

ICCC 2004

**2004 International Conference on
Computational Cybernetics**

Proceedings



Vienna, Austria - August 30 - September 1, 2004

Hungarian Fuzzy Association

Budapest Tech, Hungary

Vienna University of Technology, Austria

IEEE SMC Chapter, Hungary

IEEE NN Chapter, Hungary

IEEE Joint Chapter of IES and RAS, Hungary

IEEE R8

IEEE Systems, Man, and Cybernetics Society

EUROFUSE

*Japan Society for Fuzzy Theory and Intelligent
Informatics*

*John von Neumann Computer Society,
Budapest, Hungary*



Fractional Order Adaptive Active Vibration Damping Designed on the Basis of Simple Kinematic Considerations

József K. Tar

Institute of Mathematics and
Computational Sciences
Budapest Polytechnic

Népszínház u. 8, H-1081
Budapest
Hungary
tar.jozsef@nik.bmf.hu

Imre J. Rudas

Institute of Mathematics and
Computational Sciences
Budapest Polytechnic

Népszínház u. 8, H-1081
Budapest
Hungary
rudas@bmf.hu

János F. Bitó

Centre of Robotics and
Automation
Budapest Polytechnic

Népszínház u. 8, H-1081
Budapest
Hungary
bito.janos@nik.bmf.hu

J. A. Tenreiro Machado

Dept. of Electrotechnical
Engineering
Institute of Engineering of
Porto

Rua Dr. Antonio Bernardino
de Almeida, 4200-072 Porto
Portugal
jtm@dee.isep.ipp.pt

Abstract – In this paper a simple nonlinear, adaptive control using fractional order derivatives is applied for damping the vibration of a car forced during passing along a bumpy road. Its key idea is the partial replacement of the integer order derivatives in a traditional PD controller with a time-shift invariant, causal approximation of Caputo's fractional derivative that behaves like a Green function. Since its physical essence is rather frequency filtering than providing integer order derivatives in limit cases, the approximation applied numerically is quite convenient. In this way simple kinematic design of the desired damping becomes possible. The adaptive part of the controller guarantees the realization of this kinematic design without making it necessary for the designer to have accurate and complete dynamic model of the system to be controlled or to design a sophisticated linear "CRONE" controller that has to take the responsibility for the unknown dynamics of the system. The applicability of the approach is illustrated via simulations for a paradigm that is a rough model of a car. It was found that both adaptivity and the use of fractional order derivatives in the control are essential parts of the success of the method.

I. INTRODUCTION

Externally excited vibration normally is undesired phenomenon that occurs in various physical systems therefore its efficient damping is of great practical significance. From the point of control technology the task has the „delicate” nature that the controller cannot be provided with the “exact” system-model, and/or with complete information on the system's actual physical state. A novel branch of soft computing simultaneously using the Modified Renormalization Transformation and simple ancillary methods [1] has recently been developed that is flexible enough to incorporate various algebraic blocks like e.g. special Symplectic Transformations [2]. It was shown that in the case of a wide class of physical systems with the aid of this method quite robust adaptive controllers could be developed for the control of very inaccurately and partially modeled physical systems that can have even unmodeled internal degrees of freedom [3]. As an input the method requires the desired trajectory of the generalized coordinates of the system that has to directly be controlled.

This approach should be useful if the desired trajectory of the generalized coordinates could be prescribed with respect to an inertial system of reference. However, in the most cases the system considered is a part of a moving object that does not serve as a basis of an inertial frame, as e.g. a car proceeding along a bumpy road crossing hills and valleys. In this case some slow motion along certain

average distance between the chassis and the wheel can be prescribed for the controller because it is locally measurable quantity. A feasible kinematic formulation of this goal is the application of some „forgetting integral” that does not allow abrupt changes with respect to the former values of this distance but allows its slow variation. The desired behavior of this distance can practically be prescribed to some extent by the terms used in the traditional linear controllers as frequency filters etc. The most plausible means would be the application of a simple PD type controller to keep a finite error at bay. However, the integrating term of this controller does not „forget” the past, and for an even small but constant error it generates infinite signal for feedback. As generalizations of the concept of the derivative the concept of fractional order derivatives found more and more physical applications to describe the „longer term memory” of various physical systems like in the case of visco-elastic phenomena [4] etc. The problem of designing fractional order control systems within the frames of linear control obtained considerable attention recently, e.g. [5]. The French expression invented by Oustaloup „CRONE: *Commande Robuste d'Ordre Non Entier*” [6] almost became a „trademark” hallmarking a well-elaborated design methodology that obtained application in vibration control, too [7]. Understanding and application of this method requires deep engineering knowledge in the realm of linear systems, frequency spectrum analysis, the use of Laplace transforms and complex integrals, various typical diagrams as e.g. the Nichols plot, etc. The aim of the present paper is to propose an alternative approach not strictly restricted to the traditional way of thinking in the case of linear systems. Tackling the problem from a more general nonlinear basis requires less amount of profound and specific engineering knowledge, the application of which can be evaded by the controller's adaptive nature or learning abilities. For this purpose the „long term memory” or slowly forgetting nature of the fractional order systems is considered in a more general view.

II. FRACTIONAL ORDER DERIVATIVES AND GREEN FUNCTIONS

In the case of a normal PID-type controller the desired trajectory reproduction can be prescribed in a purely kinematics based manner. For the second time-derivative of the actual coordinate errors the desired relation can be formulated as:

$$\ddot{\mathbf{e}}^d = -P\mathbf{e} - D\dot{\mathbf{e}} - I \int_{-\infty}^t \mathbf{e}(t') dt' \quad (1)$$

The use of the integrating term in (1) is expedient whenever very accurate tracing of the nominal trajectory is needed, because even for very small permanent error sooner or later it yields quite considerable feedback. Since this typically is not the goal of a vibration-damping task in which some uncertainty may be desired in order to avoid the rigid transmission of high-frequency vibrations we omit this term. For the desired acceleration of the controlled generalized coordinates (1) then yields

$$\ddot{\mathbf{q}}^d = \ddot{\mathbf{q}}^N - P(\mathbf{q}^r - \mathbf{q}^N) - D(\dot{\mathbf{q}}^r - \dot{\mathbf{q}}^N) \quad (2)$$

If the dynamics of the system to be controlled is not very well known the feedback force calculated on the basis of (2) cannot accurately be estimated, therefore normally very big gains (i.e. P , and D coefficients) has to be applied, normally resulting in a noisy control, that is instead damping the vibration the control can introduce even more noise into the system. This situation can be improved by using fractional order derivative in (2) as

$$\ddot{\mathbf{q}}^d = \frac{d^\beta}{dt^\beta} \dot{\mathbf{q}}^N - P(\mathbf{q}^r - \mathbf{q}^N) - D \frac{d^\beta}{dt^\beta} (\mathbf{q}^r - \mathbf{q}^N) \quad (3)$$

in which the symbol d^β/dt^β denotes some fractional order derivative. In the literature, starting from the 19th century various definitions has been elaborated for this concept [8, 9], and comprehensive works were also issued recently, e.g. [10]. For our present purposes the form invented by Caputo seems to be the most expedient as

$$\frac{d^\beta}{dt^\beta} u(t) := \frac{1}{\Gamma(1-\beta)} \int_0^t \left[\frac{du(\tau)}{d\tau} \right] (t-\tau)^{-\beta} d\tau, \beta \in (0,1) \quad (4)$$

For $t>0$ (4) physically has the following simple meaning: the full 1st order derivative in the integrand removes the constant component from the signal, and this derivative is “causally reintegrated” by the use of a Green function that has a slowly forgetting nature (the contribution of the far past becomes more and more negligible in it), while its singularity in $\tau=t$ enhances the relative weight of the contribution of the $\tau \leq t$ instants. Furthermore, the relatively slowly decreasing “tail” of this function also acts as a frequency filter that rejects the high-frequency components of the traditional 1st derivative. Due to the singularity of the Green function in (4) a common finite-element numerical integration cannot accurately be done. Instead of that, we can suppose that at least $u'(\tau)$ is a relatively slowly varying function of time, therefore it can be considered as constant during the integration over a small time-interval of length δ , while the variation of the Green function can be taken into account accurately. Furthermore, to introduce symmetry against the translation of the signal in time we can omit the very long tail of the Green-function and we can go back in time only to some time $t-T$ instead of 0. The proposed approximation of (4) for finite discrete time-resolution δ in this paper was taken as

$$\begin{aligned} \frac{d^\beta}{dt^\beta} u(t) &\cong \frac{u'(t)\delta^{-\beta+1}}{\Gamma(2-\beta)} + \\ &+ \sum_{0 < s \text{ while } s\delta < T} \frac{\delta^{-\beta+1} [(s+1)^{-\beta+1} - s^{-\beta+1}]}{\Gamma(2-\beta)} u'(t-s\delta) \end{aligned} \quad (5)$$

(It is not our purpose to obtain exact integer order derivatives from this approximation when $\beta \rightarrow 1$ or $\beta \rightarrow 0$.)

In the numerical simulations in this paper $\delta=1$ ms, $T=50$ ms values were chosen, and the appropriate value for β was determined experimentally via simulations. The components of the Green function were stored in an array variable, the u' values were stored in a shift-register.

III. THE MODEL OF THE CAR AND ITS CONTROL

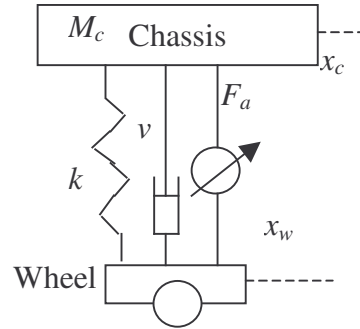


Fig. 1. The rough model of the suspension system

The model of the system considered is described in Fig. 1. The mass of the wheel is supposed to be negligible with respect to that of the chassis of mass $M_c=100$ kg the model value of which was supposed to be 150 kg. (This value cannot exactly be known *a priori* since one or more than one travelers of even 100 kg weight each can sit in the car.) The passive suspension system consisted of a spring of stiffness $k=2 \times 10^4$ N/m and viscosity of $v=1 \times 10^4$ N/(m/s). The force of the active suspension F_a was supposed to be generated according to the control law. For this purpose any motor could be used which is useful in Computed Torque Control in Robotics, e.g. voltage controlled electric DC motors the torque/force of which is proportional to their current. The coordinates x_c and x_w in m units describe the height of the chassis and the wheel, respectively, with respect to an inertial frame, i.e. with respect to the sea level, so they are not available as direct data for the controller. The nominal height of the chassis while rigidly following the wheel was prescribed to be $x_{cnom}=x_w+L_0$ with $L_0=0.5$ m. In the case of loose trajectory tracking little humps and dips need not be traced by the chassis, but climbing a higher hill or tracing a deeper valley must be performed. However, the error of this trajectory tracking is available via local measurements within the car as

$$e = x_c - x_{cnom} = (x_c - x_w) - L_0 \quad (6)$$

The 1st order time-derivative of the error can be numerically estimated by finite element methods. Because x_w and x_c are measured with respect to an inertial frame their second traditional time-derivatives also are measurable even by the use of micro-sensors developed on a chip. Therefore the appropriate representation of (3) can be written as:

$$\ddot{x}_c^d = \frac{1}{\Gamma(1-\beta)} \int_0^t \ddot{x}_w(\tau)(t-\tau)^{-\beta} d\tau - Pe - \frac{D}{\Gamma(1-\beta)} \int_0^t \dot{e}(\tau)(t-\tau)^{-\beta} d\tau \quad (7)$$

(For the numerical approximation of the integrals (5) can be used.)

Eq. (7) represents a tracing requirement expressed by the use of purely kinematic terms. The main expectation behind it is the supposition that for small proportional coefficient P some loose tracking can be achieved the accuracy of which is increased by the “filtered” integrals at low frequency (that is for hill climbing), while for the higher frequency components occurring when small dips are passed it remains loose. By the use of the approximate dynamic model of the system the appropriate active force can be estimated. Due to the approximate nature of the dynamic model exertion of this force will not result in the desired acceleration of the chassis. For the realization of (7) adaptive control is needed. Its main principles are given in the following part.

IV. PRINCIPLES OF THE ADAPTIVE CONTROL

For the adaptive control there is given an imperfect system model as a starting point. On the basis of that some excitation is calculated to obtain a desired system response \mathbf{i}^d as $\mathbf{e} = \boldsymbol{\varphi}(\mathbf{i}^d)$. This model is step by step refined in the following manner. If we apply the above approximate excitation, according to the actual system's inverse dynamics described by the unknown function a realized response $\mathbf{i}^r = \boldsymbol{\psi}(\boldsymbol{\varphi}(\mathbf{i}^d)) = \mathbf{f}(\mathbf{i}^d)$ is obtained instead of the desired one, \mathbf{i}^d . Normally one can obtain information via observation only on the function $\mathbf{f}()$ considerably varying in time, and no any possibility exists to directly “manipulate” the nature of this function: only \mathbf{i}^d as the input of $\mathbf{f}()$ can be “deformed” to \mathbf{i}^{d*} to achieve and maintain the $\mathbf{i}^d = \mathbf{f}(\mathbf{i}^{d*})$ state. [Only the *model function* $\boldsymbol{\varphi}()$ can directly be manipulated.] On the basis of the modification of the method of renormalization widely applied in Physics the following “scaling iteration” was suggested for finding the proper deformation:

$$\mathbf{i}_0; \mathbf{S}_1 \mathbf{f}(\mathbf{i}_0) = \mathbf{i}_0; \mathbf{i}_1 = \mathbf{S}_1 \mathbf{i}_0; \dots; \mathbf{S}_n \mathbf{f}(\mathbf{i}_{n-1}) = \mathbf{i}_0; \quad (8)$$

$$\mathbf{i}_{n+1} = \mathbf{S}_{n+1} \mathbf{i}_n; \mathbf{S}_n \xrightarrow{n \rightarrow \infty} \mathbf{I}$$

in which the \mathbf{S}_n matrices denote some linear transformations that map the observed response to the desired one, and the construction of each matrix corresponds to a step in the adaptive control. It is evident that if this series converges to the identity operator just the proper deformation is approached, and the controller „learns” the behavior of the observed system by step-by-step amendment and maintenance of the initial model. Since (8) does not unambiguously determine the possible applicable quadratic matrices, we have additional freedom in choosing appropriate ones. In this paper the Special Symplectic Transformations described in details e.g. in [2] were chosen as algebraic means. The main idea is the completion of the dimensions of the \mathbf{f} and \mathbf{i}_0 vectors with physically not interpreted “dummy” dimensions to avoid $\mathbf{0} \rightarrow \mathbf{0}$ or $\mathbf{0} \rightarrow \text{finite}$ or $\text{finite} \rightarrow \mathbf{0}$ mappings. Following that

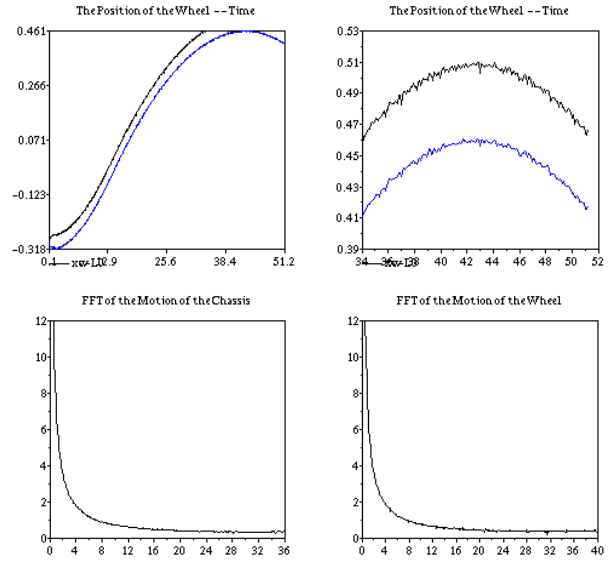


Fig. 2. Simulation results for the passive suspension only: the wheel's height (black –upper- line) and the height of the chassis minus L_0 (blue – lower- line) in m units vs. time in s units with zoomed excerpts (1st row), and the Fast Fourier Transform's absolute value vs. frequency in Hz units for the height of the wheel and of the chassis (2nd row)

by properly placing further blocks around these arrays easily invertible special matrices belonging to some special Lie Group, in this case to the Symplectic Group can be obtained, so unique proposition can be given for the \mathbf{S}_n matrices via simple computation. These matrices automatically approach the unit matrix as $\mathbf{f} \rightarrow \mathbf{i}_0$, and since they are the elements of a group, their matrix multiplication also belongs to this group, that is we remain within the mathematical frameworks of a simple Lie group the elements of which arbitrarily can approach the unit matrix. In the lack of enough free space, regarding the details we refer to [2].

V. SIMULATION RESULTS

In the simulation examples a hill of parabolic shape and height of 0.5 m was climbed by the use of a bumpy road containing bumps of the size from 0.1 to 2 m diameters and maximal height of about 1 cm. The road was modeled by a Fourier series-like construction as

$$x_w(s) = x_{w0}(s) + \sum_{d=1}^{20} A_d \left(1 - \cos \frac{2\pi s}{0.1 \times d} \right) \quad (9)$$

In (9) s measures the horizontal displacement of the car, which is obtained by constant 1 m/s^2 acceleration till achieving the 14 m/s velocity. Following that $-1 m/s^2$ deceleration happens in the horizontal direction. According to (9) this means a random-like excitation for the suspension system. In Fig. 2 typical results are given for the passive suspension when no active force is applied. It is evident that the passive suspension in this case results in a poor vibration damping quality. The vibration of the wheel is transmitted to the chassis.

If strict and accurate trajectory tracking is required the adaptive version of the outlined control using the traditional integer order derivatives can be applied. Typical results belonging to this control are given in Fig. 3 for $P=60.5 s^{-2}$ and $D=13.5 s^{-1}$ prescribed tracking behavior. It is evident

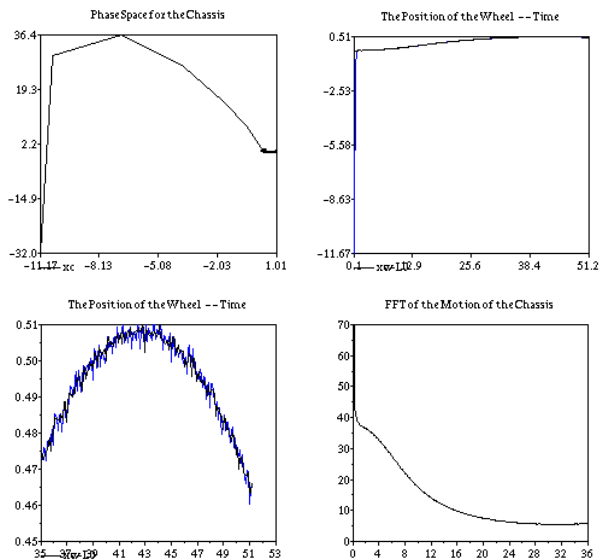


Fig. 3. Simulation results for the active adaptive suspension for rigid tracing using traditional integer order derivatives: the phase space of the motion of the chassis in m/s and m units, the wheel's height (black – upper- line) and the height of the chassis minus L_0 (blue –lower- line) in m units vs. time in s units (1st row), zoomed excerpt of trajectory tracking, and the Fast Fourier Transform's absolute value vs. frequency in Hz units for the height of the chassis (2nd row)

that such a controller strictly can trace the hill and the bumps. The presence of the 1st order derivative in general makes the control noise-sensitive by strongly amplifying the effect of fast (abrupt) changes.

To improve noise-suppression a non-adaptive active version of the above control for strongly decreased proportional feedback coefficient $P=6.05\ s^{-2}$, $D=13.5\ s^{-1}$, and non-integer order derivatives with $\beta=0.1$ can be applied (Fig. 4). The noise suppression was considerably improved due to the high-frequency active forces but the low frequency tracking is poor due to the very approximate dynamic model. To improve the situation adaptivity can be switched on

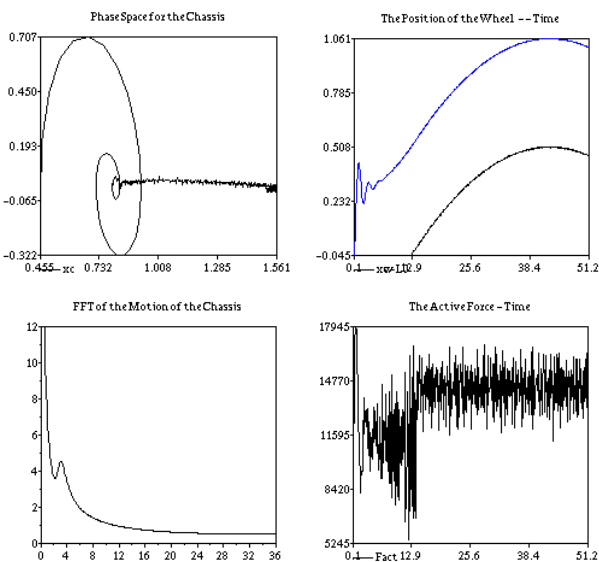


Fig. 4. Simulation results for the active non-adaptive suspension for soft tracing using $\beta=0.1$ fractional order derivatives: the phase space of the motion of the chassis in m/s and m units, the wheel's height (black – lower- line) and the height of the chassis minus L_0 (blue –upper- line) in m units vs. time in s units (1st row), the Fast Fourier Transform's absolute value vs. frequency in Hz units for the height of the chassis and the active force in N vs. time in s units (2nd row)

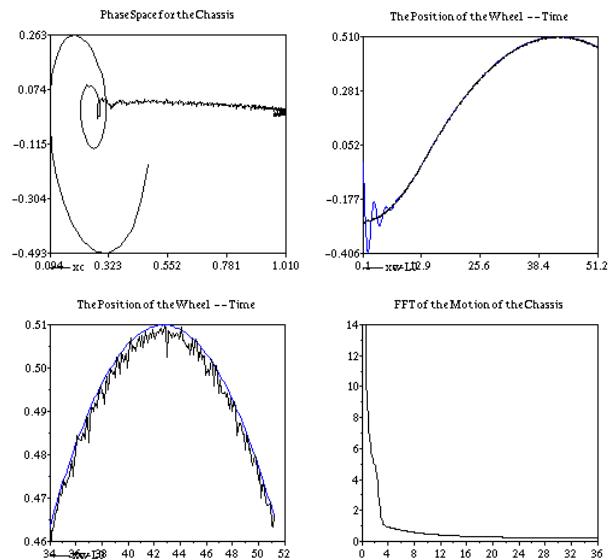


Fig. 5. Simulation results for the active adaptive suspension for soft tracing using $\beta=0.1$ fractional order derivatives: the phase space of the motion of the chassis in m/s and m units, the wheel's height (black – lower- line) and the height of the chassis minus L_0 (blue –upper- line) in m units vs. time in s units (1st row), zoomed excerpt of trajectory tracking, and the Fast Fourier Transform's absolute value vs. frequency in Hz units for the height of the chassis (2nd row)

for the same parameter-settings that were used in the case belonging to Fig. 4. The results are given in Fig. 5. While the high-frequency terms are well suppressed due to the filtered goal of the control, adaptivity makes it possible to really implement the prescribed loose tracking for the low frequency components, therefore the chassis now climbs the hill without taking the higher frequency vibrations caused by the bumpy road.

It also is clear from the Fourier spectrum of the motion of the chassis that the noise rejection is even better than in the non-adaptive case.

VI. CONCLUSIONS

In this paper the combination of the concept of fractional order derivatives and a novel branch of soft computing was applied for vibration suppression purposes in the case of a simple car model.

The main contribution of the non-integer order derivatives lies in providing the controller with appropriate causal goal functions of significantly filtered high-frequency components.

The adaptive law gives help in implementing the result of this essentially purely kinematic design without requiring *a priori* accurate information on the dynamics of the system.

In contrast to the rather traditional “CRONE” design methods in this case little profound engineering knowledge is needed. Adaptivity takes away the burden of dealing with dynamic effects from the linear fractional order controller. Furthermore, in this case no particular suppositions are needed for the nature of the vibration that assumptions used to be typical in the traditional control literature, e.g. that vibration can be treated with low order Taylor series expansion around some equilibrium position, or that the suspension system and the external excitation have characteristic eigenfrequencies or peaks in their Fourier spectra for the absorption of which the

eigenfrequency of the damping medium has to be properly tuned. It needs the possibility only for fast feedback signals.

For further research the active adaptive vibration control of physical systems having unmodeled internal degrees of freedom can be considered for which the adaptive approach was also found to be applicable.

VII. ACKNOWLEDGMENT

The authors gratefully acknowledge the support by the Hungarian National Research Fund (projects: OTKA T034651 and T034212) and that of the Hungarian-Portuguese S&T Project No. Port-6/03.

VIII. REFERENCES

- [1] J. K. Tar, J.F. Bitó, K. Kozłowski, B. Pátkai, D. Tikk: "Convergence Properties of the Modified Renormalization Algorithm Based Adaptive Control Supported by Ancillary Methods", in *Proceedings of the 3rd International Workshop on Robot Motion and Control (ROMOCO '02)*, pages 51-56, Bukowy Dworek, Poland, 9-11 November, 2002, ISBN 83-7143-429-4, IEEE Catalog Number. 02EX616.
- [2] J. K. Tar, A. Bencsik, J.F. Bitó, K. Jezernik. Application of a New Family of Symplectic Transformations in the Adaptive Control of Mechanical Systems. In *Proceedings of the 2002 28th Annual Conference of the IEEE Industrial Electronics Society*, Paper SF-001810, Nov. 5-8 2002 Sevilla, Spain, CD issue, ISBN 0-7803-7475-4, IEEE Catalog Number. 02CH37363C.
- [3] J. K. Tar, I.J. Rudas, Á. Szeghegyi, K. Kozłowski: "Adaptive Control of a Wheel of Unmodeled Internal Degree of Freedom", in *Proceedings of the Second Slovakian-Hungarian Joint Symposium on Applied Machine Intelligence (SAMI 2004)*, pages 289-300, Herlany, Slovakia, Jan. 15-17, 2004, ISBN 963-7154-23-X.
- [4] P. J. Torvik and R. L. Bagley: "On the Appearance of the Fractional Derivative in the Behaviour of Real Materials", *ASME Journal of Applied Mechanics*, vol. 51, pages 294-298, June 1984.
- [5] I. Podlubny: "Fractional-Order Systems and $PI^{\lambda}D^{\mu}$ -Controllers", *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pages 208-213, 1999.
- [6] A. Oustaloup: "La Commande CRONE: Commande Robuste d'Ordre Non Entier", *Hermes*, 1991.
- [7] P. Lanusse, T. Poinot, O. Cois, A. Oustaloup, J.C. Trigeassou: "Tuning of an Active Suspension System using a Fractional Controller and a Closed-Loop Tuning", in *Proceedings of ICAR 2003 The 11th International Conference on Advanced Robotics*, pages 258-263, Coimbra, Portugal, June 30 - July 3, 2003, ISBN. 972-96889-9-0.
- [8] J. Liouville: "Mémoire sur le calcul des différentielles a indices quelconques", *J. Ecole Polytechn.* 13, 71-162, (1832).
- [9] A. K. Grünwald: "Über 'begrenzte' Derivationen und deren Anwendung. *Zeitschrift für angewandte Mathematik und Physik*, 12, 441-480 (1867).
- [10] K. B. Oldham and J. Spanier: "The Fractional Calculus: Theory and Application of Differentiation and Integration to Arbitrary Order. Academic Press, 1974.