

ASSESSMENT EXPERIENCES IN A MATH COURSE – GRADING QUESTIONS AND DILEMMAS

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Abstract

The role of a teacher, as knowledge promoter and learning facilitator, is frequently opposed to the simultaneously inherent “validation” tasks, as far as grading and assessing are concerned, fundamentally from the students’ point of view, but, sometimes, even from our own. The generalised Math “trauma” is a difficult start up invisible barrier that we must overcome every single semester, by implementing different strategies, developing new materials, motivating with digital and technological resources (using students’ digital skills), among many other tactics and schemes.

But, in the end, the numerical grade – the knowledge and skills construction validation – must appear posted in the “system”.

As Math lecturers in a Higher Education Institution, for more than twenty years, these problems are a daily challenge we face, and the issues we intend to analyse here, emerge as a consequence of a certain "emptiness" we feel regarding the assessment we have to carry out, in the sense that we still don't have an answer to the following question: "Is it legitimate to "close your eyes" to the basic errors (some severe) when assessing learning outcomes in advanced subjects?" We teach at "end of the line", as far as General Mathematics is concerned, since our students are, essentially, from Management and Accounting Bachelor degrees.

This paper will be structured in four distinct parts, starting with the Specific and General Outcomes and skills in the Math course in question, giving also a global vision of all its syllabus components and the teaching Methodologies implemented. Subsequently, we will refer to the coherence between teaching methods and the course learning objectives as well as their connection to the syllabus items. Finally we will go through a section of small questions and answers, with their respective detailed analysis, in order to provide an objective reading material, trying to promote a fruitful and open discussion on the subject

Keywords: Assessment, Grading, Mathematics, Higher Education, Teaching and Learning strategies, Applied Math.

1 INTRODUCTION

As Mathematics professors in the School of Hospitality and Tourism (ESHT) and in the School of Accounting and Administration (ISCAP), two of the nine organic units of the Polytechnic of Porto (P.PORTO), a Higher Education Institution (HEI), we lecture Math basic courses in the first semester of the first year in degrees of Management and Accounting areas. These degrees don't have Mathematics as a nuclear course, therefore students who join these degrees do not necessarily have to take Math as a curricular unit throughout High School. Consequently, students in the first year are, in terms of mathematical knowledge, very heterogeneous.

Some of these students, as they have little (or no) Mathematics in High School, feature many gaps at the level of Math basic skills that lead them to make “serious” mistakes, not necessarily related to the advanced content taught, but with much more elementary subjects they, allegedly, should dominate, but, unfortunately, that “makes no sense” for them! We know that this problem is, unfortunately, much more common (and old) than one could expect and far from being just a personal feeling ([1],[2],[3],[4],[5])), just somehow minimized when the Mathematics courses analyzed are from degrees where the Math A National Exam is compulsory as mandatory Specific Exam to enter HEI system (as it happens for all Portuguese degrees in Engineering, for example) [6].

We don't have an ideal model to tackle this problem, but some measures could minimize the heterogeneous background impact on students' success in these Math courses ([7], [8], [9], [10]), and

some may even try to tackle this problem at an early stage, starting to work at a lower level, even at a College-level [11].

In order to contextualize and structure this small paper, we will present it divided into distinct parts that go from subjects and skills and questions in order to provide an objective scrutiny, without reading the specific themes (linked to the economy, but it could be any other), and to facilitate the understanding of the analysed issues.

2 SUBJECTS AND COMPETENCES

2.1 Global Syllabus Items

Any curriculum of the first year of a degree in the Economics and Management field has, at least, a course in Quantitative Methods (the "former" General Mathematics) where the following general subjects are covered: Differential Calculus, Integral Calculus, Matrixial Calculus and Statistics (Fig.1 and 2.).

<p>Degree: Undergraduate Degree In Accounting And Administration Course: Mathematics Academic Year: 2015/2016</p> <p>Academic year: 1 Term: Winter Semester Attendance: Mandatory Course Responsible Teacher: Doutora Filomena Soares</p> <p>Number of Credits (ECTS): 4.0 Theoretical/Practical Work (hours): 51.0 h</p> <p>Syllabus: 1. ONE VARIABLE REAL FUNCTIONS - Basic Concepts 1.1. Pre-calculus review 1.2. The logarithmic and the exponential functions 1.3. Some Economical Functions 1.4. Graphing a function, its Tangent and Normal Lines 2 DIFFERENTIATION 2.1 The meaning and interpretation of a derivative 2.2 The basic rules 2.3 The chain rule 2.4 The inverse function's derivative 2.5 Implicit Functions and Implicit differentiation 2.6 Equations of tangents and normals to curves 2.7 Differentials 2.8 Applications in Economics: Marginal Functions and Elasticity 2.9. Second and higher derivatives 2.10. Taylor's Formula and Applications 2.11. Absolute and local Extremes, Inflection Points 2.12. Limits, Asymptotes 3. MULTIVARIABLE FUNCTIONS 3.1. Definition and Domain 3.2. Limits and Continuity 3.3. Partial Differentiation 4. INTEGRAL CALCULUS 4.1. Antiderivatives and The Indefinite Integral 4.2. Direct Integration: The Rules of Integration 4.3. Integration Methods</p>	<p>Degree: Undergraduate Degree In Accounting And Administration Course: Applied Mathematics Academic Year: 2015/2016</p> <p>Academic year: 1 Term: Summer Semester Attendance: Mandatory Course Responsible Teacher: Doutora Filomena Soares</p> <p>Number of Credits (ECTS): 4.0 Theoretical/Practical Work (hours): 51.0 h</p> <p>Syllabus: 1. DEFINITE INTEGRATION 1.1. Definite integral and its properties 1.2. Areas computation 1.3. Improper integrals 1.4. Applications of the Definite integral to Business and Economics 2. SEQUENCES AND SERIES OF REAL NUMBERS 2.1. Sequences Review 2.2. Infinite Series and its Properties 2.3. Arithmetic and Geometric Sequences and Series 2.4. Applications and problems 3. LINEAR ALGEBRA 3.1. Matrices and Matrix Calculus 3.2. Determinants 3.3. Solving Systems of Linear Equations. Application</p>
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Figure 1 Syllabus examples from Accounting and Management Degree

<p>ECTS - Escola Superior de Hotelaria e Turismo Degree: Tourist Activities Management Course: Quantitative Methods I Academic Year: 2016/2017</p> <p>Academic year: 1 Term: Winter Semester Attendance: Mandatory Course Responsible Teacher: Doutora Filomena Soares</p> <p>Number of Credits (ECTS): 5.0 Theoretical/Practical Work (hours): 3.0 h</p> <p>Syllabus: 1. General and basic concepts of one variable Real functions 1.1. Pre-calculus review 1.2. Particular Functions Review: polynomial, exponential and logarithmic 1.3. Some Economical Functions 2. Differential Calculus in IR 2.1 Definition and geometric interpretation of the derivative - notion of differential 2.2 Differentiation rules 2.3 Applications in Economics: Marginal Functions and Elasticity 3. Integral Calculus in IR 3.1 Antiderivative notion - Indefinite Integral and its properties 3.2. Direct Integration: The Rules of Integration 3.3. Integration Methods 3.4 Definite Integral and its properties 3.5 Fundamental Theorem of Integral Calculus 3.5. Application of Integrals: Areas computation, Mean Value of a function; consumer and producer surplus</p>	<p>ECTS - Escola Superior de Hotelaria e Turismo Degree: Hotel Management Course: Quantitative Methods Academic Year: 2016/2017</p> <p>Academic year: 1 Term: Winter Semester Attendance: Mandatory Course Responsible Teacher: Doutora Filomena Soares</p> <p>Number of Credits (ECTS): 4.0 Theoretical/Practical Work (hours): 68.0 h</p> <p>Syllabus: 1. General and basic concepts of one variable Real functions 1.1. Pre-calculus review 1.2. Particular Functions Review: polynomial, exponential and logarithmic 1.3. Some Economical Functions 2. Differential Calculus in IR 2.1 Definition and geometric interpretation of the derivative # notion of differential 2.2 Differentiation rules 2.3 Applications in Economics: Marginal Functions and Elasticity 3. Integral Calculus in IR 3.1 Antiderivative notion - Indefinite Integral and its properties 3.2. Direct Integration: The Rules of Integration 3.3. Integration Methods 3.4 Definite Integral and its properties 3.5 Fundamental Theorem of Integral Calculus 3.5. Application of Integrals: Areas computation, Mean Value of a function; consumer and producer surplus 4. Statistics 4.1. Data Organization 4.2 Descriptive measures: 4.3 Regression and correlation 5. Probability and combinatorics 5.1 Random Experience, set of results; events 5.2 Classical Laplace Probability Definition 5.3 Combinatorial Analysis</p>
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Figure 2 Syllabus examples from Tourism Management Areas Degree

After a generic approach to contents underlying each subject, it is important to go through several applications and problems in the degrees' fundamental areas – Management and Administration.

One must notice that the common subjects, discussed in differential calculus, have a wide applicability in these fields, as, for example, they allow the study of the Economic Functions (monotony and extremes), the definition and analysis of Marginal Functions (unit and multivariate) and their

interpretation, the determination of price elasticity functions (both of supply and demand) and their interpretation, among several others.

Regarding the basic themes specific to Integral Calculus, we go through the whole panoply of immediate integrals as well as all the integration techniques (as is to be expected, the Integrals involving trigonometric functions or their inverses are not worked, since their application in the Management and Administration areas is neglectable). Afterwards, we analyse the definite integrals, their extension to improper integrals and application to plain Areas calculus. If, on the one hand, its use can be seen as a mere reversal of what was discussed in the previous topic, the areas' calculation is necessary, for example, for the determination of producer surplus and consumer surplus, as well as the medium value of functions, in the case of continuous ones.

Regarding the matrix calculus, whose main application goes through the input/output models of Leontief, we work themes ranging from the concept of a matrix and the notion of matrix determinant, through operations with (and on) arrays, condensation and reverse calculation, and all its application to linear systems resolution and discussion.

With Statistic syllabus section, we intend to develop in students the ability to interpret data, formulate and solve everyday problems, since in their future professional life they will be confronted with the need to know how to apply probabilistic and statistical techniques, promoting the use of appropriate software skills. Specifically, it aims to develop students' skills to select the best methods and techniques for a given purpose, and for a particular set of data, using a statistical software (IBM SPSS) and giving them tools and arguments to interpret, organize and present the statistical results.

In order to stay focused on the problem at stake here we will specifically work with the syllabus items regarding Differential and Integral Calculus (Quantitative Methods I – Fig 2).

2.2 Differential and Integral Calculus Learning Outputs

Globally speaking, for the degrees analysed here, the General outcomes/skills are:

- 1 To clearly structure a logical reasoning, consciously identifying all its phases, perceiving Mathematics as a tool for other Curricular Units;
- 2 Recognize and interpret new subjects of Infinitesimal and Integral Calculus required for a proper Mathematical training in a Management Degree;

When talking about the Specific outcomes/skills, we come across:

- 3 Sketching and interpreting graphs of functions with economical interest, identifying some Mathematical Models typical characteristics (exponential and logarithmic, polynomial), critically analysing the proposed solutions;
- 4 Analysing the variation of real functions, applying the notions of Differential and Integral Calculus.

2.2.1 Coherence between Syllabus Items and learning objectives of the course

The syllabus was developed in a progressive way in terms of the knowledge construction process in order to reach the presented goals and outcomes. Concretely, this joint relation - syllabus items / objective - can be analysed through a direct relation between syllabus items (SI) and objectives, in terms of specific outcomes (SO): Points 1 and 2. of the SI - Points 4 and 4 of the SO.

All syllabus items enable the stated global objectives of this curricular unit as well as the development of the students' general outcomes. It should still be pointed out that, in paragraphs 1, 2. and 3., of the Syllabus items, the models of polynomial, exponential and logarithmic functions are used to solve problems involving functions that translate revenues, profits, costs, growth stocks, interest rates, changes in scale, etc. In particular, in item 2 (Differential Calculus in IR) derivatives are applied to study a range of functions in the economic and administrative areas, namely introducing the notions of marginal functions and demand elasticity. Regarding section 3. (Integral Calculus in IR) integration is important as a reverse procedure in which differentiation and integrals are used to study the total variation of the function, the consumer and producer surpluses, the future value and present an income stream, as already stated.

2.2.2 Teaching methodologies

The classes are organised in theoretical-practical sessions (expository and monitoring teaching). The contact hours in this curricular units are developed through actual sessions that articulate the expositive method (supported by documentation and practical examples presentation) and the exploratory and practical methods, trying to focus the proposed problems, whenever possible, in the nuclear degrees' areas. We try to guide the autonomous work organization through a constant use of the Moodle platform.

The contents' presentation is developed using projections and board writing, explaining diverse examples and exhibiting application to real situations. Indication of relevant exercises to be solved by students during each class, in an autonomous way; suggestion of the main steps of resolution, clearing doubts, monitoring and validating the student's individual work. The supporting materials, the syllabus and the pedagogical dynamics of the presential sessions that involve its theoretical and practical components, the teachers' support and monitoring, the activities of clearing doubts and solving the proposed problems and tasks, along with the activities for assessing knowledge are settled and based upon the course objectives and by the development of specific and general competencies defined for each theme. The assessment moments correspond to the objectives defined for the course, focusing on the specific competencies. For each point of the syllabus, students will have available the Supporting Texts and the Exercises Book (with the respective solutions) that will be helpful during classes, allowing them to manage their own learning process in an autonomous way. The Continuous Assessment allows a constant interaction with students, promoting their autonomy and self-consciousness on reaching the established objectives, as well as the development of their responsibility for their own learning process.

3 QUESTIONS AND EXAMPLES

As already mentioned at the beginning of this text, some students enter HEI system with little (or no) Mathematics in High School, presenting many gaps at Math skills' level that lead to serious errors, not necessarily related to the advanced contents they are actually learning, in HE, but with much more basic subject they should dominate but which, unfortunately, they don't. This is the problem that concerns us, namely at the assessment level.

3.1 Questions and doubts

We will try to provide a fruitful moment for reflection around the global assessment problem.

- How to proceed?
- Are lecturers consistent in the decisions they make? Are we?
- Is "closing our eyes" to the basic errors (some severe), when evaluating advanced subjects learning performance, the correct way to act?
- Is it correct to extrapolate these past "lack of skills" to the actual teachers' responsibility?
- When students' performance is directly related with teachers' elevation, how can we differentiate these lacks? Is it our own or someone else's "responsibility"?

As we don't have straight answers to these and many other questions, having some deep problems when assessing these students, we are gladly sharing these, with an ultimate hope of guidance or "light" that can "illuminate" our way.

3.2 Examples

The examples we present below serve to illustrate, in a more tangible way, the core of this small paper and the central issues we are trying to bring to debate and discussion:

An indefinite integral resolution, during which students make the same mistake, however, as this is committed in distinct "moments" in the resolution procedure, the respective reviews and assessment are, perhaps, "too different".

Consider, for instance, the indefinite integral: $\int \frac{1}{x^2 - x - 2} dx$, whose solution, after a resolution is tricky and time-consuming, because it implies the breakdown into simple elements and application of undetermined coefficients' method, is:

$$\int \frac{1}{x^2 - x - 2} dx = \frac{1}{3} \ln \left| \frac{x-2}{x+1} \right| + C, \quad C \in \mathbb{R}$$

However, frequently, we come across the following full resolution proposal:

$$\begin{aligned} \int \frac{1}{x^2 - x - 2} dx &= \int \left(\frac{1}{x^2} - \frac{1}{x} - \frac{1}{2} \right) dx = \int \frac{1}{x^2} dx - \int \frac{1}{x} dx - \int \frac{1}{2} dx \\ &= -\frac{1}{x} - \ln|x| - \frac{1}{2}x + C, \quad C \in \mathbb{R} \end{aligned} \quad (*)$$

In cases like this our reaction is an instant "repulse". Therefore, very clear, straight and negative!

Getting the full resolution based on a "so bad" screw up, as is decomposing a fraction based on its denominator parcels, has no forgiveness and, therefore, despite the student's former correct resolution (solving obtained integrals, after having committed that "huge" fault) all the grading allegedly attributed (as a kind of mental lecturer assessment) is, "instantaneously", void.

Let's see, now, an alternative resolution of another integral where the same error type pops up, only, now, this error is committed in a different resolution procedure "moment", when compared to the last example.

Consider, for example, the indefinite integral: $\int \frac{1-2x}{(x^2 - x - 2)^2} dx$.

The correct answer to this question, after some algebraic manipulation is:

$$\int \frac{1-2x}{(x^2 - x - 2)^2} dx = -\int \frac{2x-1}{(x^2 - x - 2)^2} dx = -\frac{(x^2 - x - 2)^{-1}}{-1} + C = \frac{1}{x^2 - x - 2} + C$$

However, when dealing with this exercise, we face answers like:

$$\int \frac{-(2x-1)}{(x^2 - x - 2)^2} dx = \frac{1}{x^2 - x - 2} + C = \frac{1}{x^2} - \frac{1}{x} - \frac{1}{2} + C, \quad C \in \mathbb{R} \quad (**)$$

And, how should we "assess" this?

In cases like this, our reaction is not as bad as in the previous case. The grade that we attribute in this case is obviously not full, but we're much more benevolent, even though the error made is exactly the same: To decompose a fraction for its denominator!

It is a serious mistake that, in the previous example, was not even tolerated and here, in this case, as a matter of timing, was almost despised, as if the student had not committed it...

A conscious reflection on what we have just referred allows us to say that, without a doubt, we seem to be giving "different weights to similar situations". We can perhaps justify the main reasons, one as an emotional one and another rational one, which lead us to act this way:

The first reason, a kind of "emotional" one, is related to the timing in which the error is made, or to put it another way, the student' "timing" since that mistake as soon as he starts the full resolution versus the "luck" of another student who commits it, only at the end of the resolution, thus having more "timing", in his "emotional" sense!

The second reason has to do with the fact that the student who makes the mistake early on (*), changes completely the objective of the proposed exercise and, therefore, instead of solving an integral involving identification and decomposition in simple elements, having to deal with the

undetermined coefficients' method, he obtains three different immediate integrals, ending up by solving "small" integrals of powers of x , or very basic integrals.

When analysing the resolution (**), the student who made the mistake at the end of the resolution, did not change the objective of the integral. Despite having finalized its resolution in a disastrous way, he showed he knew how to solve the proposed integral.

But, in both resolutions, the same mistake was made – the exact same mistake. But the consequences of this one were very different when assessment is evaluated. Is this correct? Well, we don't feel comfortable expressing our view since it is not unanimous...

4 FINAL COMENTS

Is this evaluation "mode" correct and fair to students learning efforts?

Are we acting correctly?

What kind of procedures must we follow in these situations?

It is important for us to share this kind of situation we frequently face, and that affects us, daily, with as many teachers and lecturers as possible, in order to openly discuss these matters, without taboos or complexes of incompetence in order to promote the exchange of experiences and opinions which, in turn can induce a certain clarification of ideas as to the problem of evaluation in Mathematics.

As already mentioned, at the beginning of this text, some students enter the HEI system with little (or no) Mathematics in High School, presenting many gaps at Math skills' level that lead to serious errors, not necessarily related to the advanced contents they are actually learning, in HE, but with much more basic subjects they should dominate but, unfortunately, they don't. This is the problem that concerns us, namely at the assessment level.

We hope that this message could foster a fruitful discussion and help us to carry out a sustained evaluation (in its most varied aspects).

Although this problem is not a central issue, or even talked about, in this paper one must have in mind what kind of Math skills students were stimulated to (even our Math A ones), since, even actually, in HEI, several Math lecturers don't feel comfortable when working with different digital competences from students, and this is not even a new question [12].

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