Elasticity Parameter Definition and Analysis for Real-Time Pricing Remuneration Basing on Different Users Cases

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Abstract—**In the last decade Demand Response (DR) programs have been influencing loads' profiles of electric users who participate in these programs. The evolution of the simulations able to study them brought to the possibility of defining new models that can consider power consumption profiles for different types of user (MAT, AT, MT, BTE, BTN-2, BTN-1) but, in order to better match consumption and production energy curves, highly precise predictions of loads' profiles are still needed. This goal can be achieved also thanks to the study of the price elasticity factor. A way to obtain it will be examined in this paper: price and power absorption variations will be considered because elasticity is defined as the ratio of their relative variations before and after DR. This work focuses on the profiles of price variations ΔP with respect to the absorbed power variation ΔQ: users indeed are expected to vary their consumptions according to different values of remunerations. Ranges of elasticities have been evaluated to study the behavior of ΔP profiles for the more representing users. Finally, economic consequences of load regressions have been analyzed.**

***Index Terms*— Demand response, Elasticity.**

NOMENCLATURE

e	Price elasticity of demand
P	Price
Q	Quantity
ΔQ	Variation in quantity
ΔP	Variation in price
RTP	Real Time Price
MAPE	Mean Average Percentage Error

I. INTRODUCTION

Demand Response represents either the availability of an end-use consumer to change its electric usage in response to electricity price variations over time or the incentive payments designed to lower electricity consumptions (in high market prices situations or when system reliability is jeopardized).

Responsiveness and interaction of the costumers with the grid can be promoted, giving as a result an improved reliability of the power system and, in the long term, lower peak demand and reduction of overall plant and capital cost investments [1]. Every user type is defined by its elasticity value that is necessary to define in order to get a better interaction and a full information exchange between the smart grid operator and the end-consumer.

Elasticity is defined as the availability of a user to vary its power absorption after a remuneration variation: therefore, its definition is supposed to help building scenarios able to consider the impact of the long-term use of RTP remuneration (Real Time Price). Participating in the RTP program means for the consumer to change his consumption pattern with response to real-time electricity price changes [2]. As said in [3], response of consumers to price variations should not be assumed as totally flexible since constraints as maximum load reduction, price caps, load and generation balance are present. Despite that, this paper aims to focus only on elasticity definition: for that reason, these constraints are not considered.

Households' price elasticities can be increased by the energy management technologies available in a "Smart Grid" context, giving as a result positive net welfare effects of RTP and therefore a growth of business and policy interest in it [4]. It assumes lot of importance having a real-time elasticity value able to vary according to specific factors as different user's habits during the week or weekends, weather forecasts [5], time and location [6]: by defining that, a more accurate prevision for each day of the week can be done allowing to use in the most efficient way the available power in the grid for each moment of the day. As said in [7], estimation of reactions to prices will assume lot of importance in calculating volumes of load shifting, since the actual methods will not be sufficient for the balancing authorities. Interpolations of data points have been done because they allow to estimate price variations that every user type can afford or not: every regression has been marked with its MAPE (Mean Average Percentage Error) in order to define which interpolation works better. In every user profile MAPE value related to logarithmic interpolation is far greater than the other regressions, meaning that this type of interpolation is less reliable than the others.

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For this reason, logarithmic fit will be shown only for the BTN-1 consumer type. Interpolations have been realized with Microsoft Excel software. In order to plot ΔP values in function of ΔQ, data from [8] were used. Prices before and after the DR program have been calculated and the same was done with power absorptions. Starting from each graph, 4 regression lines have been plotted and their equations represented. For each one of them, new ΔP points have been calculated in order to compute MAPEs in relation to the original data set. The same procedure has been made for every interpolation line: linear, quadratic, cubic, logarithmic. To study how ΔP profiles change as the elasticity values do, some range of elasticities have been computed. Indeed, if ΔP profiles are affordable then a good long-run estimation can be made. It assumes importance then to consider new possible values of elasticity since the ones used until now are either "set/assumed" [9] and their reliability is limited to simulations.

II. METHOD

In order to define price remunerations profiles with more accuracy, some scenarios with different ranges of elasticities have been implemented. Some user types have been characterized by a different elasticity range according to their actual availability of joining DR programs. Table 1 shows the center values and the intervals for the analyzed consumers.

TABLE I: ELASTICITY VALUES RANGES FOR EACH CONSUMER TYPE.

User	MAT	AT	MT	BTE	BTN-2	BTN-1
e value	0.53	0.45	0.41	0.37	0.33	0.27
range	±0.03	± /	± /	± /	±0.02	±0.02

After computing ΔP values based on new elasticity values, the following increases (positive sign) and decreases (negative sign) have been found (Table II). It can be seen how different variations can be achieved by switching from an elasticity value to the next one, indicating how important is to well-define that parameter. Before analyzing the behavior of the interested users, it is worth of notice how it is possible to get the elasticity value from specific graphs. Since elasticity is defined as the ratio of the relative quantity variation over the relative price variation before and after DR, as reported in (1), elasticity assumes a geometrical meaning in the graphs. A plot that represents a ΔP/P trend over ΔQ/Q is characterized by an angular coefficient that is the reciprocal of the elasticity parameter (2). In Fig.1 it is shown a representation of the method used to estimate ΔP values using the best interpolation.

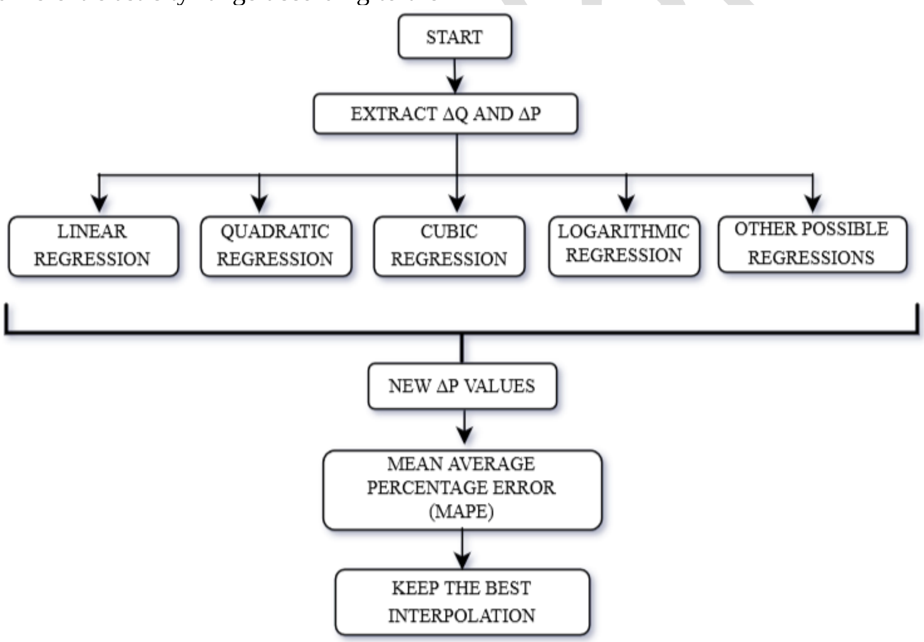

Fig.1: Scheme of the adopted method.

TABLE II: PRICE VARIATIONS FOR SOME CONSUMERS WITH RESPECT TO DIFFERENT ELASTICITIES VALUES.

MAT						
elasticities	0.50	0.51	0.52	0.54	0.55	0.56
ΔP x10^{-7}	-5.7588050638	-5.7588051395	-5.75287181	1203.6113272	1808.2963933	2412.981459447
BTN-2						
elasticities	0.31	0.32		0.34	0.35	
ΔP x10^{-7}	0.00389105958	-0.0022601286		0.000000006587096	9.269918E-8	
BTN-1						
elasticities	0.25	0.26		0.28	0.29	
ΔP x10^{-7}	15689.6879955	18414.8020269		15689.6879955825	15689.6879956	

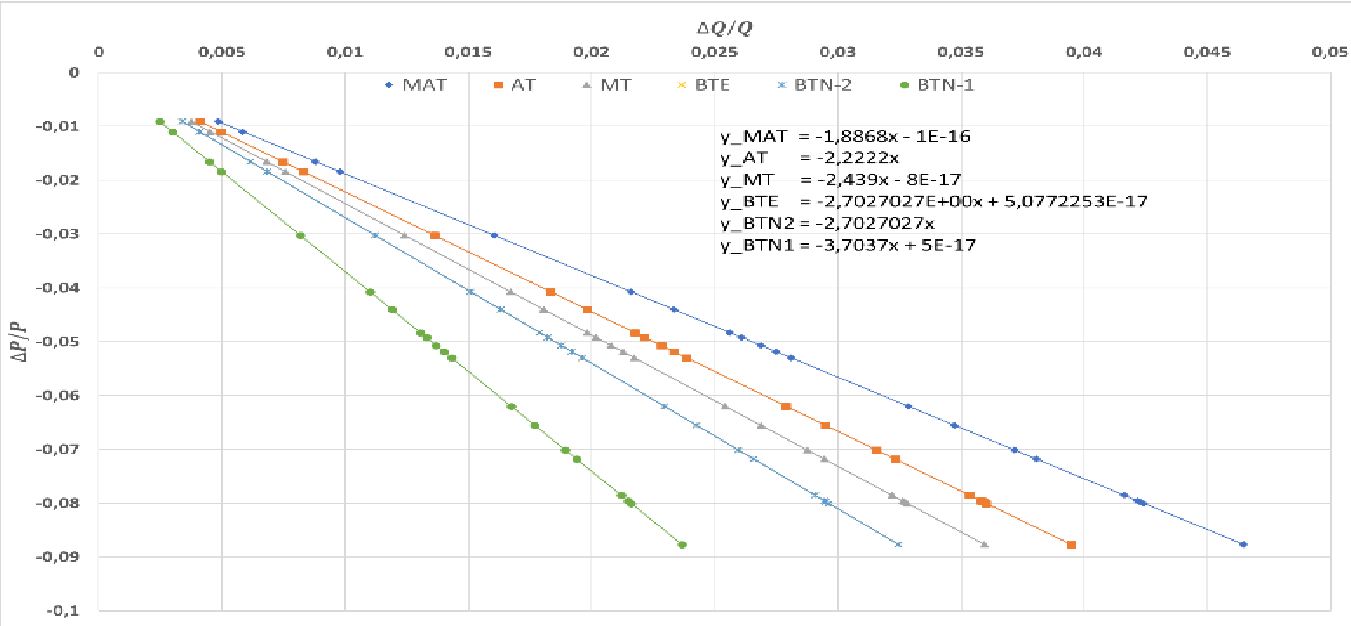

Fig.2: Plot of ΔP/P over ΔQ/Q for every user with the relative equations.

$$e = \frac{\Delta Q}{Q} \Big/ \frac{\Delta P}{P} \tag{1}$$

$$\text{slope} = \frac{\Delta P}{P} \Big/ \frac{\Delta Q}{Q} = \frac{1}{e} \tag{2}$$

$$\text{MAPE} = \frac{100\%}{n} \sum_{t=1}^{n} \frac{|At - Ft|}{At} \tag{3}$$

The reliability of this formula is demonstrated starting from the plot in Fig.2. It must be said that since BTE and BTN-2 users' consumptions are the same, their respective lines are overlying. The slope of every line is shown as the angular coefficient in each equation: according to the previous consideration their reciprocals are the elasticity values for each case. In fact, all elasticity values previously given (Tab.1) are obtained as shown in Tab. 3. As can be seen, computed values fit with good precision the given ones. A note about the BTN-2 user type must be said since its value doesn't match the given one. A possible cause is related to its power absorption values that are perfectly equal to the BTE user (that explains why they have the same elasticity values) despite they are two different types of users. In order to explain the minus signs of slopes, it's important to remind that ΔP is given by the difference of final price P_{fin} (after DR) minus initial price P_{in} (before DR): since price decreases, ΔP assumes negative sign.

TABLE III: LINES' SLOPES AND THEIR RECIPROCAL FOR EACH USER

	Slope	*|e|*	*Original e value*
MAT	-1.8868	0.5299981	0.53
AT	-2.2222	0.4500045	0.45
MT	-2.4392	0.4100041	0.41
BTE	-2.7027	0.3700000	0.37
BTN-2	-2.7027	0.3700000	0.33
BTN-1	-3.7037	0.2700003	0.27

As explained in [9],negative sign of price elasticity indicates that consumption reduces with the increase in the prices. In Tab.3 the absolute values of elasticities are reported in order to do an easier comparison with the input data (in the last column).

III. Data And Results

As said, ΔP plots in function of ΔQ quantity for each user have been made. In order to judge if a prediction is affordable or not, MAPE has been calculated for different type of interpolation. As said in [10], values under 10 are to be considered of highly accurate forecasting. The software used is Microsoft Excel and the following method was used: starting from the real data of ΔP and ΔQ, plots of ΔP in function of ΔQ were made. Starting from these graphs, linear, quadratic and cubic regression lines were graphed, and new points have been plotted thanks to the regression line equation. In two cases (BTE and BTN-1 users) logarithmic regression line was also calculated. As will be shown, MAPE value indicates that this interpolation is less accurate than the other three, therefore it has not been calculated for the other consumer types. MAPE formula, expressed by (3), considers the actual values (called "At") and the forecast values (called "Ft") both averaged on the total number n of elements. Each forecast value is obtained from the equation of the interpolation line that is written in the corner of the plots.

$$\mathbf{MAPE} = \frac{\mathbf{100\%}}{\mathbf{n}} \sum_{\mathbf{t=1}}^{\mathbf{n}} \frac{\mathbf{|At - Ft|}}{\mathbf{At}} \tag{3}$$

A. *MAT Consumer*

The three interpolations for MAT consumer are here presented.

B. *Other Consumer types*

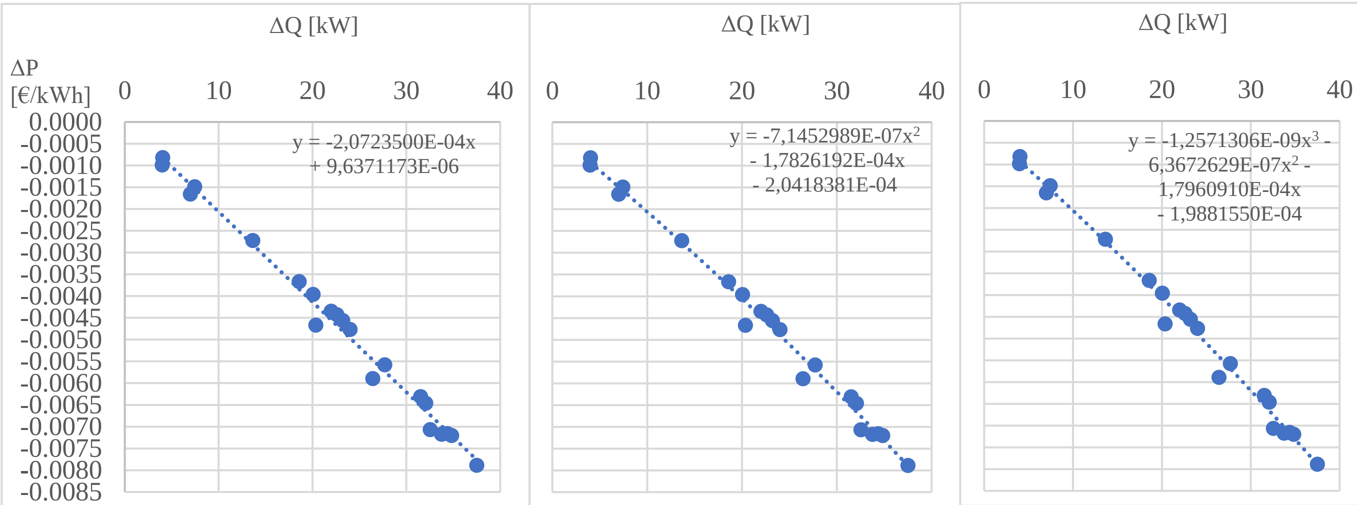

Fig.3: Linear, quadratic and cubic interpolations of BTE user type with their respective equations.

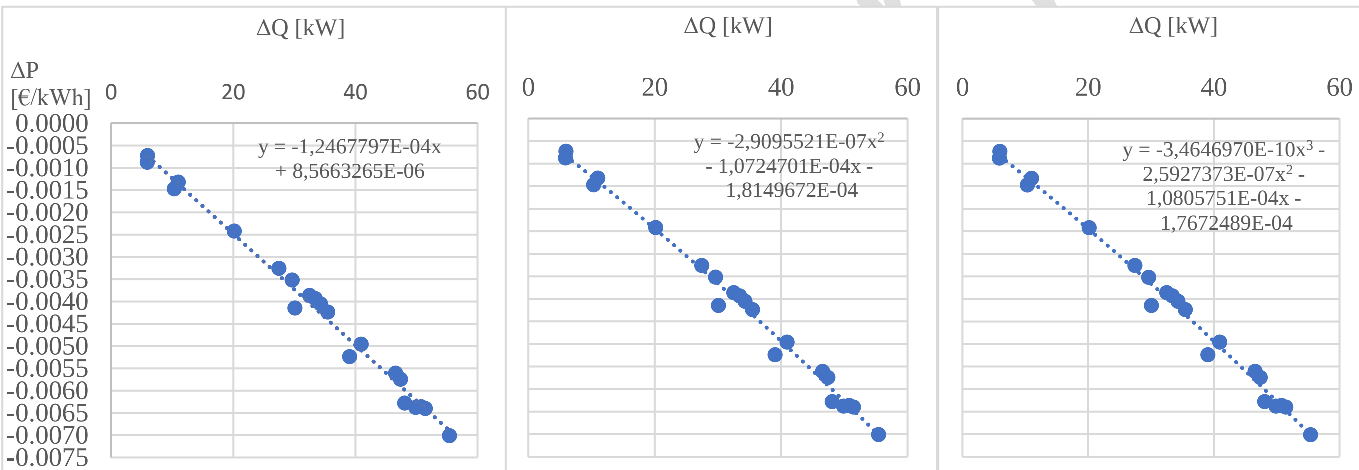

Fig.4: Linear, quadratic and cubic interpolations of MT user type with their respective equations.

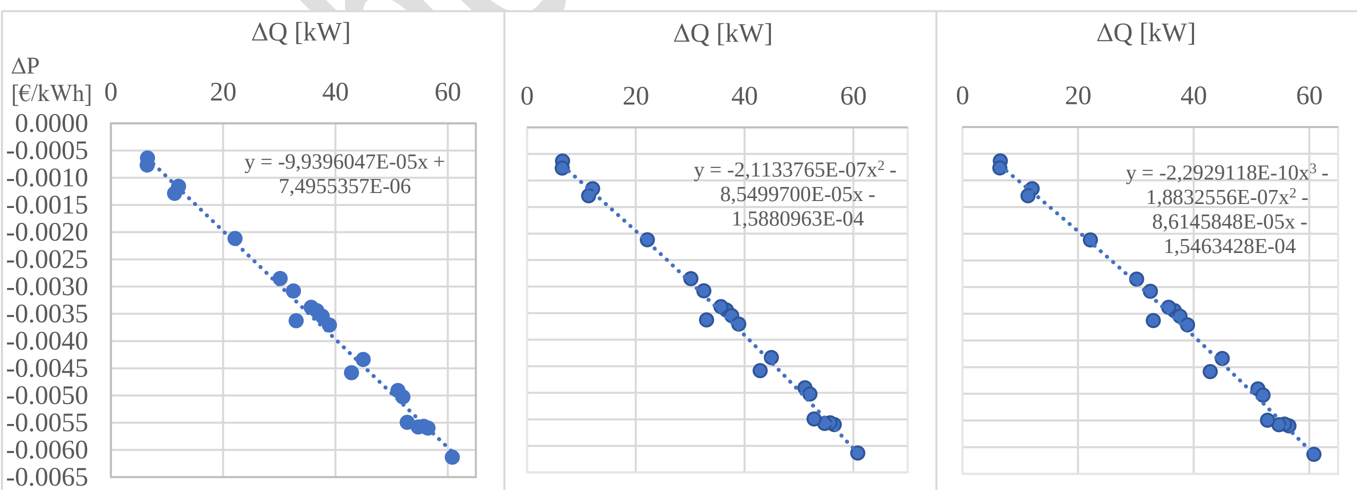

Fig.5: Linear, quadratic and cubic interpolations of AT user type with their respective equations.

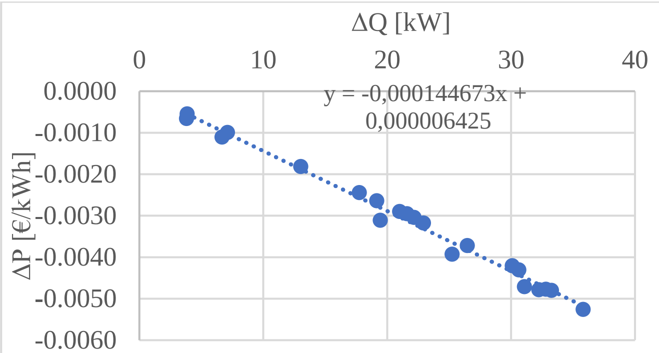

Fig.6: ΔP over ΔQ (points) and their 1st grade regression line.

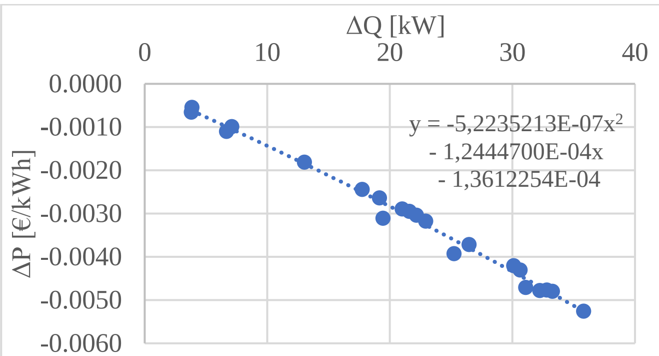

Fig.7: ΔP over ΔQ (points) and their 2nd grade interpolation line.

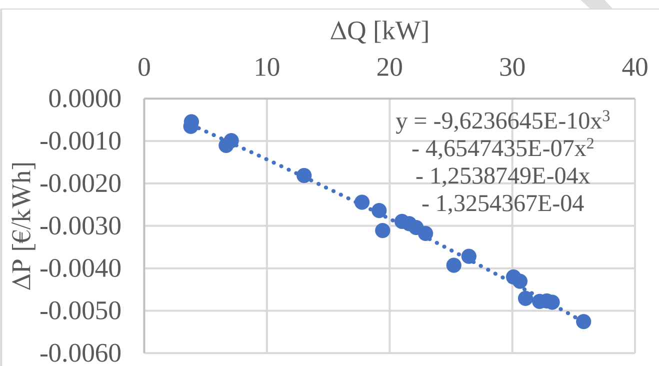

Fig.8: ΔP over ΔQ (points) and their 3rd grade interpolation line.

C. MAPEs values and BTN-1 logarithmic interpolation

Table IV shows MAPE values for all users calculated for each interpolation done. All values are under 10, meaning that each interpolation can be considered accurate. For higher voltage levels quadratic interpolations are slightly more accurate: that's the proof that a higher interpolation grade (that brings higher computational costs) doesn't bring necessarily to smaller errors.

TABLE IV: MAPES VALUES FOR ALL USERS IN EACH INTERPOLATION.

Mapes	Linear	Quadratic	Cubic
MAT	4.88944	4.3930201	4.396962
AT	4.88417	4.3930200	4.396961
MT	4.88417	4.3930201	4.396961
BTE	4.88417	4.3930211	4.396960
BTN-2	4.88417	4.5605391	4.233762
BTN-1	4.88417	4.3930196	4.396961

For the BTN-1 user only, a logarithmic interpolation has been studied too. As one could predict, this kind of fit is not as good as the others (Fig.9). For that reason, the plot has been reported only for this type of consumer. In this case MAPE=21.1365. Instead, as can be seen in all cases, linear, quadratic and cubic regressions represent a good solution to fit this kind of data.

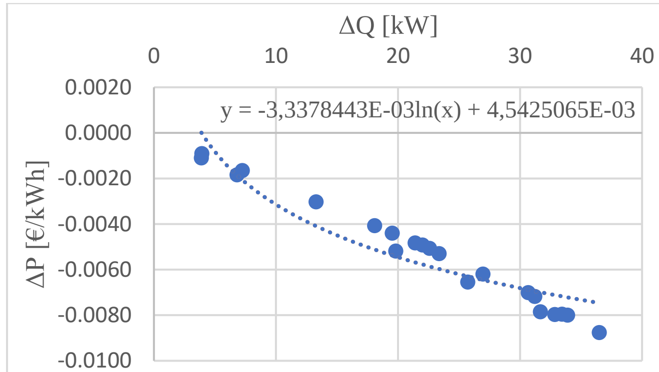

Fig.9: ΔP over ΔQ (points) and their logarithmic interpolation line.

IV. Data Evaluation Method

It has been showed how consumption-price points can be interpolated in different ways. Those interpolations bring errors, that may result in wrong estimations of the power available for the D.N.O. (Distribution Network Operator) or different price remunerations expected by end-users. Since a model able to represent and predict data is needed, a useful operation is to valuate which regression brings the smallest error and how much it influences DR programs both in terms of power and remuneration as mentioned above. As demonstrated in the previous section, logarithmic interpolation should not be used due to its high MAPE. For that reason, a comparison only between the first, second and third grade interpolations has been made: for each case it was assumed that the DNO wanted to remunerate the end-user with a specific amount; a comparison between implemented powers in each case was studied. The adopted method is shown in figure 10. Starting from the equations of ΔQ(ΔP) (the inverse plots of the showed previously, fig.11), a value of ΔP taken from the original data was used as input for the equations. Each case brought a different value of ΔQ, as shown in table 5. Those values represent the actual power the D.N.O. receives from DR users. The chosen reference value of ΔP in this case is -0,003930763045 while -0,0039 and -0,0040 [€/kWh] values have been considered as inputs. An incrementation of 2,6% of ΔP (0,0001 €/kWh, i.e. from 0,0039 to 0,0040) brings to an incrementation of average deviation of 36,2%.

A more important consideration concerns the actual available power. As previously said, D.N.O. expects to have precise quantities of power available: due to interpolations of load profiles, that quantity can be different from the theoretic one. This brings either to difficult real-time power management or different values of remunerations ΔP, depending on D.N.O.'s choice to use ΔP or ΔQ as inputs respectively. For example, taking ΔP=0,004 [€/kWh] as study case, an average deflection of |ΔQ|=2,4667 [MW] from the real values causes a loss of 545,88 [€/h] (applying the highest tariff scenario).

TABLE V: ΔQ WITH DEVIATIONS FROM REAL ΔQ.

ΔP = -0,0039 [€/kWh]	**ΔQ**	**Deviation from original ΔQ [MW]**
Linear interpolation	26,94652527	-1,6940980211
Quadratic interpolation	27,16229970	-1,9098724593
Cubic interpolation	27,08780488	-1,8353776303
ΔP = -0,0040 [€/kWh]	**ΔQ**	**Deviation from original ΔQ [MW]**
Linear interpolation	27,63050168	-2,3780744329
Quadratic interpolation	27,80148071	-2,5490534669
Cubic interpolation	27,72538433	-2,4729570872

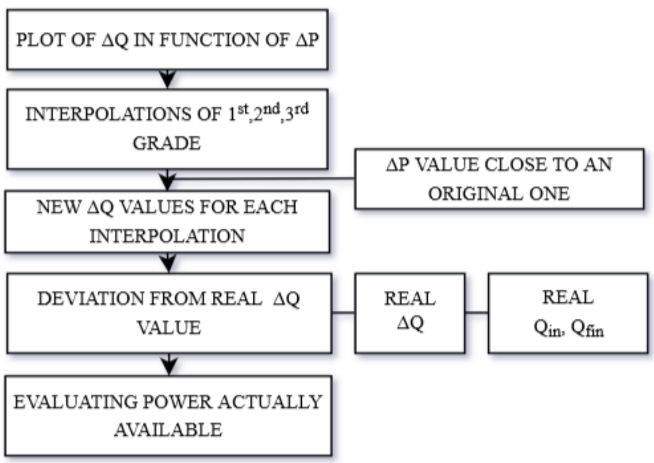

Fig.10: Representation of the method adopted to evaluate money loss.

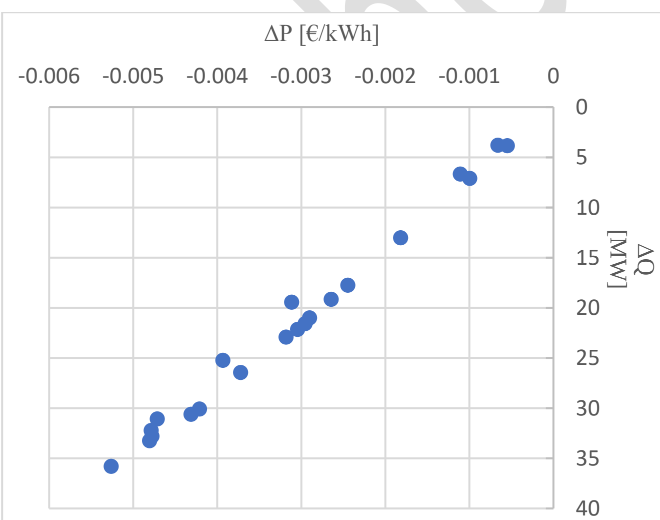

Fig.11: Example of plot of ΔQ in function of ΔP.

V. CONCLUSIONS

This paper introduced the elasticity factor explaining why its definition will assume importance in future scenarios. A good estimation of elasticity indeed allows to deal with environmental, technical and social issues relative to the production and consumption of electric energy. In addition, "there are a number of factors that have triggered increasing interest in this matter over the recent years: energy deregulation; a large increase in the price of certain primary energy products; policies to correct the environmental damage caused by energy (in particular, those related to global warming); and the growing promotion of energy efficiency" [11]. A way to obtain elasticity parameter by analyzing price-power variations graphs has been explained, showing its reliability by using real elasticity values as a comparison. Finally, 3 types of data interpolations have been computed for each type of consumer: each one has been characterized by its MAPE value in order to judge in an objective way the best possible interpolation. There's not a sensible difference between MAPE values, especially between close voltage levels users. At the end, a single plot dedicated to the logarithmic fit has been computed, demonstrating how bad this fit is for this kind of data.

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