Complex dynamics of financial indices

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Abstract This paper presents a novel method for the analysis of nonlinear financial and economic systems. The modeling approach integrates the classical con- cepts of state space representation and time series re- gression. The analytical and numerical scheme leads to a parameter space representation that constitutes a valid alternative to represent the dynamical behavior. The results reveal that business cycles can be clearly revealed, while the noise effects common in financial indices can elegantly be filtered out of the results.

Keywords Time series,Complex dynamics, Financial analysis, State space, Trendlines

1 Introduction

Financial indices play a fundamental role in mankind activities. The time series reveal complex dynamical phenomena and considerable efforts have been made to involve mathematical tools usual in the analysis of nonlinear dynamics [2, 18, 26, 27]. In spite of the ef- forts that have been devoted to this topic, the true is that "randomness" plays still a fundamental role when working with these kind of objects [3, 9, 20, 22, 25]. This state of affairs somehow points toward adopting statistical or stochastic analytical methods and, con- sequently, precluding modeling perspectives closer to those common in electrical, mechanical, thermal, and other types of physical systems. This paper emerges from realizing that this classical paradigm for system modeling needs to be adapted to financial systems and proposes a new approach for overcoming some of the aforementioned limitations.

Financial signals reflect the dynamics of a complex system where the concepts of measure, variable, pa- rameter, and model are not clearly defined as occurs in physics or engineering. The "financial system," under- lying the index evolution, is composed by a multitude of different agents, exhibiting a plethora of phenom- ena with distinct nature and size that interplay both between themselves and the "economical system." Be- sides these difficulties of defining an assertive mod- eling paradigm, financial indices reveal a noisy be- haviour with chaotic characteristics. This fact poses numerical problems for calculating derivatives and, therefore, it is not straightforward adopting tools usual in dynamical systems such as the state space repre- sentation. Phase variables constitute a common choice for state variables, since they require the consecutive time derivatives and are a solid option for construct- ing trajectories representative of the system dynam- ics. Nevertheless, such option is avoided in the case of financial dynamics due to the heavy noise present in the indices. The noise filtering is often discharged a priori since a strong intervention perturbs not only noise, but also the signals that are under evaluation.

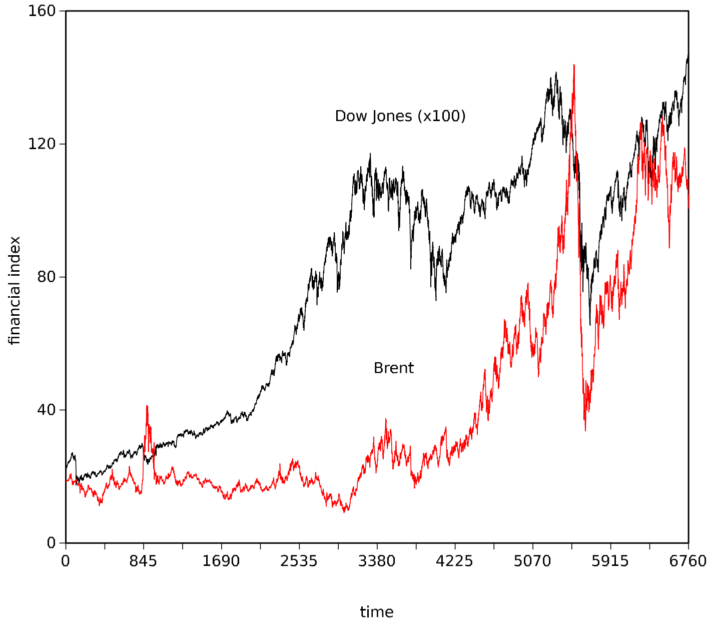


Fig. 1 Time evolution of the Dow Jones Industrial Average and the Europe Brent Spot Price FOB, from 18 May 1987 up to 12 April 2013 (6,760 points)

Several other options were proposed to overcome that problem, namely the use of pseudo state space [4], the adoption of fractional derivatives [8], the analy- sis using transforms [11, 15, 17], the study by means of visualization tools [12, 13], the formulation of fil- tering as an inverse optimization problem [10], or de- scribing the dynamics in the viewpoint of power law regressions [14, 16]. This paper adopts a new strat- egy by reformulating the problem of calculating the derivative. The newly proposed method discharges the noise and preserves the trend of the financial dynam- ics by considering regressions at several time scales. The resulting parameters are then used for represent- ing the dynamics in a multidimensional plot that be- haves similarly to the classical phase space. Further- more, this strategy leads to an algorithm establishing a compromise between time resolution and filtering, while leading to the direct analysis of the resulting plots.

Bearing these ideas in mind this paper is organized as follows. Section 2 formulates the new method, de- velops several experiments with two financial indices and discusses the results. Finally, Sect. 3 draws the main conclusions.

2 Modeling approach and experiments

In the standard procedure for calculating the deriva- tive of a function, we start by constructing a trian- gle and by obtaining its slope. Such slope represents the average over the interval. Consequently, reducing the interval length approximates the slope of the tri- angle up to the value of the derivative. Nevertheless, at small time scales, the effect of noise predominates. Often in financial analysis we are interested in the trend over a given time window and we simply look at the global signal evolution without paying atten- tion to small (noisy) phenomena. Following this line of thought, for a given signal $f(t)$, where t denotes time, the proposed method calculates a trendline $g(t)$ that approximates $f(t)$ over an interval $t \in T$. The derivative now can be obtained from $\frac{dg}{dt}$. This is the standard method of calculating numerical derivatives when adopting polynomials for *g(t)* and a few points for *T* [5, 7, 23]. In the proposed algorithm, we shall preserve the initial idea of having a considerable num- ber of points in *T* embedded with a scheme for defin- ing the appropriate time scale. Therefore, we divide iteratively the domain of *f(t)* into 2^{n-1}, $n=1, 2, \ldots$, time windows of identical size. Obviously, the higher the value of *n*, the smaller the number of points in the corresponding interval T_n and the stronger the effect of noise upon the calculations. The choice for a par- ticular value of *n*, that is, the choice of a given time scale, is for the user to decide based upon a compro- mise between instantaneous behavior and noise limita- tion. The second aspect of the algorithm is the choice of *g(t)*. In the present case, we consider "robust" func-

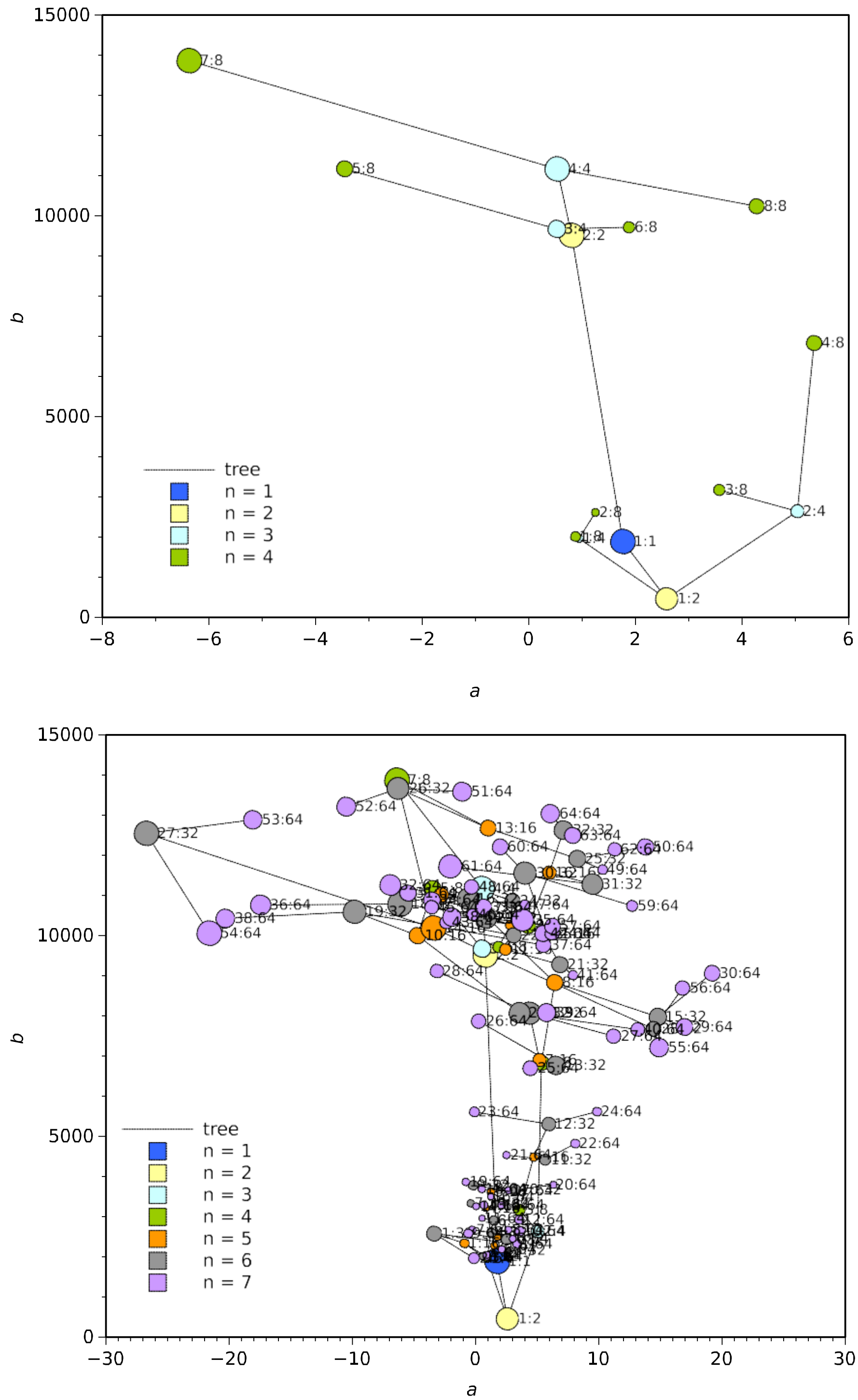
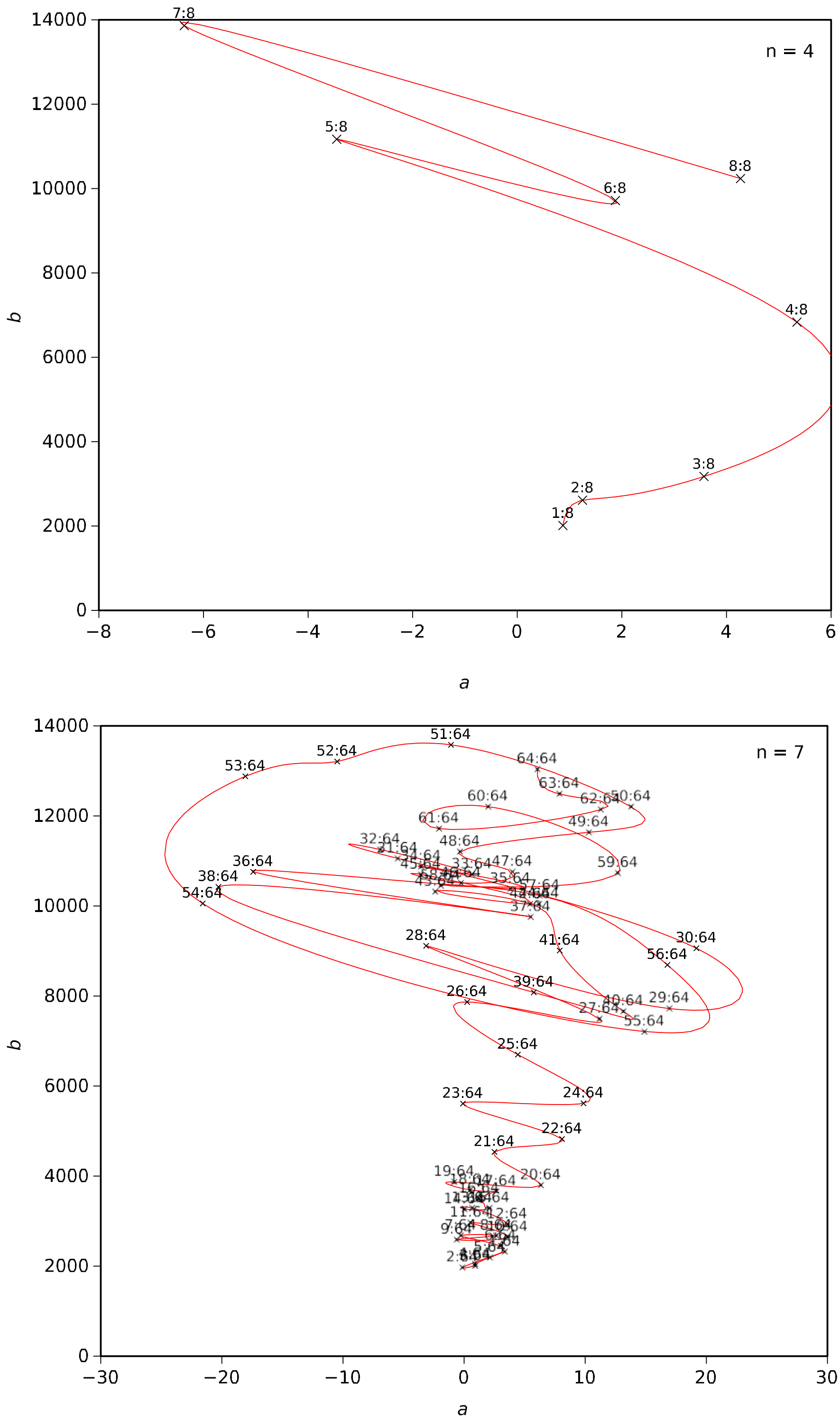


Fig. 2 Dependency tree in the locus {*a, b*} for the Dow Jones Industrial Average, *n* = 4 and *n* = 7, linear trendline

Fig. 3 Locus {*a, b*} for the Dow Jones Industrial Average, $n=4$, and $n=7$, linear trendline



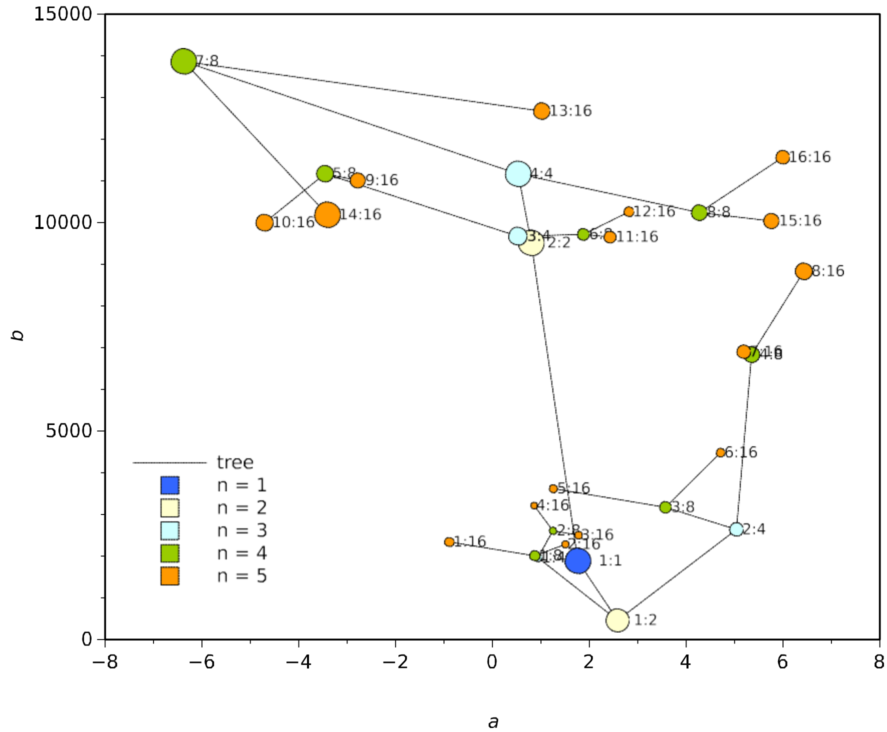


Fig. 4 Dependency tree in the locus $\{a, b\}$ for the Dow Jones Industrial Average, $n = 5$, linear trendline

tions, where robustness means that they do not present singularities and that they include parameters simple to associate with the time series and its dynamics. In other words, while in standard calculations the deriva-
tive is yielded by $\frac{dg}{dt}$ in the proposed algorithm, we substitute $f(t)$ and its derivatives by the parameters of $g(t)$. In the sequel, we choose $g(t) = b_k + a_k t$ and $g(t) = b_k \exp(a_k t)$, $a_k, b_k \in R$, $k = 1, \ldots, n$. There- fore, for the straight line and exponential functions,
we can loosely say that a_k and b_k reflect the mean val-
ues of $\frac{df}{dt}$ and f over each time window T_n. More-

over, we substitute the representation in the phase space by an alternative one consisting of the *param- eter space*.

We adopt numerical experiments for two financial indices, representing the Dow Jones Industrial Aver- age and the Europe Brent Spot Price FOB (in dollars per barrel) available at the websites of the “Yahoo! Fi- nance” and the “US Energy Information Administra- tion,” respectively. The time series include daily val- ues in period T_1 starting at 18 May 1987 and ending at 12 April 2013. Special

Figure 1 shows the time evolution of the Dow Jones Industrial Average and the Europe Brent Spot Price FOB over period T_1.

Figure 2 depicts the dependency tree in the locus $\{a, b\}$ for the Dow Jones Industrial Average, $n = 4$ and $n = 7$, linear trendline, in period T_1. The circles are proportional to the mean root mean square error over each time window and the labels $p : q$ mean the pth time window for a total of q intervals. The a-axis
reflects the slope of the financial index and, therefore,

the first/second quadrant means a positive/negative cy-
days and holidays that lead to some lack of data in the original time series, were estimated by simple interpolation of the neighbor val- ues, so that all weeks include 5 days, making a total of 6,760 points.

cle. On the other hand, the *b*-axis reflects the value of the index itself. It is clear that the larger the number of intervals *n*, the more intricate the tree becomes and the larger the dispersion particularly for *a*. Another as- pect that deserves to be highlighted is the choice of a succession of time bisections. That algorithm would lead to the requirement of a total number of points per- forming a power of two. To alleviate this restriction, during the time scale formulation, several limit points are not considered so that each one has an even num- ber of points. Since the number of points remaining in each interval is still large, the effect of such truncation

upon the result is negligible. Figure 3 shows the pa- rameter locus {*a, b*} (i.e., a mimic of the phase plane). The dots represent the estimated parameters a_k and b_k,

Fig. 5 Locus {*a*, *b*}, *n* = 5, of the Dow Jones Industrial Average, for linear and exponential trendlines

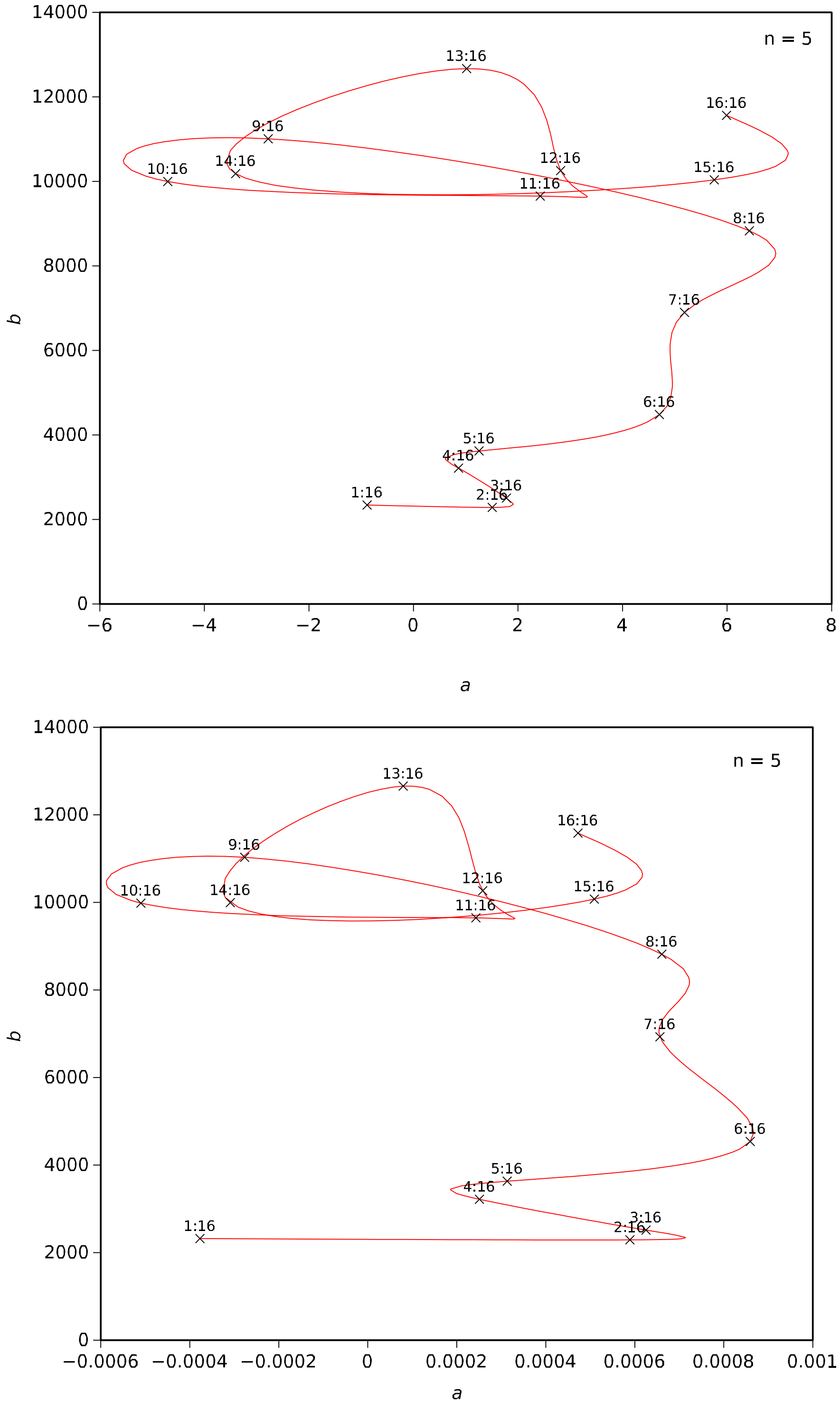
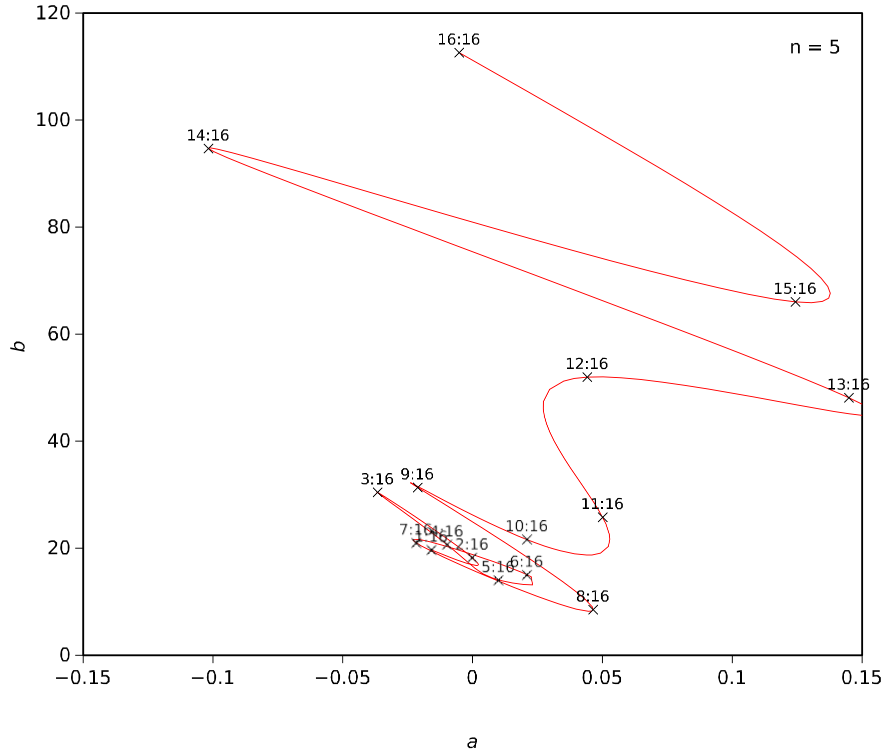


Fig. 6 Locus {*a, b*}, *n* = 5, of the Europe Brent Spot Price FOB, for linear and exponential trendlines



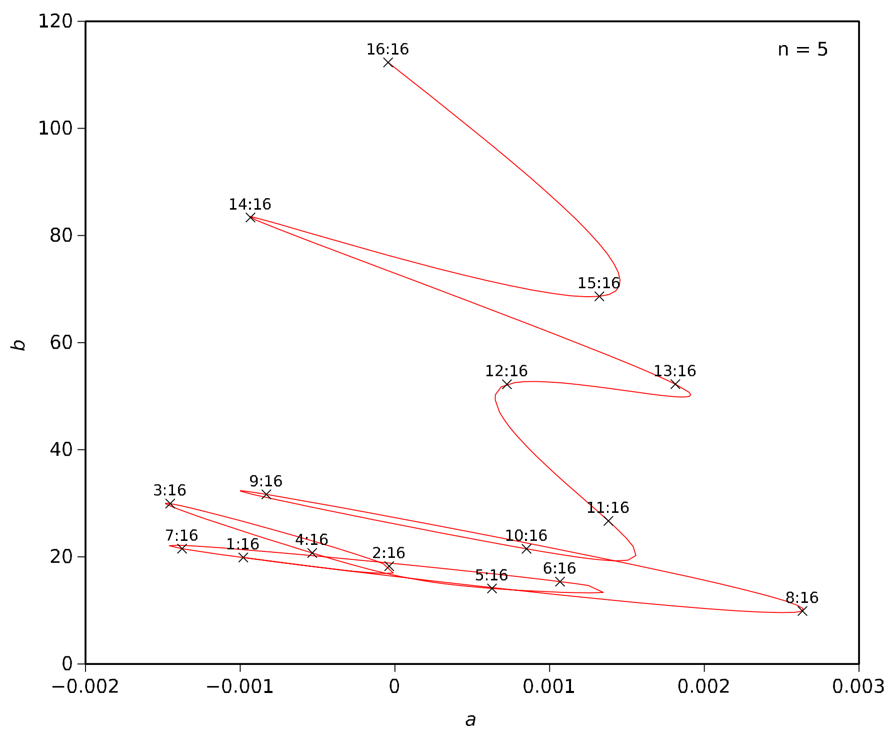
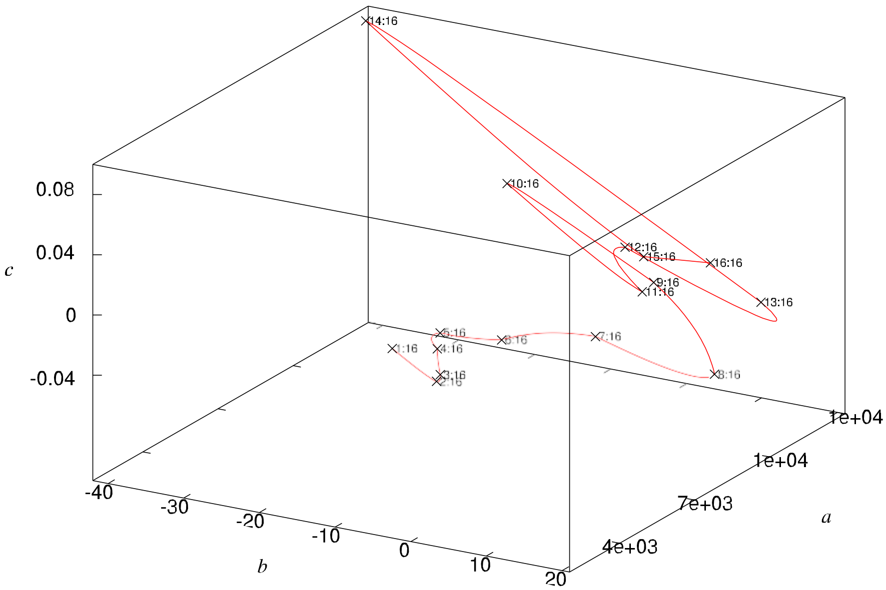


Fig. 7 Locus {*a, b, c*}, *n* =5, of the Dow Jones Industrial Average, for parabolic trendline



$k = 1, \ldots, n$ for each period and the connecting trajectories. We observe that $n = 3$ leads to an oversimplification, while the case $n = 7$ reflects the noisy behavior of the original time series.

Several experiments demonstrated that $n = 5$ establishes a good compromise between precision and noise. For example, Fig. 4 depicts the dependency tree in the locus {*a, b*} for the Dow Jones Industrial Average, $n = 5$, and linear trendline. Figures 5 and 6 show the locus {*a, b*}, $n = 5$, for the Dow Jones Industrial Average and the Europe Brent Spot Price FOB, respectively, when adopting the linear and exponential trendlines.

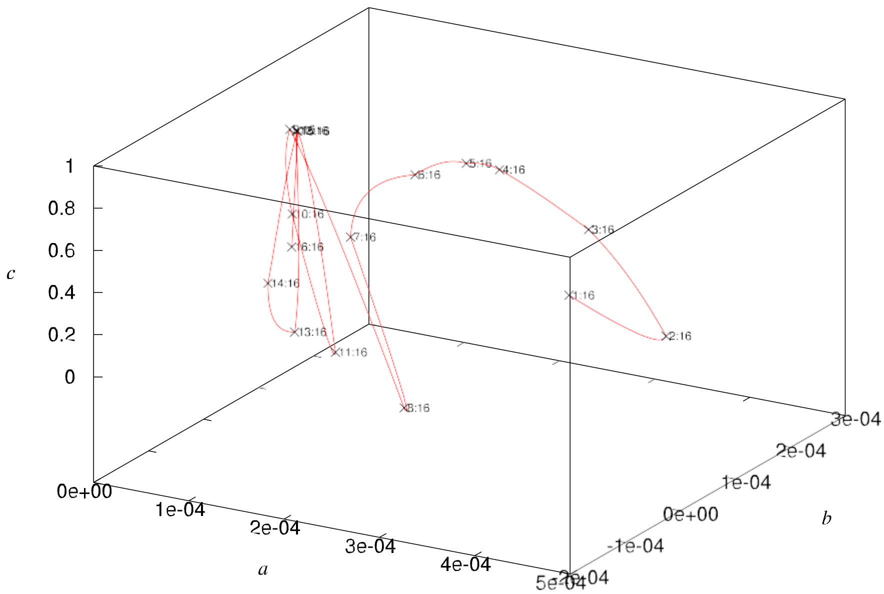
We should note that the meaning of parameters *a* and *b* vary with the trendline *g*. Nevertheless, the two plots are of the same type and lead to identical con- clusions. This means also that there is no special rea- son for selecting one particular type of trendline. In what concerns the dynamics of the Dow Jones In- dustrial Average, we observe four phases: a first increasing trajectory {1 : 16} → {8 : 16}, two repetitive cycles {8 : 16}→ {11 : 16} and {11 : 16}→ {15 : 16}, and finally a (still) indeterminate trajectory for {15 : 16}→ {16 : 16} that presently is in a positive state. For the Europe Brent Spot Price FOB, we observe a distinct behavior, namely three small initial cy- cles for {1 : 16}→ {9 : 16}, an increasing trajectory for {9 : 16}→ {13 : 16}, and an oscillatory behavior, composed of alternative and positive trajectories, for {13 : 16}→ {14 : 16}→ {15 : 16}→ {16 : 16}. This behavior can be recognized directly at the time evolu- tion represented in Fig. 1, but as occurs in state space representations, we obtain a much clear picture of the overall dynamics.

In the analysis of dynamical systems, often is re- quired the adoption of more than two state variables. However, the use of a second-order derivative in the present time series is clearly a problematic option. Therefore, it is relevant to investigate if the proposed method can be generalized for a larger number of di- mensions. In this line of thought, we consider a 3-dimensional representation supported by the parabolic and Harris trendlines $g(t) = a_k + b_k t + c_k t^2$ and $g(t) = (a_k + b_k t^{c_k})^{-1}$, $a_k, b_k, c_k \in R$, $k = 1, \ldots, n$, respectively. Both trendlines involve three parameters reflecting distinct "dynamical properties" and, there- fore, making them suitable for a three-dimensional representation. Nevertheless, it is possible to adopt other types of functions revealing better/worst proper- ties for each specific type of time series. In particular, the parameter c required by the Harris model, reflects the power law behavior known in fractional order dy- namics [1, 6, 19, 21, 24].

Figures 7 and 8 depict the locus $\{a, b, c\}$, $n = 5$, for the Dow Jones Industrial Average when adopting the parabolic and Harris regressions.

The Harris model seems to be slightly superior to the parabolic regression because we verify that the third dimension is useful is discriminating the complex dynamics that appears in the final period of time. Fur-

Fig. 8 Locus {*a, b, c*}, *n* =5, of the Dow Jones Industrial Average, for Harris trendline



thermore, we observe a smooth evolution at the first part and two large loops at the final part of the trajec- tory. The large loops demonstrate a strong dynamical instability and a kind of strange attractor influencing present day financial dynamics. These results are in ac- cordance with those depicted by the two-dimensional charts, but provide a better visualization.

In conclusion, we verified that the parameter space constitutes a valid alternative to the classical state space representation, namely by avoiding noise effects that are present in financial time series.

3 Conclusions

This study addressed the analysis of complex and nonlinear dynamics in financial systems. Markets are characterized by means of indices with considerable noise making difficult the application of state space representations. The proposed methodology reformu- lates the classical methods leading to a new model based in the trajectory evolution in the parameter space. Financial cycles and crises are clearly visible since noise effects are eliminated.

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