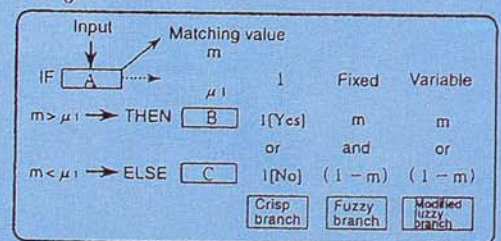
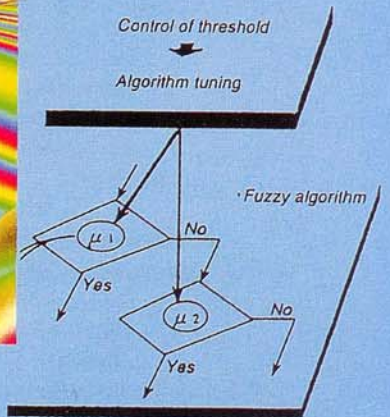
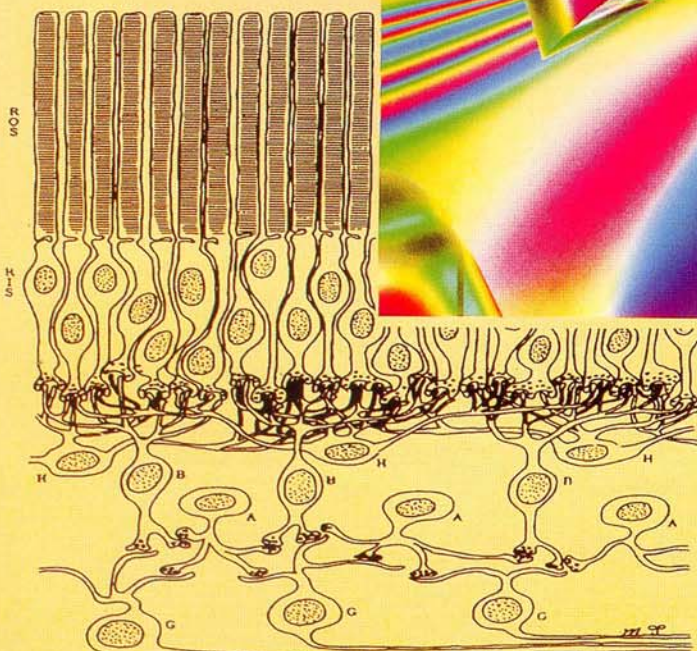
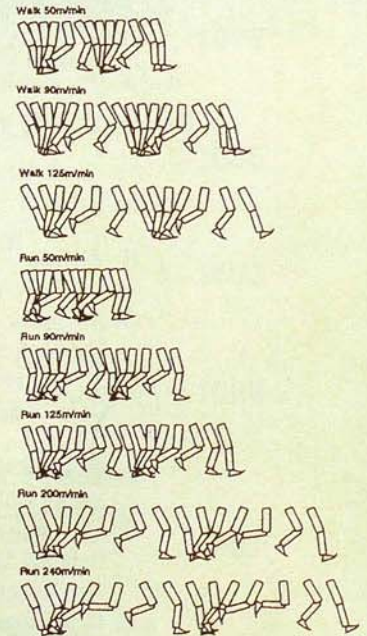
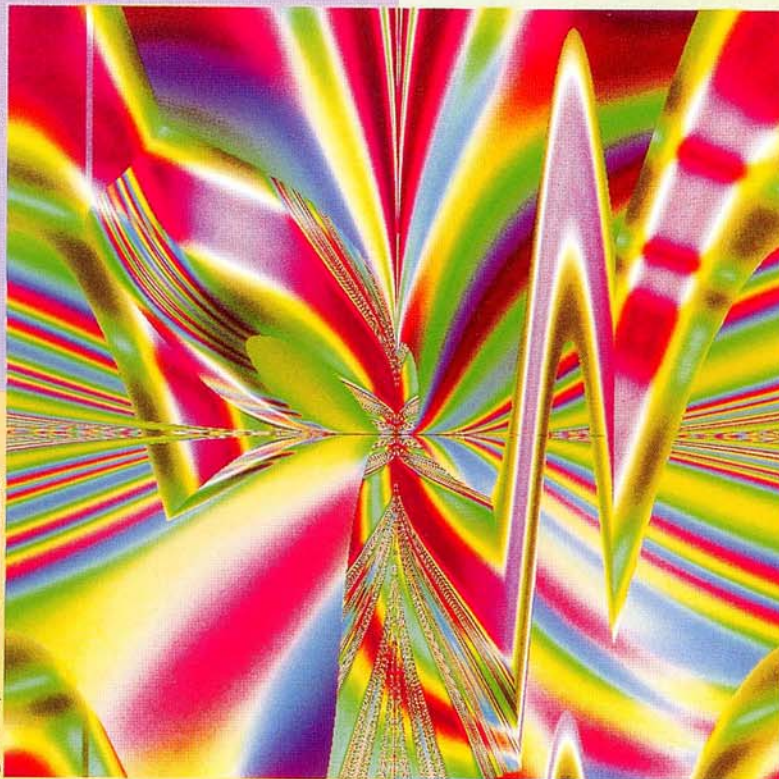
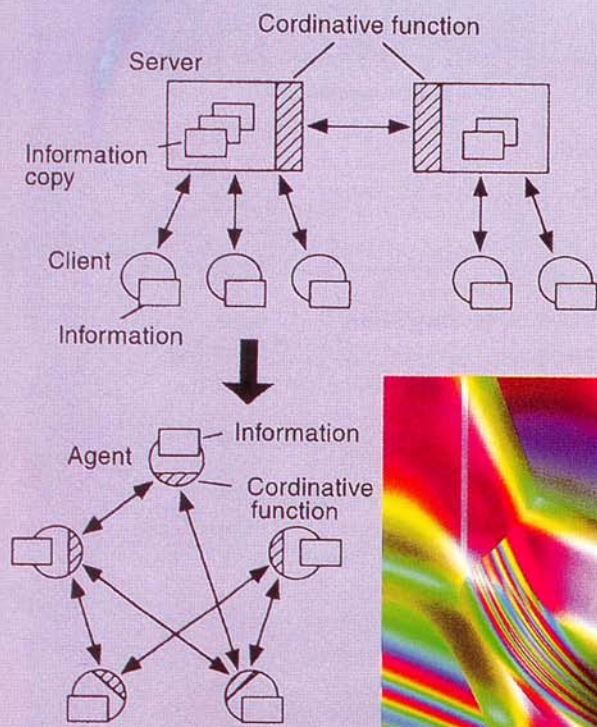


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Paper:

Fractional Control of Coordinated Manipulators

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When two robots execute a coordinated motion it is required specification not only of the desired trajectory of each robot, but also of the forces exerted by the end effectors. This article discusses the fractional-order position and force control of two co-operative robots handling one object. The system robustness and performance is analyzed and compared with other control approaches. The experiments reveal that fractional algorithms lead to performances superior to classical integer-order controllers.

Keywords: control, fractional calculus, robotics, cooperating robots

1. Introduction

Two robots carrying a common object are a logical alternative to the case in which a single robot is not able to handle the load. The choice of a robotic mechanism depends on the type of work to be performed and, consequently, is determined by the position of the robots and by their dimensions and structure. In general, the selection is done through experience and intuition, but it is important to measure the manipulation capability of the robotic system [1] useful in the robot operation. In this perspective it was proposed the concept of manipulability [2, 3] or, the statistical evaluation of manipulation [4]. Other related aspects such as the coordination of two robots handling an object, collision avoidance and the path planning have been also investigated [5].

With two cooperative robots the resulting interaction forces have to be accommodated and consequently, in addition to position feedback, force control is also required to accomplish adequate performances [6, 7]. There are two basic methods for force control, namely the hybrid position/force and the impedance schemes. The first method [8] separates the task into two orthogonal subspaces corresponding to the force and the position controlled variables. Once established the subspace decomposition two independent controllers are designed. The second method [9] requires the definition of the arm mechanical impedance. The impedance accommodates the

interaction forces that can be controlled to obtain an adequate response. This paper addresses the control of two arm system, using fractional-order (*FO*) algorithms [10–13] inserted in position/force cascade structure.

Bearing these facts in mind this article is organized as follows. Section two presents the controller architecture for the position/force control of two arms. Based on these concepts, section three develops several experiments for the time analysis and the performance evaluation of *FO* and classical controllers, for robots having several types of dynamic phenomena at the joints. Finally, section four presents the main conclusions.

2. Modeling and Control

The dynamics of a robot with n links interacting with the environment is modeled as:

$$\mathbf{T} = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) - \mathbf{J}^T(\mathbf{q})\mathbf{F} + \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} \quad . . . \quad (1)$$

where \mathbf{T} is the $n \times 1$ vector of actuator torques, \mathbf{q} is the $n \times 1$ vector of joint coordinates, $\mathbf{H}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is the $n \times 1$ vector of centrifugal / Coriolis terms and $\mathbf{G}(\mathbf{q})$ is the $n \times 1$ vector of gravitational effects. The matrix $\mathbf{J}^T(\mathbf{q})$ is the transpose of the Jacobian and \mathbf{F} is the force that the load exerts in the robot gripper. For a simple *RR* manipulator ($n = 2$) the dynamics yields:

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -m_2 r_1 r_2 S_2 \dot{q}_2^2 - 2m_2 r_1 r_2 S_2 \dot{q}_1 \dot{q}_2 \\ m_2 r_1 r_2 S_2 \dot{q}_1^2 \end{bmatrix} \quad . \quad (2)$$

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} g(m_1 r_1 C_1 + m_2 r_1 C_1 + m_2 r_2 C_{12}) \\ g m_2 r_2 C_{12} \end{bmatrix} \quad . \quad (3)$$

$$\mathbf{J}^T(\mathbf{q}) = \begin{bmatrix} -r_1 S_{12} - r_2 S_{12} & r_1 C_{11} + r_2 C_{12} \\ -r_2 S_{12} & r_2 C_{12} \end{bmatrix} \quad . \quad (4)$$

$$\mathbf{H}(\mathbf{q}) = \begin{bmatrix} (m_1 + m_2)r_1^2 + m_2 r_2^2 & m_2 r_2^2 \\ +2m_2 r_1 r_2 C_2 & +m_2 r_1 r_2 C_2 \\ +J_{1m} + J_{1g} & \\ +m_2 r_2^2 & m_2 r_2^2 \\ +m_2 r_1 r_2 C_2 & +J_{2m} + J_{2g} \end{bmatrix} \quad . \quad (5)$$

where $C_{ij} = \cos(q_i + q_j)$ and $S_{ij} = \sin(q_i + q_j)$.

We consider two *RR* robots with identical dimensions (**Fig. 1**). The contact of the robot gripper with the load is

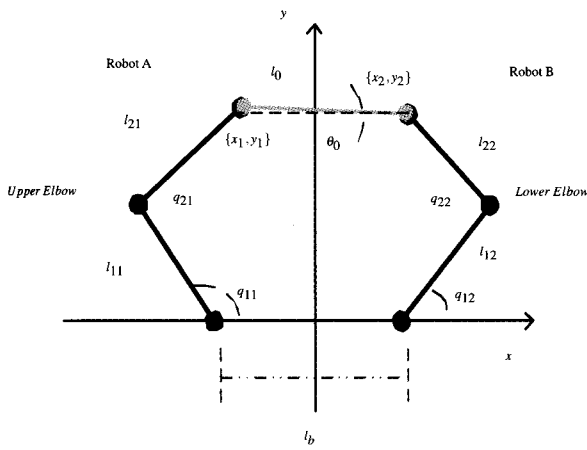


Fig. 1. Two RR robots, with distance l_b between the shoulders, working in cooperation for the manipulation of an object with length l_0 and orientation θ_0 .

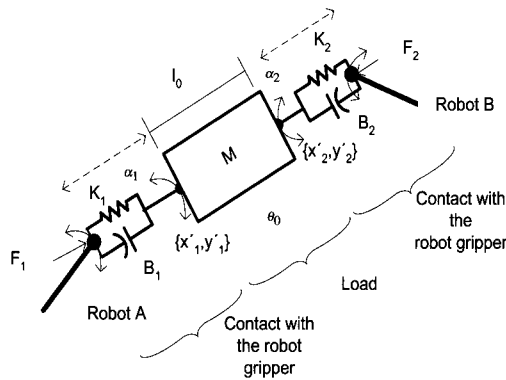


Fig. 2. The contact between the robot gripper and the object.

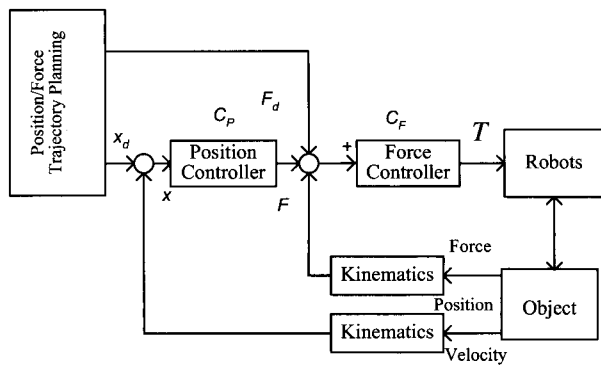


Fig. 3. The position/force cascade controller.

modeled through a linear system (Fig. 2) with a damping B and a stiffness K . The numerical values adopted for the robots and the object are $m_1 = m_2 = 1.0$ kg, $l_1 = l_2 = l_b = l_0 = 1.0$ m, $\theta_0 = 0^\circ$, $B_1 = B_2 = 1$ Nsm $^{-1}$ and $K_1 = K_2 = 10^4$ Nm $^{-1}$.

The controller architecture (Fig. 3), consisting on a cascade controller, is inspired on the impedance and compliance schemes. Therefore, we establish a cascade of force and position algorithms as internal an external feedback

Table 1. The parameters of the FO controllers.

(a) Position controller			
i	K_p	K_α	α
1	0.1259	1.55×10^{-3}	$\frac{1}{2}$
2	0.1259	1.55×10^{-3}	$\frac{1}{2}$
(b) Force controller			
i	K_p	K_β	β
1	10.59	2.0×10^{-3}	$-\frac{1}{5}$
2	10.59	2.0×10^{-3}	$-\frac{1}{5}$

Table 2. The parameters of the PD – PI controller.

(a) Position controller		
i	K_p	K_d
1	25.0×10^3	25.0×10^1
2	25.0×10^3	25.0×10^1
(b) Force controller		
i	K_p	K_i
1	5.0×10^2	10.0×10^2
2	5.0×10^2	10.0×10^2

loops, respectively, where x_d and F_d are the payload desired position coordinates and contact forces.

In the position and force control loops we consider FO controllers [8, 10] of the type $C(s) = K_p + K_\alpha s^\alpha$, $-1 < \alpha < 1$, that are approximated by 4th-order discrete-time Pade expressions ($a_i, b_i \in R, k = 4$):

$$C(z) \approx K \frac{a_0 z^k + a_1 z^{k-1} + \dots + a_k}{b_0 z^k + b_1 z^{k-1} + \dots + b_k}, \quad K > 0. \quad (6)$$

To analyze the system performance we consider not only robots with ideal transmissions, but also robots with joint having backlash, flexibility and nonlinear friction. Moreover, we compare the response of FO and classical PD – PI (P-Proportional, D-Derivative, I-Integral), algorithms:

$$C(s) = K_p + K_d s \quad (7)$$

$$C(s) = K_p + K_i \frac{1}{s} \quad (8)$$

for the position and force loops, respectively.

Both algorithms were tuned, having in mind getting almost similar performances in the two cases (Tables 1 and 2). In order to study the system dynamics we apply separately small amplitude rectangular pulses δy_d and $\delta F y_d$, separately, at the position and force references, and we analyze the system response.

The experiments adopt a controller sampling frequency $f_c = 10$ kHz, a contact forces of the grippers $\{F x_j, F y_j\} \equiv \{0.5, 5\}$ Nm, $j = \{A, B\}$, a operating point of the center of the object $\{x, y\} \equiv \{0, 1\}$ m and an object orientation $\theta_0 = 0^\circ$.

In a first phase we consider robots with ideal transmissions at the joints. In a second phase we analyze the re-

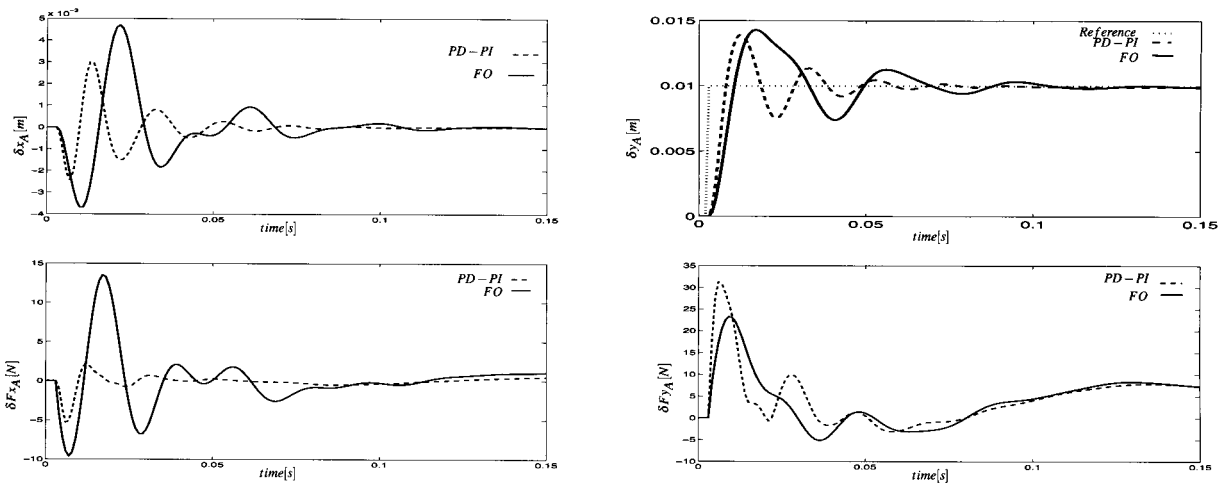


Fig. 4. Time response of robot A with ideal joints, under the action of the FO and the PD-PI algorithms, for a pulse perturbation at the position reference $\delta y_d = 0.1$ m and a payload with $M = 1$ kg, $B_i = 10$ Nsm⁻¹ and $K_i = 10^3$ Nm⁻¹.

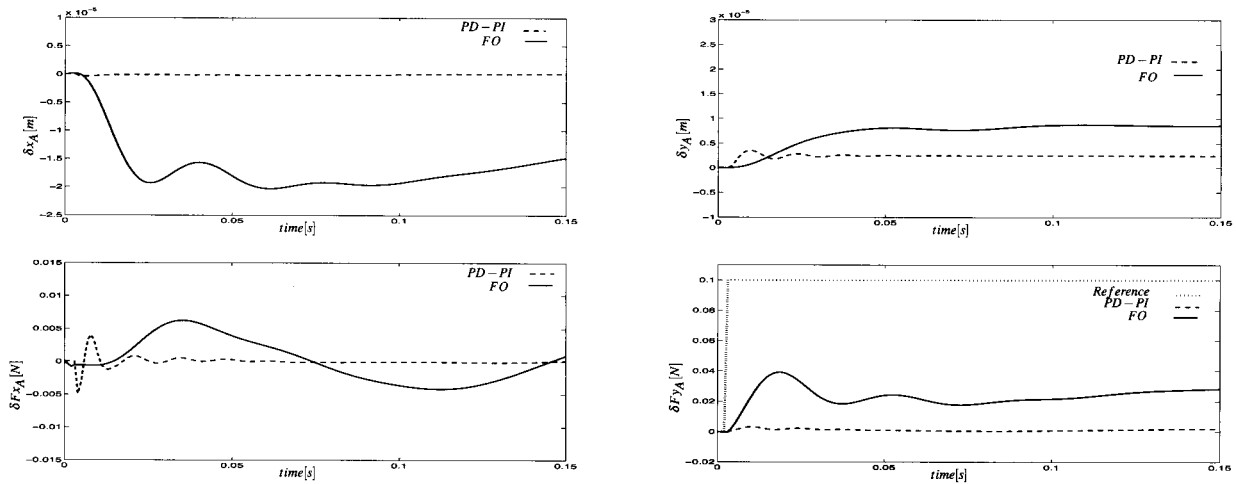


Fig. 5. Time response of robot A with ideal joints, under the action of the FO and the PD-PI algorithms, for a pulse perturbation at the force reference $\delta F_{y_d} = 0.1$ N and a payload with $M = 1$ kg, $B_i = 10$ Nsm⁻¹ and $K_i = 10^3$ Nm⁻¹.

sponse of robots with dynamic backlash at the joints. For the i^{th} joint gear ($i = 1, 2$), with clearance h_i , the backlash reveals impact phenomena between the inertias, which obey the principle of conservation of momentum and the Newton law:

$$\dot{q}'_i = \frac{\dot{q}_i(J_{ii} - \epsilon J_{im}) + \dot{q}_{im}J_{im}(1 + \epsilon)}{J_{ii} + J_{im}} \dots \dots \dots (9)$$

$$\dot{q}'_{im} = \frac{\dot{q}_i J_i(1 + \epsilon) + \dot{q}_{im}(J_{im} - \epsilon J_{ii})}{J_{ii} + J_{im}} \dots \dots \dots (10)$$

where $0 \leq \epsilon \leq 1$ is a constant that defines the type of impact ($\epsilon = 0$ inelastic impact, $\epsilon = 1$ elastic impact). the variables \dot{q}_i and \dot{q}_{im} (\dot{q}'_i and \dot{q}'_{im}) represent the velocities of the i^{th} joint and motor before (after) the collision, respectively. The parameter J_{ii} (J_{im}) stands for the link (motor) inertias of joint i . The numerical values adopted are $h_i = 1.8 \times 10^{-4}$ rad and $\epsilon_i = 0.8$.

In a third phase we study the RR robot with compliant joints. For this case the dynamics corresponds to model (1) augmented by the equations:

$$\mathbf{T} = \mathbf{J}_m \ddot{\mathbf{q}}_m + \mathbf{B}_m \dot{\mathbf{q}}_m + \mathbf{K}_m (\mathbf{q}_m - \mathbf{q}) \dots \dots \dots (11)$$

$$\mathbf{K}_m (\mathbf{q}_m - \mathbf{q}) = \mathbf{J}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) \dots \dots (12)$$

where \mathbf{J}_m , \mathbf{B}_m and \mathbf{K}_m are the $n \times n$ diagonal matrices of the motor and transmission inertias, damping and stiffness, respectively. In the simulations we adopt $K_{mi} = 2.0 \times 10^6$ Nm·rad⁻¹ and $B_{mi} = 10^4$ Nms·rad⁻¹ ($i = 1, 2$).

In a fourth phase we study the inclusion of nonlinear friction of the robot joints, given by the expression.

$$T_{friction} = \begin{cases} B_{ai} + K_{ai} \dot{q}_i & q_i > 0 \\ 0 & q_i = 0 \\ B_{ai} - K_{ai} \dot{q}_i & q_i < 0 \end{cases} \dots \dots \dots (13)$$

where the parameters, B_{ai} , K_{ai} ($i = 1, 2$) represent the viscosity coefficient and the Coulomb friction, respectively.

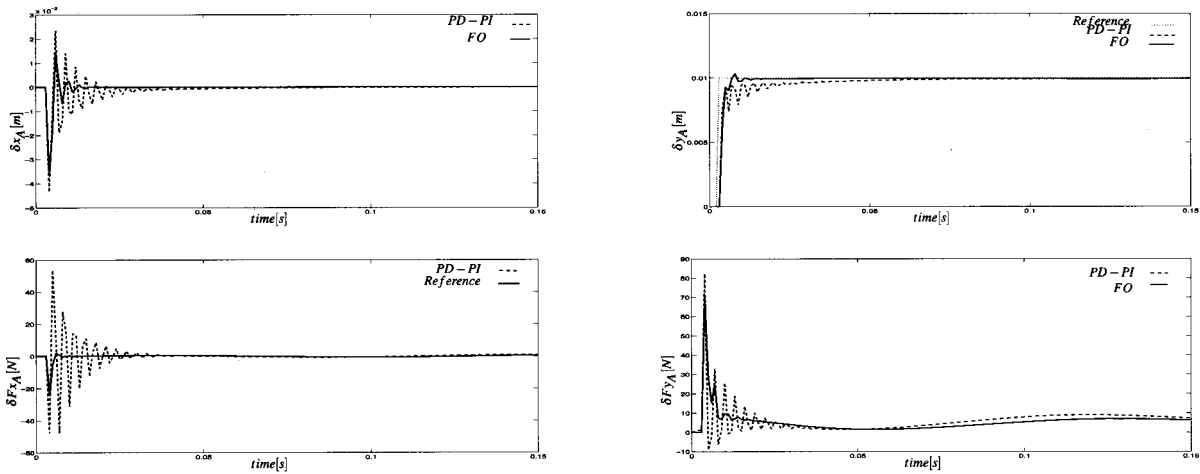


Fig. 6. Time response of robot A with joints having backlash, under the action of the *FO* and the *PD-PI* algorithms, for a pulse perturbation at the position reference $\delta y_d = 10^{-3}$ m and a payload $M = 1$ kg, $B_i = 1$ Nsm $^{-1}$ and $K_i = 10^3$ Nm $^{-1}$.

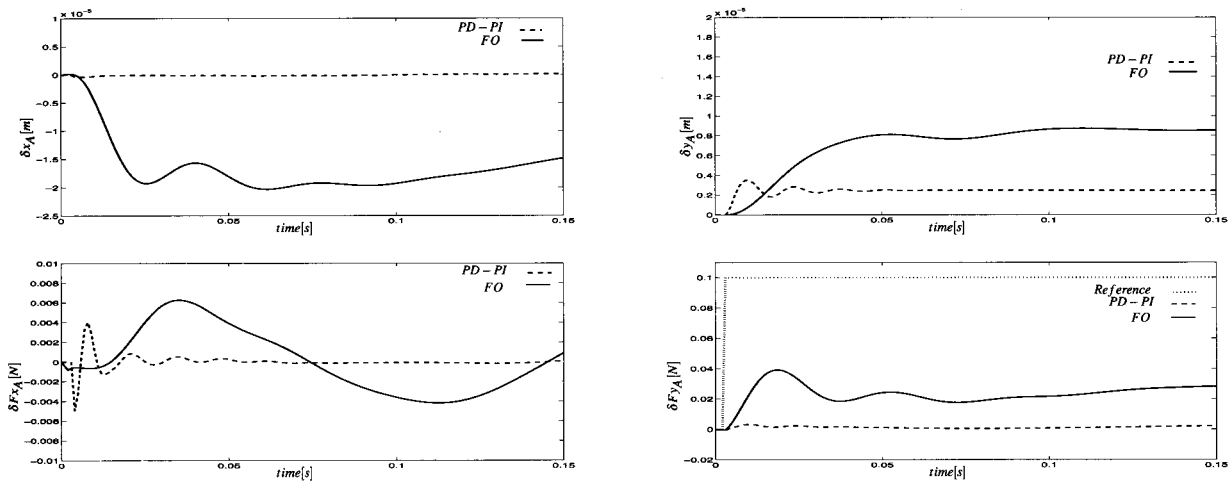


Fig. 7. Time response of robot A with joints having backlash, under the action of the *FO* and the *PD-PI* algorithms, for a pulse perturbation at the force reference $\delta F y_d = 0.1$ N and a payload $M = 1$ kg, $B_i = 1$ Nsm $^{-1}$ and $K_i = 10^3$ Nm $^{-1}$.

Table 3. Time response characteristics for a pulse δy_d at the robot A position reference.

No	C(s)	PO%	e_{ss} [mm]	T_p [s]	T_s [s]
1	<i>PD-PI</i>	39.0	5.0×10^{-3}	1.1×10^{-2}	25.0×10^{-2}
	<i>FO</i>	43.0	0.9×10^{-3}	1.6×10^{-2}	15.0×10^{-2}
2	<i>PD-PI</i>	0.2	2.7×10^{-2}	37.0×10^{-2}	5.0×10^{-1}
	<i>FO</i>	0.2	3.5×10^{-3}	4.0×10^{-2}	4.0×10^{-2}
3	<i>PD-PI</i>	0.3	64.0×10^{-2}	38.0×10^{-2}	45.0×10^{-2}
	<i>FO</i>	0.3	50.0×10^{-3}	25.0×10^{-2}	19.0×10^{-2}
4	<i>PD-PI</i>	45.1	5.0×10^{-3}	1.2×10^{-2}	26.0×10^{-2}
	<i>FO</i>	43.0	0.9×10^{-3}	1.7×10^{-2}	15.0×10^{-2}

Table 4. Time response characteristics for a pulse $\delta F y_d$ at the robot A force reference.

No	C(s)	PO%	e_{ss} [mm]	T_p [s]	T_s [s]
1	<i>PD-PI</i>	400.0	9.8×10^{-1}	1.1×10^{-3}	2.0×10^{-1}
	<i>FO</i>	115.0	77.0×10^{-3}	25.0×10^{-3}	5.0×10^{-1}
2	<i>PD-PI</i>	400.0	9.8×10^{-1}	1.1×10^{-3}	2.0×10^{-1}
	<i>FO</i>	100.0	77.0×10^{-3}	20.0×10^{-3}	4.0×10^{-1}
3	<i>PD-PI</i>	100.0	9.8×10^{-1}	1.1×10^{-3}	1.0×10^{-1}
	<i>FO</i>	100.0	77.0×10^{-3}	20.0×10^{-3}	4.0×10^{-1}
4	<i>PD-PI</i>	100.0	9.8×10^{-1}	1.1×10^{-3}	1.0×10^{-1}
	<i>FO</i>	60.0	95.0×10^{-3}	20.0×10^{-3}	1.0×10^{-1}

3. Time Response Analysis

Figures 4 and 5 depict the time response of robot A for *FO* and classical algorithms. We considered robots with ideal joints and perturbations at the position and force ref-

erences $\delta y_d = 0.1$ m and $\delta F y_d = 0.1$ N, respectively.

Figures 6 to 11 show the time responses for robots having dynamic backlash, flexibility and friction at the joints.

The time response characteristics (Table 3 and 4), namely the percent overshoot *PO*%, the steady-state er-

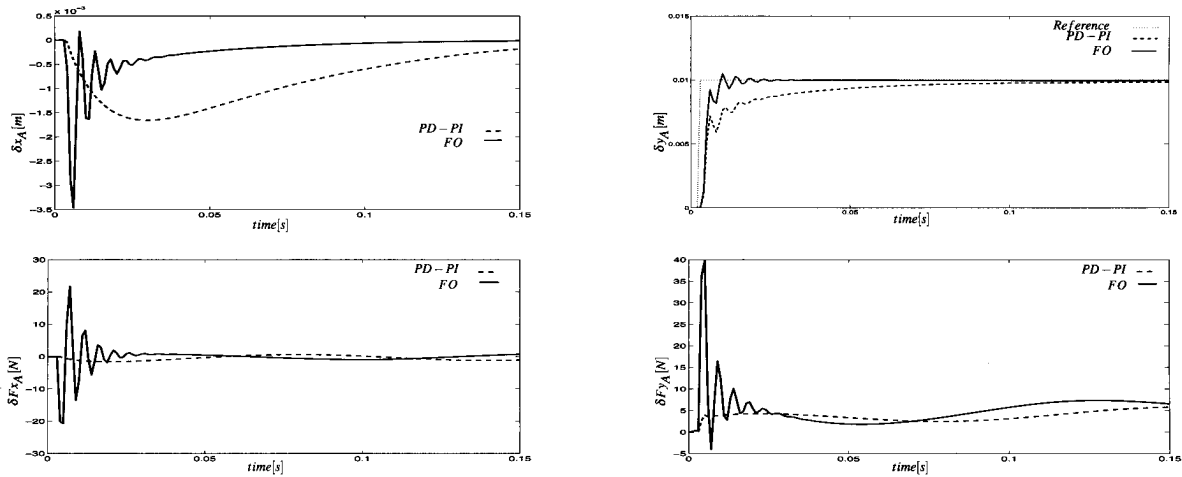


Fig. 8. Time response of robot A with joints having flexibility, under the action of the FO and the PD – PI algorithms, for a pulse perturbation at the position reference $\delta y_d = 10^{-3}$ m and a payload $M = 1$ kg, $B_i = 1$ Nsm $^{-1}$ and $K_i = 10^3$ Nm $^{-1}$.

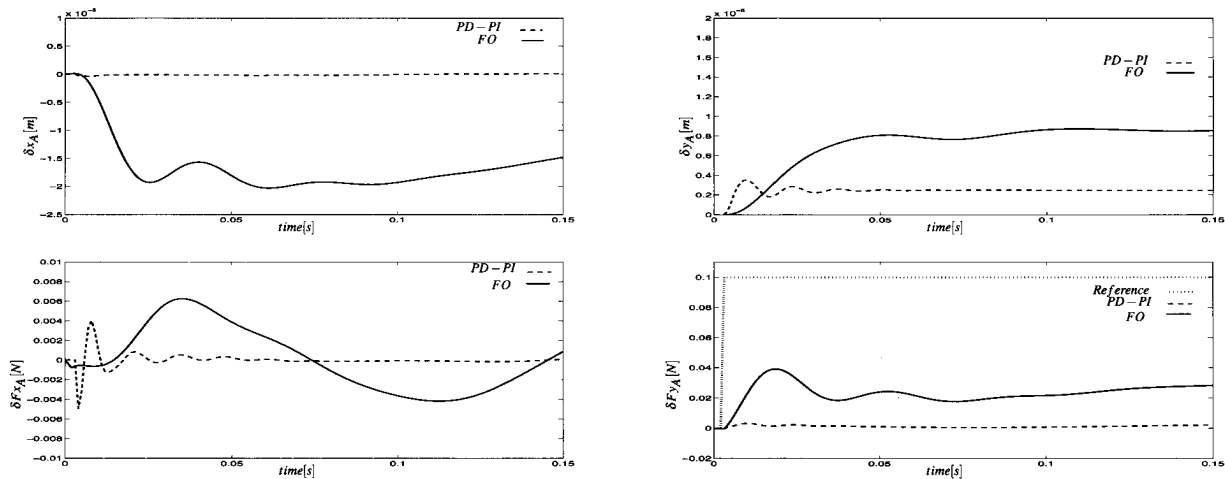


Fig. 9. Time response of robot A with joints having flexibility, under the action of the FO and the PD – PI algorithms, for a pulse perturbation at the force reference $\delta F_{y_d} = 0.1$ N and a payload $M = 1$ kg, $B_i = 1$ Nsm $^{-1}$ and $K_i = 10^3$ Nm $^{-1}$.

ror e_{ss} , the peak time T_p and the settling time T_s reveal that, although tuned for almost similar performances in the first case, the FO is superior to the PD – PI in the cases of robots with joint dynamic phenomena.

It is clear that the fractional controller demonstrates better performance for joints having nonlinearities.

4. Conclusions

This paper studied the position/force control of two robots working in cooperation using fractional and integer order control algorithms. The system time response was analyzed for manipulators having several types of dynamical phenomena at the joints. The transient response of the system shows the superior performance of the FO controller.

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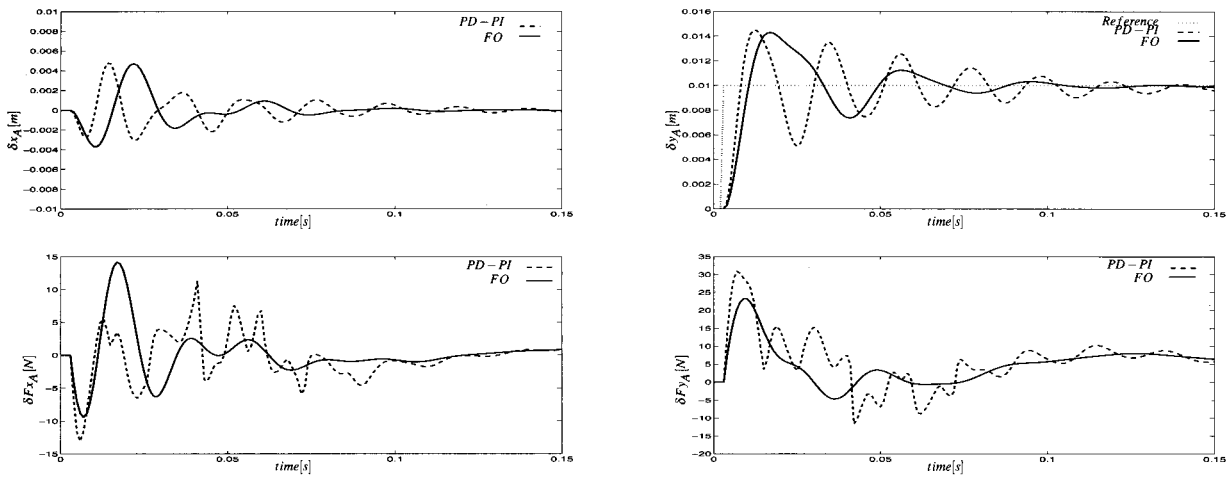


Fig. 10. Time response of robot A with joints having nonlinear friction, under the action of the FO and the PD – PI algorithms, for a pulse perturbation at the position reference $\delta y_d = 10^{-3}$ m and a payload $M = 1$ kg, $B_i = 1$ Nsm $^{-1}$ and $K_i = 10^3$ Nm $^{-1}$.

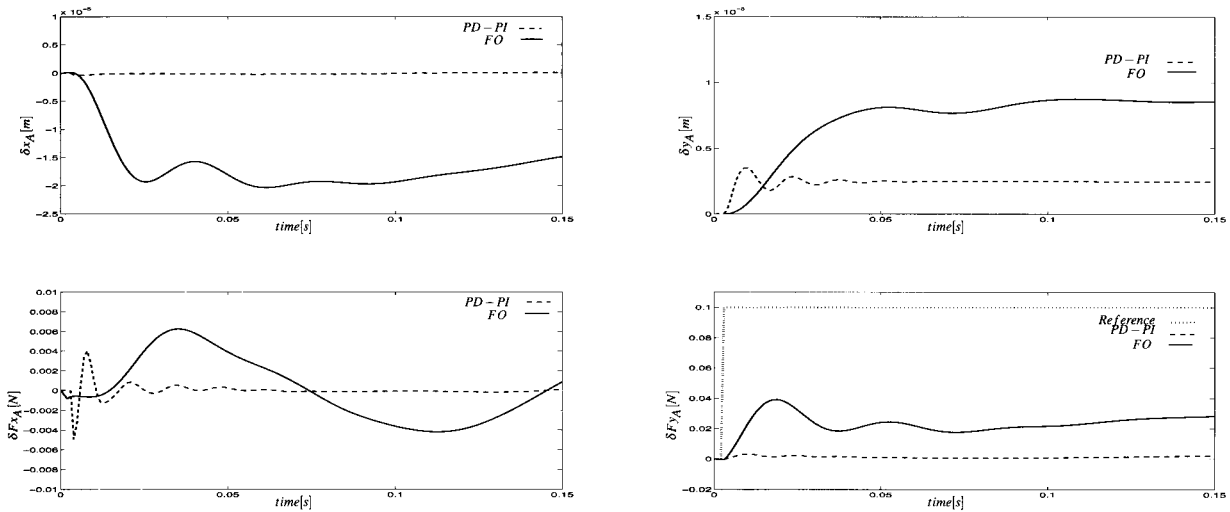


Fig. 11. Time response of robot A with joints having nonlinear friction, under the action of the FO and the PD – PI algorithms, for a pulse perturbation at the force reference $\delta F y_d = 0.1$ N and a payload $M = 1$ kg, $B_i = 1$ Nsm $^{-1}$ and $K_i = 10^3$ Nm $^{-1}$.

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