SAMS, 1998, Vol. 33, pp. 373–384
Reprints available directly from the publisher
Photocopying permitted by license only

© 1998 OPA (Overseas Publishers Association) N.V.
Published by license under
the Gordon and Breach Science
Publishers imprint.
Printed in India.

A STATISTICAL PERSPECTIVE TO THE FOURIER ANALYSIS OF MECHANICAL MANIPULATORS

J. A. TENREIRO MACHADO*
and ALEXANDRA M. S. F. GALHANO

Faculty Engineering of the University of Porto, Dept. of Electrical and Computer Engineering, 4099 Porto Codex, Portugal

(Received 30 July 1997)

A new approach to the analysis and design of robot manipulators is presented. The novel feature resides on a non-standard formulation to the modelling problem. Usually, system descriptions are based on a set of differential equations which, in general, require laborious computations and may be difficult to analyse. These facts motivate the need of alternative models based on different mathematical concepts. The proposed statistical approach to the Fourier modelling gives clear guidelines towards the optimisation of the robot kinematics and point out structural characteristics of the trajectory planning algorithms.

Keywords: Robots; manipulators; modelling; kinematics; Fourier analysis; statistics

1. INTRODUCTION

Mechanical manipulators are developed according to engineering and scientific principles that are based on fundamental concepts such as those arising from mathematics and physics. Based on these formulations, the first step on the study of a physical phenomena is the development of an adequate model. Usually the fundamental concepts are the differential and matrix calculus and the classical newtonian physics, while the model consists on a set of differential equations. Nevertheless, several phenomena, such as quantum physics

*Corresponding author.

and thermodynamics, may be studied through different mathematical tools namely using statistical methods. These facts suggest that, for a given problem, we may adopt different mathematical models, each with its own merits and drawbacks. The second step on the study of the physical phenomena is the analysis of the properties revealed by the model. For a model consisting on a set of linear differential equations we can adopt simple strategies (e.g., the Fourier analysis) but, for a non-linear model these tools are not adequate and the analysis becomes complex and difficult to generalise. In fact, experience demonstrates that for a large number of cases, such as the kinematic and dynamic models arising in robotics, efficient tools capable of rendering clear results are still lacking.

This paper presents a framework where it is developed a new modelling formalism based on the embedding of statistical and Fourier transform concepts. These concepts are then illustrated on several experiments. The examples reveal not only the capabilities of the new method but also the limitations of standard robot structures and path planning algorithms. Consequently, in order to develop the new formalism the paper is organised as follows. Section two starts by presenting the fundamental modelling concepts. Based on the new concepts, section three illustrates its application on the kinematic and trajectory planning analysis of mechanical manipulators. Finally, section four outlines the main conclusions.

2. EMBEDDING STATISTICS AND FOURIER TRANSFORM TOWARDS THE MODELLING OF ROBOT MANIPULATORS

For a robot having n degrees of freedom (dof) the classical direct kinematic model is described by a set of equations $\boldsymbol{p} = \psi(\boldsymbol{q})$ relating the operational and the joint spaces, where $\boldsymbol{p} = [p_1, \ldots, p_n]^T$ and $\boldsymbol{q} = [q_1, \ldots, q_n]^T$ are the $n \times 1$ vectors of position in the operational and joint spaces, respectively. Based on these equations considerable research has been done on issues such as the optimisation of the manipulator structure [1, 2] and the development of efficient path planning algorithms [3, 4]. However, the kinematic equations usually are non-linear and reveal a plethora of variables and parameters that give rise to a cumbersome work both in the analysis and design stages. Therefore, in order to overcome these problems alternative concepts are required. Statistics is a mathematical tool well adapted to handle a large volume of data that has already been used in some restricted classes of robotic problems [5–7]. Nevertheless, for the kinematic modelling, statistics is not capable of dealing with time-dependent relations. Therefore, to overcome the limitations of statistics [7–9], the new method [10] will also take advantage of the Fourier transform by embedding both tools in a broader formalism. In this line of thought, the first stage of the new modelling formalism starts by comprising:

- A set of input variables (*ivs*), that is, variables that are free to change independently.
- A set of output variables (*ovs*), that is, variables that depend on the *ivs*.
- A set of parameters to be optimised in the design stage.

As usually, in the kinematics the *ivs* and *ovs* are established by the relation $\psi : \{\boldsymbol{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}\} \rightarrow \{\boldsymbol{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}}\}$, while for the inverse kinematics we get the reverse relation $\psi^{-1} : \{\boldsymbol{p}, \dot{\mathbf{p}}, \ddot{\mathbf{p}}\} \rightarrow \{\boldsymbol{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}\}$. In both cases, the set of parameters depend on the manipulator structure and the time/space evolution of the trajectories.

The second stage of the formalism consists on the embedding of the statistical analysis into the Fourier transform through the algorithm:

i) A statistical sample for the variables is obtained by driving the manipulator through a large number of trajectories (generated according with adequate statistics) having appropriate time/space evolutions. All the variables (i.e., the *ivs* and the *ovs*) are calculated, sampled in the time domain, and the resulting numerical values are stored in arrays.

ii) For each of the previous arrays, the Fourier transform is computed (numerically) and the corresponding frequency spectrum is stored in a second class of arrays.

iii) After concluding the statistical sample of trajectories, for all the variables and for each frequency within the spectrum range under study, several statistical indices (e.g., percentiles) of the amplitudes and/or phases of the arrays obtained in ii) are calculated.

The statistics of the frequency spectrum is stored in a third class of arrays.

iv) For all variables and for each frequency, the values of the statistical indices calculated in iii) are collected on a 'composite' frequency spectrum and stored in a forth class of arrays.

v) The procedure i) to iv) is repeated for different numerical values of the link lengths. The numerical results for the forth class of arrays obtained in iv) are compared and analytical expressions, that fit the numerical data, are extrapolated.

vi) The algorithm i) to v) is repeated for different time/space trajectories.

vii) The algorithm i) to vi) is repeated for different robot structures.

viii) The partial conclusions drawn by the analytical expressions obtained in v), vi) and vii) are integrated and final conclusions are drawn.

In order to illustrate the new method in the next section we analyse the kinematics of planar manipulators and we compare the results with the classical direct and inverse kinematic equations.

3. A NEW MODEL FOR THE KINEMATICS OF PLANAR ROBOTS

The application of the formalism defined in the previous section requires the development of numerical calculations for the statistics. Therefore, before proceeding, we need to establish the different experiments according with the following guidelines:

i) Modelling case:
$IK = \{\text{Inverse Kinematics}\}$,
$DK = \{\text{Direct Kinematics}\}$

ii) Type of trajectory
1) Time acceleration profile:
$O = \{\text{On/Off acceleration}\}$
$T = \{\text{Triangular acceleration}\}$,
$P = \{\text{Parabolic acceleration}\}$,
$S = \{\text{Sinusoidal acceleration}\}$

2) Total time definition:
   MAL = {Maximum Acceleration Limitation},
   MVL = {Maximum Velocity Limitation},
   RAL = {Random Acceleration Limitation}
3) Space evolution:
   SL = {Straight Line}
   DP = {Direct Parabolic},
   IP = {Inverse Parabolic}

iii) Type of robot mechanical structure:
   RR = {joint 1 Rotational, joint 2 Rotational},
   RP = {joint 1 Rotational, joint 2 Prismatic}

The type of "time acceleration profile" leads to different formulae for the "total time definition" according with:

Acceleration $O \propto$

$$MAL: t_{\max} = 2\sqrt{dist/A_{\max}};\; MVL: t_{\max} = 2dist/V_{\max};$$
$$RAL: t_{\max} = 2\sqrt{dist/random(A_{\max})}$$

Acceleration $T \propto$

$$MAL: t_{\max} = \sqrt{8dist/A_{\max}};\; MVL: t_{\max} = 2dist/V_{\max};$$
$$RAL: t_{\max} = \sqrt{8dist/random(A_{\max})}$$

Acceleration $P \propto$

$$MAL: t_{\max} = \sqrt{8dist/A_{\max}};\; MVL: t_{\max} = 2dist/V_{\max};$$
$$RAL: t_{\max} = \sqrt{8dist/random(A_{\max})}$$

Acceleration $S \propto$

$$MAL: t_{\max} = \sqrt{2\pi dist/A_{\max}};\; MVL: t_{\max} = dist/V_{\max};$$
$$RAL: t_{\max} = \sqrt{2\pi dist/random(A_{\max})}$$

where $0 \le random(A_{\max}) \le A_{\max}$, *dist* is the total distance along the trajectory, $t_{\max}$ is the total time of duration of the movement and $A_{\max}$ and $V_{\max}$ are the maximum allowed values for the acceleration and the velocity, respectively. For example, Figures 1 and 2 show the Statistics

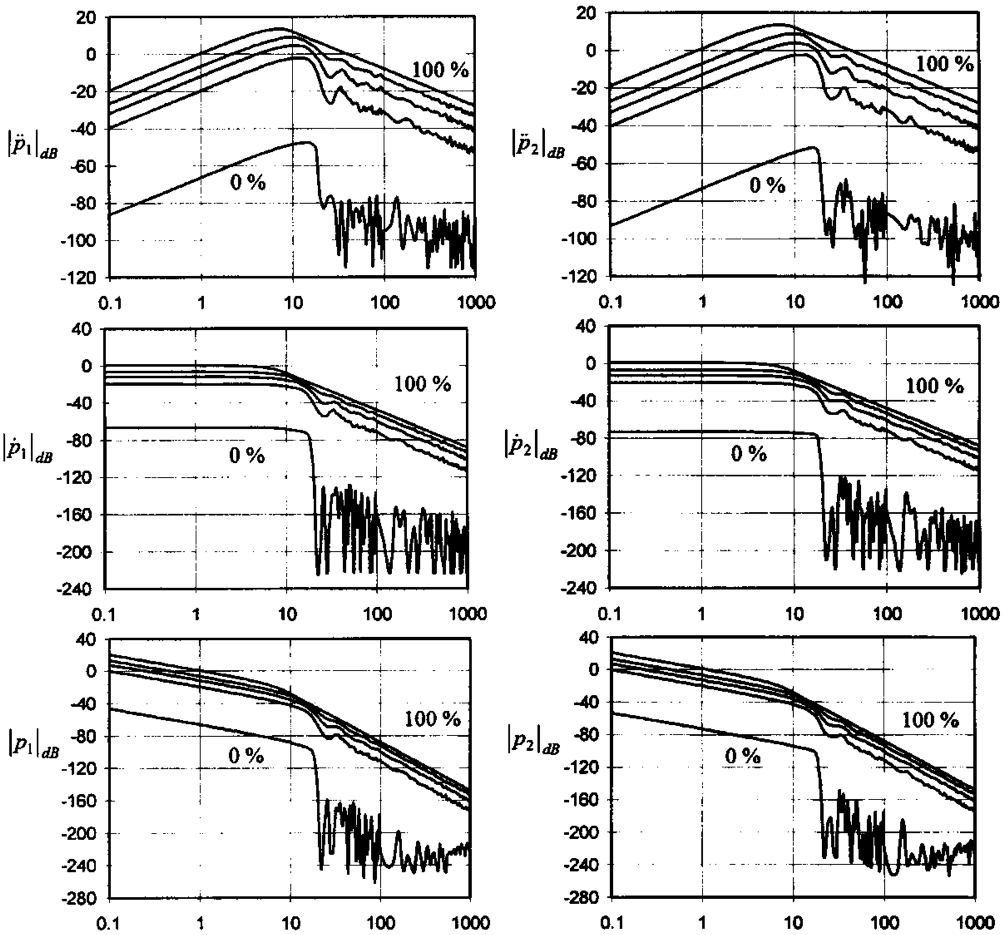

FIGURE 1 Percentiles of the *SHC* of the *ivs* in the experiment {*IK-RR*, *O*, *MAL*, *SL*} with $l_1 = 1$, $l_2 = 0.1$, $A = 10$.

of the Harmonic Content (SHC) of *ivs* and *ovs* for a sample of 500 trajectories and the experiment {*IK-RR*, *O*, *MAL*, *SL*}.

Applying a frequency-domain identification algorithm [11] to estimate the *SHCs* of the results we get numerical data of the gain and poles/zeros for the different experiments. Integrating heuristically these numerical values into analytical expressions (i.e., defining parametric formulae that fit in the numerical results) we get the results depicted on Table I and we conclude the properties:

- Numerical convergence – after repeating a large number of numerical experiments the charts with the *SHC* of the variables do not change significantly.
- Derivative/integral sensitivity – Although being composite curves, the *SHC* still obey the 'standard' $j\omega$ operator for variables that are related by the derivative operator in the time domain.

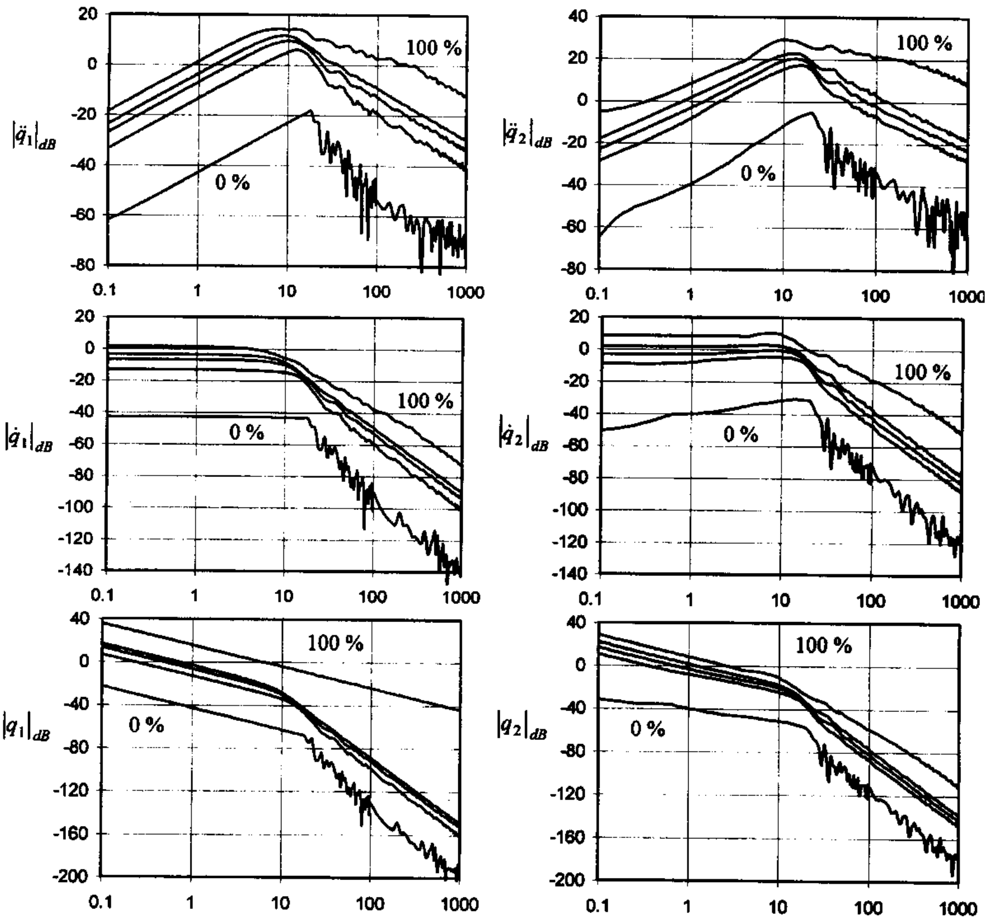

FIGURE 2 Percentiles of the *SHC* of the *ivs* in the experiment {*IK-RR*, *O*, *MAL*, *SL*} with $l_1 = 1$, $l_2 = 0.1$, $A = 10$.

- Analytical coherence – The numerical data that results from the experiments 'fits' the analytical expressions that lead to clear conclusions. For example, the *DK*, that calculates the trajectories in the joint space, leads to expressions for the poles that do not depend on the length of the workspace. On the other hand, the *IK*, that calculates the trajectories in the operational space, leads to poles that are sensitive to the length of the workspace. Therefore, trajectory planning algorithms on the joint space are more 'robust', that is, lead to a constant bandwidth.
- Generality – While for the classical deteriministic models we can not find clear relations between different robot structures, with the *SHC* general characteristics are highlighted. For example, joint 1 of the *RR* robot reveals a *SHC* that follows closely the *SHC* of joint 1

TABLE I Poles and Gain of the 50%-*SHC* for the Kinematics of Planar Manipulators

Experiment	$F\{\ddot{p}_1\}, F\{\ddot{p}_2\}$	$F\{\ddot{q}_1\}$	$F\{\ddot{q}_2\}$	
IK-RR *O, MAL, SL*	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.492 \pm j1.042)$	$\sqrt{A_{\max}}(\frac{0.644}{\sqrt[4]{l_1 l_2}} \pm j\frac{1.396}{\sqrt[6]{l_1^2 l_2}})$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.545 \pm j1.827)$	Poles
	$0.738\sqrt{l_1 l_2}$	$1.421 - 0.202\frac{l_1-l_2}{l_1+l_2} - 1.071\left\lvert\frac{l_1-l_2}{l_1+l_2}\right\rvert$	0.678	Gain
IK-RR *S, MAL, SL*	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.401 \pm j1.803)$	$\sqrt{A_{\max}}(\frac{0.408}{\sqrt[4]{l_1 l_2}} \pm j\frac{2.169}{\sqrt[6]{l_1^2 l_2}}),$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.392 \pm j2.367)$	Poles
	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}0.564$	$\frac{\sqrt{A_{\max}}}{\sqrt[6]{l_1^2 l_2}}0.695$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}1.231$	
	$0.888\sqrt{l_1 l_2}$	$1.623 - 0.287\frac{l_1-l_2}{l_1+l_2} - 1.158\left\lvert\frac{l_1-l_2}{l_1+l_2}\right\rvert$	0.758	Gain
IK-RR *T, MAL, SL*	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.445 \pm j1.660)$	$\sqrt{A_{\max}}(\frac{0.469}{\sqrt[4]{l_1 l_2}} \pm j\frac{2.001}{\sqrt[6]{l_1^2 l_2}}),$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.419 \pm j2.149)$	Poles
	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}0.583$	$\frac{\sqrt{A_{\max}}}{\sqrt[6]{l_1^2 l_2}}0.794$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}1.261$	
	$0.883\sqrt{l_1 l_2}$	$1.611 - 0.290\frac{l_1-l_2}{l_1+l_2} - 1.154\left\lvert\frac{l_1-l_2}{l_1+l_2}\right\rvert$	0.759	Gain
IK-RR *P, MAL, SL*	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.566 \pm j0.824),$	$\sqrt{A_{\max}}(\frac{0.612}{\sqrt[4]{l_1 l_2}} \pm j\frac{1.072}{\sqrt[6]{l_1^2 l_2}}),$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.575 \pm j1.406),$	Poles
	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.938 \pm j3.176),$	$\sqrt{A_{\max}}(\frac{1.069}{\sqrt[4]{l_1 l_2}} \pm j\frac{4.133}{\sqrt[6]{l_1^2 l_2}}),$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(1.194 \pm j3.941),$	
	$0.806\sqrt{l_1 l_2}$	$1.456 - 0.237\frac{l_1-l_2}{l_1+l_2} - 1.068\left\lvert\frac{l_1-l_2}{l_1+l_2}\right\rvert$	0.691	Gain
IK-RR *O, MVL, SL*	$\frac{V_{\max}}{\sqrt{l_1 l_2}}(0.525 \pm j0.843),$	$V_{\max}(\frac{0.614}{\sqrt{l_1 l_2}} \pm j\frac{1.061}{\sqrt[3]{l_1^2 l_2}}),$	$\frac{V_{\max}}{\sqrt{l_1 l_2}}(0.522 \pm j1.431),$	Poles
	$0.795\sqrt{l_1 l_2}$	$1.465 - 0.221\frac{l_1-l_2}{l_1+l_2} - 1.082\left\lvert\frac{l_1-l_2}{l_1+l_2}\right\rvert$	0.709	Gain
IK-RR *O, RAL, SL*	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.338 \pm j0.600)$	$\sqrt{A_{\max}}(\frac{0.472}{\sqrt[4]{l_1 l_2}} \pm j\frac{0.855}{\sqrt[6]{l_1^2 l_2}})$	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.441 \pm j1.161)$	Poles
	$0.741\sqrt{l_1 l_2}$	$1.444 - 0.244\frac{l_1-l_2}{l_1+l_2} - 1.062\left\lvert\frac{l_1-l_2}{l_1+l_2}\right\rvert$	0.713	Gain

DK-RR *O, MAL, SL*	$\sqrt{A_{\max}}(0.338 \pm j1.459)$,	$\sqrt{A_{\max}}(0.353 \pm j0.841)$,	$\sqrt{A_{\max}}(0.337 \pm j0.830)$,	Poles
	$\sqrt{l_1 l_2}\left(0.924 + 2.093\left\|\frac{l_1-l_2}{l_1+l_2}\right\|^3\right)$	1.665	0.827	Gain
IK-RR *O, MAL, DP*	$\ddot{p}_1 \sim \frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.495 \pm j0.884)$, $\ddot{p}_2 \sim \frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.348 \pm j1.373)$	$\sqrt{A_{\max}}\left(\frac{0.632}{\sqrt[4]{l_1 l_2}} \pm j\frac{1.147}{\sqrt[6]{l_1^2 l_2}}\right)$,	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.479 \pm j2.065)$,	Poles
	$\ddot{p}_1 \sim 2.24\sqrt{l_1 l_2}$, $\ddot{p}_2 \sim 1.373\sqrt{l_1 l_2}$	$2.469 - 1.586\left\|\frac{l_1-l_2}{l_1+l_2}\right\|$	0.749	Gain
IK-RR *O, MAL, IP*	$\ddot{p}_1 \sim \frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.348 \pm j1.1373)$, $\ddot{p}_2 \sim \frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.495 \pm j0.884)$	$\sqrt{A_{\max}}\left(\frac{0.632}{\sqrt[4]{l_1 l_2}} \pm j\frac{1.147}{\sqrt[6]{l_1^2 l_2}}\right)$,	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.479 \pm j2.065)$,	Poles
	$\ddot{p}_1 \sim 1.373\sqrt{l_1 l_2}$, $\ddot{p}_2 \sim 2.24\sqrt{l_1 l_2}$	$2.469 - 1.586\left\|\frac{l_1-l_2}{l_1+l_2}\right\|$	0.749	Gain
IK-RP *O, MAL, SL*	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.510 \pm j1.028)$,	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.580 \pm j1.188)$,	$\frac{\sqrt{A_{\max}}}{\sqrt[4]{l_1 l_2}}(0.464 \pm j1.800)$,	Poles
	$0.772\sqrt{l_1 l_2}$	$1.441 - 1.289\left\|\frac{l_1-l_2}{l_1+l_2}\right\|$	$\frac{1}{\sqrt{l_1 l_2}}\left(0.470 - 0.343\left\|\frac{l_1-l_2}{l_1+l_2}\right\|\right)$	Gain
DK-RP *O, MAL, SL*	$\sqrt{A_{\max}}(0.447 \pm j1.535)$	$\sqrt{A_{\max}}(0.457 \pm j0.948)$	$\sqrt{A_{\max}}(0.378 \pm j0.883)$	Poles
	$\sqrt{l_1 l_2}\left(0.704 + 2.502\left\|\frac{l_1-l_2}{l_1+l_2}\right\|^3\right)$	1.671	$\frac{1}{\sqrt{l_1 l_2}}\left(0.521 - 0.435\left\|\frac{l_1-l_2}{l_1+l_2}\right\|\right)$	Gain

Note: For the *RP* robot $l_{\max} = l_1 + l_2$ and $l_{\min} = |l_1 - l_2|$ where $l_{\max}$ and $l_{\min}$ are maximum and minimum lengths of the workspace, respectively. For the *RR* robot l_1 and l_2 are the length of the first and second links, respectively.

of the *RP* structure which reflects that, in both cases, we have a rotational joint. On the other hand, for joint 2, we have different mechanical articulations in the two cases and, therefore, the experiments lead to distinct results (Fig. 3).

- Compatibility – The conclusions based on the analysis of the *SHC* are coherent with the results of previous studies using different mathematical tools [1, 2, 9]. For example if $l_1 + l_2 =$ constant we verify that the maximum gain and bandwidth of the *SHC* occurs for $l_1 = l_2$. Also, the lower and upper elbow solutions for the *IK* where analysed revealing, as expected, similar properties in both cases.

Another aspects of interest is that the *SHC* of *ovs* depends, not only of the system, but also on the excitations that, for the kinematics, is related with the type of trajectory planning algorithm. For example, Figure 4 shows that the *SHC* of the *ivs* are identical for the *SL* trajectories while the symmetry is not preserved for the *DP* experiment. Moreover, the *SL* trajectories 'avoid' the singular points near the boundary of the robot workspace in constrast with the *DP* case where we may get very high amplitudes for the *ovs* as can be observed in the 100%-percentile in Figure 5.

The aforementioned experiments were performed with the kinematics of planar robots. Therefore, two possible developments are the study of the dynamics and the application of the new method to robots with more *dofs*. In both cases, the major point to be further investigated consists on the complexity of the heuristic equations of

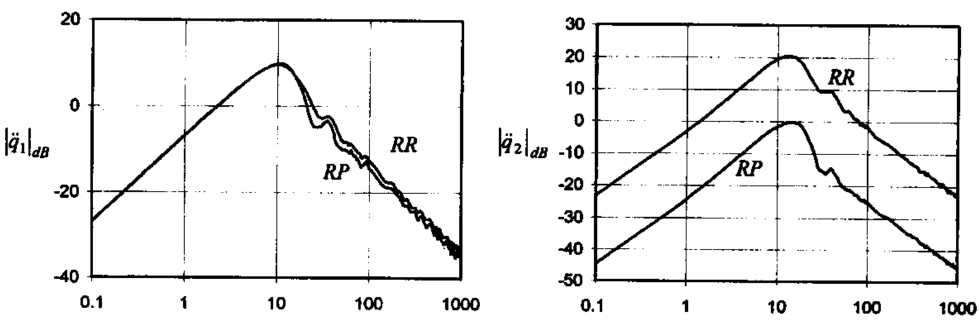

FIGURE 3 Comparison the 50%-percentiles of the *SHC* of the *ovs* in the experiments {*IK-RR, O, MAL, SL*} with $l_1 = 1$, $l_2 = 0.1$, $A = 10$ and {*IK-RR, O, MAL, SL*} with $l_{max} = 1.1$, $l_{min} = 0.9$, $A = 10$.

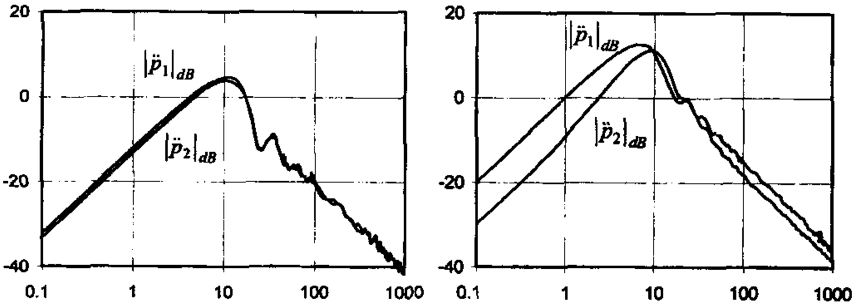

FIGURE 4 Comparison of the 50%-percentiles of the *SHC* of the *ivs* in the experiments {*IK-RR, O, MAL, SL*} and {*IK-RR, O, MAL, DP*} with $l_2=1$, $l_2=10$.

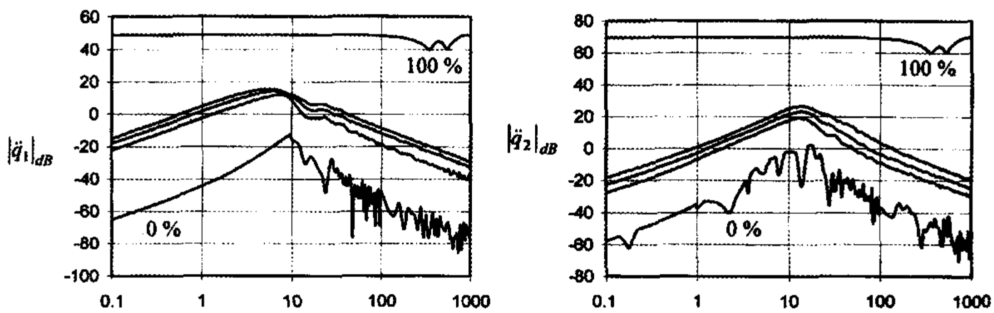

FIGURE 5 Percentiles of the *SHC* of the *ovs* in the experiment {*IK-RR, O, MAL, DP*} with $l_1=1$, $l_2=0.1$, $A=10$.

the *SHC*. On the other hand, the analysis of the histogram for each *iv* or *ov* remains almost immediate.

4. CONCLUSIONS

A new method to the analysis and design of robot manipulators was announced. The novel feature resides on a non standard approach to the modelling problem. Usually, system descriptions are based on a set of differential equations that can be very complex and hard to tackle. This motivates the need of models based on alternative concepts having distinct characteristics. The proposed method, by embedding the statistical analysis into the Fourier transform, provides a framework giving clear guidelines towards the optimisation both of the trajectory planning algorithm and the robot structure and a deeper understandding of the actuator requirements.

References

[1] Tsai, Y. C. and Soni, A. H., Accessible Region and Synthesis of Robot Arms, *ASME J. Mech. Design*, **103**, 803–811, October, 1981.

[2] Yoshikawa, T., Manipulability of Robotic Mechanisms, *The Int. J. Robotics Research*, **4**, 3–9, Summer, 1985.

[3] Sahar, G. and Hollerbach, J. M., (1986). Planning of Minimum-Time Trajectories for Robot Arms, *The Int. J. Robotics Research*, **5**, 90–100.

[4] Scheinman, V. and Roth, B. (1984). On the Optimal Selection and Placement of Manipulators, RoManySy'84: *The Fifth CISM-IFToMM Symposium*, Udine, pp. 39–45, Italy.

[5] Mooring, B. W. and Pack, T. J. (1987). Aspects of Robot Repeatability, *Robotica*, **5**, 223–230.

[6] Singh, J. R., Rastegaar, J. and Snee, S. (1992). Determination of Manipulator Kinematic Characteristics: A Probabilistic Approach, *Int. Journal of Robotics and Automation*, 7(4), 161–170.

[7] Galhano, A. M. S. F., Martins de Carvalho, J. L. and Tenreiro Machado, J. A. (1990). The Statistical Study of Robot Manipulators, *IEEE Int. Symp. on Intelligent Control*, Philadelphia, Pennsylvania, USA, pp. 424–429.

[8] Galhano, A. M. S. F., de Carvalho, J. L. M. and Tenreiro Machado, J. A. (1991). On the Statistical Modelling of Mechanical Manipulators, IFAC Symp. on Robot Control SYROCO'91, Vienna, Austria, pp. 15–20.

[9] Tenreiro Machado, J. A. and Galhano, A. M. S. F. (1993). Statistical Modelling of Robotic Manipulators, *J. Systems Analysis-Modelling-Simulation*, **12**, 101–123.

[10] Tenreiro Machado, J. A. and Galhano, A. M. S. F. (1997). A Statistical and Harmonic Model for Robot Manipulators, *IEEE Int. Conf. on Robotics an Automation*, Albuquerque, New Mexico, USA.

[11] Lopes dos Santos, P. and Martins de Carvalho, J. L. (1990). Automatic Transfer Function Synthesis From a Bode Plot, *29th IEEE Conf. on Decision and Control*, Honolulu, Hawaii, USA.