Strategic Particle Swarm Inertia Selection for the Electricity Markets Participation Portfolio Optimization Problem

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ABSTRACT

The portfolio optimization is a well-known problem in the areas of economy and finance. This problem has also become increasingly important in electrical power systems, particularly in the area of electricity markets, mostly due to the growing number of alternative/complementary market types that are being introduced to deal with important issues, such as the massive integration of renewable energy sources in power systems. The optimization of electricity market players' participation portfolio comprises significant time constraints, which cannot be satisfied by the use of deterministic techniques. For this reason, meta-heuristic solutions are used, such as particle swarm optimization. The inertia parameter is one of the most important, and it is the main focus of this paper. This paper studies eighteen popular inertia calculation strategies, by comparing their performance in the portfolio optimization problem. A strategic methodology for the automatic selection of the best inertia calculation method for the needs of each optimization is also proposed.

KEYWORDS: Artificial Intelligence; Electricity Markets; Inertia Parameter; Particle Swarm Optimization; Portfolio Optimization.

1. Introduction

Worldwide electricity markets have experienced major changes with the restructuring of the power system, which has been largely potentiated by environmental and economic policies (Sioshansi 2013). At the same time, with the massive integration of renewable energy sources, characterized by a high associated variability, the uncertainty of electricity prices has also increased (Sharma, Bhakar, and Tiwari 2014). The need for the involved entities to understand and deal with this constantly changing environment has led to development of several simulation tools. The use of Artificial Intelligence (AI) methods, particularly multi-agent technology, has proven to be of crucial importance in this domain, by allowing the distributed modelling of complex and dynamic systems with a high level of interactions between the involved players (Velik and Nicolay 2014). Some of the most relevant electricity market simulators are the Electricity Market Complex Adaptive System (EMCAS) (Koritarov 2004), the Agent-based Modelling of Electricity Systems (AMES) (Li, Sun, and Tesfatsion 2011) and the Multi-Agent Simulator for Competitive Electricity Markets (MASCEM) (Santos et al. 2015).

MASCEM has been developed by the authors' research team, and it facilitates the study of electricity markets by representing the entities that typically participate in electricity markets through software agents. Moreover, MASCEM is integrated with another multi-agent system, which detains the sole purpose of providing decision support to players' actions in electricity market negotiations. The Adaptive Decision Support for Electricity Markets Negotiations (AiD-EM) system provides decision support to negotiations in auction based markets (Pinto et al. 2014) and also for bilateral contract negotiations (Pinto, Vale, et al. 2015). Moreover, AiD-EM is equipped with a portfolio optimization methodology, which identifies the market opportunities in which market players should negotiate at each moment in order to maximize their outcomes from market participation (Pinto, Morais, et al. 2015).

The typical portfolio optimization problem consists in finding the optimum way of investing a particular amount of money in a given set of securities or assets (Fernández and Gómez 2007). Traditionally, the optimal management of a portfolio of assets is solved by minimizing the investment risk while guaranteeing a given level of returns. Markowitz (Markowitz 1952) introduced this concept and formulated the fundamental theorem of a mean-variance portfolio framework, which explains the trade-off between mean and variance, representing the expected returns and risk of a portfolio, respectively. Risk refers to the possibility of suffering harm or loss, as result from uncertainty. However, there is a difference between risk and uncertainty: risk is something that usually can be controlled while uncertainty is beyond players' control (Conejo, Carrión, and Morales 2010). The results of players in electricity markets are influenced by many uncertain factors, e.g. other players' bidding strategy, penetration of renewable energy sources and change of demand. These uncertainties bring along risks in electricity pricing. The main reason for this may be attributed to the particular characteristic of non-mass storage of electricity (M. Liu and Wu 2007). Four different risk measures for the portfolio optimization problem in electricity markets are presented by Chang (Chang, Yang, and Chang 2009). This author considers the Mean-variance model, Semi-variance model, Mean absolute deviation model and Variance with skewness, all these models derive from the initial risk measure that was proposed by Markowitz for application in the financial markets. These models for risk minimization consider the history of the markets. The author used Genetic Algorithm (GA) to solve the portfolio optimization problem; Chang's project concludes that GA is an effective method, for solving the portfolio optimization problem with different risk measures. Particle Swarm Optimization (PSO) is used in (Cura 2009) and (Zhu et al. 2011) as an alternative method to solve the portfolio optimization problem; the used risk measures are also derived from the Markowitz mean-variance model. Meta-heuristic techniques are, in fact, a common choice for the resolution of this optimization problem, as detailed in (Ponsich, Jaimes, and Coello 2013) where a rather complete survey on the use of evolutionary algorithms to handle with the portfolio optimization problem is presented. Additionally, variations using methods such as fuzzy logic also present promising solutions, e.g. (Y.-J. Liu and Zhang 2015).

Different random search algorithms have been applied to this problem by the authors, namely the PSO (Faia et al. 2015), GA (Faia, Pinto, and Vale 2017) and Simulated Annealing (SA) (Faia, Pinto, and Vale 2016a). Due to the results obtained in the aforementioned publications and in order to obtain better results in the problem of portfolio optimization in the electricity markets, the PSO is the one that presents the most promising results and thus it is the approach that is being optimized by this work, by exploring the PSO search parameters, namely the inertia calculation strategy. Thus, in this work, PSO has been used, as it has proven its advantage in solving the envisaged problem. As it is demonstrated in the experimental findings section, the deterministic methods have a very long execution time, which compromises the negotiation process. Thus, the use of the meta-heuristic (PSO) is an essential point in the application of the problem in a real context.

PSO is a stochastic population based algorithm which was originally introduced by Kennedy and Eberhart (Eberhart and Kennedy 1995). This algorithm is motived by the intelligent behaviour of some animals, e.g. bird flocking and fish schooling in search of food. The PSO algorithm has been widely used, because of its easy implementation, high degree of flexibility in the used parameters and computation efficiency compared to other heuristic algorithms. In PSO, the swarm consists of individuals, called particles, which change their position over time. Each particle represents a potential solution to the problem. During the optimization process, the particles fly around in a multi-dimensional search space. In the search process each particle adjusts its position according to its own experience and the experience of its neighbouring particles, making use of the best position encountered by itself and its neighbours.

When applying meta-heuristic optimization methods, however, equally importantly to the choice of the most appropriate method, is the selection of the parameters that provide the best chances for the success of the optimization problem in each different application context. This is essential to take advantage on the stochastic nature of meta-heuristic methods to reach an adequate balance between exploration (of the complete search space) and exploitation (of the most promising regions of the search space).

In the PSO algorithm there are many parameters that can be modelled in order to obtain better results in the end. The term of inertia is one of them and will be explored throughout this work. Inertia in the PSO research is a mechanism that controls an exploration and exploitation of the swarm, working as a mechanism that does not allow the particle to settle in a position. The inertia defines a weight that controls the movement of the particle in the next iteration by the contribution of the previous solution. With large values of inertia will facilitate exploration, on the contrary the smaller values will promote the exploitation (Palupi Rini, Mariyam Shamsuddin, and Sophiyati Yuhaniz 2011).

This paper studies the influence of different inertia calculation strategies in the outcomes of the PSO process applied to the resolution of the market participation portfolio optimization problem. For this, eighteen alternative inertia calculation methods are compared, and their advantages are evaluated. Additionally, a strategic methodology for the automatic selection of the best inertia calculation method for the needs of each optimization is also proposed. The proposed method uses a utility function that considers the influence of several factors in the decision process, namely: (i) the objective function results achieved by using each inertia calculation strategy, (ii) the number of iterations required to reach the final solution, and (iii) the execution time associated to the PSO approach when using each inertia calculation. By using the proposed methodology, it is possible to select the most appropriate inertia calculation strategy for each distinct optimization, considering the requirements in terms of execution time and quality of the final result. In addition, the proposed method presents an important contribution, since it will provide a greater variety of results, which may surpass those obtained in the previous works (Faia et al. 2015; Faia, Pinto, and Vale 2016a, 2016b), thus it is a benchmark in solving the problem of optimization of portfolios in the electric energy market.

After this introductory section, section 2 presents the formulation of the considered portfolio optimization problem. Section 3 presents the proposed methodology, including the application of the PSO meta-heuristic, the different inertia calculation strategies, and the proposed adaptive inertia selection method. Section 4 presents the experimental findings achieved when using the proposed PSO approach with the different inertia calculation methods to solve the portfolio optimization problem, and a demonstration of the results achieved when using the proposed method for automatic selection of the inertia calculation strategy. Finally, section 5 presents a discussion on the most relevant conclusions of this work.

2. Problem formulation

This present section will be used to present the objective function of the problem as well as the restrictions applied to it.

2.1. Objective function

Considering the expected production of a market player for each period of each day, the amount of power to be negotiated in each market is optimized to get the maximum income that can be achieved. Eq. (1) is used to optimize players' market participation portfolio, as proposed in (Pinto, Morais, et al. 2015).

$$f(Spow_{M \dots NumS}, Bpow_{S1 \dots NumS}) = Max \begin{bmatrix} \sum_{M=M1}^{NumM} (Spow_{M,d,p} \times ps_{M,d,p} \times Asell_M) - \\ \sum_{S=S1}^{NumS} (Bpow_S \times ps_{S,d,p} \times Abuy_S) \end{bmatrix} \quad (1)$$

$\forall d \in Nday, \forall p \in Nper, Asell_M \in \{0,1\}, Abuy \in \{0,1\}$

$ps_{M,d,p} = Value(d, p, Spow_M, M)$

$ps_{S,d,p} = Value(d, p, Bpow_S, S)$

In Eq. (1) d represents the weekday, $Nday$ represent the number of days, p represents the negotiation period, $Nper$ represent the number of negotiation periods, $Asell_M$ and $Abuy_S$ are boolean variables, indicating if this player can enter negotiations in each market type, M represents the referred market, $NumM$ represents the number of markets, S represents a session of the balancing market, and $NumS$ represents the number of sessions. Variables $ps_{M,d,p}$ and $ps_{S,d,p}$ represent the expected (forecasted) prices of selling and buying electricity in each session of each market type, in each period of each day. The outputs are $Spow_M$ representing the amount of power to sell in market M, and $Bpow_S$ representing the amount of power to buy in session S.

This formulation considers the expected production of a market player for each period of each day. As explained before, the price of electricity in some market types depends on the power amount to trade. With the application of a clustering mechanism (*Value* function in Eq. (1)) it is possible to apply a fuzzy approach to estimate the expected prices depending on the negotiated amount. Eq. (2) defines this condition, where *Data* refers to the historical data that correlates the amount of transacted power, the day, period of the day and the particular market session.

$$Value(day, per, Pow, Market) = Data(fuzzy(pow), day, per, Market) \quad (2)$$

2.2. Constraints

Eq. (3) represents the main constraint to be applied in this type of problems, and imposes that the total power that can be sold in the set of all markets is never higher than the total expect production (TEP) of the player, plus the total of purchased power (Pinto, Morais, et al. 2015). Further constrains depend on the nature of the problem itself, e.g. type of each market, negotiation amount, type of supported player (renewable based generation, cogeneration, etc.).

$$\sum_{M=M1}^{NumM} Spow_M \leq TEP + \sum_{S=S1}^{NumS} Bpow_S \tag{3}$$

$$TEP = \sum Energy_{prod}\, , Energy_{prod} \in \{Renew_{prod}, Therm_{prod}\} \tag{4}$$

$$0 \leq Renew_{prod} \leq Max_{prod} \tag{5}$$

$$Min_{prod} \leq Therm_{prod} \leq Max_{prod}, if\ Therm_{prod} > 0 \tag{6}$$

From the restrictions and considerations presented we can see that the energy produced comes from renewable sources and non-renewable sources (thermoelectric), by Eq. (4). If the player is a producer of thermoelectric power, you have to set your production at a minimum since it is not feasible to completely turn off the production plant, Eq. (6). In Eq. (5) the producer, have to comply the only restriction is the maximum production capacity.

3. Proposed methodology

This paper studies and compares different formulas for the calculation of PSO's inertia parameter. The inertia weight is the PSO parameter that allows balancing the exploration and exploitation characteristics of PSO. Inertia is, thereby, one of the main characteristics that allows the algorithm to achieve good performances. The characteristics and particularities of each inertia calculation method lead to the need of developing an automatic methodology that is able to select the most appropriate calculation method given the needs of each optimization process (balance between the expected quality of results and the required execution time). Usually in works done where the authors try to compare different methods in solving the same problem, they aim to analyse the value of the objective function and the value of number of iterations, since the number of iterations is proportional to the execution time and this can depend on the machine in question (Bansal et al. 2011). What is evaluated is the objective function value that can be obtained in a given number of iterations, because having a small number of measurements means that the method converges efficiently, that is, that it is really directing demand in the right direction. It can be said that the number of iterations is directly connected with the quality of the method in solving the problem.

Therefore, this paper also proposes a methodology that selects the best inertia weight calculation depending on the objective function value, convergence time and number of iterations. The proposed methodology allows the user to choose the importance (weight) to give to each of the variables in order to achieve the solution that most fits the needs of each simulation.

3.1. PSO approach

The two main Equations that are used in PSO are the velocity update Eq. (7) and position update Eq. (8).

$$v_{id}^{k+1} = w.v_{id}^{k} + c_1.r_1^k.\left(Pbest_{id}^{k} - x_{id}^{k}\right) + c_2.r_2^k.\left(Gbest_{id}^{k} - x_{id}^{k}\right) \quad (7)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \quad (8)$$

In PSO's equation the term v_{id}^{k} represent the velocity of particle *i*, for variable *d* and iteration *k*, x_{id}^{k} is position of particle *i*, for variable *d* and iteration *k*, k is the current iteration, $Pbest$ is personal best, $Gbest$ is global best, w is the inertia term, c_1 is the local attraction term, c_2 is the global attraction term, and r_1, r_2 are random numbers between [0,1].

Parameters w, c_1 and c_2 provide the flexibility to the algorithm towards a better performance in the search process. Section 3.2 presents an overview of several strategies that are found in the literature for the inertia parameter calculation.

3.2. Inertia calculation methods

The inertia parameter determines the contribution rate of a particle's previous velocity to its velocity at the current time step. This parameter has been proposed in (Shi and Eberhart 1998) as an improvement of the standard PSO, presented by Eberhart and Kennedy in 1995 (Kennedy and Eberhart 1995). Table I summarizes a total of eighteen Eqs. that allow calculating the inertia weight to be applied in PSO.

TABLE I – INERTIA CALCULATION METHODS

The several inertia calculation strategies presented in Table I can be categorized into three classes, depending on the calculation nature. A brief discussion on the characteristics of each type of calculation is provided as follows.

1) Constant and random inertia weights

The first introduction of the inertia parameter in the PSO algorithm was in 1998 by Shi and Eberhart (Shi and Eberhart 1998). The concept of Inertia is proposed by introducing a constant Inertia Weight (w1). In this first approach, the inertia value is constant throughout all iterations. The same characteristic is also verified in Eq. (w2). In this Eq., the value of the inertia weight is a uniform random variable in the range [0.5 1], since variable *rand()* is a random number between [0 1] with uniform distribution.

2) Time varying inertia weights

Most of the PSO variants use time-varying inertia weight strategies, in which the value of the inertia weight is determined based on the iteration number. These methods can be either linear or non-linear and increasing or decreasing. Eqs. (w3) to (w14) are of this type. Eqs. (w3) to (w12) require the definition of the range in which the inertia weight will vary. This means that the inertia value is w_{max} at the beginning, and w_{min} at the end of the number of iterations. The exception is Eq. (w9), where the opposite is verified, since this function increases the inertia value throughout the number of iterations. Eq. (w13) adds a chaotic random component to the inertia calculation; while Eq. (w14) represents a non-linear variance of the inertia value throughout the number of iterations.

3) Adaptive inertia weights

The adaptive inertia weight category tries to adapt the value of inertia based on parameters that provide status information of where the particles are in the search space at each time. Eqs. (w15) to (w18) are examples of adaptive inertia weight. In (w15), (w16) and (w17) the inertia weight is adapted to each particle, therefore in each iteration there as many inertia values as particles in the swarm. This adaptation in dependent on the difference between particles' current position, in each iteration, and the personal and global best positions. The inertia weight value in Eq. (w18) is different for each variable, taking into account the global best position and the value of each specific variable in the personal best position of each particle.

3.3. Strategic selection of the inertia calculation method

Given the diversity of inertia calculation methods that are available in the literature, and the importance of the appropriate definition of this parameter to ensure the proper balance between exploration and exploitation of the search space, it becomes essential to develop a suitable methodology that allows the automatic definition of the best inertia calculation method to apply, given the needs of each optimization process.

As mentioned before, the use of heuristics to solve optimization problems is crucial when a fast response is required and deterministic methods are not able to provide the optimal solution in an acceptable time frame. For this reason, the selection of the inertia calculation parameter must take into account the user's needs in terms of required execution time of the optimization process, and also in what concerns the associated expected quality of results (objective function value). In the Insert Fig. **1** is represented the flow chart of the methodology to be applied. The methodology can be applied to problems of different natures, as it is not specific to the optimization of portfolio of electricity market participation. As already mentioned, the PSO is used as a resolution method. Following the flowchart, the methodology performs 10 simulations for each inertial strategy applied. As the methodology created is intended to be versatile, it can be applied to different types of inertia not being stuck to them. Another fact that can show a greater confidence in the choice, is the times that each inertia is executed, this step is at the discretion of the user. After completing the simulations of each inertia strategy, the values for the parameters are obtained.

Insert Fig. 1

With the results of the different strategies of inertia it is possible to calculate the utility function (Eq. 9), so that it can be calculated the user must define the weights to be given to each parameter. After the utility function is calculated, the methodology will select the one that obtained the highest value.

In this way, the proposed utility function is presented in Eq. (12), which maximizes the expected added value of each inertia calculation method, taking into account three parameters: the objective function vale (9), the execution time (10), and the number of iterations (11). This utility function enables the user to define the desired balance between the three parameters, by including weights that give more or less influence to each parameter in the definition of the best solution (inertia calculation method to be applied).

$$parameter_{objective} = objective_{max} + objective_{min} + objective_{mean} - objective_{std} \quad (9)$$

$$parameter_{time} = -(time_{max} + time_{min} + time_{mean} - time_{std}) \quad (10)$$

$$parameter_{iterations} = -(iterations_{max} + iterations_{min} + iterations_{mean} - iterations_{std}) \quad (11)$$

$$\begin{aligned} Utility\ function \\ &= \max\big(parameter_{objective} \times weight_{objective} + parameter_{time} \times weight_{time} \\ &+ parameter_{iterations} \times weight_{iterations}\big) \end{aligned} \quad (12)$$

Since the target problem (optimization of electricity market players' participation portfolio) is a (profits) maximization problem, as presented in Eq. (1), the goal of the proposed utility function is to achieve the maximum objective function value in the minimum amount of execution time and in as less iterations as possible. These three parameters are calculated taking into account historical values from previous executions of the PSO approach using each inertia calculation method. The variables that are taken into account for the definition of each of the three parameters, for each inertia calculation method, are: the maximum achieved value (objective function, execution time and number of iterations, respectively), the minimum value, the mean value, and the standard deviation. While the maximum, minimum and mean achieved values assume a positive contribution towards the definition of each parameter, the standard deviation assumes a negative value (a smaller variability represents a positive evidence).

The values in Eqs. (9-12) should be normalized. The weight values that are attributed to each parameter should be in the range between [0,1] and are subject to (13).

$$weight_{objective} + weight_{time} + weight_{iterations} = 1 \quad (13)$$

The definition of the weight values allows the user to gives equal weight to the three variables, meaning that they all have the same importance in the choice of the best inertia calculation method. If a higher weight is given to one of the parameters, it means that this parameter is more important for the requirements of the current optimization, thereby, the corresponding inertia weight will be more suitable for results with that feature.

4. Experimental Findings

This section presents a case study with the goal of demonstrating the performance of the proposed methodology for the automatic selection of the best inertia calculation method. This is achieved through the application of the presented PSO approach to solve the portfolio optimization problem, as presented in section 2. For the portfolio optimization problem, five different types of markets are considered: day-ahead spot market, balancing market, bilateral contracts, forwards market, and a smart grid market (Morais et al. 2012). The historic of real electricity market prices and amounts of transacted power from the Iberian Market – MIBEL (MIBEL 2007) is used, concerning the time range from January, 2002 to October, 2012. The used data is extracted from the MIBEL website (MIBEL 2007). The data used to model the smart grid market, including the historic log of negotiations, is based on previous works from the authors (Morais et al. 2012). Using this data, the PSO approach is executed considering the following configurations, 10 number of particles maximum number of iterations equally to 10000, stopping criterion is $Sum(var(Pbest)) = 1 \times 10^{-8}$, c_1 and c_2 are 1.

4.1. Statistical analysis

Sampling values are almost always somewhat different, and the problem is whether the different observed samples actually suggest differences between populations or whether they are just random variations that can be expected between random samples from the same population. When the assumptions of normality and homoscedasticity are violated, the result of a traditional analysis of variance can not be relied upon. The non-parametric alternative for ANOVA at one criterion is the Kruscal-Wallis test.

The Kruscal-Wallis test is the nonparametric test used to compare three or more independent samples. Indicates if there is a difference between at least two of them. This is used to test the null hypothesis that all populations have equal distribution functions against the alternative hypothesis that at least two of the populations have different distribution functions. In this way it is assumed that equality of averages when equality of equal distributions exists (Theodorsson-Norheim 1986).

By the test Kruscal-Wallis it is possible to obtain the value of p = 0 that gives us indication of rejects the null hypothesis that all eighteen data samples come from the same distribution at a 1% significance level. Given the result of the test that gives the indication of the null hypothesis, the comparison between the pairs of groups is made in order to verify which of the samples differ from each other.

The Bonferroni procedure is performed in order to make the comparison in pairs. Through execution in Matlab you can get the Insert Fig. **2**. In the graph, we have represented the 95% confidence interval for all sample groups (18 inertia calculation methods). By selecting each group it is possible to see which groups differ in the value of the average, using the Bonferroni procedure.

Insert Fig. 2.

As we can see in this case, group 16 does not have any groups in which it can be said that the mean is equal, but we can say that there are 17 groups in which the mean differs significantly.

The analysis for group 3 results in Fig. 3.

Insert Fig. 3.

By analysing the graph of Insert Fig. **3**, it is possible to observe that there are 10 groups with significantly different mean values. Table II is constructed so as to uncover the results of this analysis.

TABLE II – BONFERRONI PROCEDURE

Since the p-value is equal to 1 in all these group tests, the null hypothesis where the groups are considered to have similar means with an error of 5% is accepted. Taking into account this analysis, it is concluded that the applied inertia strategies obtain different results of objective function value. We can conclude that there are similarity strategies when comparing the value of their objective function as in the case of the group (inertia strategies) 3, 6, 7, 8, 9, 11, 12 and 17, by the Bonferroni procedure.

4.2. PSO Results

In the first case the initial methodology is used where the user has the possibility of defining a larger number of parameters. The case study presented are the results of the optimization for the first period of the day. Some total of 1000 simulations has been executed using each of the inertia calculation methods presented in section 3.2.

TABLE III presents the results of PSO for the different inertia calculation methods. The results are divided by three parameters: objective function value, execution time and number of iterations. For each of the parameters, the minimum, maximum and mean values are presented, as well as the standard deviation (STD).

TABLE III - SUMMARY OF PSO RESULTS WITH THE ALTERNATIVE INERTIA CALCULATION METHODS FOR 1ST PERIOD

As can be observed by the analysis of TABLE III, regarding the objective function parameter, inertia calculation method w17 presents the best solution for maximum, mean and STD measures. However, this does not mean that, when applied, this methodology will reach the best solution, since the maximum value of min measure is achieved by w17. Analysing the execution time parameter, it is not possible to identify the best solution; however, there are two methods that detach from the rest: w16 because it presents the lowest minimum average values, and w10 because it presents the lowest maximum and STD. As expected, the execution time and the number of iterations are closely linked. By analysing the number of iterations parameter, values indicate the same best solutions: w16 and w10.

Insert Fig. **4** presents a visual comparison of the results achieved by the different inertia calculation methods regarding the objective function results, including the mean, maximum and STD values.

Insert Fig. 4.

From Insert Fig. **4** it is visible that, when considering the objective function results by themselves, it is not easy to directly choose the best inertia calculation method, since the achieved results are similar when using several of the methods. However, some of the alternatives can be easily excluded, such as w2, w10 and w16, s they present high STD values, and lower global objective function results.

TABLE IV presents the results of the proposed methodology for the automatic selection of the best inertia calculation method, with the maximum importance for the results of the objective function, by defining $weight_{objective} = 1$, and consequently only considering the objective function parameter, Eq. (9). TABLE IV shows the best five solutions.

TABLE IV – UTILITY FUNCTION RESULTS CONSIDERING ONLY THE OBJECTIVE FUNCTION PARAMETER

From TABLE IV it is visible that when opting for a solution in which the most important is the value of the objective function, then the best inertia calculation method to be used by PSO to solve the portfolio optimization problem is w12 (higher utility function value). However, it is also visible that the results achieved by the five best methods considering this parameter, are very close to each other.

Insert Fig. **5** shows the results achieved by applying the different inertia calculation methods to the PSO approach, regarding their execution time, including the mean, minimum and STD values.

Insert Fig. 5.

From Insert Fig. **5** it is visible that w2, w11 and w12 can be easily excluded, as these are, by far, the ones with the highest mean execution time. In order to be able to reach a solution for the best inertia calculation method to apply when considering only the execution time parameter, the maximum importance is given to the execution time parameter, $weight_{time} = 1$, thus applying only Eq. (10) to define the utility function results. The achieved results of the best five inertia calculation methods are presented in TABLE V.

TABLE V – UTILITY FUNCTION RESULTS CONSIDERING ONLY THE EXECUTION TIME PARAMETER

As can be seen by TABLE V, w10 is the best inertia calculation method when considering only the execution time parameter (higher utility function value). The results achieved by applying the different inertia calculation methods to the PSO approach, regarding the number of iterations required to reach the solution, including the mean, minimum and STD values can be seen graphically in Insert Fig. **6**.

Insert Fig. 6.

As can be seen by Insert Fig. **6**, w2, w3, w9 and w17 can be discarded, since they present quite high mean and STD values when compared to the other inertia calculation methods. TABLE VI presents the utility function results of the best five inertia calculation methods regarding the best inertia calculation method to apply when considering only the number of iterations parameter, *i.e.* the maximum importance is given to the number of iterations parameter, $weight_{iterations} = 1$, thus applying only Eq. (11) to define the utility function results.

TABLE VI - UTILITY FUNCTION RESULTS CONSIDERING ONLY THE NUMBER OF ITERATIONS PARAMETER

From TABLE VI it is visible that w14 is the best inertia calculation method to be applied when giving total preference for the number of iterations parameter. This result reinforces the idea of the relationship between execution time and number of iterations, since the best result from applying Eq. (10) and Eq. (11) is the same.

The combination of the three parameters is crucial to achieve a solution that most fits the user's requirements regarding the balance between objective function results, execution time, and number of iterations. TABLE VII presents the utility function results for different combinations of these parameters.

TABLE VII –UTILITY FUNCTION RESULTS FOR DIFFERENT COMBINATIONS OF WEIGHTS FOR 1ST PERIOD

Scenarios 1, 2 and 3 of TABLE VII are the ones that have been previously discussed, regarding the total preference for each of the parameters separately. In scenario 4, the weight has been distributed equally by three parameters. In this case, the best inertia calculation method is w6. This inertia calculation has achieved very good results concerning the objective function results (second best, as shown by TABLE IV), and has also required a relatively low execution time in a small number of iterations (as shown in Insert Fig. **5** and in Insert Fig. **6** respectively). Thereby, w6 may be considered the most balanced inertia calculation method. Scenarios 5 and 6 consider a slightly larger weight for the objective function in comparison to the execution time and number of iterations parameters. The best inertia calculation method resulting from these two scenarios is also w6, since in these cases, the very good results of w6 in terms of objective function results present an even larger influence over the utility function calculation. Finally, as can be seen by the results of scenario 7, with the further increase of the weight for the objective function parameter, solutions tend to select w12, because this is the best inertia calculation method considering the objective function parameter.

5. Conclusions

This paper presented a comparative study of the performance of different PSO inertia calculation methods. An adaptive methodology for the selection of the most appropriate inertia calculation method for different optimization requirements has also been proposed. The proposed method uses a utility function, which considers three main parameters: the objective function results achieved by the PSO approach when using each inertia calculation method, the associated execution time, and the number of iterations.

In this sense, what was done in this work was not to adapt an algorithm to the problem resolution, but rather to take up different algorithms, since each strategy can be considered a different algorithm, and in this sense an automatic methodology was created that allowed to select one of the strategies without them suffering any adaptation to the problem. Of course, the constraints of the problem were applied and in this way the solution would be directed to optimum, but this was done in all strategies of inertia.

Results are evaluated concerning the application of the alternative inertia calculation methods and of the proposed automatic inertia selection method to the problem of electricity market players' participation portfolio optimization. The experimental findings, using real data from the Iberian electricity market operator –MIBEL – show that the proposed method is able to accomplish its purpose, since it is able to identify the most suitable inertia calculation method depending on the defined preferences. Results show that, when the objective function result is the most important factor, the inertia that is able to achieve the better results is the one selected. On the other hand, when the preference is given to the minimization of the execution time and/or the number of iterations, the inertia calculation methods that allow the PSO approach to achieve the faster results are the chosen ones. For different combinations of parameters, the method has shown its ability to adapt its choice in order to meet the preference weight on the different parameters (e.g. by choosing inertia calculation methods that present slightly lower objective function results, but in a much faster execution time).

From the achieved results, it is also possible to conclude that the strategies belonging to the time varying group have showed better results. Additionally, it can be concluded that w6 (Oscillating Inertia Weight) is the more balanced inertia to be implemented in the PSO algorithm, since it the one that is capable of providing PSO with the means of achieving the best objective function results in the least execution time and number of iterations.

By achieving its purpose, the proposed methodology also enables the adaption of the portfolio optimization process to the requirements of AiD-EM's Efficiency and Effectiveness (2E) balance mechanism (Pinto, Morais, et al. 2015). This method automatically adapts the execution time of the decision making process of the AiD-EM decision support system, depending the user's requirements in terms of balance between execution time and quality of results.

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Conflict of interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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Figures Capitation

Fig.1 Flowchart methodology
Fig.2 Bonferroni confidence interval by 95%
Fig.3 Bonferroni procedure for group 3
Fig.4 Objective function results of the alternative inertia calculation methods
Fig.5 Execution time results of the alternative inertia calculation methods
Fig.6 Number of iterations results of the alternative inertia calculation methods

Figures

Fig. 1

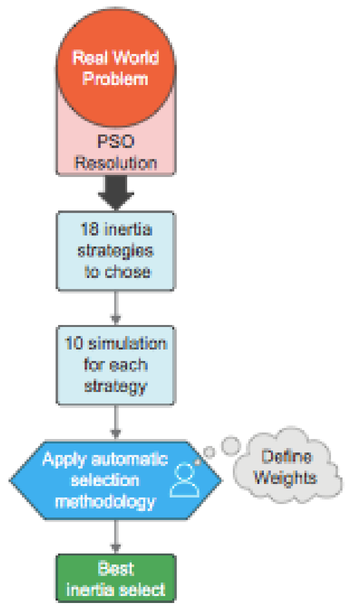

Fig.2

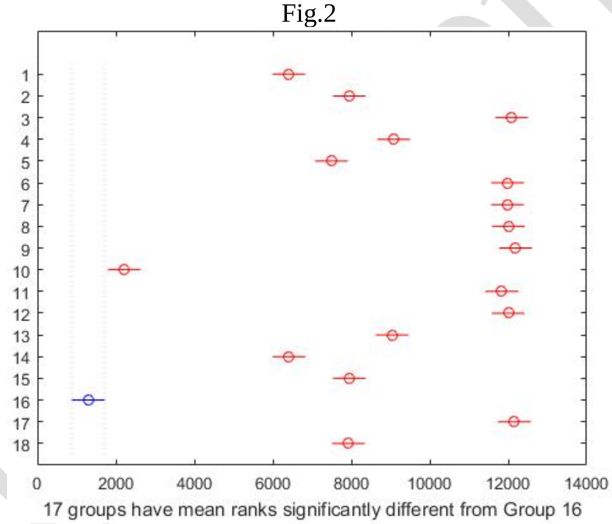

Fig.3

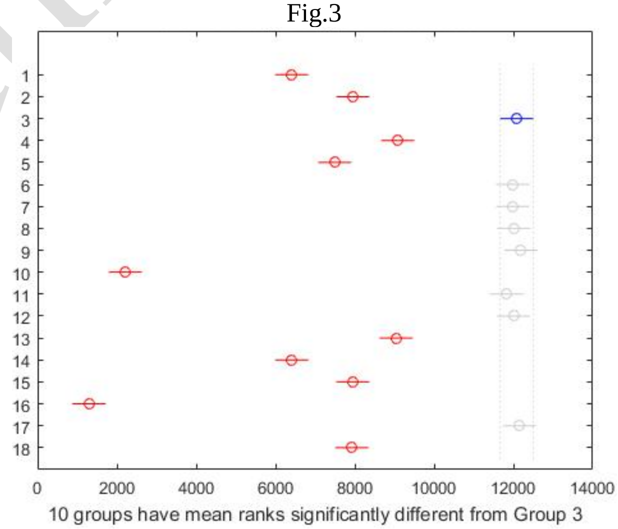

Fig. 4

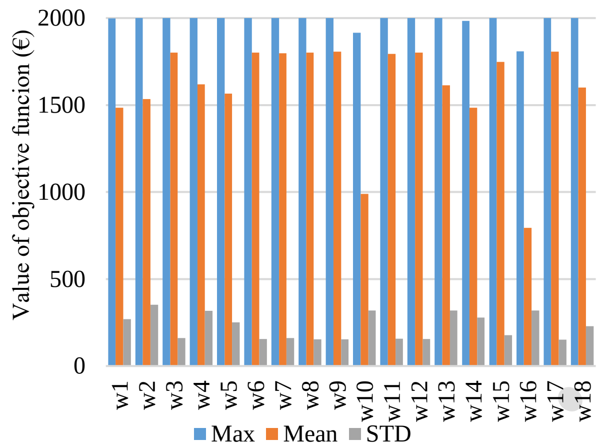

Fig. 5

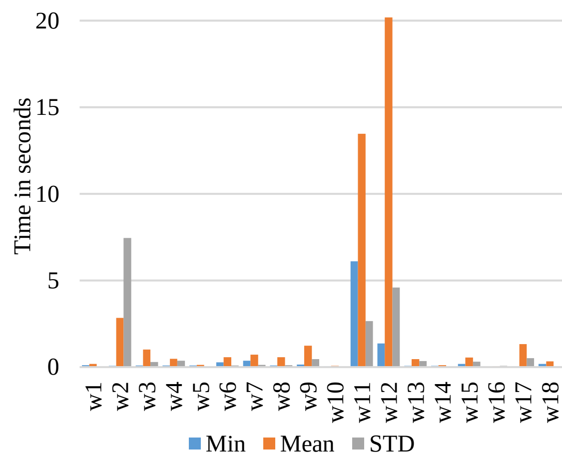

Fig.6

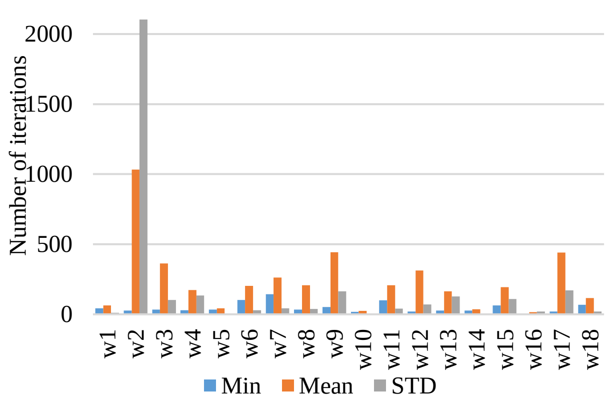

TABLE I

Name	Label	Inertia weight strategy	Source
Constant Inertia Weight	w1	$w = c$	(Shi and Eberhart 1998)
Random Inertia Weight	w2	$w = 0.5 + \frac{Rand()}{2}$	(Eberhart and Shi 2001)
Linear Decreasing Inertia Weight	w3	$w_k = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times k$	(Xin, Chen, and Hai 2009)

Name	Label	Inertia weight strategy	Source
The Chaotic Inertia Weight	w4	$z = 4 \times z \times (1 - \times)$ $w_k = (w_{max} - w_{min}) \times \frac{iter_{max} - k}{iter_{max}} + w_{min} \times z$	(Feng et al. 2007)
Simulated Annealing Inertia Weight	w5	$w_k = w_{min} + (w_{max} - w_{min}) \times \lambda^{k-1}$	(Al-Hassan, Fayek, and Shaheen 2006)
Oscillating Inertia Weight	w6	$T_k = \frac{2S_1}{3 + 2k^*}$ $w_k = \frac{w_{min} + w_{max}}{2} + \frac{w_{max} - w_{min}}{2} cos\left(\frac{2\pi k}{T_k}\right)$	(Kentzoglanakis and Poole 2009)
Exponent Decreasing Inertia Weight	w7	$w_k = (w_{max} - w_{min} - d_1) exp\left(\frac{1}{1 + \frac{d_2 k}{iter_{max}}}\right)$	(H. R. Li and Gao 2009)
Logarithm Decreasing Inertia Weight	w8	$w_k = (w_{max} - w_{min} - d_1) log_{10}\left(a + \frac{10k}{iter_{max}}\right)$	(Gao, An, and Liu 2008)
Sigmoid Increasing Inertia Weight	w9	$u = 10^{(log(iter_{max})-2)}$ $w_k = \frac{(w_{min} - w_{max})}{(1 + e^{u(k - n \times iter_{max})})} + w_{max}$	(Malik et al. 2007)
Sigmoid Decreasing Inertia Weight	w10	$u = 10^{(log(iter_{max})-2)}$ $w_k = \frac{(w_{min} - w_{max})}{(1 + e^{-u(k - n \times iter_{max})})} + w_{max}$	(Malik et al. 2007)
Natural Exponent Inertia Weight Strategy 1	w11	$w_k = w_{min} + (w_{max} - w_{min}) e^{\left(-\frac{k}{\frac{iter_{max}}{10}}\right)}$	(Chen et al. 2006)
Natural Exponent Inertia Weight Strategy 2	w12	$w_k = w_{min} + (w_{max} - w_{min}) e^{\left(-\frac{k}{\frac{iter_{max}}{4}}\right)^2}$	(Chen et al. 2006)
Chaotic Random Inertia Weight	w13	$z = 4 \times z \times (1 - \times)$ $w_k = 0.5 \times rand() + 0.5 \times z$	(Eberhart and Shi 2001)
Nonlinear time-varying Inertia	w14	$w_k = \left(\frac{2}{k}\right)^{0.3}$	(Lei, Qiu, and He 2006)
Adaptive Inertia 1	w15	$m_k^p = \frac{Gbest_k - current_k}{Gbest_k + current_k}$ $w_k^p = w_{min} + (w_{k-1} - w_{min}) \times \frac{e^{m_k^p} - 1}{e^{m_k^p} + 1}$	(Eberhart and Shi 2001)
Adaptive Inertia 2	w16	$ISA_k^p = \frac{\lvert current_k^p - Pbest_k^p \rvert}{\lvert current_k^p - Gbest_k \rvert + \varepsilon}$ $w_k^p = 1 - \alpha \left(\frac{1}{1 + e^{ISA_k^p}}\right)$	(Qin et al. 2006)

Name	Label	Inertia weight strategy	Source
Adaptive Inertia 3	w17	$w_k^p = w_{min} + (w_{max} - w_{min}) \frac{Rank_p^i}{Total\ Population}$	(Panigrahi, Ravikumar Pandi, and Das 2008)
Adaptive Inertia 4	w18	$w_k^v = 1.1 - \frac{Gbest}{average(Pbest_v)}$	(Arumugam and Rao 2008)

TABLE II

Group pairs		p-value
3	6	1
3	7	1
3	8	1
3	9	1
3	11	1
3	12	1
3	17	1

TABLE III

Inertia	Objective function				Execution time				No. iterations			
	Min	Max	Mean	STD	Min	Max	Mean	STD	Min	Max	Mean	STD
w1	571,4824	1998,601	1483,835	270,3166	0,1172	0,4369	0,1836	0,0353	42	119	64	10,9145
w2	444,8956	2000,646	1535,16	351,6517	0,0759	30,5615	2,8343	7,4635	27	10000	1033	2726,893
w3	935,0451	2000,646	1802,21	160,4235	0,0911	2,1218	1,0118	0,2879	33	790	364	101,3678
w4	448,531	2000,646	1618,602	316,5736	0,0870	1,9378	0,4865	0,3753	30	669	173	134,383
w5	665,6891	2000,305	1565,889	251,3641	0,0924	0,2722	0,1238	0,0151	33	62	43	4,2325
w6	1227,777	2000,646	1801,005	155,9684	0,2739	0,8723	0,5654	0,0856	102	292	203	28,5653
w7	1227,777	2000,646	1798,605	159,9215	0,3647	1,2876	0,7284	0,1310	144	392	262	43,5402
w8	1145,403	2000,646	1802,178	153,8245	0,0888	1,2147	0,5809	0,1116	34	342	208	37,4837
w9	1113,985	2000,646	1806,297	154,0475	0,1495	3,0664	1,2289	0,4589	53	1104	443	164,413
w10	255,5252	1915,381	989,5448	318,2123	0,0489	0,2092	0,0752	0,0127	17	41	25	2,9147
w11	1123,781	2000,646	1794,183	157,4724	6,1068	22,1113	13,4719	2,6573	99	337	208	39,7237
w12	1227,777	2000,646	1800,646	155,079	1,3630	32,8944	20,1776	4,5962	21	521	313	70,6490
w13	493,989	2000,646	1614,062	319,8066	0,0817	2,1791	0,4588	0,3529	27	701	163	126,7996
w14	473,1102	1983,957	1484,764	278,2137	0,0822	0,2753	0,1064	0,0129	28	51	37	3,3480
w15	930,7379	2000,645	1747,196	176,4664	0,1834	3,0088	0,5600	0,3146	64	972	193	108,3896
w16	77,55506	1809,204	793,7311	318,4755	0,0204	1,9284	0,0488	0,0694	6	506	16	19,4170
w17	1154,965	2000,646	1807,083	151,7726	0,0625	3,5692	1,3244	0,5119	21	1232	440	170,1553
w18	776,7735	1999,369	1600,276	229,6418	0,1841	0,5197	0,3319	0,0568	67	185	116	19,4069

TABLE IV

Inertia	w17	w6	w7	w8	w12
Equation (9)	2.42101	2.45352	2.45022	2.41246	2.4538

TABLE V

Inertia	w1	w18	w5	w10	w14
Equation (10)	-0.02059	-0.029	-0.01362	-0.00897	-0.0129

TABLR VI

Inertia	w1	w18	w5	w10	w14
Equation (11)	-0.02086	-0.03432	-0.0128	-0.00747	-0.01067

TABLE VII

Scenario	Weight	Utility Function (12)	Inertia	Equation (9)	Equation (10)	Equation (11)
1	[1 0 0]	2,4538	w12	2,4538	0	0
2	[0 1 0]	-0,00897	w10	0	-0,008974	0
3	[0 0 1]	-0,00747	w10	0	0	-0,007469
4	[0.33 0.33 0.33]	0,782847	w6	0,817841	-0,016225	-0,018769
5	[0.5 0.25 0.25]	1,200517	w6	1,226762	-0,012169	-0,014077
6	[0.7 0.15 0.15]	1,70172	w6	1,717467	-0,007301	-0,008446
7	[0.95 0.025 0.025]	2,291289	w12	2,33111	-0,037873	-0,001948