not so much in evaluate the correct functioning of the algorithm, once throughout these sets there is no pallet in which the load is not stable, so it would not make any sense to use the instances of these three sets to find out if the algorithm correctly identifies when the load on the pallets is unstable, because this never happens in any of the instances. In this sense, the application of the algorithm to these three sets was carried out not only to confirm the stability in each of the pallets of the various instances present in these sets but also to be able to evaluate the efficiency of the algorithm, checking whether the algorithm, in addition to being valid, was also fast in all the situations/instances.

4.1. Set BR

Starting with the set BR, it only has 3 instances: BR100, BR101, and BR102. The three instances are quite similar, presenting the arrangement of various boxes on a single pallet, being this pallet identical in all instances, and also the arrangement of the boxes is quite similar.

The instances of this set are composed of two files. In the problems associated with these instances, an attempt was made to find the best possible arrangement for a set of boxes on a single pallet, being the number of boxes of this set greater than the number of boxes that the pallet could support, this is, it would not be possible to allocate all the boxes to the pallet, so some of them would be left out. In this sense, these instances were then formed by two files, the first showing the characteristics of the boxes and the pallet itself, while the second one would contain the disposition of the boxes on the pallet after their allocation.

However, the BR101 and BR102 instances are inspired by the BR100, with only minor changes in the arrangement of the boxes on the pallets, so the file that presents the characteristics of the boxes and the pallet is the same for these three instances. The content of this file can then be seen in figure 12.

BR_1_100 - Bloco de notas

Ficheiro Editar Formatar Ver Ajuda

```
3
587	233	220
1	78	1	49	1	47	1	70
2	46	1	45	1	43	1	75
3	98	0	44	1	36	1	69
```

Figure 12 - File that presents the characteristics of the boxes and the pallet in BR instances

Looking at the content of this file, it can be divided into three parts: number of box types, pallet dimensions, and dimensions of box types.

Starting with the number of types of boxes, this is shown on the first line. In this case, the number of box types would then be there.

Moving on to the pallet dimensions, these are displayed on the second line. In this line, the length, width, and height of the pallet are presented, in this same order, which means that the pallet used in these three instances will have a length equal to 587, width equal to 233, and height equal to 220.

Finally, after the second line, the dimensions of the different types of boxes are presented, being assigned one line of the file for each type. It is known that the dimensions of a box usually come down to length, width, and height. However, in these problems, there is the possibility of rotating the boxes, so it does not make sense to speak of length, width, and height, because in a certain situation one of the sides of a box type can assume the position of the length of that box and in another situation, this same side can assume the position of width. Because of that, it is then more appropriate to speak first of the length of the three different sides of the boxes, not specifying which is the length, width, and height. In this sense, each line in this section contains the length of one side of the boxes of that type, the possibility of that side of the box be facing up, the length of the other side of the boxes of that type, the possibility of that other side be facing up, the length of the third side of boxes of that type, the possibility of that third side be facing up and the number of boxes of that type, in this exact same order. Regarding the possibility of the sides be facing up, these possibilities are binary variables, so 1 represents that effectively that side can be facing up and 0 represents the opposite, this is, that side can never be facing up. In this sense, and as an example, in the third line of the file shown in figure 12, the dimensions of the first type of boxes of this instance are presented, and the boxes of this type have a side with length equal to 78, another with length equal to 49 and the third with length equal to 47. In terms of the possibility that these sides are oriented upwards, considering that 1 indicates the possibility of this happening, it can be concluded that all sides of this type of box can be facing up. Finally, it is also indicated that there are 70 boxes of this type. However, as already mentioned, these may or may not be allocated to the pallet, depending on the ideal arrangement.

It is also important to point out, for future reference, that in the total of the three types, 214 boxes were available to be allocated to the pallet.

This ends the presentation of the files that present the characteristics of the boxes and pallets, so it is now time to proceed to the files that contain the arrangement of the boxes on the pallet after their allocation. At this point, it is once again important to mention that the BR101 and BR102 instances are inspired by the BR100 instance, with only minor changes being made to it. As such, the content of the BR100 instance will be presented first and then the modifications applied for the creation of the other two instances will be explained.

SolStab_BR_1_100 - Bloco de notas

Ficheiro Editar Formatar Ver Ajuda

29961138		28438284				
0	0	0	98	36	44	2
0	0	44	98	36	88	2
0	0	88	98	36	132	2
0	0	132	98	36	176	2
0	0	176	98	36	220	2
0	36	0	98	72	44	2
0	36	44	98	72	88	2
0	36	88	98	72	132	2
0	36	132	98	72	176	2
0	36	176	98	72	220	2
0	72	0	98	108	44	2
0	72	44	98	108	88	2
0	72	88	98	108	132	2

Figure 13 - File that presents the arrangement of the box in BR instances

The content of this file can be divided into two sections.

In the first one, which only corresponds to the first line of the file, the values of the total volume are presented, which corresponds to the value of the true volume occupied in the pallet, and of the total volume occupied, which, in turn, corresponds to the value of the sum of the volume of all the boxes disposed on the pallet.

Then, the second section of this file will appear, in which the arrangement of the various boxes on the pallet is shown. In this sense, the number of lines in this section will be equal to the number of boxes arranged on the pallet, with each line showing the location of a box. To do so, on each line will appear the initial x coordinate of that box, the initial y coordinate, the initial z coordinate, the final x coordinate, the final y coordinate, the final z coordinate, and the type of that box, in this same exact order. Thus, the second line present in figure 13 exposes the location of a box that has initial x coordinate equal to 0, initial y coordinate equal to 0, initial z coordinate equal to 0, final x coordinate equal to 98, final y coordinate equal to at 36, final z-coordinate equals 44 and which is of type 2.

In the case of the instance BR100, this section of the file is composed of 203 lines, which means that from the 214 available boxes, only 203 were arranged on the pallet.

Moving on now to the analysis of the changes made to create the BR101 and BR102 instances, in both cases, those changes consisted in the removal of only one of the boxes arranged on the pallet. In this sense, the structure of the files is exactly the same, changing only the number of lines in the second section of this file, which is now composed of 202 lines in both cases.

As mentioned above, BR100 presents the arrangement of 203 boxes on a pallet, each box being stable and, consequently, the load in general is also stable. The instance BR101 is exactly like the BR100 with only one change, which, in this case, was the removal of a box that completely supported another, to verify if the algorithm identified this lack of support and, taking into account that this would lead the box to become unstable, if it would return as a result that the load was not stable. Finally, BR102 is also an adaptation of BR100, presenting the same layout, only with the change of having removed a box that partially supported another. However, this change would make the center of gravity of the other box to be out of its support polygon, which means that it would become unstable. In this sense, it was also expected that the algorithm would return, once more, that the load was not stable.

Looking at appendix H, which shows the output window of the BR100 instance, it can be seen that the algorithm worked correctly, having gone through all 203 boxes and, in the end, indicated that there was no problem with the stability.

As for instance BR101, the output window generated with the application of the algorithm to this instance can be found in appendix I, in which it is shown that the algorithm stopped at box 198 and then indicated that there was a problem regarding the stability of the load. In this appendix can also be verified that the execution was interrupted after counting the number of boxes that supported the box in question, this is, box 198, and it can be seen that the algorithm counted a total of 0 boxes supporting it. Therefore, can be concluded that the algorithm worked correctly, having covered the load until it reached a box, in this case, box 198, for which there was no support, being, as such, obviously unstable. In this sense, the algorithm interrupted its execution and returned that there was a problem with the stability of the load.

Finally, regarding the instance BR102, it can be seen from its output window, present in appendix J, that the algorithm was interrupted in the analysis of box 193, returning that something was wrong with the stability of the load disposed in the pallet. The execution was terminated after verifying that the result of the application of the function "Poligono" to box 198 was "False". It can also be verified, by the output window, that before the application of the function "Poligono" to this box, the number of boxes that were supporting it had been counted, having verified that there was only a single box supporting it. Taking into account this information, it can be concluded that the problem that the algorithm found in this instance was different from the one found in the last instance because this time the problem was not related to the lack of boxes supporting the box in question, but to the application of the function "Poligono". Therefore, can be concluded that the algorithm correctly identified that the projection of the center of gravity of the box in question was located outside its support polygon, whereby the function "Poligono" returned "False" as a result, and the algorithm was interrupted, having, as already mentioned, returned that something was wrong with the stability of the load.

Table 4 - Results from set BR

Instance	Expected Result	Final Result
BR100	Stable Load	Stable Load
BR101	Unstable Load	Unstable Load
BR102	Unstable Load	Unstable Load

4.2. Remaining Sets

Moving now to the instances of the RunByOccupation, RunByVolume, and RunByWeight sets, the problem observed in these instances is very different from that of the instances of set BR. This is because, if, on the one hand, in the instances of the set BR there was a maximization problem, in which there was a limited number of pallets, in this case only one, and an attempt is made to allocate the maximum number of boxes to that pallet, on the other hand, in the instances of these three sets there are minimization problems, because this time the number of pallets is unlimited, so the objective is to use the smallest number of pallets to load all the boxes presented.

Of these three sets, the first, RunByOccupation, is the one that contains the fewest number of files. Each of these sets is equivalent to a folder in which several files are stored. Each file corresponds to an order from a store and all the relevant information regarding that same order is summarized within that file. Inside the set RunByOccupation are presented orders from a total of twelve stores: 1751; 2306; 2659; 2702; 2723; 2725; 2727; 2731; 2742; 3417; 3510; 4125. On the other hand, in terms of the other two sets, in addition to the orders from these twelve stores, there is also one more order, which comes from the store marked with the code 2732.

It is important to note that the order of each store will be exactly the same for all sets, this is, the order of store 1751 will be the same for both the set RunByOccupation, set RunByVolume, and the set RunByWeight. However, although the order is the same for the three sets, which means that the number and dimensions of the boxes to be placed on the pallets, as well as the number of pallets to be used and the dimensions of these pallets will remain the same from one set to the other, yet the arrangement of the boxes on the pallets will be different in each set. This is because before allocating the boxes to the pallets, it is known that this same allocation must be made with a certain purpose and this purpose is different in the three sets. Starting with the set RunByOccupation, the allocation is carried out taking into account the occupancy values of the pallets. As for the set RunByVolume, the allocation is, in its turn, carried out taking into account the values of the sum of the volumes of all the boxes arranged on the pallet. Finally, in terms of the set RunByWeight, the allocation of boxes is guided by the weight carried on each pallet.

Considering the differences between the instances of the set BR and the instances of these three sets, the content of the files that represent them are evidently also quite different. To begin with, if the instances of the set BR were composed of two files, the instances of these sets are composed of only one file, similar to the one shown in figure 14.

```
Output_loja_1751 - Bloco de notas
Ficheiro  Editar  Formatar  Ver  Ajuda
[order_id]
1
[pallet_quantity]
3
[pallet_id      x       y       z       weight          total_load]
0       1200    800     1491    25000   432160
[total_volume    total_occupied_volume   m1      m2]
1431360000      1215332220      1.2459  28.169
[item_quantity]
71
[xmin    ymin    zmin    xmax    ymax    zmax    sequence        item_type       weight          k       irregular]
0       0       0       294     340     196     0       69      11120   0       0
294     0       0       588     340     196     1       69      11120   0       0
588     0       0       882     340     196     2       69      11120   0       0
882     0       0       1176    340     196     3       69      11120   0       0
0       0       196     596     294     596     4       0       3460    0       0
0       340     0       594     616     398     5       6       3180    0       0
0       616     0       384     738     206     6       40      8180    0       0
384     616     0       584     736     130     7       29      2170    0       0
```

Figure 14 - File that present the information of instances of the remaining sets

The file structure is similar for all instances of these three sets, and this structure can be divided into several parts. To begin with, all files start with the order_id, indicating the number of the order exposed in that instance. Then appears the pallet_quantity section, in which the number of pallets that were needed to load all the boxes required in that order is displayed. After these two initial sections, the information regarding each of the pallets used is presented, and for each of the pallets there are also several sections:

1) Pallet dimensions: This section displays the pallet number (pallet_id), the length, width, and height of the pallet, its own weight, and the weight of the load allocated to it. Thus, looking at the data of the instance shown in figure 14, pallet 0 has length equal to 1200ml, width equal to 800ml, height equal to 1491ml, its own weight equal to 25000mg and that the weight of the load it carries ascends to a value equal to 432160mg;
2) Volumes and stability measures: In this section not only are presented the values of the total volume and the total volume occupied of this pallet, but also the values obtained for the stability measures m1 and m2. Thus, looking again at figure 14, it can be concluded that the total used volume of pallet 0 was equal to 1431360000ml^3, corresponding to the multiplication of the length by the height by the width of this same pallet, which were presented in the previous section. The total volume occupied, which in turn corresponds, as already explained, to the sum of the volume of each of the boxes arranged on this pallet, has a value of 1215332220ml^3. Regarding the stability measures, the value obtained for m1 was equal to 1.2459 boxes, while the value obtained for m2 was equal to 28.169%.
3) Number of boxes: This is the section that displays the number of boxes that were placed on that pallet, so it can be concluded that pallet 0, of the instance represented in the file of figure 14, received a total of 71 boxes.
4) Arrangement of boxes: This section shows the arrangement of boxes allocated to that pallet. This section, in terms of structure, is very similar to the second section of the files that contains the arrangement of boxes in BR instances. In this sense, also in this section, each line is equivalent to the presentation of the location of a box, so the number of lines is equal to the number of boxes arranged on the pallet. Thus, in each line of this sector will be found the initial x coordinate of that box, the initial y coordinate, the initial z coordinate, the final x coordinate, the final y coordinate, the final z coordinate, the position in the sequence of loading, the type, the weight, the value of the variable k and the number of irregularities of that box, in this exact same order. As such, looking at figure X , it can be concluded that the first box of pallet 0 has initial x coordinate equal to 0, initial y coordinate equal to 0, initial z coordinate equal to 0, final 14 coordinate equal to 294, final y coordinate equal to 340, final z coordinate equal to 196, position in the loading sequence equal to 0, type equal to 69, weight equal to 11120, the value of variable k equal to 0 and 0 irregularities. Bearing in mind that in the previous section it is mentioned that this pallet had 71 boxes, after this line, there were still 70 more lines in this section.

These four sections will be repeated several times, depending on the number of pallets used in the instance in question. In the case of the instance of the file shown in figure X, it has already been analyzed that three pallets were needed to respond to the entire order of that store, so these four sections will then be repeated three times, although different values are presented in each one of them, as the arrangement of boxes on each pallet is always different.

In this subchapter will now be presented all the important information related to each of the instances studied from these three sets. To organize the information so that its perception and analysis were easier, it was arranged in tables. Thereby, for each set were created two tables with all the essential information regarding each of the instances included on it.

In the first tables will be presented a group of nine variables that adequately summarize the order data, as well as the disposition of the boxes on the pallets that were used to load the boxes of that order. Thereby, belong to this group:

1) Pallet Quantity: Indicates the total number of pallets used to load the boxes required in that order.
2) Pallet X: Represents the length of the pallet in question. The pallets used will all be identical, therefore having similar dimensions, so this variable will always present the same value, in this case, 1200.
3) Pallet Y: Represents the width of the pallet. Like the length, the width of the pallets is also a constant, assuming a value of 800ml.
4) Pallet Z: Represents the height of the pallet. Unlike the width and length of the pallets, which are constant, the height is variable, according to the number and dimensions of the boxes that are loaded on the pallets. That is, the more boxes loaded on a pallet, the more height it will gain. Even so, pallets have a height limit, never exceeding a height of 1500ml on any pallet.
5) Pallet Weight: Indicates the pallet weight, not including the load weight. Thereby, and taking into account that the pallets used will always be identical, as well as their length and width, their weight will also be constant, always assuming a value of 25000mg.
6) Pallet Total Load: Counts the total weight loaded on the pallet, this is, the weight of the load placed on the pallet. Unlike Pallet Weight, which is constant for all pallets, Pallet Total Load is variable, depending on the weight of the boxes that are allocated to the pallet in question. In this sense, the calculation of the value of this variable for a given pallet consists of the sum of the weight of each of the boxes that are allocated to that same pallet.
7) Total Volume: Represents the volume used in the pallet. It is known that the pallets have all the same dimensions, with a length equal to 1200ml, width equal to 800ml, and height equal to a maximum of 1500, so the maximum volume of the pallets will be the same for all pallets, being equal to 1440000000ml^3 (1200*800*1500). However, it is also known that situations in which the entire volume of the pallets is used will be very rare. This is because in most situations the pallets are not loaded to their maximum height, so the volume totally used will depend exclusively on this variable. For this, the length of the pallets, equal to 1200ml, will be multiplied by their width, equal to 800ml, by their height, which will then be variable, depending on how high the pallet is loaded. This is, the value of Total Volume on each pallet will be equal to the multiplication of the value of Pallet X by the value of Pallet Y by the value of Pallet Z. Bearing in mind that the length of the pallets and their width are constant, and consequently also Pallet X and Pallet Y are constant, the variation of this variable will be directly proportional to the variation of Pallet Z, that is, the height of the pallets.
8) Total Occupied Volume: Represents the total occupied volume of the pallet. To understand this variable, it is important to first explain the difference between volume used and volume occupied. It is known that when loading the boxes on the pallets, in most situations, empty spaces were left between the boxes, empty spaces that are too small to allocate other boxes in them. It is then considered that these spaces are being used, as it is not possible to allocate any more boxes to them, but not occupied, since they are empty. In this sense, the total occupied volume of a pallet will never be greater than its used volume. May be equal, in cases there are no empty spaces left between the boxes, or lower, in cases where there are empty spaces. Regarding the value of this variable, it is calculated through the sum of the individual volume of each of the boxes loaded on that pallet.

9) Item Quantity: Represents the total number of boxes loaded on each pallet.

Table 5 - Orders data from the set RunByOccupation

Store	Pallet Id	Pallet X	Pallet Y	Pallet Z	Pallet Weight	Pallet Total Load	Total Volume	Total Occupied Volume	Item Quantity
1751	1	1200	800	1491	25000	432160	1431360000	1215332220	71
	2			1498		246360	1438080000	1117709218	49
	3			598		56840	574080000	369556208	15
2306	1			1500		555879	1440000000	1246078124	88
	2			1492		406406	1432320000	1144868492	76
	3			1489		260239	1429440000	1059053898	55
	4			282		10940	270720000	68896632	3
2659	1			1500		549590	1440000000	1236456385	88
	2			1496		374440	1436160000	1137014171	73
	3			884		101500	848640000	359306572	28
2702	1			1497		612349	1437120000	1247776336	111
	2			1499		350810	1439040000	1196676320	69
	3			590		81550	566400000	1130701830	13
2723	1			1500		256036	1438080000	754465853	75
	2			1498		387106	1438080000	1232444185	77
	3			1500		297080	1440000000	1180264724	51
	4			1047		144000	1005120000	754465853	33
2725	1			1499		547339	1439040000	1232444185	117
	2			1497		308158	1437120000	1180264724	68
	3			598		78570	574080000	444827398	22
2727	1			1496		485830	1436160000	1260250341	66
	2			1488		307260	1428480000	1139707087	62
	3			592		75780	568320000	445561684	17
2731	1			1500		677150	1440000000	1259509077	91
	2			1500		401010	1440000000	1081306348	56
	3			398		29120	382080000	196587308	8
2742	1			1496		410760	1436160000	1255754992	63
	2			1498		222880	1438080000	1121701236	45
	3			444		43300	426240000	277238320	12
3417	1			1500		628810	1440000000	1215266017	108
	2			1496		355440	1436160000	1134371426	70
	3			396		20220	380160000	182840168	6
3510	1			1500		568879	1440000000	1203110396	87
	2			1498		304240	1438080000	1140841296	50
	3			708		35280	679680000	497792408	15
4125	1			1500		655280	1440000000	1269996437	94
	2			1498		310580	1438080000	1211754240	59
	3			1346		168840	1292160000	934426252	30

Table 6 - Orders data from the set RunByVolume

Store	Pallet Id	Pallet X	Pallet Y	Pallet Z	Pallet Weight	Pallet Total Load	Total Volume	Total Occupied Volume	Item Quantity
1751	1	1200	800	1350	25000	314200	924788646	924788646	46
	2			1494		320050	902052360	902052360	45
	3			1494		258680	875756640	875756640	44
2306	1			1495		525970	1278080430	1162280942	79
	2			1496		377359	1288039868	1161787852	70
	3			1499		404140	1194828352	1100526856	69
	4			294		26100	94301496	94301496	4
2659	1			1500		343520	1082133955	1082133955	66
	2			1497		383640	995942985	995942985	59
	3			1489		354110	918250178	918250178	64
2702	1			1349		425789	949089615	949089615	70
	2			1478		343470	899958614	899958614	58
	3			1489		358530	861539987	861539987	65
2723	1			1499		297420	1119816154	1102164058	61
	2			1500		293572	1118627734	1100257174	67
	3			1500		277746	1105220926	1086134750	61
	4			1488		264810	1031064357	1031064357	47
2725	1			1346		357347	988643094	988643094	77
	2			1479		274269	953153594	953153594	57
	3			1497		316420	915739619	915739619	73
2727	1			1496		301580	1058177786	1058177786	45
	2			1498		297000	934090263	934090263	50
	3			1141		280850	853251063	853251063	50
2731	1			1348		386380	939257810	939257810	51
	2			1498		462150	914280959	914280959	66
	3			1001		205190	682863964	682863964	38
2732	1			1494		467833	1356729432	1180496856	49
	2			1494		455540	1339485402	1145853354	64
	3			1494		310359	1213551980	1104332572	56
	4			490		27820	109219408	109219408	5
2741	1			1344		235070	990216068	990216068	45
	2			1498		214500	862925516	862925516	24
	3			1495		233440	801552964	801552964	41
3417	1			1199		367270	895868593	895868593	73
	2			1432		349290	848088248	848088248	48
	3			1496		368800	788520770	788520770	63
3510	1			1476		351250	971481068	971481068	50
	2			1500		409719	947504064	947504064	62
	3			1468		321579	922758968	922758968	40
4125	1			1497		404270	1123359796	1123359796	53
	2			1500		470020	1230752196	1130381744	62
	3			1500		389140	1162435389	1128081069	66
	4			222		14920	34354320	34354320	2

Table 7 - Orders data from the set RunByWeight

Store	Pallet Id	Pallet X	Pallet Y	Pallet Z	Pallet Weight	Pallet Total Load	Total Volume	Total Occupied Volume	Item Quantity
1751	1	1200	800	1180	25000	317390	840963597	840963597	48
	2			1472		298880	822697055	822697055	47
	3			1484		276660	938946994	938946994	40
2306	1			1496		481600	1298885024	1138836116	67
	2			1492		437689	1151507766	1068752166	83
	3			1494		351510	1311308864	1142130176	63
	4			286		52770	169178688	169178688	9
2659	1			1500		418630	1096309555	1096309555	55
	2			1500		343320	943330672	943330672	68
	3			1498		319320	856686891	856686891	66
2702	1			1346		452109	927788956	927788956	71
	2			1498		348710	961549743	961549743	70
	3			1492		326870	821249517	821249517	52
2723	1			1476		291006	1008698004	1008698004	62
	2			1492		284720	982048317	982048317	56
	3			1500		273962	1143870626	1121930386	55
	4			1490		243060	1216943632	1148797072	60
	5			328		30800	68146560	68146560	3
2725	1			1344		336830	901899372	901899372	73
	2			1494		314838	955353407	955353407	72
	3			1500		296368	1000283528	1000283528	62
2727	1			1494		351730	1112122640	1112122640	45
	2			1036		289510	771468458	771468458	49
	3			1349		238190	916827534	916827534	51
2731	1			1348		413230	945721774	945721774	55
	2			1486		395580	924766751	924766751	57
	3			1002		244910	666914208	666914208	43
2732	1			1492		519570	1408224916	1177386044	50
	2			1488		389893	1117487512	1054567496	67
	3			1492		309529	1307948650	1137105514	50
	4			396		42560	170843136	170843136	7
2741	1			1153		402300	800814792	800814792	42
	2			1489		244740	917670676	917670676	37
	3			1490		338320	936209080	936209080	41
3417	1			1192		389610	887290901	887290901	86
	2			1495		352879	828028124	828028124	48
	3			1472		338220	817158586	817158586	50
3510	1			1494		389610	1012220436	1012220436	62
	2			1499		352879	913272408	913272408	43
	3			1495		340059	916251256	916251256	47
4125	1			1492		428060	1243377284	1132735804	57
	2			1498		476940	1098612168	1098612168	67
	3			1493		359970	1184828957	1140539741	57
	4			252		13380	44289216	44289216	2

Heading now to the second tables, these will contain the information related to the results of the application of the algorithm, showing now only the result of the evaluation of the stability for every pallet of each order, but also the time that took for the algorithm to present the results.

In this sense, these tables will be formed for a group of three variables, to which, besides Pallet Id, belongs two more variables:

1) Stability: Indicates whether the load placed on that pallet is stable or not, taking into account the result of the application of the developed algorithm. The ✓ symbol indicates that the algorithm did not find any problem with the stability of the load, and therefore it can be considered that it is stable. On the other hand, the symbol **X** indicates that the developed algorithm found a problem in the arrangement of the boxes that affected the stability of the load, so it should not be considered that it is stable.
2) Time: Displays the real time it took to execute the algorithm. Unlike the previous variables that referred to each of the pallets individually, this variable, like the Pallet Quantity, refers to the instance as a whole, exposing the execution time for the entire order and not for each of the pallets individually.

Table 8 - Results from RunByOccupation

Store	1751			2306				2659			2702				2723			2725		
Pallet Id	1	2	3	1	2	3	4	1	2	3	1	2	3	4	1	2	3	1	2	3
Stability	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time	0.358739s			0.39902s				0.228967s			0.441585s				0.308203s			0.475818s		

Store	2727			2731			2732			3417			3510			4125		
Pallet Id	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Stability	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time	0.156233s			0.129125s			0.300683s			0.356297s			0.259702s			0.1993s		

Table 9 - Results from RunByVolume

Store	1751			2306				2659			2702				2723			2725			2727		
Pallet Id	1	2	3	1	2	3	4	1	2	3	1	2	3	4	1	2	3	1	2	3	1	2	3
Stability	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time	0.151166s			0.380573s				0.400658s			0.386421s				0.259288s			0.411754s			0.199814s		

Store	2731			2732				2742			3417			3510			4125			
Pallet Id	1	2	3	1	2	3	4	1	2	3	1	2	3	1	2	3	1	2	3	4
Stability	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time	0.302489s			0.193364s				0.128222s			0.230485s			0.320289s			0.416056s			

Table 10 - Results from RunByWeight

Store	1751			2306				2659			2702			2723					2725			2727		
Pallet Id	1	2	3	1	2	3	4	1	2	3	1	2	3	1	2	3	4	5	1	2	3	1	2	3
Stability	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time	0.303254s			0.494812s				0.176128s			0.237785s			0.5314847s					0.194098s			0.177444s		

Store	2731			2732				2742			3417			3510			4125			
Pallet Id	1	2	3	1	2	3	4	1	2	3	1	2	3	1	2	3	1	2	3	4
Stability	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Time	0.2745s			0.416316s				0.253925s			0.382506s			0.366917s			0.20061s			

To extend the study of the stability of the load in the pallets, it was decided to analyze more metrics referring to this factor. To this end, a set of eight metrics was created, all of them referring to the support area of the boxes. Support area of the boxes is the area of the boxes that is supported, this is, that has another box underneath. Although in some situations the boxes are 100% supported, with the entire base having other boxes underneath, there are also situations where this is not the case and only part of the base is on top of other boxes.

It then becomes evident that a relation between the support area of the boxes and their stability could be possibly made. Thus, if a box has a support area much smaller than the total area of its base, that is, if the percentage of the base that is supported is very small, it is unlikely that that box will be stable. The same is true in the opposite direction, that is, if the percentage of the base that is supported is big, then it is almost guaranteed that this box is stable. However, this is not a decisive factor, as often a box with 60% of the supported base can be unstable and, on the other hand, another with only 40% can be stable. Even so, taking into account that it was already known that all the boxes of these instances were stable, it was then interesting to analyze these variables, to reinforce the analysis of their stability.

In this sense, the following metrics were created:

1) Average: Represents the average of area supported of all the boxes on that pallet;
2) Min: Represents the minimum value of area supported among all the boxes arranged on this pallet;
3) Max: Represents the maximum value of area supported among all boxes allocated to this pallet;
4) Median: Represents the median of the area supported values of all boxes;
5) Average %: Represents the average of the percentage of supported area of all the boxes on that pallet. To calculate this variable, first was calculated the percentage of supported area of each box. Having calculated this percentage for all the boxes arranged on that pallet, its average was then calculated;
6) Min %: Represents the minimum value of the percentage of supported area of the boxes;
7) Max %: Represents the maximum value of the percentage of supported area of the boxes;
8) Median %: Represents the median of the values of the percentage of supported area of each one of the boxes of that pallet.

In addition to all these metrics, the metrics “M1” and “M2”, that are shown in the files that presents the initial data from the instances, were also analyzed.

“M1” is a stability measure that quantify the average number of boxes that support each of the boxes present in a pallet that are not in contact with the base of that pallet. To calculate this metrics only the boxes allocated to the pallet that are not disposed in the base of the pallet will be considered, that is, boxes with z0 different from 0. Then, the number of boxes that supports them will be calculated for each of those boxes. And later, the average of all those values is determined and that will be the value of “M1” for that pallet.

“M2” is another stability measure, but this one measures the percentage of boxes on a pallet that do not have at least 3 of its sides in contact with other boxes or with the limits of the pallet. As it known, every box was 4 sides, or 4 lateral faces, which in a pallet can be in contact with other boxes or with the limits of the pallet or not. So, to calculate this measure first is seen how many boxes in that pallet have at least 3 of its sides in contact with other boxes or with the limits of that pallet. Then this value is transformed into percentage, dividing it by the total number of boxes in the pallet. And then, it is calculated the opposite value, to obtain the percentage of boxes that do not have at least 3 of its sides in contact with another surface.

The values of all these variables were calculated and grouped in the following table:

Table 11- Stability data from the set RunByWeight

Store	Pallet ID	Average	Min	Max	Average	Average %	Min %	Max %	Median %	M1	M2
1751	1	65039.81	24000	165688	56700	87.32	37.99	100.00	100.00	1.25	31.25
	2	65600.88	18796	170456	58896	85.35	34.45	100.00	100.00	1.42424	36.1702
	3	75996.73	20000	186576	70840	83.45	43.44	100.00	100.00	1.16129	30
2306	1	59573.93	10692	187200	55776	88.54	34.29	100.00	99.51	1.28846	38,806
	2	52910.09	11288	154400	46400	86.87	31.78	100.00	100.00	1.29231	32.5301
	3	64649.13	9656	163944	55500	85.62	43.89	100.00	100.00	1.13462	39.6825
	4	84754.66	46864	161160	77218	100.00	100.00	100.00	100.00	nan	22.2222
2659	1	69934.26	16068	211600	55449	83.46	35.17	100.00	85.50	1.08511	40
	2	51364.47	10200	165800	46400	85.85	33.81	100.00	100.00	1.46	25
	3	55781.61	9156	168636	54064	86.32	30.89	100.00	100.00	1.27451	27.2727
2702	1	54345.86	11288	187200	46400	88.10	36.06	100.00	100.00	1.56863	29.5775
	2	53292.76	8588	186576	43456	88.68	31.48	100.00	100.00	1.33333	34.2857
	3	58891.83	8588	164496	46800	82.57	36.03	100.00	89.09	1.12195	40.3846
2723	1	54306	23000	152064	49764	84.15	38.62	100.00	100.00	1.28571	27.4194
	2	52000.82	18480	150500	43296	79.12	31.54	100.00	100.00	1,175	37.5
	3	59749.51	18944	163944	49840	81.74	31.03	100.00	81.70	1.17073	34.5455
	4	57399.96	9600	179256	45936	81.45	33.01	100.00	93.62	1.32	33.3333
	5	87392	54560	152064	55056	94.59	83.78	100.00	91.89	1	66.6667
2725	1	48826.33	8064	163944	50893	87.68	48.90	100.00	100.00	1.23636	28.7671
	2	48798.32	8880	150400	50240	85.51	29.89	100.00	100.00	1.30508	37.5
	3	54906.13	9180	204180	43296	84.66	33.74	100.00	95.56	1.36	35.4839
2727	1	68839.63	8700	186576	57330	83.49	33.93	100.00	86.98	1.18421	31.1111
	2	50864.47	18600	168636	39800	81.65	29.68	100.00	88.13	1.26316	30.6122
	3	57787.29	16214	275224	46400	86.78	27.63	100.00	97.13	1.11905	33.3333
2731	1	66817.37	6156	167440	70112.50	91.36	45.97	100.00	100.00	1.32558	34.5455
	2	58223.16	11288	142400	60434	87.82	40.51	100.00	100.00	1.14286	33.3333
	3	54470.79	8588	126280	46400	86.62	39.52	100.00	100.00	1.14286	30.2326
2732	1	92331.88	20370	211600	54000	84.96	29.46	100.00	96.23	1.13636	36
	2	57893.48	17000	161304	46400	82.67	37.86	100.00	88.71	1.12281	32.3433
	3	68326.46	11288	165760	55440	83.57	45.64	100.00	87.18	1.09524	32
	4	103720	72168	156800	90306	95.20	66.43	100.00	100.00	1	42.8571
2742	1	62800.76	6500	150500	55720	87.38	49.68	100.00	100.00	1.46875	40.4762
	2	73051.13	8880	158976	59840	83.04	44.00	100.00	98.23	1.11538	29.7297
	3	67671.32	16000	164496	56250	86.07	34.18	100.00	91.72	1,125	53.6585
3417	1	46007.48	12760	187200	41536	92.47	18.39	100.00	100.00	1.72881	22,093
	2	60602.08	160	163944	55250	89.16	33.80	100.00	100.00	1.05556	45.8333
	3	58315.76	16200	163944	50424	87.97	27.14	100.00	100.00	1.27778	36
3510	1	62013.80	10200	157528	58752	84.83	35.27	100.00	100.00	1.31373	22.5806
	2	79535.58	13680	211600	65025	83.89	37.05	100.00	100.00	1.02941	34.8837
	3	71974.85	11664	163944	57907	84.53	29.45	100.00	98.50	1.30303	31.9149
4125	1	64552	19008	177320	54910	86.76	33.28	100.00	94.87	1.28889	29.8246
	2	64461.22	17110	168636	64690	83.34	27.94	100.00	91.60	1.18182	35.8209
	3	70606.17	20060	163944	55552	86.11	38.07	100.00	100.00	1.42222	31.5789
	4	105536	46240	164832	46240	100.00	100.00	100.00	100.00	nan	50

As can be seen from the table caption, these data belong to the set RunByWeight, and only the data from this set will be analyzed in this chapter. However, equal tables were also made for the set RunByOccupation and RunByVolume, which are present in the appendix K and appendix L, respectively.

Looking at the data presented in Table 11, there are some data that are definitely worth noting.

Starting with the Average variable, it is worth highlighting the fact that for most of the pallets this variable presents a value between 45000ml^2 and 70000ml^2, which fully matches the values found for the minimum and the maximum, with the minimum value found over all the pallets being 160ml^2 and the maximum being 275224ml^2. This 160ml^2 immediately draws attention as it is very low, thus raising the question of whether this box would actually be stable, as indicated by the algorithm. In this sense, it was important to understand better what was happening with this box. To that, the data provided in the file that present all the instance data was analyzed. It was then found that this box had 1ml^2 of length and 160ml^2 of width, so the area of its base was equal to 160ml^2. Therefore, although 160ml^2 is a value too small, this box is supported at 100%.

Moving on now to the analysis of the percentages, in all the pallets the average is always quite high, never being less than 79%. This data validates the assessment made earlier by the algorithm, as these percentages of supported base are a very strong indication that the boxes are stable. Some pallets even showed an average of 100%, which means that all the boxes had 100% supported bases, as can be seen by the minimum and maximum values on these pallets. This is justified by the fact that these are pallets with very few boxes allocated, being all of them at the base of the pallet and therefore 100% supported. Looking at the minimum values, it appears that on the first pallet used by store 3417 there was a box that had only 18.39% of its base supported. Although this value seems very low, this does not invalidate the validity of the stability assessment, as this 18% can be distributed over crucial areas, such as the corners of the box, thus ensuring its stability. As for the median, like the average, the values obtained for this variable were also always very high.

4.3. Analysis of results

After presenting the relevant data for each of the sets studied, the results obtained with the application of the algorithm created to assess the stability of the load on the pallets will be analyzed. This analysis will focus not only on the effectiveness of the algorithm but also its efficiency, evaluating not only the result returned by the algorithm but also its execution time.

Starting with the set BR, in this one was confirmed the effectiveness of the algorithm, having verified that the algorithm correctly evaluated the presented instances, not only when the load was stable, but also when it was unstable for different reasons. In this sense, this set of instances validated the effectiveness of the algorithm in the most diverse situations, having proven that it would always return the correct result.

Proceeding to the set RunByOccupation, it can be seen from the data presented in the tables of the previous chapter that stability was always verified in all the pallets loaded in the different instances. Since the data provided belong effectively to pallets whose load was stable, this proves that the algorithm worked correctly, having always carried out the right evaluation. Regarding the execution times, it is verified that the algorithm is quite fast, not exceeding half a second in any of the twelve instances that are part of this set, being the longest execution time obtained for the sixth instance, corresponding to store 2725, in the which execution took 0.475818s.

Moving on to the second set, RunByVolume, also in this one it is verified that the stability is proven in all the pallets, this is, that the storage of the boxes in the pallets was done respecting the stability, which means that in the end all the load is stable on all pallets. Again, the created algorithm always returns the expected results, evaluating the load as being stable on all pallets, as expected, proving once again its effectiveness in this set. In terms of execution time, also in this set there is no instance in which it exceeds half a second, the maximum time being less than the one checked in the RunByOccupation, being also achieved also in the sixth instance, corresponding to store 2725, but ascending this time to just 0.411754s. It should also be noted that this set contains the instance that presented the lowest execution time of the aggregate of the 3 last sets, this being the instance referring to the order from store 2742, to which the execution took only 0.128222s.

Finally, concerning RunByWeight, the load arranged on the various pallets of each instance was always stable and the algorithm evaluated each of the pallets accordingly, thus proving, once again, its effectiveness. In terms of algorithm efficiency, in this set was found the only instance for which the execution time was greater than half a second, being the instance referring to store 2723, which presented an execution time equal to 0.531847s. For all other instances, the execution time was less than half a second. Even so, despite having found an instance whose execution time was longer than half a second, it cannot be considered that the algorithm is not effective, since 0.531847s is still a very fast time.

This proves the validity of the algorithm created to evaluate the stability of the load placed on a pallet, being verified that the characteristics of the storage of the boxes on the pallets do not interfere with the effectiveness of the algorithm, which is capable of correctly evaluating the stability of the load regardless of the differences in the arrangement, confirming that the algorithm correctly evaluated all analyzed instances. Regarding the efficiency of the algorithm, can be said that the characteristics of the arrangement of the boxes influence it since different execution times were obtained for the various instances. Even so, this influence was too small, with very short execution times in all instances, so it can be said that the algorithm is globally effective.

The results obtained allowed not only to prove the stability of the load of each of the pallets of the various instances worked on but also, as previously mentioned, the efficiency and effectiveness of the algorithm developed to evaluate this factor.

5. CONCLUSION

5.1. Final Conclusions

Finished this phase of the project and taking into account that its main objective was to create an algorithm that would allow evaluation of the stability of the boxes arranged on a pallet, it can be said that the objectives for this phase of the project were fully achieved, having effectively presented an algorithm that efficiently evaluates this issue.

To achieve this objective, the concept of support polygon was used in an algorithm programmed in C++ language, being for this purpose the “Jarvis March” algorithm also applied, to create the convex hull that would shape the support polygon.

After creating this algorithm, it was applied in a series of instances. For each instance, three possible configurations of the organization of the pallets were presented, being them organized according to:

- Occupancy Maximization;
- Weight Maximization;
- Volume Maximization;

In this sense, the algorithm was applied to each of the three configurations of the different instances and the results returned were always the intended, which means that it was verified that the algorithm was effectively capable of, being provided an arrangement of a pallet to it, this is, the position of the all the boxes present on a pallet, correctly assess whether each of those boxes and, consequently, the entire load arranged on the pallet was stable.

However, this algorithm proved to be not only effective, having correctly evaluated the stability of the pallets, but also efficient, since the time spent for its execution was always quite reduced, regardless of the instance and configuration of the pallets.

5.2. Limitations and future works

Turning now the attention to the evaluation of the limitations and future work related to this project, undoubtedly one of the attractive aspects of this project is the fact that it presents much more future work than limitations.

Starting with the limitations, it is worth noting the fact that this algorithm needs to receive the positioning of the boxes on the pallets to be able to assess the stability of the pallet, being useless without such information. Another important limitation is the fact that this project only used fictional information regarding the arrangement of the pallets, this is, information previously created, instead of having collected real information on the arrangement of the pallets from any organization, which would have been a lot more interesting.

As for future work, as already mentioned, this project was just the initial phase of a much more complex project to be carried out at the Instituto de Engenharia de Sistemas e Computadores, Tecnologia e Ciência, thus existing a long list of future works that still need to be carried out, all of them with the intention of fully automate and autonomize palletizing operations in a warehouse. Among these future works, can be highlighted the need to create an algorithm that can identify the best order to load the boxes to the pallets where they should be disposed, as well as the need to create another algorithm that ensures a good distribution of the weight of the load by the volume of the pallet so that the products do not damage each other.

BIBLIOGRAPHIC REFERENCES

Abdou, G., & Lee, E. (1992). Contribution to the development of robotic palletization of multiple box sizes. *Journal of Manufacturing Systems*, *11*(3), 160–166.

Arcanjo, M. C. P. (2017). *Reorganização do processo de paletização numa indústria do ramo alimentar*.

Barbosa, J. P. W. (2018). *Restrições de Garras Manipuladoras Dentro Do Problema De Empacotamento De Paletes Do Produtor ? Uma Abordagem Heurística*.

Bayer, F. M., Eckhardt, M., & Machado, R. (2011). *Automação de Sistemas*.

Beamon, B. M. (1998). Supply Chain Design and Analysis: Models and Methods. *International Journal of Production Economics*, *55*, 281–294.

Bhattacharya, A., & McGlothlin, J. D. (2012). *Occupational Ergonomics: Theory and Applications*. CRC Press.

Bischoff, E. E., & Ratcliff, M. S. W. (1995a). Loading Multiple Pallets. *The Journal of the Operational Research Society*, *46*(11), 1322–1336.

Bischoff, E. E., & Ratcliff, M. S. W. (1995b). Issues in the development of approaches to container loading. *Omega*, *23*(4), 377–390. https://doi.org/10.1016/0305-0483(95)00015-G

Bortfeldt, A., & Gehring, H. (2001). A hybrid genetic algorithm for the container loading problem. *European Journal of Operational Research*, *131*(1), 143–161. https://doi.org/10.1016/S0377-2217(00)00055-2

Bortfeldt, A., & Wäscher, G. (2013). Constraints in container loading – A state-of-the-art review. *European Journal of Operational Research*, *229*(1), 1–20. https://doi.org/10.1016/J.EJOR.2012.12.006

Christensen, S. G., & Rousøe, D. M. (2009). Container loading with multi-drop constraints. *International Transactions in Operational Research*, *114*(3), 509–527.

CSCMP. (2010). CSCMP Supply Chain Management Definitions and Glossary. https://cscmp.org/CSCMP/Educate/SCM_Definitions_and_Glossary_of_Terms/CSCMP/Educate/SCM_Definitions_and_Glossary_of_Terms.aspx

De Castro Silva, J. L., Soma, N. Y., & Maculan, N. (2003). A greedy search for the three-dimensional bin packing problem: The packing static stability case. *International Transactions in Operational Research*, *10*(2), 141–153. https://doi.org/10.1111/1475-3995.00400

Djuric, A. M., Rickli, J. L., & Urbanic, R. J. (2016). A Framework for Collaborative Robot (CoBot) Integration in Advanced Manufacturing Systems. *SAE International Journal of Materials and Manufacturing*, *9*(2), 457–464. https://doi.org/10.4271/2016-01-0337

Dos Santos, A. M. (1995). *Paletização Retangular-Caraterização e Métodos de Resolução*.

Dyckhoff, H. (1990). A typology of cutting and packing problems. *European Journal of Operational Research*, *44*(2), 145–159. https://doi.org/10.1016/0377-2217(90)90350-K

Dzitac, P., & Mazid, A. M. (2008). An efficient control configuration development for a high-speed robotic palletizing system. *2008 IEEE International Conference on Robotics, Automation and Mechatronics, RAM 2008*, 140–145. https://doi.org/10.1109/RAMECH.2008.4681379

Edelsbrunner, H., & Maurer, H. A. (1981). On the intersection of Orthogonal objects. *Information Processing Letters*, *13*(4–5), 177–181. https://doi.org/10.1016/0020-0190(81)90053-3

Faccio, A. P. (2008). *Propostas de Solução para o Problema de Corte de Estoque Bidimensional de uma Fábrica de Móveis de Pequeno Porte*. *017*, 137.

Filipe, N., Gomes, N., Manuel, A., & Ferrolho, P. (2013). *Sistema Robótico de Paletização para Caixas com Grandes Volumes de Produção. Caso de Estudo: Cerutil*.

Foltz, L. (2004). Palletizing Of Cases , Trays Or Bundles ; Equipment Types and Operation. *Hixson Manufacturing Engineering Department*, 1–16.

Franklin, C., Dominguez, E., Fryman, J., & Lewandowski, M. (2020). Collaborative robotics: New era of human–robot cooperation in the workplace. *Journal of Safety Research*, *74*, 153–160.

Fuellerer, G., Doerner, K. F., Hartl, R. F., & Iori, M. (2010). Metaheuristics for vehicle routing problems with threedimensional loading constraints. *European Journal of Operational Research*, *201*(3), 751–759.

Gauss, L., & Sellito, M. A. (2017). Product Family Architecture: A Case Study on the Design of a Floor Level Palletizer. *THE INTERNATIONAL JOURNAL OF PRODUCTION ECONOMICS*, *2001*.

Gilmore, P. C., & Gomory, R. E. (1963). A Linear Programming Approach to the Cutting Stock Problem—Part II. *Operations Research*, *11*(6), 863–888. https://doi.org/10.1287/opre.11.6.863

Groover, M. P. (2019). *Automation, Production Systems, and Computer-Integrated Manufacturing* (5ª). Pearson.

Gupta, A. K., Arora, S. K., & Westcott, J. R. (2016). *Industrial Automation and Robotics* (Har/Cdr). Mercury Learning and Information.

Haines, E. (1994). Point in polygon strategies. In P. Heckbert (Ed.), *Graphics Gems IV* (pp. 24–46). Academic Press.

Hibbeler, R. C. (2010). *Engineering Mechanics: Statics*. Prentice Hall.

ISO - International Organization for Standardization. (2021). *Robotics Vocabulary*. *2021*, 14.

Junqueira, L., Morabito, R., & Sato Yamashita, D. (2011). MIP-based approaches for the container loading problem with multi-drop constraints. *Annals of Operations Research*, *199*(1), 51–75.

Kiran, D. R. (2019). *Production Planning and Control: A Comprehensive Approach*.

Liu, W. Y., Lin, C. C., & Yu, C. S. (2011). On the three-dimensional container packing problem under home delivery service. *Asia-Pacific Journal of Operational Research*, *28*(5), 601–621. https://doi.org/10.1142/S0217595911003466

Mack, D., Bortfeldt, A., & Gehring, H. (2004). A parallel hybrid local search algorithm for the container loading problem. *International Transactions in Operational Research*, *11*(5), 511–533.

Maya, M., Castillo, E., Lomelí, A., González-Galván, E., & Cárdenas, A. (2013). Workspace and payload-capacity of a new reconfigurable Delta parallel robot regular paper. *International Journal of Advanced Robotic Systems*, *10*. https://doi.org/10.5772/54670

McGuire. (2015). *Handbook of Humanitarian Health Care Logistics* (3ª).

Min, H., & Zhou, G. (2002). Supply chain modeling: past, present and future. *Computers & Industrial Engineering*, *43*(1–2), 231–249. https://doi.org/10.1016/S0360-8352(02)00066-9

Moura, A., & Oliveira, J. F. (2007). *An integrated approach to the vehicle routing and container loading problems, Greece. In EURO XX-20th European conference on operational research.* 1–21.

Ngoi, B. K. A., Tay, M. L., & Chua, E. S. (1994). Applying spatial representation techniques to the container packing problem. *International Journal of Production Research*, *32*(1), 111–123.

Noberto Pires, J. (2002). Das máquinas gregas à moderna robótica industrial. *Público*.

Nof, S. Y. (2009). Automation: What It Means to Us Around the World. In *Springer Handbook of Automation* (pp. 13–52). Springer.

Ollero, A., Boverie, S., Goodall, R., Sasiadek, J., Erbe, H., & Zuehlke, D. (2006). Mechatronics, robotics and components for automation and control: IFAC milestone report. *Annual Reviews in Control*, *30*(1), 41–54.

Poldi, K. C. (2004). *"Algumas extensões do problema de corte de estoque."* http://www.teses.usp.br/teses/disponiveis/55/55134/tde-22102004-125652/

Popple, R. A. (2009). The Science of Palletizing. In *Angewandte Chemie International Edition, 6(11), 951–952.*

Ramos, A. G., Oliveira, J. F., & Lopes, M. P. (2016). A physical packing sequence algorithm for the container loading problem with static mechanical equilibrium conditions. *International Transactions in Operational Research*, *23*(1–2), 215–238. https://doi.org/10.1111/itor.12124

Reddy Vemula, B. (2015). *Evaluation of Robot Structures For Applications That Require High Performance, Safety And Low Energy Consumption* (Issue 223). http://mdh.diva-portal.org/smash/get/diva2:859697/FULLTEXT02.pdf

SHASTRI, R. (2010). 10 Good Reasons to Invest in Robots. *Chemical Engineering World*, *45*(9), 62–63. http://cat.inist.fr/?aModele=afficheN&cpsidt=23329537

Shimrat, M. (1962). Algorithm 112: position of point relative to polygon. *Communications of the ACM*, *5*(8), 434.

Silva, E., Oliveira, J. F., & Wäscher, G. (2016). The pallet loading problem: A review of solution methods and computational experiments. *International Transactions in Operational Research*, *23*(1–2), 147–172.

Silveira, S., & Lima, W. Q. (2003). *Um breve histórico conceitual da Automação Industrial e Redes para Automação Industrial*.

Vaidya, S., Ambad, P., & Bhosle, S. (2018). Industry 4.0 – A Glimpse. *Procedia Manufacturing*, *20*, 233–238. https://doi.org/10.1016/J.PROMFG.2018.02.034

Wäscher, G., Haußner, H., & Schumann, H. (2007). An improved typology of cutting and packing problems. *European Journal of Operational Research*, *183*(3), 1109–1130. https://doi.org/10.1016/J.EJOR.2005.12.047

Yam, K. L. (2009). *The Wiley Encyclopedia of Packaging Technology* (3ª). Wiley.

APPENDIX A - OUTPUT WINDOW FOR THE INSTANCE 2725 IN RUNBYOCCUPATION

```
Console de Depuração do Microsoft Visual Studio
Conjunto:RunByOccupation
Nome:2725
Order_Id:1
Pallet_Quantity:3
Pallet_Id:0
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1499
-->Pallet_Weight:25000
-->Pallet_TotalLoad:547339
-->Total_Volume:1439040000
-->Total_Occupied_Volume:1232444185
-->Item_Quantity:117
-->Stability: Check!
Pallet_Id:1
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1497
-->Pallet_Weight:25000
-->Pallet_TotalLoad:308158
-->Total_Volume:1437120000
-->Total_Occupied_Volume:1180264724
-->Item_Quantity:68
-->Stability: Check!
Pallet_Id:2
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:598
-->Pallet_Weight:25000
-->Pallet_TotalLoad:78570
-->Total_Volume:574080000
-->Total_Occupied_Volume:444827398
-->Item_Quantity:22
-->Stability: Check!

Acabou
```

APPENDIX B - OUTPUT WINDOW FOR THE INSTANCE 2725 IN RUNBYVOLUME

```
Console de Depuração do Microsoft Visual Studio
Conjunto:RunByVolume
Nome:2725
Order_Id:1
Pallet_Quantity:3
Pallet_Id:0
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1346
-->Pallet_Weight:25000
-->Pallet_TotalLoad:357347
-->Total_Volume:988643094
-->Total_Occupied_Volume:988643094
-->Item_Quantity:77
-->Stability: Check!
Pallet_Id:1
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1479
-->Pallet_Weight:25000
-->Pallet_TotalLoad:274269
-->Total_Volume:953153594
-->Total_Occupied_Volume:953153594
-->Item_Quantity:57
 -->Stability: Check!
Pallet_Id:2
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1497
-->Pallet_Weight:25000
-->Pallet_TotalLoad:316420
-->Total_Volume:915739619
-->Total_Occupied_Volume:915739619
-->Item_Quantity:73
-->Stability: Check!

Acabou
```

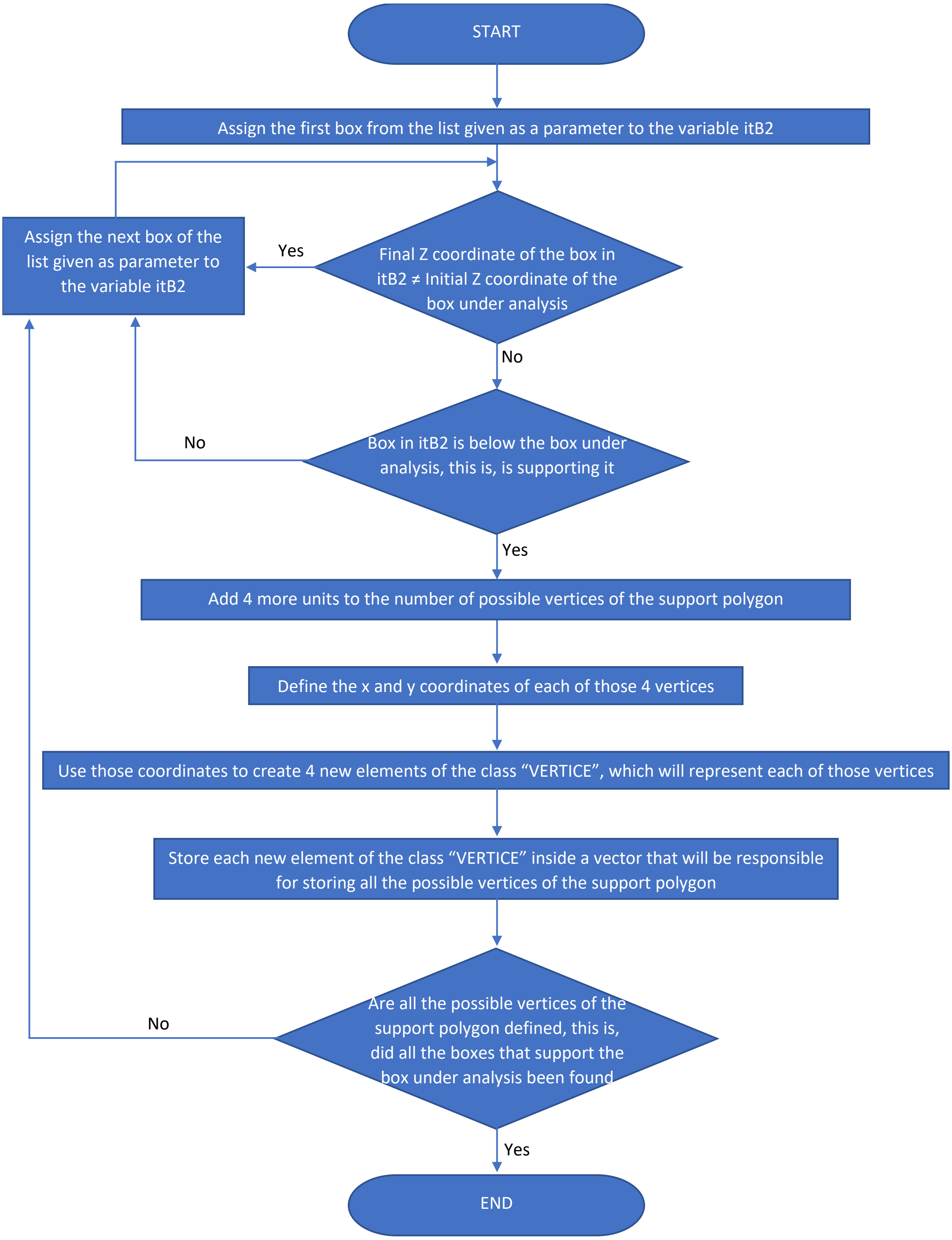
APPENDIX C - OUTPUT WINDOW FOR THE INSTANCE 2742 IN RUNBYVOLUME

```
Console de Depuração do Microsoft Visual Studio
Conjunto:RunByVolume
Nome:2742
Order_Id:1
Pallet_Quantity:3
Pallet_Id:0
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1344
-->Pallet_Weight:25000
-->Pallet_TotalLoad:235070
-->Total_Volume:990216068
-->Total_Occupied_Volume:990216068
-->Item_Quantity:45
-->Stability: Check!
Pallet_Id:1
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1494
-->Pallet_Weight:25000
-->Pallet_TotalLoad:214500
-->Total_Volume:862925516
-->Total_Occupied_Volume:862925516
-->Item_Quantity:34
-->Stability: Check!
Pallet_Id:2
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1495
-->Pallet_Weight:25000
-->Pallet_TotalLoad:233440
-->Total_Volume:801552964
-->Total_Occupied_Volume:801552964
-->Item_Quantity:41
-->Stability: Check!

Acabou
```

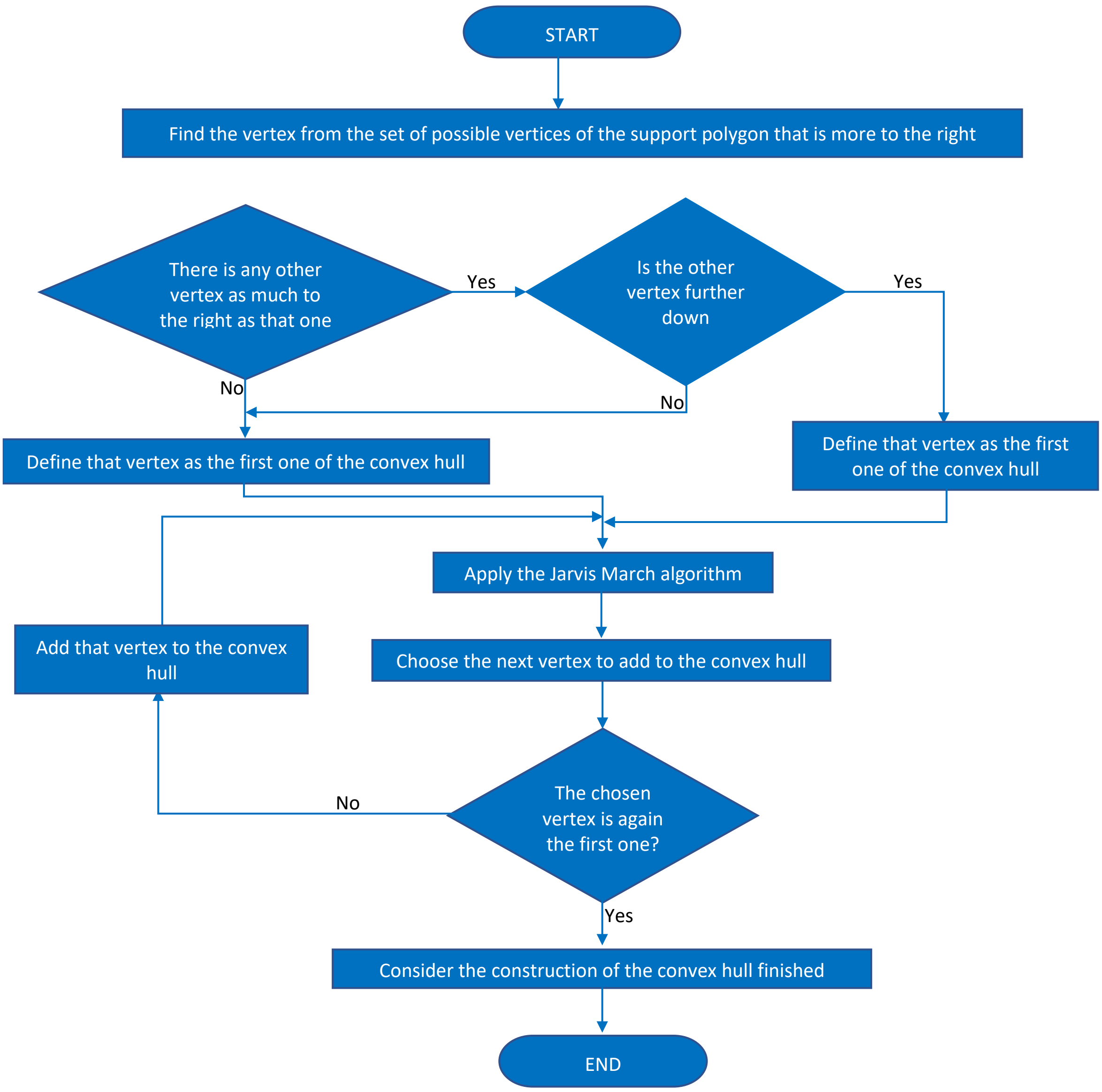
APPENDIX D - OUTPUT WINDOW FOR THE INSTANCE 2723 IN RUNBYWEIGHT

```
Console de Depuração do Microsoft Visual Studio
Conjunto:RunByWeight
Nome:2723
Order_Id:1
Pallet_Quantity:5
Pallet_Id:0
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1476
-->Pallet_Weight:25000
-->Pallet_TotalLoad:291006
-->Total_Volume:1008698004
-->Total_Occupied_Volume:1008698004
-->Item_Quantity:62
-->Stability: Check!
Pallet_Id:1
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1492
-->Pallet_Weight:25000
-->Pallet_TotalLoad:284720
-->Total_Volume:982048317
-->Total_Occupied_Volume:982048317
-->Item_Quantity:56
-->Stability: Check!
Pallet_Id:2
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1500
-->Pallet_Weight:25000
-->Pallet_TotalLoad:273962
-->Total_Volume:1143870626
-->Total_Occupied_Volume:1121930386
-->Item_Quantity:55
-->Stability: Check!
Pallet_Id:3
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:1490
-->Pallet_Weight:25000
-->Pallet_TotalLoad:243060
-->Total_Volume:1216943632
-->Total_Occupied_Volume:1148797072
-->Item_Quantity:60
-->Stability: Check!
Pallet_Id:4
-->Pallet_X:1200
-->Pallet_Y:800
-->Pallet_Z:328
-->Pallet_Weight:25000
-->Pallet_TotalLoad:30800
-->Total_Volume:68146560
-->Total_Occupied_Volume:68146560
-->Item_Quantity:3
-->Stability: Check!

Acabou
```

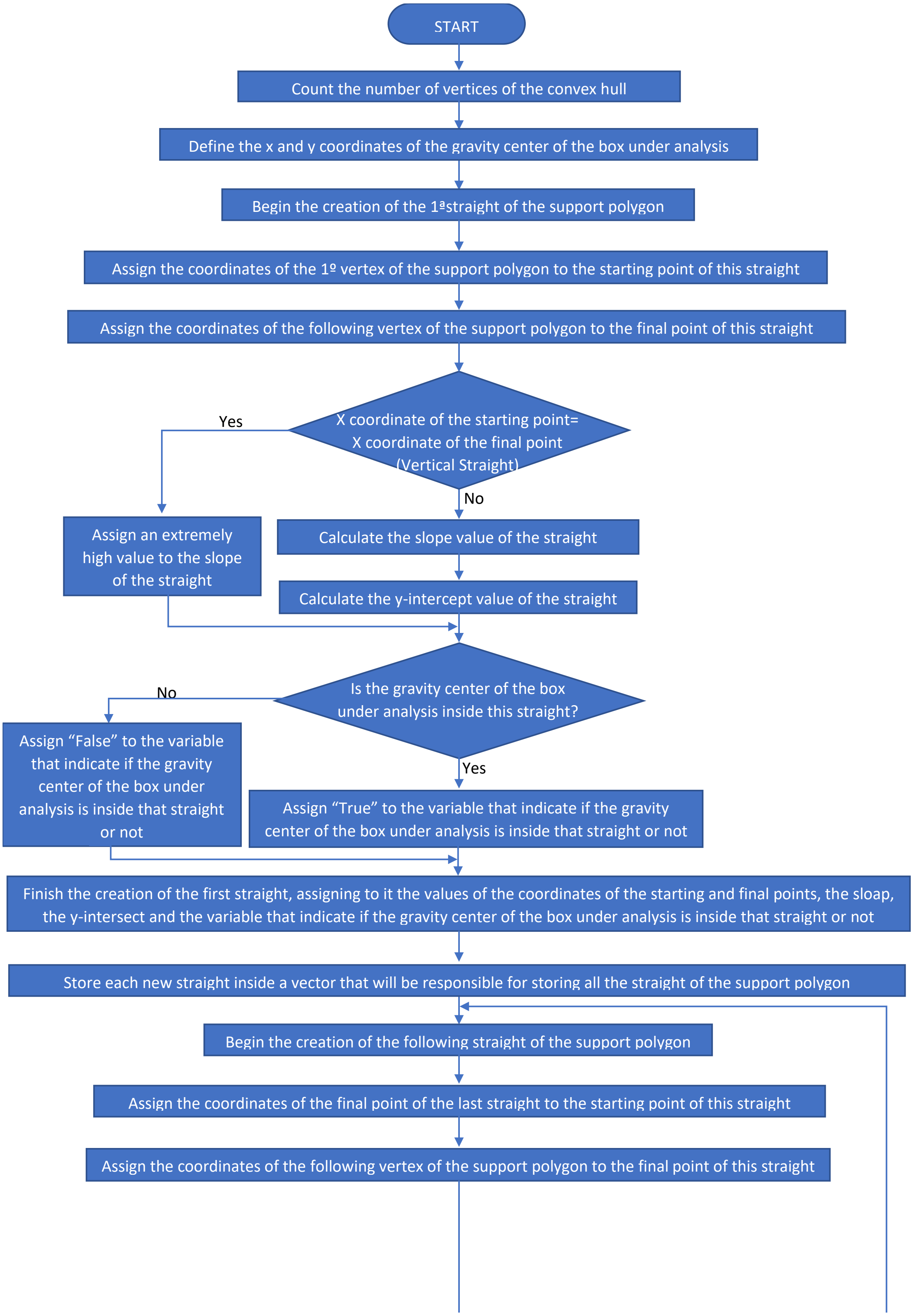
APPENDIX E – FLOWCHART OF THE QUANTIFICATION OF POSSIBLE VERTICES FOR THE SUPPORT POLYGON IN FUNCTION “POLIGONO”

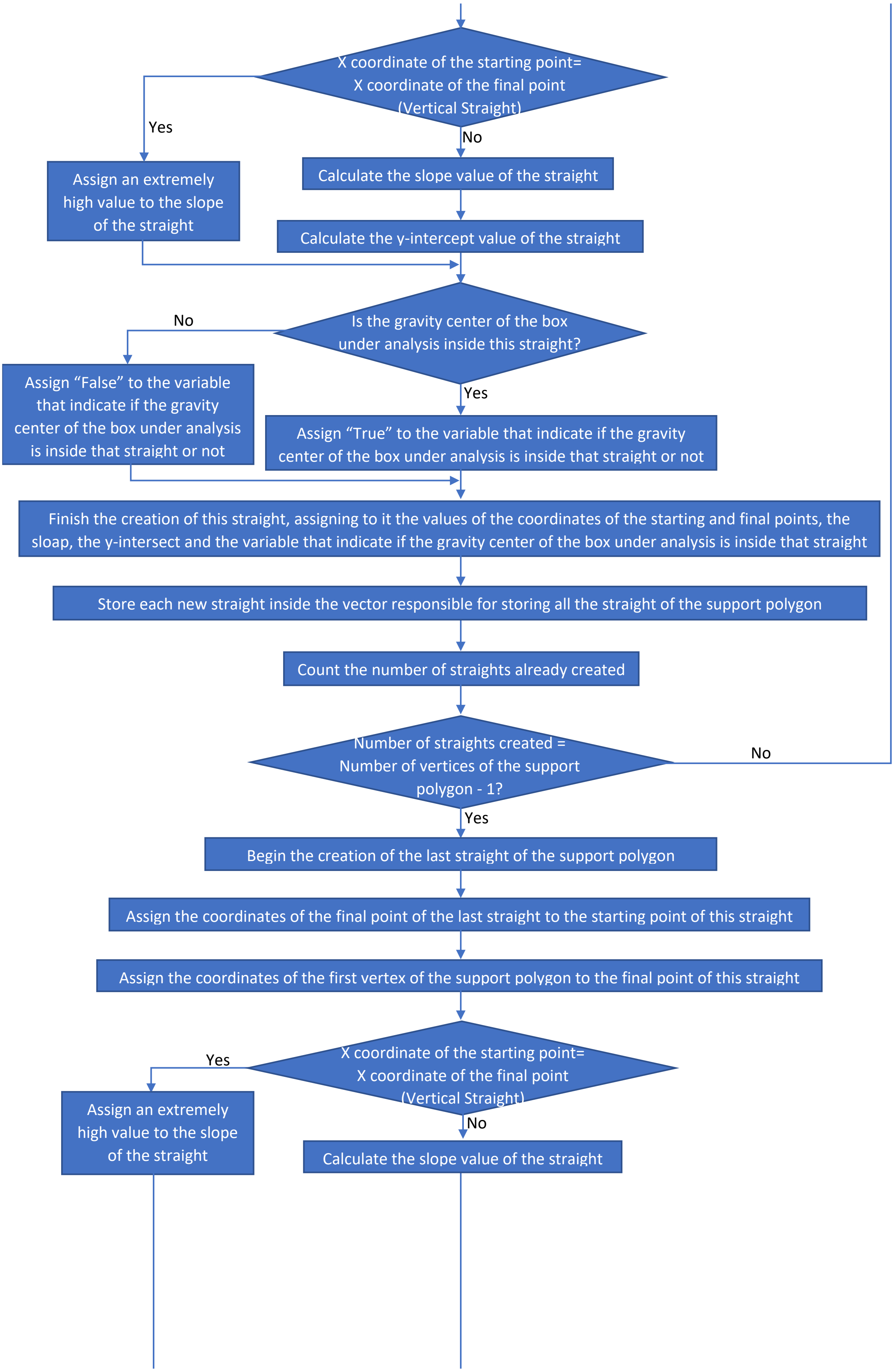


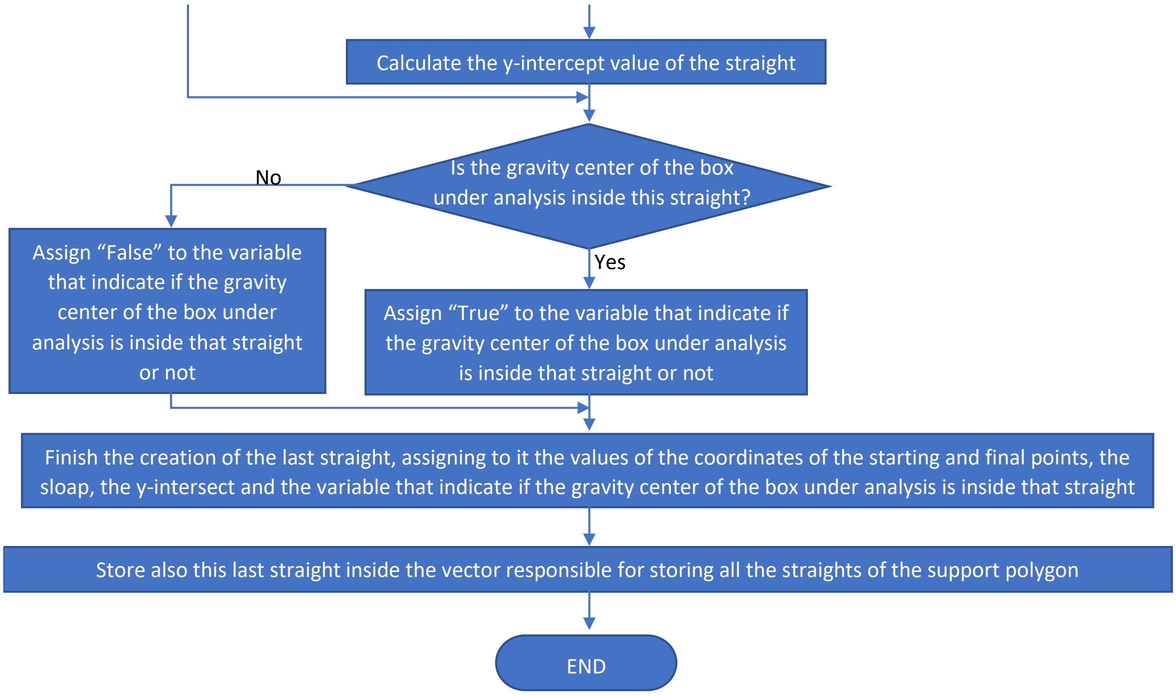
APPENDIX F – FLOWCHART OF THE GENESIS OF THE CONVEX HULL IN FUNCTION “POLIGONO”



APPENDIX G – FLOWCHART OF THE CREATION OF THE SUPPORT POLYGON IN FUNCTION "POLIGONO"




X coordinate of the starting point=
X coordinate of the final point
(Vertical Straight)
Yes
No
Assign an extremely high value to the slope of the straight
Calculate the slope value of the straight
Calculate the y-intercept value of the straight
No
Is the gravity center of the box under analysis inside this straight?
Assign "False" to the variable that indicate if the gravity center of the box under analysis is inside that straight or not
Yes
Assign "True" to the variable that indicate if the gravity center of the box under analysis is inside that straight or not
Finish the creation of this straight, assigning to it the values of the coordinates of the starting and final points, the sloap, the y-intersect and the variable that indicate if the gravity center of the box under analysis is inside that straight
Store each new straight inside the vector responsible for storing all the straight of the support polygon
Count the number of straights already created
Number of straights created =
Number of vertices of the support
polygon - 1?
No
Yes
Begin the creation of the last straight of the support polygon
Assign the coordinates of the final point of the last straight to the starting point of this straight
Assign the coordinates of the first vertex of the support polygon to the final point of this straight
Yes
X coordinate of the starting point=
X coordinate of the final point
(Vertical Straight)
No
Assign an extremely high value to the slope of the straight
Calculate the slope value of the straight


Calculate the y-intercept value of the straight
Is the gravity center of the box under analysis inside this straight?
No
Yes
Assign “False” to the variable that indicate if the gravity center of the box under analysis is inside that straight or not
Assign “True” to the variable that indicate if the gravity center of the box under analysis is inside that straight or not
Finish the creation of the last straight, assigning to it the values of the coordinates of the starting and final points, the sloap, the y-intersect and the variable that indicate if the gravity center of the box under analysis is inside that straight
Store also this last straight inside the vector responsible for storing all the straights of the support polygon
END

APPENDIX H - OUTPUT WINDOW FOR THE INSTANCE BR100

```
Console de Depuração do Microsoft Visual Studio
X Vertice1: 581
X Vertice2: 534
Y Vertice1: 184
Y Vertice2: 184
Gravity Center Inspetion: Inside
X Vertice1: 534
X Vertice2: 534
Y Vertice1: 184
Y Vertice2: 135
Gravity Center Inspetion: Inside
Result: true

Box: 203
Initial X Coordinate: 534
Final X Coordinate: 581
Initial Y Coordinate: 184
Final Y Coordinate: 233
Initial Z Coordinate: 141
Final Z Coordinate: 219
NumberOfBoxesSupportingThisOne: 1
X Gravity Center: 557.500000
Y Gravity Center: 208.500000
X Vertice1: 534
X Vertice2: 581
Y Vertice1: 184
Y Vertice2: 184
Gravity Center Inspetion: Inside
X Vertice1: 581
X Vertice2: 581
Y Vertice1: 184
Y Vertice2: 233
Gravity Center Inspetion: Inside
X Vertice1: 581
X Vertice2: 534
Y Vertice1: 233
Y Vertice2: 233
Gravity Center Inspetion: Inside
X Vertice1: 534
X Vertice2: 534
Y Vertice1: 233
Y Vertice2: 184
Gravity Center Inspetion: Inside
Result: true

Everything Right
```

APPENDIX I - OUTPUT WINDOW FOR THE INSTANCE BR101

```
Console de Depuração do Microsoft Visual Studio
Result: true

Box: 197
Initial X Coordinate: 440
Final X Coordinate: 487
Initial Y Coordinate: 135
Final Y Coordinate: 184
Initial Z Coordinate: 141
Final Z Coordinate: 219
NumberOfBoxesSupportingThisOne: 1
X Gravity Center: 463.500000
Y Gravity Center: 159.500000
X Vertice1: 440
X Vertice2: 487
Y Vertice1: 135
Y Vertice2: 135
Gravity Center Inspetion: Inside
X Vertice1: 487
X Vertice2: 487
Y Vertice1: 135
Y Vertice2: 184
Gravity Center Inspetion: Inside
X Vertice1: 487
X Vertice2: 440
Y Vertice1: 184
Y Vertice2: 184
Gravity Center Inspetion: Inside
X Vertice1: 440
X Vertice2: 440
Y Vertice1: 184
Y Vertice2: 135
Gravity Center Inspetion: Inside
Result: true

Box: 198
Initial X Coordinate: 440
Final X Coordinate: 487
Initial Y Coordinate: 184
Final Y Coordinate: 233
Initial Z Coordinate: 141
Final Z Coordinate: 219
NumberOfBoxesSupportingThisOne: 0
Result: false

Something is Wrong
```

APPENDIX J - OUTPUT WINDOW FOR THE BR102

Console de Depuração do Microsoft Visual Studio

```
X Vertice1: 498
X Vertice2: 451
Y Vertice1: 135
Y Vertice2: 135
Gravity Center Inspetion: Inside
X Vertice1: 451
X Vertice2: 451
Y Vertice1: 135
Y Vertice2: 86
Gravity Center Inspetion: Inside
Result: true

Box: 193
Initial X Coordinate: 498
Final X Coordinate: 545
Initial Y Coordinate: 37
Final Y Coordinate: 86
Initial Z Coordinate: 141
Final Z Coordinate: 219
NumberOfBoxesSupportingThisOne: 1
X Gravity Center: 521.500000
Y Gravity Center: 61.500000
X Vertice1: 498
X Vertice2: 503
Y Vertice1: 37
Y Vertice2: 37
Gravity Center Inspetion: Inside
X Vertice1: 503
X Vertice2: 503
Y Vertice1: 37
Y Vertice2: 86
Gravity Center Inspetion: Outside
X Vertice1: 503
X Vertice2: 498
Y Vertice1: 86
Y Vertice2: 86
Gravity Center Inspetion: Inside
X Vertice1: 498
X Vertice2: 498
Y Vertice1: 86
Y Vertice2: 37
Gravity Center Inspetion: Inside
Result: false

Something is Wrong
```

APPENDIX K - STABILITY DATA FROM THE SET RUNBYOCCUPATION

Store	Pallet ID	Average	Min	Max	Median	Average %	Min %	Max %	Median %	M1	M2
1751	1	63377.35	16560	175224	51992	88.08	38.15	100.00	100.00	1.2459	28.169
	2	76512.24	22680	164832	67384	83.08	37.06	100.00	88.04	1.025	36.7347
	3	82793.34	39728	158360	79344	85.23	32.53	100.00	100.00	1	13.3333
2306	1	52552.25	9400	146400	46400	88.73	31.59	100.00	100.00	1.69333	26.1364
	2	53145.58	11704	128480	53200	81.00	34.67	100.00	84.07	1.04762	28.9474
	3	64966.96	13464	151200	61712	80.08	28.74	100.00	82.37	1.06667	32.7273
	4	103012	91756	108640	100198	100.00	100.00	100.00	100.00	nan	33.3333
2659	1	60077.50	8632	211600	46400	88.05	29.38	100.00	100.00	1.17808	34.0909
	2	55196.92	12800	112112	53200	83.04	27.17	100.00	98.27	1.25862	30.137
	3	61124-82	15873	107440	59280	83.23	29.14	100.00	100.00	1.23529	32.1429
2702	1	52014.70	8588	156000	46400	89.08	50.89	100.00	100.00	1.53333	23.4234
	2	59480.19	17385	167440	49908	81.87	37.11	100.00	90.85	1.05	36.2319
	3	87464.92	44896	187200	68220	94.09	60.81	100.00	100.00	1	38.4615
2723	1	55616.57	11776	175224	50424	90.04	35.47	100.00	100.00	1.22951	25.3333
	2	54200.69	17664	186576	43428	84.63	35.97	100.00	90.52	1.23077	31.1688
	3	58776.09	19600	167440	47037.50	77.23	25.57	100.00	82.89	1.07143	31.3725
	4	68706.55	25080	152064	60760	82.32	36.17	100.00	87.69	1.08333	30.303
2725	1	46213.68	8880	137600	43296	89.02	42.29	100.00	100.00	1.21	32.4786
	2	56354.68	14800	246000	54720	85.79	14.93	100.00	99.15	1.08929	39.7059
	3	74537.73	24000	187200	60760	93.36	57.00	100.00	100.00	1	36.3636
2727	1	59182.33	10200	175224	46400	89.46	27.41	100.00	100.0	1.32075	36.3636
	2	56649.24	17160	170456	40800	82.86	32.90	100.00	91.05	1.08163	29.0323
	3	83932.82	34496	172928	69226	87.88	48.28	100.00	100.00	1	23.5294
2731	1	56361.29	8588	154780	46732	91.94	42.58	100.00	100.00	1.35526	20.8791
	2	63938.45	13600	155216	55500	80.54	35.55	100.00	100.00	1.06383	32.1429
	3	89913.50	58000	165688	70800	96.84	76.38	100.00	100.00	1	50
2742	1	67352.57	6500	152064	53450	87.54	34.07	100.00	100.00	1.09615	41.2698
	2	71526.11	26136	170456	58752	84.89	40.81	100.00	90.01	1.11111	26.6667
	3	88227.66	48500	155800	84584	97.70	83.62	100.00	100.00	1	41.6667
3417	1	48537.45	160	168636	41760	89.97	38.13	100.00	100.00	2.32911	13.8889
	2	53915.70	14740	187200	46000	80.29	25.86	100.00	85.95	1.18966	242857
	3	105156	52000	170456	105868	100.00	100.00	100.00	100.00	nan	33.3333
3510	1	57987.10	8008	211600	56250	82.67	34.34	100.00	94.19	1.10526	27.5862
	2	80603.40	23183	163944	68808	83.28	36.54	100.00	85.60	1.16667	32
	3	100139.34	25480	167440	88804	82.48	33.68	100.00	93.56	1.125	33.3333
4125	1	61114.47	14384	167440	55552	89.10	41.52	100.00	100.00	1.37805	20.2128
	2	72128.64	20995	164832	55552	88.58	45.34	100.00	100.00	1.08	28.8136
	3	88707.34	28000	187200	89152	80-25	29.59	100.00	82.35	1.08696	40

APPENDIX L - STABILITY DATA FROM THE SET RUNBYVOLUME

Store	Pallet ID	Average	Min	Max	Median	Average %	Min %	Max %	Median %	M1	M2
1751	1	71966.02	20240	186576	60000	87.33	43.61	100.00	97.35	1.05556	41.3043
	2	69024.74	16500	123228	56700	85.44	39.47	100.00	99.51	1.15625	37.7778
	3	65847.91	23520	159984	66992	86.02	39.50	100.00	100.00	1.09375	31.8182
2306	1	51692.01	5340	160080	51240	86.27	15.84	100.00	96.50	1.37097	24.0506
	2	59497.06	9656	164832	54400	84.02	36.15	100.00	95.51	1.24138	31.4286
	3	60968.10	12420	131596	53822	84.64	39.93	100.00	92.23	1.16949	33.3333
	4	100789	89496	110148	94872	100.00	100.00	100.00	100.00	nan	50
2659	1	60558.15	17440	168636	47104	86.70	31.84	100.00	99.46	1.31373	30.303
	2	62212.70	10080	211600	49728	85.48	33.31	100.00	100.00	1.04082	38.9831
	3	54383.22	10200	159850	55552	87.69	27.81	100.00	100.00	1.32653	26.5625
2702	1	54936.90	8588	164496	46400	88.56	40.40	100.00	100.00	1.87755	12.8571
	2	61219.66	8588	150500	56250	86.89	32.26	100.00	98.90	1.31818	34.4828
	3	52578.43	11288	164496	36747	85.75	42.30	100.00	100.00	1.15094	30.7692
2723	1	56707.82	15200	179256	46561.50	83.76	32.17	100.00	92.40	1.10909	32.7869
	2	56473.15	19600	163944	45020	89.04	39.67	100.00	100.00	1.21569	31.3433
	3	56057.33	21185	163944	49812	83.46	30.56	100.00	100.00	1.38636	27.8689
	4	65958.11	20952	186576	52514	82.14	37.11	100.00	100.00	1.05128	51.0638
2725	1	50535.64	10368	163944	53000	87.75	23.42	100.00	100.00	1.14062	37.6623
	2	56131.56	8920	152256	52008	84.23	27.69	100.00	100.00	1.33333	35.0877
	3	48080.18	8880	148500	43296	87.28	22.32	100.00	100.00	1.15517	30.137
2727	1	63742.71	13200	186576	50562	83.14	16.54	100.00	86.99	1.13889	37.7778
	2	63291.38	13920	175224	46400	86.65	47.93	100.00	93.93	1.25581	28
	3	57795.30	10200	170456	40000	88.49	46.08	100.00	100.00	1.175	22
2731	1	63446.21	10692	165688	62988	87.45	26.08	100.00	100.00	1.44737	33.3333
	2	57112.53	8588	155216	55400	90.12	37.46	100.00	100.00	1.0566	39.3939
	3	63870.21	7800	163944	52640	86.77	50.00	100.00	100.00	1.03704	31.5789
2732	1	89712.06	14040	211600	54924	85.75	38.08	100.00	100.00	1.02326	36.7347
	2	65614.09	11288	167440	51480	85.45	37.23	100.00	93.24	1.31373	31.25
	3	63839.45	16920	156950	61292	81.54	26.00	100.00	92.17	1.02174	30.3571
	4	77627.20	41648	163944	59488	94.05	70.21	100.00	100.00	1	60
2742	1	66817.31	6500	152064	50130	86.54	52.50	100.00	100.00	1.48649	31.1111
	2	72111.53	14800	155800	67896	86.13	41.05	100.00	100.00	1.125	44.1176
	3	65541.66	8880	152064	57860	86.25	37.16	100.00	97.73	1.09375	41.4634
3417	1	48117.75	11264	167440	33712	89.72	41.29	100.00	100.00	1.89362	26.0274
	2	63926.98	11700	187200	59200	88.45	39.70	100.00	100.00	1.08108	35.4167
	3	49985.08	160	168636	46400	88.70	30.58	100.00	100.00	1.81395	19.0476
3510	1	66962.04	10032	163944	60088	81.99	31.10	100.00	95.00	1.17073	32
	2	58660	10200	150150	56250	81.59	30.75	100.00	100.00	1.30612	32.2581
	3	79819.58	13680	211600	71280	83.68	32.40	100.00	100.00	1.125	40
4125	1	77851.28	20838	187200	66264	84.53	35.00	100.00	100.00	1.38298	28.3019
	2	68025.71	21576	147280	66528	89.16	39.05	100.00	100.00	1.17308	30.6452
	3	62811.88	20400	173692	50150	88.32	27.16	100.00	100.00	1.52727	30.303
	4	84420	57120	111720	57120	100.00	100.00	100.00	100.00	nan	50