FRACTIONAL-ORDER CONTROL OF COOPERATIVE ROBOTS

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Abstract: In this paper it is studied the implementation of fractional-order algorithms in the position/force control of two cooperating robotic manipulators. The system performance is analyzed in the time and frequency domains.

We analyze and compare different control methodologies such as the integer and the fractional-order *PID* presenting the simulation results assessing the performance of the proposed fractional-order algorithms.

Keywords: control, fractional order, robots, cooperation, position, force, task.

1. INTRODUCTION

Robots that are built to imitate the upper body of human beings represent a good solution for the two essential skills in the domain of anthropic robotics: a skillful manipulation and grasping [1]. Humanoid robot arms can be used to assist men in a large variety of environments (*e.g.*, homes, hospitals, offices). The human arm forms the basis for humanoid arm designs. According to the movement sciences of kinesiology and biomechanics, the human arm, along with the shoulder girdle form part of the human upper limb [2].

Two robots carrying a common object are a logical alternative for the case in which a single robot is not able to handle the load. Nevertheless, with two cooperative robots the resulting interaction forces have to be accommodated and, in addition to position feedback, force control is also required [3, 4]. There are two basic methods for force control, namely the hybrid position/force and the impedance schemes. The first method, proposed by Raibert and Craig [5], separates the task into two orthogonal subspaces corresponding to the force and the position controlled variables. Once established the subspace decomposition two independent controllers are designed. The second method was first proposed by Siciliano *et al.* [4, 6]. In this scheme, by a proper choice of the arm mechanical impedance the interaction forces can be controlled to obtain an adequate response. This paper studies the position/force control of two cooperative manipulators, using fractional-order (*FO*) algorithms [7-10]. The *FO* algorithms constitute a useful tool to describe several physical phenomena, such as heat, flow, electricity, magnetism, mechanics, or fluid dynamics. Presently, the *FO* theory is applied in almost all areas of science and engineering: it as the ability to improve the modeling and control of many dynamical systems [11].

In this line of thought the paper is organized as follows. Section two presents the advantages of cooperative robotics focusing on the controller architecture for the position/force control of two robotic manipulators. Section three develops several experiments for the performance evaluation of the controllers. Finally, section four outlines the main conclusions.

2. COOPERATION AND CONTROL OF TWO ROBOTIC MANIPULATORS

In many cases, increasing the efficiency of robotic systems can be achieved through coordination and cooperation between the robots and the development of new control strategies. Like humans, the achievement of certain tasks is only possible using their arms and the efficiency can be significantly increased if using both arms (*e.g.*, carrying a heavy and/or large object). In robotics things are not so different: using two robotic manipulators can increase the efficiency of the system by reducing the effort in performing those tasks.

When the robots interact with objects in the workspace it is often necessary to perform manipulation tasks with large objects and if they are only supported by one side, they get difficult to handle.

Nevertheless, this difficulty can be overcome if they are transported and manipulated by the two ends of the object distributing the load among robots reducing the effort of each one of them.

However, the existence of a closed dynamic chain, corresponding to the various links and load, represents a challenge for the motion control of robots as well as the control of the internal forces that occur in each of the robots.

When two robots grasp an object (Fig. 1), that is moved it from one location to another, a coordinated motion is required. In order to get good performances it is necessary to specify not only the desired motion of each robot but also the corresponding handling force. In the system under study the contact of the robot gripper with the load is modeled through a linear system with a mass M, a damping B and a stiffness K (Fig. 2). It is also considered that the load has length l_0 and orientation θ_0. On the other hand, we consider two manipulators each with two rotational joints the robots have link lengths l_1 and l_2 and the shoulders are separated by the distance l_b. The dynamics of a robot with n links interacting with the environment is modeled as:

$$\tau = H(q)\ddot{q} + C(q,\dot{q}) + G(q) - J^T(q)F \tag{1}$$

where $\boldsymbol{\tau}$ is the n x 1 vector of actuator torques, $\boldsymbol{q}$ is the n x 1 vector of joint coordinates, $\boldsymbol{H(q)}$ is the n x n inertia matrix, $\boldsymbol{C(q,\dot{q})}$is the n x 1 vector of centrifugal/*Coriolis* terms and $\boldsymbol{G(q)}$ is the n x 1 vector of gravitational effects. The matrix $\boldsymbol{J^T(q)}$ is the transpose of the Jacobian matrix and $\boldsymbol{F}$ is the force that the load exerts in the robot gripper.

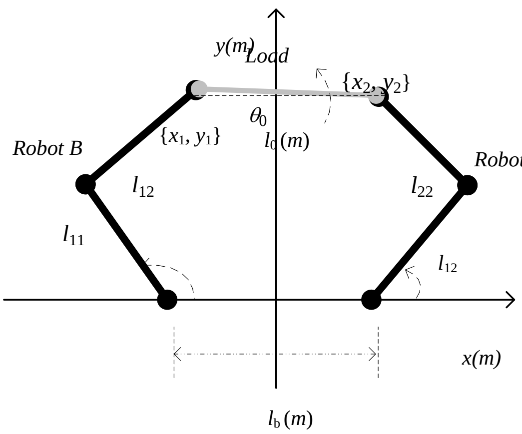

Fig. 1. Two cooperating robots for the manipulation of an object.

The numerical values adopted for the robots and the object are m_1 = 5.0 *kg*, m_2 = 5.0 *kg*, r_1 = 1.0 *m*, r_2 = 0.8 *m*, $J_{1m} = J_{2m}$ = 1.0 kgm^2, $J_{1g} = J_{2g}$ = 4.0 kgm^2 $l_b = l_0$ = 1.0 *m* and θ_0 = 0 *deg*, $B_1 = B_2$ = 10.0 $Ns.m^{-1}$ and $K_1 = K_2 = 10^4$ Nm^{-1}.

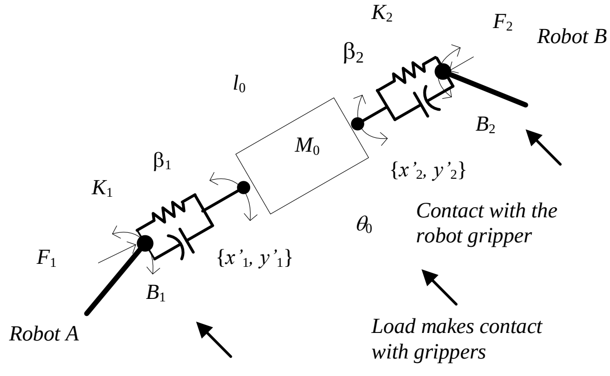

Fig. 2. The contact between the robot gripper and the object.

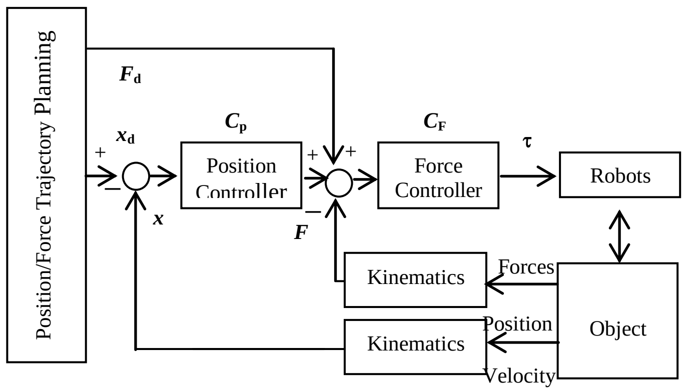

Fig. 3. The position/force controller architecture.

The controller architecture (Fig. 3) is inspired on the impedance and compliance schemes. Therefore, we establish a cascade of force and position algorithms as internal and external feedback loops, respectively, where x_d and F_d are the payload desired position coordinates and contact forces [12].

We consider fractional controllers (*FO*) both in the position and force control loops we adopt the Grünwald-Letnikov definition:

$$D^{\alpha}[x(t)] = \lim_{h\to 0}\left[\frac{1}{h^{\alpha}}\sum_{k=1}^{\infty}\frac{(-1)^k\Gamma(\alpha+1)}{\Gamma(k+1)\Gamma(\alpha-k+1)}x(t-kh)\right] \tag{2}$$

where Γ is the gamma function and h is the time increment. In our case, for implementing the algorithms of the type $C(s) = K\ s^{\alpha}$, $-1 < \alpha < 1$, we adopt discrete-time approximations using 4th –order Pade fractions.

The *FO* algorithms has emerged as a useful tool for the control and management systems and complex industrial processes, entertainment systems and specific applications such as control of flight platforms such as helicopters, planes or biologically inspired robots [13, 14].

3. SYSTEMS PERFORMANCES

This section analyzes the system performance for a robot with ideal transmissions comparing the response of integer and fractional algorithms, namely $C_P(s) = K_{P0} s^{\alpha_P}$: $C_P(s) = K_p\ (1 + T_d\, s)$ and $C_F(s) = K_{F0} s^{\alpha_F}$: $C_F(s) = K_F\ [1 + (T_i\, s)^{-1}]$, in the position and force loops, respectively [15, 16]. Both algorithms were tuned using the *PSO* (Particle Swarm Optimization) method getting a similar performance in the two cases. The resulting parameters yield for the integer and fractional algorithm are shown in the next tables.

Table 1. Integer *PID* Controller Parameters tuned with the *PSO* method

	K_p	K_i	K_d
q_1	616×10^3	120×10^3	3.00×10^3
q_2	345×10^3	751×10^3	5.55×10^3

	K_p	K_i	K_d
F_X	1×10^{-3}	0	1×10^{-3}
F_Y	1×10^{-3}	0	1×10^{-3}

Table 2. Fractional *PID* Controller Parameters tuned with the *PSO* method

	K_p	K_i	K_d	α	β
q_1	600×10^3	80×10^3	2.50×10^3	0.3	0.9
q_2	350×10^3	500×10^3	4.50×10^3		

	K_p	K_i	K_d	α	β
F_X	1×10^{-3}	0	1×10^{-3}	0.3	0.9
F_Y	1×10^{-3}	0	1×10^{-3}		

In all experiments the controller sampling frequency is $f_c = 10$ kHz for the operating point A of the object and a contact force of each gripper of $\{Fx_j, Fy_j\} \equiv \{0.5, 5\}$ *Nm* for the j^{th} ($j = 1, 2$) robot.

In order to validate the performance of the system we establish a trajectory to be accomplished by the two robot manipulators while handling an object.

The robot base of the manipulator A and B are located at the points (0.5, 0) and (-0.5, 0) respectively. The initial position of the circular path for each manipulator is located in the points (0.5, 1) and (-0.5, 1) respectively for the manipulator *A* and *B*. Each manipulator has two rotational joint positions defined by the angles q_1 and q_2, which corresponds (using the direct kinematics) to an x and y position of the workspace. To initialize the trajectory, it is necessary to determine the initial angles that corresponds to the known positions x and y using the inverse kinematics. In this scenario the angles for the initial positions are:

Table 3. Initial angles for both robot manipulators.

	q_1 (rad)	q_2 (rad)
Robot A	-1,047	2,093
Robot B	1,047	-2,093

In the following figure we can see the system response, illustrating the trajectory of each manipulator. Both manipulators need to synchronizingly move the object and, at the same time, keep it horizontally aligned, *i.e.*, $\theta_0 = 0$ *rad*.

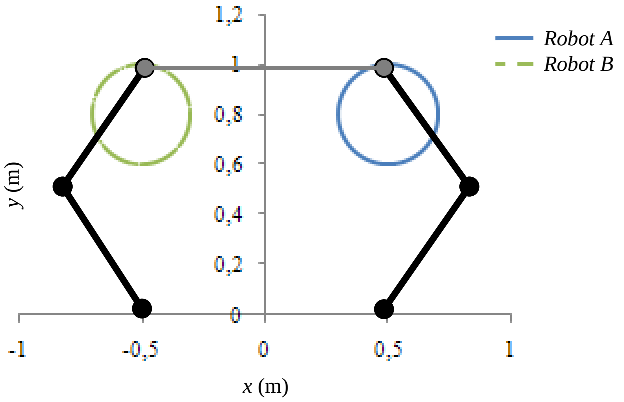

Fig. 4. Trajectory realized by the two Robots.

When considering the challenges found in controlling a nonlinear system, it becomes hard to evaluate the best kind of controller that gives a "good" performance. While one controller may be superior operating with a fast and aggressive control action, another may be best suited for a slow and smooth response.

To check the system performance we simulate a trajectory as shown in Fig. 4 and, after several few seconds, we apply a small step disturbance with amplitude of 0.05 for the duration of 1.5 seconds (Fig. 5).

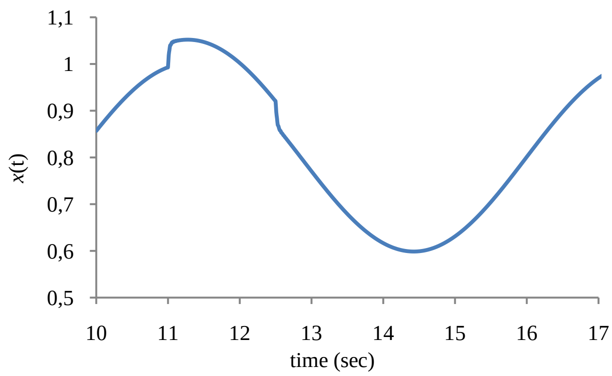

Fig. 5. *Robot B* x-axis trajectory.

To analyze more clearly the dynamical response to the step perturbation we subtract the dynamic response without perturbation as we can see in Figures 6 and 7.

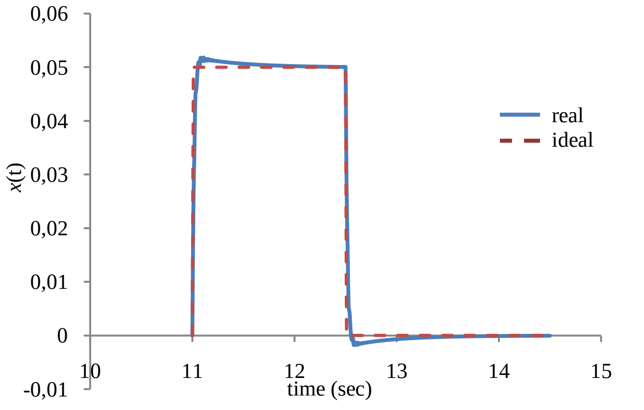

Fig. 6. Step response of the *Robot B* to a disturbance by using the integer *PID*.

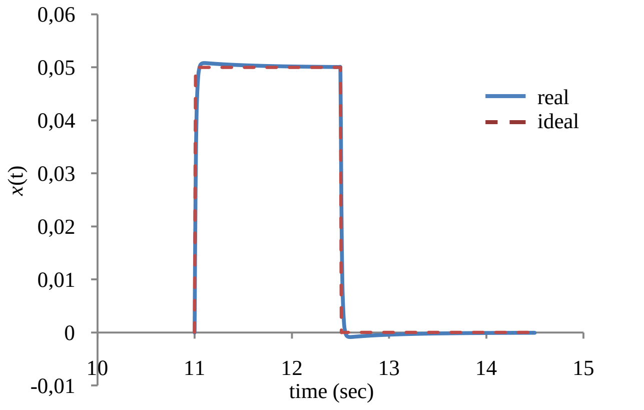

Fig. 7. Step response of the *Robot B* to a disturbance by using the fractional *PID*.

In order to perform a controller evaluation comparing the time response characteristics of the integer and fractional order controller, the following criteria where calculated: rise time T_R (seconds), peak time T_P (seconds), settling time T_S (seconds) and overshoot *OV* (%) (Table 4).

Table 4. Time response characteristics of the system to a step input.

	OV (%)	T_R (seconds)	T_P (seconds)	T_S(seconds)
PID	3,665	0,055	0,11	1,43
$PI^{\alpha}D^{\beta}$	1,667	0,055	0,1	1,42

The step responses reveal a large diminishing of the overshoot and the rise time when compared with the integer *PID*, showing a good transient response and a zero steady-state error. The $PI^{\alpha}D^{\beta}$ leads to better results than the integer *PID* controller tuned through the *ZNOL* rule. These results demonstrate the effectiveness of the fractional algorithms when used for the control of fractional-order systems.

We now analyze two indices that measure the response error, namely the integral square error (*ISE*) and the integral time square error (*ITSE*) criteria defined as:

$$ISE = \int_0^{\infty} [r(t) - c(t)]^2 \, dt \tag{3}$$

$$ITSE = \int_0^{\infty} t \, [r(t) - c(t)]^2 \, dt \tag{4}$$

We can use other performance criteria such as the integral absolute error (*IAE*) or the integral time absolute error (*ITAE*); however, in the present case the *ISE* and the *ITSE* criteria have produced the best results and are adopted in the study.

To study those measures we made three experiments in which the gains, (*Kp*, *Ki*, *Kd*) and the {α, β} have been changed for each robot accordingly to three different optimization methods.

The pattern search is a method for direct search and follows a robust framework that allows the proof of convergence to a stationary point. The pattern search (*GPS*) algorithms [17] are derivative free methods for the minimization of smooth functions, possibly with linear inequality constraints. Standard search operates by finding a set of dots called default, which increases or decreases depending on whether any point within the pattern has a lower objective function than the current point. The search stops after a minimum standard is achieved.

One of the simplest methods for determining a local minimum is known as the gradient descent. Computationally it is not a good algorithm, because it can find a local minimum before the global minimum, which is the ultimate goal. The algorithm is then "tied" in this least until some noise is added, thus moving the algorithm out of this false minimum, i.e., the convergence tends to be extremely slow and the convergence to the global minimum is not guaranteed.

The last method was the already mentioned *PSO* which has been successfully used in many applications such as robotics [18-20] and electrical systems [21]. This optimization technique, based on a population search, is inspired by social behaviour of bird flocking fish schooling. An analogy is established between a particle and an element of swarm. These particles fly trough the search space following current optimum particles.

The following tables show the gains for the three experiments E1(*Pattern search*), E2(*Gradient descent*), and E3(*PSO*), for the two types of controllers discussed herein.

Table 5. Integer *PID* controller parameters tuned with the pattern search.

E1	K_p	K_i	K_d
q_1	650x10^3	128x10^3	2.50x10^3
q_2	350x10^3	740x10^3	5.50x10^3

E1	K_p	K_i	K_d
F_X	1x10^{-3}	0	1x10^{-3}
F_Y	1x10^{-3}	0	1x10^{-3}

Table 6. Integer *PID* controller parameters tuned with the gradient descent.

E2	K_p	K_i	K_d
q_1	$615\text{x}10^3$	$120\text{x}10^3$	$2.80\text{x}10^3$
q_2	$340\text{x}10^3$	$740\text{x}10^3$	$5.50\text{x}10^3$

E2	K_p	K_i	K_d
F_X	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$
F_Y	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$

Table 7. Integer *PID* controller parameters tuned with the *PSO*.

E3	K_p	K_i	K_d
q_1	$616\text{x}10^3$	$100\text{x}10^3$	$3.00\text{x}10^3$
q_2	$345\text{x}10^3$	$751\text{x}10^3$	$5.55\text{x}10^3$

E3	K_p	K_i	K_d
F_X	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$
F_Y	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$

Table 8. Fractional *PID* controller parameters tuned with the pattern search.

E1	K_p	K_i	K_d
q_1	$650\text{x}10^3$	$80\text{x}10^3$	$2.50\text{x}10^3$
q_2	$400\text{x}10^3$	$500\text{x}10^3$	$4.50\text{x}10^3$

E1	K_p	K_i	K_d
F_X	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$
F_Y	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$

Table 9. Fractional *PID* controller parameters tuned with the gradient descent.

E2	K_p	K_i	K_d
q_1	$550\text{x}10^3$	$80\text{x}10^3$	$2.50\text{x}10^3$
q_2	$300\text{x}10^3$	$500\text{x}10^3$	$4.50\text{x}10^3$

E2	K_p	K_i	K_d
F_X	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$
F_Y	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$

Table 10. Fractional *PID* controller parameters tuned with the *PSO*.

E3	K_p	K_i	K_d
q_1	$600\text{x}10^3$	$80\text{x}10^3$	$2.50\text{x}10^3$
q_2	$350\text{x}10^3$	$500\text{x}10^3$	$4.50\text{x}10^3$

E3	K_p	K_i	K_d
F_X	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$
F_Y	$1\text{x}10^{-3}$	0	$1\text{x}10^{-3}$

Fig. 8 depicts the *y*-axis indices *ISE* and *ITSE* for the *Robot B* considering the integer *PID* controller.

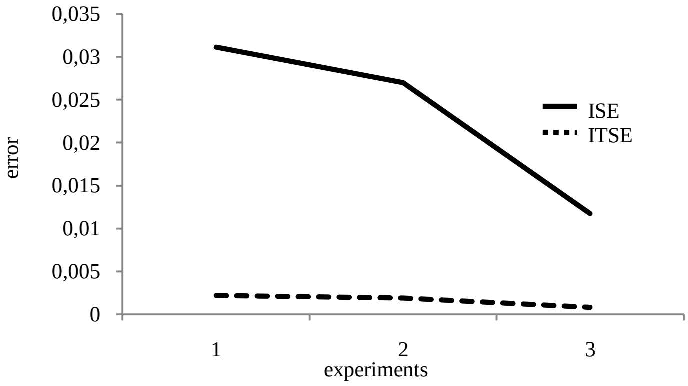

Fig. 8. Performance criteria of the system under the action of the *PID* controller.

Analyzing the *ISE* we can observe that the system response under the action of the *FO* controller is more favorable than the integer controller.

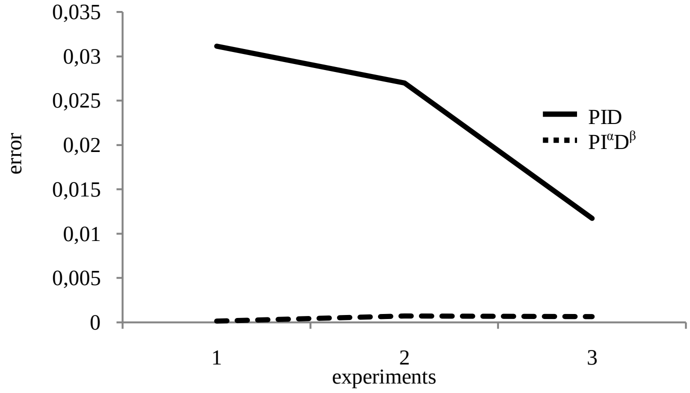

Fig. 9. Step response *ISE* criteria under the action of the integer and *FO* controller.

Making the same analysis for the *ITSE* we obtained the following figure.

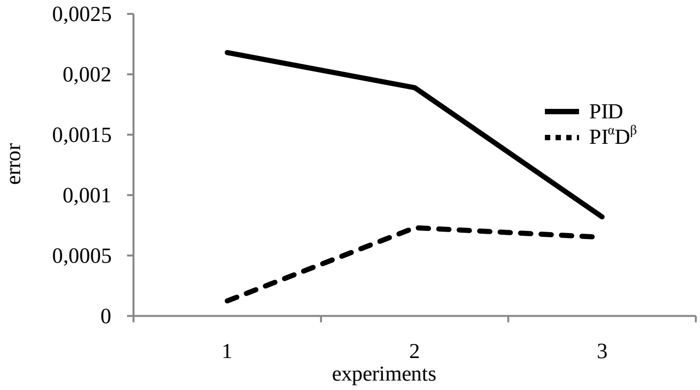

Fig. 10. Step response *ITSE* criteria under the action of the integer and *FO* controller.

Analyzing the results we can highlight that the use of the fractional controller results in the minimization of the *ITSE* criteria.

4. SUMMARY AND CONCLUSIONS

This paper compares the position/force control of two cooperative robot manipulators using fractional and integer order algorithms. The dynamic performance of the two arms holding an object was analyzed in the time domain.

The results demonstrate that the fractional-order algorithm is far superior, revealing a good performance and a high robustness.

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