Reply to: Comments on "Particle Swarm Optimization with Fractional-Order Velocity"

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We agree with Ling-Yun et al. [5] and Zhang and Duan comments [2] about the typing error in equation (9) of the manuscript [8]. The correct formula was initially proposed in [6,7]. The formula adopted in our algorithms discussed in our papers [1,3,4,8] is, in fact, the following:

$$\begin{aligned} v_{t+1} = \alpha v_t + \frac{1}{2}\alpha(1-\alpha)v_{t-1} + \frac{1}{6}\alpha(1-\alpha)(2-\alpha)v_{t-2} \\ + \frac{1}{24}\alpha(1-\alpha)(2-\alpha)(3-\alpha)v_{t-3} \\ + \phi_1(b-x) + \phi_2(g-x) \end{aligned} \quad (1)$$

The results in [5] and [8] are not directly comparable. In [5], the charts are obtained by running a single test; while in [8], the charts are obtained by the median of 201 simulations tests. It should be noted that we use several simulations, because particle swarm optimization (PSO) is a stochastic algorithm. Therefore, each time it is executed it leads to a distinct trajectory. Due to this fact, a test group of 201 simulation was considered, and the median adopted as the final output. For example, for the Easom function, $f_4(x)$ (2), with $\alpha = 0.5$, the 201 evolution trajectories obtained are depicted in Fig. 1 by blue lines. The corresponding median is depicted by the red line.

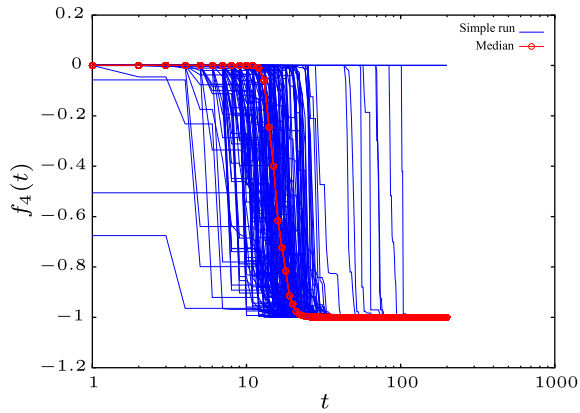

Fig. 1 Evolution of 201 simulations for the Easom function, with $\alpha = 0.5$, and its median

In conclusion, with exception of the typing error in the formula the implemented algorithm and the published results are correct.

$$\begin{aligned} f_4(x) &= -\cos(x_1)\cos(x_2)\mathrm{e}^{-(x_1-\pi)^2-(x_2-\pi)^2} \\ f^*(x) &= -1.0, \quad x_1, x_2 \in [-100, 100] \end{aligned} \quad (2)$$

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